CLASS TEST

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# ENGINEERING MATHEMATICS

## CIVIL ENGINEERING

**Date of Test : 25/09/2025**

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (d) | 7. | (b) | 13. | (a) | 19. | (a) | 25. | (c) |
| 2. | (b) | 8. | (b) | 14. | (b) | 20. | (c) | 26. | (a) |
| 3. | (a) | 9. | (a) | 15. | (a) | 21. | (a) | 27. | (b) |
| 4. | (a) | 10. | (b) | 16. | (d) | 22. | (a) | 28. | (b) |
| 5. | (a) | 11. | (b) | 17. | (b) | 23. | (c) | 29. | (a) |
| 6. | (a) | 12. | (d) | 18. | (c) | 24. | (a) | 30. | (c) |

## DETAILED EXPLANATIONS

1. (d)

For function to be differentiable i.e. continuous $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$$f(0^-) = \lim_{x\to 0^-} \frac{\sin(3p-1)x}{3x} \times \frac{(3p-1)}{(3p-1)}$$

$$= \lim_{x\to 0^-} \frac{\sin(3p-1)x}{(3p-1)x} \times \frac{(3p-1)}{3} = \frac{(3p-1)}{3}$$

$$f(0^+) = \lim_{x\to 0^+} \frac{\tan(3p+1)x}{2x} \times \frac{(3p+1)}{(3p+1)}$$

$$= \lim_{x\to 0^+} \frac{\tan(3p+1)x}{(3p+1)x} \times \frac{3p+1}{2} = \frac{3p+1}{2}$$

For function to be continuous,

$$\frac{3p-1}{3} = \frac{3p+1}{2}$$

By solving, we get, $p = -\frac{5}{3}$

2. (b)

We have $y = e^x(A\cos x + B\sin x)$

$$y' = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

$$= y + e^x[-A\sin x + B\cos x]$$

$$y'' = y' + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x)$$

$$= y' + y' - y - y$$

$$= 2y' - 2y$$

$\Rightarrow$ Order = 2

Degree = 1

3. (a)

$$\frac{dy}{dx} = e^{ax} \times e^{by}$$

$$\frac{dy}{e^{by}} = e^{ax} \times dx$$

$$\frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c$$

$y(0) = 0$

$\Rightarrow$ $c = -\left[\frac{1}{b} + \frac{1}{a}\right] = -\left[\frac{a+b}{ab}\right]$

4. (a)

$$\nabla \cdot \vec{F} = 0 \qquad \text{[For solenoidal vector]}$$

$$\frac{\partial(y^2 - z^2 + 3yz - 2x)}{\partial x} + \frac{\partial(3xz + 2xy)}{\partial y} + \frac{\partial(2xy - axz + 2z)}{\partial z} = 0$$

$$-2 + 2x - ax + 2 = 0$$

From here, $a = 2$

5. (a)

Greatest rate of increase of $\phi$ is magnitude of directional derivative at that point.

$$\nabla\phi = (2xyz + 4z^2)\hat{i} + x^2 z\hat{j} + (x^2 y + 8xz)\hat{k}$$

$$\nabla\phi|_{(1,-2,1)} = \hat{j} + 6\hat{k}$$

Greatest rate of increase $= \sqrt{1^2 + 6^2} = \sqrt{37} = 6.08$

6. (a)

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ -6 & 4.5 & -90 \end{bmatrix}$$

$R_2 \leftarrow R_2 + 2R_1$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ 0 & 19.5 & -78 \end{bmatrix}$$

$$19.5y = -78$$

or $y = -4$

$$3x + 7.5y = 6$$

$$3x + 7.5(-4) = 6$$

$$3x = 36$$

$\Rightarrow$ $x = 12$

$\therefore$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$

7. (b)

$$\lim_{x \to 0} \frac{\ln(1+5x)}{e^{7x} - 1} \qquad \left(\frac{0}{0} \text{ indeterminate form}\right)$$

Applying L' Hospitals rule

$$\lim_{x \to 0} \frac{\ln(1+5x)}{e^{7x} - 1} = \lim_{x \to 0} \frac{5}{(1+5x)7e^{7x}} = \frac{5}{7}$$

8. (b)

Probability of first item being defective,

$$P_1 = \frac{15}{50}$$

Probability of second item being defective,

$$P_2 = \frac{14}{49}$$

Probability of third item being defective,

$$P_3 = \frac{13}{48}$$

Probability that all three are defective,

$$P = P_1P_2P_3 = \frac{15}{50}\times\frac{14}{49}\times\frac{13}{48} = \frac{13}{560}$$

9. (a)

Given, $x = b(2 - \cos\theta)$, $y = b(\sin\theta + \theta)$

$$\therefore \quad \frac{dx}{d\theta} = b\sin\theta,$$

$$\frac{dy}{d\theta} = b(\cos\theta + 1)$$

$$\frac{dx}{dy} = \frac{dx/d\theta}{dy/d\theta} = \frac{b\sin\theta}{b(\cos\theta+1)}$$

$$= \frac{2b\sin\left(\frac{\theta}{2}\right).\cos\left(\frac{\theta}{2}\right)}{b\times 2\cos^2\left(\frac{\theta}{2}\right)} = \tan\left(\frac{\theta}{2}\right)$$

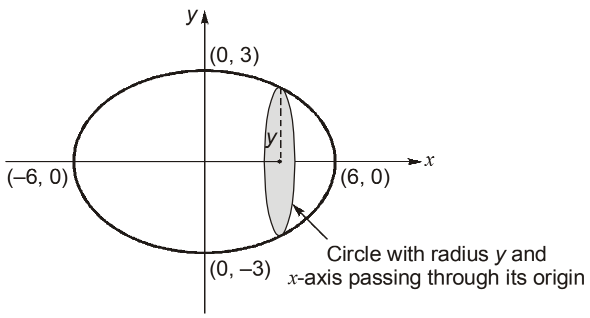
10. (b)

$$P(T) = 0.5$$

Probability of getting tails exactly 6 times is

$$8C_6(0.5)^6(0.5)^2 = \frac{7}{64}$$

11. (b)




$$\text{Volume generated} = \int_{-6}^{6}\pi y^2 dx = \int_{-6}^{6}\pi\left(\frac{36-x^2}{4}\right)dx$$

$$= \frac{\pi\times 2}{4}\int_{0}^{6}(36-x^2)dx = \frac{\pi}{2}\left[36x - \frac{x^3}{3}\right]_0^6$$

$$= 72\pi \text{ unit}^3$$

12. (d)

$$IF = e^{\int f'(x)dx} = e^{f(x)}$$

Solution of differential equation,

$$y \times IF = \int IF \cdot f(x) \cdot f'(x)dx$$

$$y \times e^{f(x)} = \int e^{f(x)} \cdot f(x) \cdot f'(x)dx$$

Let $f(x) = t$

$$f'(x)\ dx = dt$$

$$y \times e^t = \int e^t \cdot t dt$$

$$y.e^t = t.e^t - e^t + c$$

$$y = t - 1 + ce^{-t}$$

$$\log(y + 1 - t) = -t + c'$$

$$\log [y + 1 - f(x)] + f(x) = c'$$

13. (a)

For particular integral,

$$PI = \frac{96x^2}{D^2(D^2+4)} = 96\frac{1}{4D^2\left(1+\frac{D^2}{4}\right)}x^2 = \frac{96}{4}\left[\frac{\left(1-\frac{D^2}{4}\right)x^2}{D^2}\right]$$

$$= 24\frac{\left(x^2 - \frac{1}{2}\right)}{D^2}$$

$$PI = 24\left[\frac{x^4}{4\times 3} - \frac{x^2}{4}\right] = 2x^2(x^2 - 3)$$

$$PI\big|_{x=2} = 2 \times 2^2(4-3) = 8$$

14. (b)

$$u(x, y) = 2x(1 - y)$$

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy = -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$$

$$dv = (2x)dx + 2(1 - y)dy$$

$$v = x^2 + 2y - y^2 + c$$

15. (a)

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx + 6k)dx = 1$$

$$\left.\frac{kx^2}{2}\right|_0^2 + 2kx|_2^4 + \left.\left(\frac{-kx^2}{2}+6kx\right)\right|_4^6 = 1$$

$$\frac{k}{2}(2^2-0)+2k(4-2)-\frac{k}{2}(6^2-4^2)+6k(6-4) = 1$$

$$2k + 4k - 10k + 12k = 1$$

$$8k = 1 \quad \Rightarrow k = \frac{1}{8}$$

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)dx = \int_0^2 \frac{1}{8}x^2dx + \int_2^4 \frac{1}{4}xdx + \int_4^6\left(-\frac{1}{8}x^2+\frac{3}{4}x\right)dx$$

$$= \left.\frac{1}{8}\frac{x^3}{3}\right|_0^2 + \left.\frac{1}{4}\frac{x^2}{2}\right|_2^4 - \left.\frac{1}{8}\frac{x^3}{3}\right|_4^6 + \left.\frac{3}{4}\frac{x^2}{2}\right|_4^6$$

$$= \frac{1}{3}+\frac{3}{2}-\frac{19}{3}+\frac{15}{2} = 3$$

16. (d)

$$I = \int_0^{\pi/2}\sqrt{1+\sec x}\,dx = \int_0^{\pi/2}\sqrt{1+\frac{1}{\cos x}}\,dx$$

$$= \int_0^{\pi/2}\frac{\sqrt{1+\cos x}}{\sqrt{\cos x}}\,dx = \int_0^{\pi/2}\frac{\sqrt{2}\cos(x/2)}{\sqrt{1-2\sin^2(x/2)}}\,dx$$

Let $\sin\frac{x}{2} = t,$ $\quad\begin{cases} x = 0, & t = 0 \\ x = \frac{\pi}{2} & t = \frac{1}{\sqrt{2}}\end{cases}$

$$\frac{1}{2}\cos\frac{x}{2}dx = dt$$

$$I = \int_0^{1/\sqrt{2}}\frac{2\sqrt{2}dt}{\sqrt{1-2t^2}}$$

$$= 2\sin^{-1}\left(\sqrt{2}t\right)\Big|_0^{1/\sqrt{2}} = 2\sin^{-1}\left(\sqrt{2}\times\frac{1}{\sqrt{2}}\right) - 2\sin^{-1}(0)$$

$$= 2\times\frac{\pi}{2} = \pi = 3.14$$

17. (b)

$$(2y-3x)dx + xdy = 0$$

$$\frac{dy}{dx}+\frac{2}{x}y = 3$$

$$IF = e^{\int\frac{2}{x}dx} = e^{2\ln x} = x^2$$

$$y.x^2 = 3\int x^2dx = x^3 + c$$

For $x = 0$, $y = 0$

$\Rightarrow$ $\quad 0 = 0 + c$

$\Rightarrow$ $\quad c = 0$

For $x = 2$, $\quad y\times 2^2 = 2^3$

$$y = 2$$

18. (c)

$$\frac{4C_1 \cdot 4C_1 \cdot 4C_1 \cdot 4C_1}{52C_4} = \frac{4\times4\times4\times4}{(52\times51\times50\times49)/(4\times3\times2\times1)}$$

$$= \frac{4\times4\times4\times4\times3\times2}{52\times51\times50\times49} = \frac{256}{270725}$$

19. (a)

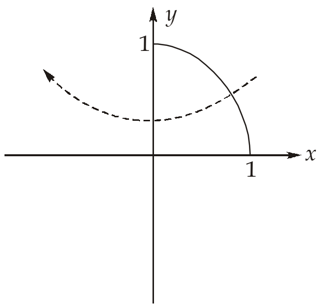
$x = \sin\left(\frac{\pi k}{2}\right), y = \cos\left(\frac{\pi k}{2}\right)$

Just by seeing, we can know that it represents a circle in $x - y$ plane, given by

$$x^2 + y^2 = 1$$

Given $0 \le k \le 1$, which gives $0 \le x \le 1$; $0 \le y \le 1$

or $$0 \le \frac{\pi k}{2} \le \frac{\pi}{2}$$




So we get a quarter circle, when this is rotated with respect to y-axis by 360 degree, it creates a hemisphere of radius 1.

Surface area of hemisphere,

$$A_S = 2\pi r^2$$
$$= 2\pi(1)^2 = 2\pi$$

20. (c)

$$f(y) = y^2 e^{-y}$$
$$f'(y) = y^2\left(-e^{-y}\right) + e^{-y} \times 2y$$
$$= e^{-y}\left(2y - y^2\right)$$

Putting $f'(y) = 0$

$$e^{-y}\left(2y - y^2\right) = 0$$
$$e^{-y} y(2 - y) = 0$$

$y = 0$ or $y = 2$ are the stationary points

Now, $$f''(y) = e^{-y}(2 - 2y) + \left(2y - y^2\right)\left(-e^{-y}\right)$$
$$= e^{-y}\left(2 - 2y - 2y + y^2\right)$$
$$= e^{-y}\left(y^2 - 4y + 2\right)$$

At $y = 0$, $$f''(0) = e^{-0}(0 - 0 + 2) = 2$$

Since $f''(0) = 2$ is > 0 at $y = 0$ we have a minima

$$\text{Now, at } y = 2 f''(2) = e^{-2}\left(2^2 - 4 \times 2 + 2\right)$$
$$= e^{-2}(4 - 8 + 2)$$
$$= -2e^{-2} < 0$$

∴ At $y = 2$ we have a maxima.

21. (a)

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$
$$P(x < 3) = P(0) + P(1) + P(2)$$
$$= \frac{\mu^0 e^{-\mu}}{0!} + \frac{\mu^1 e^{-\mu}}{1!} + \frac{\mu^2 e^{-\mu}}{2!}$$
$$= \frac{1}{e^\mu} + \frac{\mu}{e^\mu} + \frac{\mu^2}{2e^\mu}$$

As $\mu(\text{mean}) = 6.8$

$$\therefore \quad P(x < 3) = \frac{1 + 6.8 + \left(\frac{6.8^2}{2}\right)}{e^{6.8}} = \frac{30.92}{897.85} \simeq 0.034$$

22. (a)

$$\sin x \cos y dx + \cos x \sin y dy = 0$$

Divide by $\cos x \cos y$, we get ,

$$\tan x \, dx + \tan y dy = 0$$

Integrating the equation,

$$\log \sec x + \log \sec y = C_1$$
$$\log \frac{1}{\cos x \cos y} = C_1$$
$$\cos x \cos y = C$$

Since it passes through $\left(0, \frac{\pi}{3}\right)$

$$\cos(0) \cos\left(\frac{\pi}{3}\right) = C$$
$$\frac{1}{2} = C$$

⇒ The equation of curve is,

$$\cos x \cos y = \frac{1}{2}$$

23. (c)

$$P(x) = x^5 + x + 2$$

It has a real root at $x = -1$

$$\Rightarrow \quad P(x) = (x^4 - x^3 + x^2 - x + 2)(x + 1)$$

Now, $x^4 - x^3 + x^2 + x + 2$ will give other 4 roots

To find roots,

$$\Rightarrow \quad x^4 - x^3 + x^2 - x + 2 = 0$$

$\Rightarrow \quad x^3(x-1) + x(x-1) + 2 = 0$

$\Rightarrow \quad x(x^2+1)(x-1) + 2 = 0$

In the above expression, $x^2 + 1$ is always positive. So, either '$x$' or '$x - 1$' should be negative in order to satisfy the equation.

For $x > 1$, both $(x)$ and $(x - 1)$ are positive and,

For $x < 0$, both $(x)$ and $(x - 1)$ are negative

$\therefore$ $x$ should lie within 0 and 1 in order to have real roots.

As $x \in (0, 1)$

$\Rightarrow \quad |x| < 1$

$\Rightarrow \quad |x^2 + 1| < 2, |x| < 1$ and $|x - 1| < 1$

$\therefore$ The product of these three will be less than 2 and hence, no real value of '$x$' can satisfy the equation

$$x^4 - x^3 + x^2 - x + 2 = 0$$

$\therefore$ The equation will have four imaginary roots apart from one real roots.

24. (a)

$$I = \int \sec^3\theta\, d\theta = \int \sec\theta.\sec^2\theta\, d\theta$$

$$= \sec\theta \int \sec^2\theta\, d\theta - \int \tan\theta(\sec\theta\tan\theta)\, d\theta$$

$$= \sec\theta\tan\theta - \int \tan^2\theta\sec\theta\, d\theta$$

$$\Rightarrow \quad I = \sec\theta\tan\theta - \int(\sec^2\theta - 1)\sec\theta\, d\theta$$

$$= \sec\theta\tan\theta - \int \sec^3\theta\, d\theta + \int \sec\theta\, d\theta$$

$$\Rightarrow \quad I = \sec\theta\tan\theta - I + ln|\sec\theta + \tan\theta| + c$$

$$\Rightarrow \quad I = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2} ln|\sec\theta + \tan\theta| + c$$

$$\therefore \quad a + b = \frac{1}{2} + \frac{1}{2} = 1$$

25. (c)

$$\int_C \bar{F}.\overline{dr} = \int_C x^2y^2dx + y.dy$$

For curve $C$, $\quad y^2 = 4x$

and $\quad 2y\,dy = 4\,dx$

$$\Rightarrow \quad \int_C \bar{F}.\overline{dr} = \int_0^4 x^2(4x)dx + 2dx$$

$$= \int_0^4 (4x^3 + 2)dx = 264$$

26. (a)

To obtain maximum value of $f(x)$, first $f'(x)$ should be equated to zero.

$\Rightarrow \quad f'(x) = 6x^2 - 6x - 36 = 0$

$\Rightarrow \quad x^2 - x - 6 = 0$

$\Rightarrow \quad (x-3)(x+2) = 0$

$\therefore \quad f'(x) = 0 \quad$ at $x = 3$ and $-2$

Now, $\quad f''(x) = 12x - 6$

$f''(3) = 30 > 0$

at $x = 3$, there is local minima

and $\quad f''(2) = -30 < 0$

$\therefore$ at $x = -2$, a local maxima is observed.

27. (b)

$$\text{Length of curve} = \int_0^1 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

Curve: $3x^2 = y^3$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{\sqrt{3y}}{2}$$

$$\therefore \quad \text{Length} = \int_0^1 \sqrt{1+\frac{3y}{4}}\, dy$$

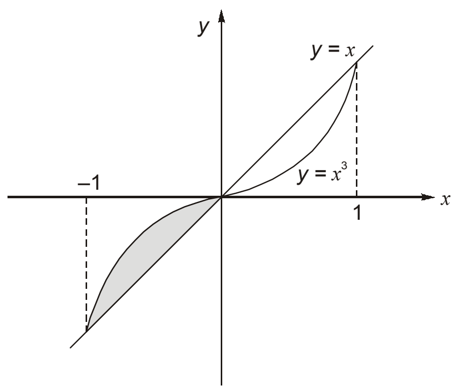
$$= \frac{1}{2}\int_0^1 \sqrt{4+3y}\, dy$$

$$= \frac{1}{2}\left[\frac{(4+3y)^{3/2}}{\frac{3}{2}\times 3}\right]_0^1$$

$$= \frac{1}{9}\left(7\sqrt{7}-8\right)$$

28. (b)

Point of inter-section of the two curves are $x = 0, 1, -1$




$$\text{Area} = \int_{-1}^{0} \left(x^3 - x\right) dx = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} = \frac{0-(-1)^4}{4} - \frac{0-(-1)^2}{2} = \frac{1}{4}$$

29. (a)

$$(x+1)\frac{dy}{dx}+1 = 2e^{-y}$$

$$\Rightarrow \quad (x+1)\frac{dy}{dx} = (2e^{-y}-1)$$

$$\Rightarrow \quad \frac{dy}{(2e^{-y}-1)} = \frac{dx}{x+1}$$

$$\Rightarrow \quad \frac{e^y dy}{2-e^y} = \frac{dx}{x+1}$$

$$\Rightarrow \quad -\log(2-e^y) = \log(x+1)+c$$

$$\Rightarrow \quad (x+1)(2-e^y) = k$$

30. (c)

$$\frac{d^2y}{dx^2} = y$$

$$\Rightarrow \quad D^2y = y \qquad (\therefore d/dx = D)$$

$$(D^2-1)y = 0$$

$$D^2-1 = 0$$

$$D = \pm 1$$

$$y = C_1 e^x + C_2 e^{-x}$$

Given point passes through origin

$$\Rightarrow \quad 0 = C_1 + C_2$$

$$C_1 = -C_2 \qquad \text{...(i)}$$

Also, point passes through (ln 2, 3/4)

$$\Rightarrow \quad \frac{3}{4} = C_1 e^{\ln 2} + C_2 e^{-\ln 2}$$

$$\frac{3}{4} = 2C_1 + \frac{C_2}{2}$$

$$\Rightarrow \quad C_2 + 4C_1 = 1.5 \qquad \text{...(ii)}$$

From (i) $C_1 = -C_2$, putting in (ii), we get

$$\Rightarrow \quad -3C_2 = 1.5$$

$$C_2 = -0.5$$

$$\therefore \quad C_1 = 0.5$$

$$\Rightarrow \quad y = 0.5(e^x - e^{-x})$$

$$y = \frac{e^x - e^{-x}}{2}$$

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