CLASS TEST

Sl.: 01SK_EE_JKLMNOPQ_301025

MADE EASY

Leading Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

ELECTROMAGNETICS THEORY

ELECTRICAL ENGINEERING

Date of Test : 30/10/2025

ANSWER KEY

1.	(a)	7.	(a)	13.	(c)	19.	(a)	25.	(d)
2.	(b)	8.	(a)	14.	(b)	20.	(c)	26.	(a)
3.	(d)	9.	(a)	15.	(a)	21.	(a)	27.	(b)
4.	(c)	10.	(a)	16.	(d)	22.	(a)	28.	(b)
5.	(d)	11.	(c)	17.	(b)	23.	(c)	29.	(c)
6.	(a)	12.	(b)	18.	(c)	24.	(b)	30.	(a)

DETAILED EXPLANATIONS

1. **(a)**

In cylindrical coordinate system:

$$\nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \rho\sin 2\phi & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[\hat{a}_\rho(0) - \rho\hat{a}_\phi(0) + \hat{a}_z\frac{\partial}{\partial \rho}(\rho\sin 2\phi)\right] = \frac{1}{\rho}(\sin 2\phi)\hat{a}_z$$

at point $P(4, \pi/6, 0)$,

$$\nabla \times \vec{A} = \frac{1}{4}\sin\left(2\times\frac{\pi}{6}\right)\hat{a}_z = \frac{1}{4}\cdot\frac{\sqrt{3}}{2}\hat{a}_z = \frac{\sqrt{3}}{8}\hat{a}_z$$

2. **(b)**

The given vector $\vec{A}$ is in spherical coordinates

$$\vec{A} = 2r\cos\theta\cdot\cos\phi\hat{a}_r + r^{1/2}\hat{a}_\phi$$

$$\text{div.}\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\cdot A_r) + \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial\theta}(\sin\theta\cdot A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(2r^3\cos\theta\cos\phi\right) + 0 + \frac{1}{r\sin\theta}(0)$$

$$= 6\cos\theta\cos\phi$$

At point $\left(1, \frac{\pi}{4}, \frac{\pi}{3}\right)$,

$$\nabla\cdot\vec{A} = 6\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) = 2.121$$

3. **(d)**

The incremental work is given by:

$$dW = -Q\cdot\bar{E}\cdot dl$$

Now dl in the direction of $\hat{a}_\phi$;

$$d\bar{l} = r\,d\phi\hat{a}_\phi = 6\times10^{-6}\hat{a}_\phi$$

Thus

$$\vec{E}\cdot d\bar{l} = -200\times6\times10^{-6} = -1200\times10^{-6}$$

$$dW = -40\times10^{-6}\times(-1200\times10^{-6})$$

$$= 48\text{ nJ}$$

5. **(d)**

As we know;

$$\nabla\times\vec{A} = \vec{B}$$

Where $\vec{B}$ is magnetic flux density:

$$\text{Units of } \vec{B} = \frac{Wb}{m^2}$$

Therefore,
$$\text{unit of } \vec{A} = \frac{Wb}{m}$$

$\because$
$$\text{units of } \nabla = \frac{1}{m}$$

6. **(a)**

$$C = 4\pi\varepsilon a \qquad \text{where } \varepsilon = \varepsilon_0,$$
$$a = 18 \text{ cm}$$
$$C = 4\pi \times \frac{10^{-9}}{36\pi} \times 18 \times 10^{-2}$$
$$= 2 \times 10^{-11} = 20 \text{ pF}$$

7. **(a)**

At the centre of the loop, assuming loop is horizontal,

$$\vec{H} = \frac{I}{2r}\hat{a}_z$$

Magnetic flux density,
$$\vec{B} = \frac{\mu_0 I}{2r} = \frac{4\pi \times 10^{-7} \times 1 \times 10^{-3}}{2 \times 2}\hat{a}_z$$
$$\vec{B} = 0.314\hat{a}_z \text{ (n Wb/m}^2)$$

9. **(a)**

$$\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t} \text{ is continuity equation}$$

10. **(a)**

$$I = \int \vec{J}.d\vec{s} = \int_{-0.01}^{0.01}\int_{-\pi/4}^{\pi/4} 100\cos 2y\hat{a}_x d_y d_z \ \hat{a}_x$$
$$= [0.01 + 0.01] \times 100 \times \left.\frac{\sin 2y}{2}\right|_{-\pi/4}^{\pi/4}$$
$$= (50 \times 0.02) \times \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{-\pi}{2}\right)\right] = 2\text{A}$$

11. **(c)**

The potential is given by:

$$V_{AB} = -\int_B^A \overline{E} \cdot d\overline{l}$$

Now, we know that
$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho \text{ for infinite line charge}$$
$$\vec{E} = \frac{10^{-9}}{2\pi\left(\frac{10^{-9}}{36\pi}\right)\rho}\hat{a}_\rho = \frac{18}{\rho}\hat{a}_\rho \text{ V/m}$$

$$d\bar{l} = d\rho \cdot \hat{a}_\rho + \rho d\phi \hat{a}_\phi + dz \hat{a}_z$$

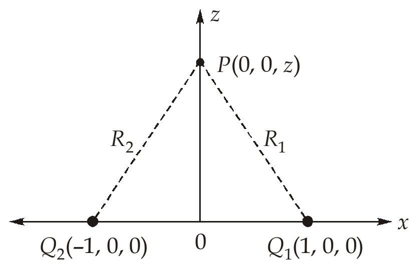
$$\bar{E} \cdot d\bar{l} = \frac{18}{\rho} d\rho$$

$$V_{AB} = -\int_4^2 \frac{18}{\rho} d\rho = \left[-18 \ln \rho\right]_4^2 = -18\left[\ln 2 - \ln 4\right]$$

$$= 18 \ln 2 = 12.48 \text{ volts}$$

12. (b)

Consider the diagram shown below:



Distance of Q_1 and Q_2 from point P are:

$$R_1 = R_2 = \sqrt{z^2 + 1}$$

Since, $Q_1 = Q_2$

and $R_1 = R_2$

Now, potential at P is twice that of single charge

$$V = 2 \times \frac{Q}{4\pi\varepsilon_0 R_1}$$

$$= 2 \times \frac{8 \times 10^{-9}}{4\pi \times \left(\frac{10^{-9}}{36\pi}\right)\sqrt{z^2+1}} = \frac{144}{\sqrt{z^2+1}} \text{ Volts}$$

$$\frac{dV}{dz} = \frac{d}{dz}\left(\frac{144}{(z^2+1)^{1/2}}\right)$$

$$= 144 \times \left[-\frac{1}{2}(z^2+1)^{-3/2}\right] \times 2z$$

$$\frac{dV}{dz} = \frac{-144z}{(z^2+1)^{3/2}};$$

$$\left|\frac{dV}{dz}\right| = \frac{144z}{(z^2+1)^{3/2}} \text{ V/m}$$

13. (c)

$$E = \frac{\Delta V}{\Delta Z} = \frac{250 - 100}{5 \times 10^{-3}} = 3 \times 10^4 \text{ V/m}$$

$$\vec{E} = -\nabla V = -3 \times 10^4 \hat{a}_z \text{ V/m}$$

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E} = \left(\frac{10^{-9}}{36\pi}\right) \times 2.4 \times (-3 \times 10^4) \hat{a}_z$$

$$\vec{D} = -6.366 \times 10^{-7}\,\hat{a}_z \text{ C/m}^2$$

Since $\vec{D}$ is constant between the disks, and $D_n = \rho_s$ at a conductor surface.

$\therefore$ $$\rho_s = \pm 6.366 \times 10^{-7} \text{ C/m}^2$$

Positive sign on the upper plate and negative sign on the lower plate.

14. (b)

Magnitude of electrical flux density,

$$|\vec{D}| = \frac{Q}{4\pi |R|^2}$$

$$R = \sqrt{1^2 + (3)^2 + (-4)^2} = 5.099 \text{ m}$$

$$|\vec{D}| = \frac{40 \times 10^{-9}}{4\pi (5.099)^2} = 122.43 \text{ pC/m}^2$$

15. (a)

The given vector is in cylindrical coordinates,

$$\vec{A} = \rho z \sin\phi \hat{a}_\rho + 3\rho z^2 \cos\phi \hat{a}_\phi$$

$$\vec{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

$$\text{div } \vec{A} = \nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \cdot A_\rho) + \frac{1}{\rho}\cdot\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \times \rho z \sin\phi) + \frac{1}{\rho}\cdot\frac{\partial}{\partial \phi}(3\rho z^2 \cos\phi) + 0$$

$$= \frac{1}{\rho}(2\rho z \sin\phi) - \frac{1}{\rho}(3\rho z^2 \sin\phi)$$

$$= 2z \sin\phi - 3z^2 \sin\phi$$

$$= (2 - 3z) z \sin\phi$$

At point $\left(5, \frac{\pi}{2}, 1\right)$

$$\text{div} \vec{A} = (2 - (3 \times 1)) \times 1 \sin\frac{\pi}{2}$$

$$= -1$$

16. (d)

With in the conductor,

$$\vec{J} = \nabla \times \vec{H}$$

$$= -\frac{\partial}{\partial z}\left(\frac{Ir}{2\pi a^2}\right)\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{Ir^2}{2\pi a^2}\right)\hat{a}_z$$

$$\vec{J} = \frac{1}{r}\frac{I}{2\pi a^2}\cdot 2r\,\hat{a}_z$$

$$\vec{J} = \frac{I}{\pi a^2}\hat{a}_z$$

Outside the conductor, $\vec{J} = \nabla \times \vec{H}$

$$= \frac{-\partial}{\partial z}\left(\frac{I}{2\pi r}\right)\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{I}{2\pi}\right)\hat{a}_z$$

$$\vec{J} = 0$$

17. (b)

$$\text{Magnetic flux} = \Phi = \int \vec{B}.\vec{ds}$$

$$= \int_0^{2.0}\int_{0.5}^{2.5}\left(\frac{2.0}{r}\right)\hat{a}_\phi .dr\,dz\,\hat{a}_\phi$$

$$= 2 \times 2 \times [\ln 2.5 - \ln 0.5]$$

$$\Phi = 4 \times \ln\left(\frac{2.5}{0.5}\right) = 6.4377 \text{ Wb}$$

18. (c)

$$\text{The turns per unit length} = \frac{400}{0.5} = 800$$

So, the axial field is,

$$B = \mu_0 \text{H} = \mu_0\, 800\, I \text{ (Wb/m}^2)$$

$$\frac{L}{l} = \frac{N\Phi}{I} = N\left(\frac{B}{I}\right)\text{A}$$

$$\frac{L}{l} = \frac{400(800I \times (4\pi \times 10^{-7}))}{I} \times \pi(0.02)^2$$

Inductance, $L = 505.32\ \mu\text{H}$

19. (a)

$$\mu_1 = 2\,\mu_0;$$

$$\mu_2 = 5\,\mu_0$$

$$B_2 = 10\hat{a}_\rho + 15\hat{a}_\phi - 20\hat{a}_z \text{ mWb/m}^2$$

$$B_{1n} = B_{2n} = 15\hat{a}_\phi$$

$$H_{1t} = H_{2t}$$

$$B_{1t} = \frac{\mu_1}{\mu_2}B_{2t} = \frac{2}{5}(10\hat{a}_\rho - 20\hat{a}_z)$$

$$B_{1t} = (4\hat{a}_\rho - 8\hat{a}_z)\text{mWb/m}^2$$

$$W_{m1} = \frac{1}{2}B_1 \cdot H_1 = \frac{B_1^2}{2\mu_1}$$

$$= \frac{(4^2 + 15^2 + 8^2) \times 10^{-6}}{2 \times 2 \times 4\pi \times 10^{-7}}$$

$$= \frac{305}{16\pi} \times 10 = 60.68 \text{ J/m}^3$$

20. (c)

$$E = \int (v \times B) \cdot dl$$

$$v \times B = -0.15 \sin 10^3 t\, u_x \text{ V/m}$$

$$E = \int_0^{0.25} -0.15 \sin 10^3 t\, dx$$

$$E = -0.15 \sin 10^3 t [x]_0^{0.25}$$

$$E = -0.0375 \sin 10^3\, t \text{ V}$$

21. (a)

$$M = \chi_m H$$

$$= \chi_m \frac{B}{\mu} = \frac{(\mu_r - 1)}{\mu_r \mu_0} B = \left(1 - \frac{1}{4.5}\right) \times \frac{4y}{\mu_0} \times 10^{-3} \hat{a}_z$$

$$= \frac{28y}{9\mu_0} \times 10^{-3} \hat{a}_z$$

$$J = \nabla \times M$$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & \frac{28y}{9\mu_0} \end{vmatrix} = \frac{28}{9\mu_0} \times 10^{-3} \hat{a}_z = 2475.7 \text{ A/m}^2$$

22. (a)

$$E = \frac{\rho_s}{2 \epsilon_0} \hat{a}_n$$

$$E = E_1 + E_2 + E_3$$

$$= \left(\frac{10}{2 \epsilon_0} \hat{a}_x + \frac{(-20)}{2 \epsilon_0} \hat{a}_y + \frac{30}{2 \epsilon_0} (-\hat{a}_z)\right) \times 10^{-6}$$

$$= \frac{10}{2 \epsilon_0} \left[\hat{a}_x - 2\hat{a}_y - 3\hat{a}_z\right] \times 10^{-6}$$

$$|E| = \frac{10}{2 \epsilon_0} \sqrt{1 + 2^2 + 3^2} \times 10^{-6}$$

$$= \frac{10}{2 \epsilon_0} \sqrt{14} = \frac{18.71}{\epsilon_0} \times 10^{-6} = 2.11 \times 10^6 \text{ V/m}$$

23. (c)

$$H = H_x + H_y$$

$$H = \frac{I}{4\pi\rho} [\cos \alpha_2 - \cos \alpha_1] \hat{a}_\phi$$

$$H_x = \frac{5}{4\pi \times 2} (\cos 0° - \cos 90°)(-\hat{a}_x \times \hat{a}_z) = \frac{5}{8\pi} \hat{a}_y \text{A/m}$$

$$H_y = \frac{5}{4\pi \times 2}(\cos 0° - \cos 90°)(\hat{a}_y \times \hat{a}_z) = \frac{5}{8\pi}\hat{a}_x$$

$$\vec{H} = \frac{5}{8\pi}(\hat{a}_x + \hat{a}_y) \text{ A/m}$$

24. (b)

$$H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\hat{a}_\phi$$

$$\rho = \sqrt{5^2 + 5^2} = \sqrt{50}\text{ m}$$

$$H = \frac{2}{4\pi\sqrt{5^2+5^2}}[\cos\alpha_2 - \cos\alpha_1]\hat{a}_\phi$$

$$\cos\alpha_2 = \frac{10}{\sqrt{50+100}} = \frac{10}{\sqrt{150}}$$

$$\cos\alpha_1 = \cos 90° = 0$$

$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho = \hat{a}_z \times \left(\frac{5\hat{a}_x + 5\hat{a}_y}{5\sqrt{2}}\right) = \left(\frac{-\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)$$

$$\vec{H} = \frac{2}{4\pi \times 5\sqrt{2}} \times \left(\frac{10}{\sqrt{150}} - 0\right) \times \left(\frac{-\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)$$

$$= \frac{1}{20\pi}\left(-\hat{a}_x + \hat{a}_y\right) \times \frac{10}{5\sqrt{6}}$$

$$= \frac{1}{10\pi\sqrt{6}}\left(-\hat{a}_x + \hat{a}_y\right)\text{A/m}$$

25. (d)

$$\nabla^2 V = \frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\rho dV}{d\rho}\right) + \frac{1}{\rho^2}\frac{d^2V}{d\phi^2} + \frac{d^2V}{dz^2} = 0$$

$$= \frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\rho dV}{d\rho}\right) = 0$$

$$\frac{dV}{d\rho} = \frac{A}{\rho}$$

$$V = A \ln \rho + B$$

$$V(\rho = 4) = A \ln 4 + B = 0$$

$$V(\rho = 12) = A \ln 12 + B = V_0$$

$$V_0 = A \ln 12 - A \ln 4 = A \ln 3$$

$$E = -\nabla V = -\left[\frac{dV}{d\rho}\hat{a}_\rho\right] = -\frac{A}{\rho}\hat{a}_\rho$$

$$E(\rho = 8) = -6\,\hat{a}_\rho \text{ kV/m}$$

$$A = 48$$

$$V_0 = 48 \ln 3 \text{ V}$$

26. (a)

$$\nabla^2 V = \frac{1}{\rho}\frac{d}{d\rho}\frac{\rho dV}{d\rho} + \frac{1}{\rho^2}\frac{d^2V}{d\phi^2} + \frac{d^2V}{dz^2} = \frac{-\rho_v}{\epsilon} = \frac{-\rho_v}{\epsilon_0\epsilon_r}$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\rho dV}{dp}\right) = \frac{-\rho_v}{\epsilon} = \frac{-10}{\rho}\times\frac{10^{-12}}{3.6\times 1}\times 36\pi\times 10^9$$

$$\nabla^2 V = \frac{-\rho_v}{\epsilon}$$

$$\frac{d}{d\rho}\frac{\rho dV}{d\rho} = -0.1\,\pi$$

$$\frac{\rho dV}{d\rho} = -0.1\,\pi\rho + A$$

$$V = -0.1\,\pi\rho + A\ln\rho + B$$

$$-0.1\,\pi\times 2 + A\ln 2 + B = 0$$

$$A\ln 2 + B = 0.2\,\pi$$

$$-0.1\,\pi\times 5 + A\ln 5 + B = 60$$

$$A\ln 5 + B = 60 + 0.5\,\pi$$

$$A\ln 2.5 = (60 + 0.3\,\pi)$$

$$A = \left(\frac{60+0.3\pi}{\ln 2.5}\right) = 66.51$$

$$E = -\nabla V$$

$$= -\left(-0.1\pi + \frac{A}{\rho}\right)\hat{a}_\rho$$

At $\rho = 1$,

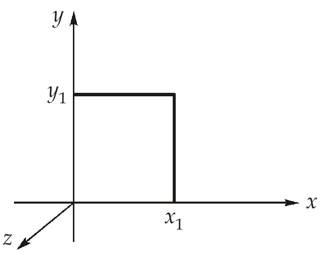
$$E = -\left(-0.1\pi + \frac{66.51}{1}\right)\hat{a}_\rho = -66.19\hat{a}_\rho \text{ V/m}$$

27. (b)

$$x_1 = y_1 = 1 \text{ m}$$

$$B_0 = \sin\pi x\sin\pi y\ T$$

$$B = B_0\cos\omega_0 t$$



$$E(t) = \iint_s \frac{-dB}{dt}\cdot\hat{n}\,dS$$

$$= \iint_s \omega_0 B_0\sin\omega t\,\hat{n}\,dS$$

$$E_{max} = \omega_0\int_{y=0}^{1}\int_{x=0}^{1}\sin\pi x\sin\pi y\,dx\,dy$$

$$E_{max} = 1000\times 2\pi\times\frac{4}{\pi^2} = \frac{8000}{\pi}\text{ V/turns}$$

$$E_{rms} = \frac{1}{\sqrt{2}}\times\frac{8000}{\pi}\times 10 = 18 \text{ kV}$$

28. (b)

$$C = 4\pi \in_0 \left(\frac{ab}{a-b}\right)$$

$$A_a = 4\pi a^2$$

$$A_b = 4\pi b^2$$

$$a = \sqrt{\frac{A_a}{4\pi}}\ ;\ b - \sqrt{\frac{A_b}{4\pi}}$$

$$ab = \frac{\sqrt{A_a A_b}}{4\pi};$$

$$a - b = \frac{\sqrt{A_a} - \sqrt{A_b}}{\sqrt{4\pi}}$$

$$C = 4\pi \in_0 \left[\frac{\sqrt{A_a A_b}}{4\pi} \times \frac{\sqrt{4\pi}}{\left(\sqrt{A_a} - \sqrt{A_b}\right)}\right]$$

$$= \sqrt{4\pi} \in_0 \frac{\sqrt{A_a A_b}}{\sqrt{A_a} - \sqrt{A_b}}$$

29. (c)

Capacitance, $$C = \frac{\varepsilon A}{d}$$

$$\frac{C_1}{C_2} = \frac{\varepsilon_1 A_1 d_2}{d_1 \varepsilon_2 A_2}$$

As the area of both the plates and the distance between the plates is same for both the capacitor.

$$\Rightarrow \quad \frac{C_1}{C_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\Rightarrow \quad \frac{C_1}{C_2} = \frac{12}{48} \Rightarrow C_1 : C_2 = 1 : 4$$

30. (a)

$$V_{AB} = -\int_B^A \vec{E} \cdot \vec{dl}$$

where, $$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_o r} \hat{a}_r$$

$$V_{AB} = -\int_B^A \frac{x \times 10^{-9}}{(2\pi\varepsilon_o r)} dr = \frac{-x \times 10^{-9} \times 36\pi}{2\pi \times 10^{-9}} \int_B^A \frac{1}{r} dr$$

$$= -18x[\ln r]_4^2$$

$$= -18x[\ln 2 - \ln 4]$$

$$= 18x[\ln 4 - \ln 2]$$

$$= 18 \ln 2$$

$$= 12.48x \text{ V}$$

■■■■