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STRUCTURE ANALYSIS

CIVIL ENGINEERING

Date of Test: 24/08/2025

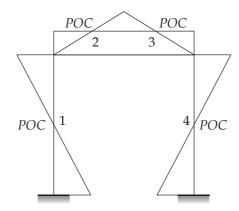
ANSWER KEY >

1.	(b)	7.	(c)	13.	(c)	19.	(a)	25.	(c)
2.	(c)	8.	(b)	14.	(b)	20.	(b)	26.	(b)
3.	(b)	9.	(a)	15.	(d)	21.	(c)	27.	(b)
4.	(b)	10.	(c)	16.	(c)	22.	(d)	28.	(a)
5.	(c)	11.	(b)	17.	(d)	23.	(c)	29.	(b)
6.	(b)	12.	(b)	18.	(b)	24.	(a)	30.	(c)

DETAILED EXPLANATIONS

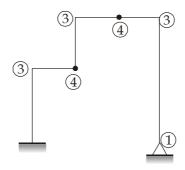
$$F_{FB} = 0$$
 At joint C ,
$$F_{BC} = 0$$

$$F_{CD} = 0$$
 At joint A ,
$$F_{AB} = 0$$



4 points of contra-flexure.

3. (b)



Kinematic indeterminacy = 3 + 3 + 3 + 4 + 4 + 1 = 18

4. (b)

$$\Sigma F_{y} = 0$$

$$\Rightarrow \qquad V_{A} - 10 = 0$$

$$\Rightarrow \qquad V_{A} = 10 \text{ kN (\uparrow)}$$

$$\Sigma M_{G} = 0$$

$$\Rightarrow \qquad H_{A} \times 3 - 10 \times 9 = 0$$

$$\Rightarrow \qquad H_{A} = \frac{90}{3} = 30 \text{ kN (\leftarrow)}$$

Resultant reaction,
$$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{10^2 + 30^2}$$

$$= 31.62 \text{ kN}$$



5. (c)

Given

Stiffness matrix

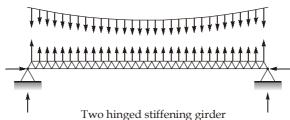
$$[k] = \frac{3EI}{L} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

Flexibility matrix is the inverse of stiffness matrix:

$$[f] = \frac{L}{3EI[3\times3-1\times1]} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

$$[f] = \frac{1}{24EI} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

6. (b)



$$D_s = r_e - 3$$

= 4 - 3 = 1

Indeterminate to one degree.

7. (c)

Let the parts are x and (1500 - x)

$$\therefore \frac{x \times 10 \times 5}{100} = \frac{(1500 - x) \times 12.5 \times 4}{100}$$

$$\Rightarrow x = 1500 - x$$

$$\Rightarrow 2x = 1500$$

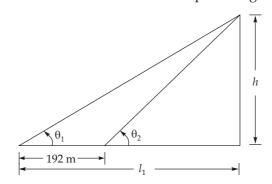
$$\Rightarrow \qquad \qquad x = 750$$

$$\Rightarrow \qquad \qquad x = 750$$

Hence, the sum lent out at 12.5% per annum = 1500 - 750 = Rs. 750

8.

Let the height of tower is h and its distance from the point of ground is l_1



$$\therefore \qquad \tan \theta_1 = \frac{h}{l_1} = \frac{5}{12} \qquad \dots (i)$$

$$\tan \theta_2 = \frac{h}{l_1 - 192} = \frac{3}{4}$$
 ...(ii)

From (i) and (ii)

$$\frac{l_1}{l_1 - 192} = \frac{3}{4} \times \frac{12}{5} = \frac{9}{5}$$

$$5l_1 = 9l_1 - 9 \times 192$$

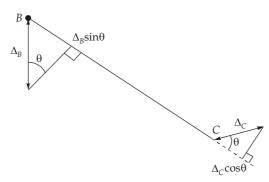
$$4l_1 = 9 \times 192$$

$$l_1 = \frac{9 \times 192}{4}$$

$$l_1 = 432 \text{ m}$$

Hence, height of tower = $\frac{5l_1}{12} = \frac{5}{12} \times 432 = 180$ meters

9. (a)



As length of member will not change because members are inextensible,

$$\begin{array}{rcl} \Delta_B \sin\theta &=& \Delta_C \cos\theta \\ \Rightarrow & \Delta_C &=& \Delta_B \tan\theta \\ \Rightarrow & \Delta_C &=& \frac{4\Delta_B}{3} = \frac{4}{3}\Delta \end{array}$$

10. (c)

Slope deflection equation for moment of A in the beam AB is given by

$$M_{AB} = (M_F)_{AB} + \frac{2EI}{L} \left(2\theta_A + \theta_B - \frac{3\Delta}{L} \right)$$

11. (b)

$$M_A = 0;$$
 $V_B \times L = w \frac{L}{4} \times \frac{L}{8}$ $V_B = \frac{wL}{32}$ $V_B \times \frac{L}{2} = H_B \times h$ $W_B \times \frac{L}{2} = H_B \times h$ $W_B \times \frac{L}{2} = H_B \times h$

$$\Rightarrow H_B = \frac{wL^2}{64h}$$
So,
$$k = \frac{1}{64}$$

12. (b)

Applying Betti's theorem

$$25 \times 0.002 + 15 \times \frac{9}{1000} = 15 \times \theta_A + 22 \times 0.004$$

 $\theta_A = 0.00647 \text{ radian}$

13. (c)

$$M_{FCB} = -\frac{12 \times 4^2}{12} = -16 \text{ kNm}$$

$$M_{CB} + M_{CA} = 0$$

$$\Rightarrow \qquad -16 + \frac{2EI}{4} \times (2\theta_C) + \frac{2EI}{4} \times (2\theta_C) = 0$$

$$\Rightarrow \qquad \theta_C = \frac{8}{EI}$$

14. (b)

Using slope deflection method

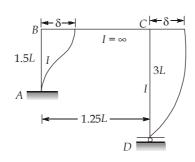
Equilibrium equation at joint B is:

$$M_{BA} + M_{BC} + 10 \times 3 + 12 \times 3 \times 1.5 = 0$$

$$\Rightarrow \frac{2EI}{4}(2\theta_B) + \frac{2EI}{4}(2\theta_B) + 84 = 0$$

$$\Rightarrow \theta_B = \frac{-42}{EI}$$

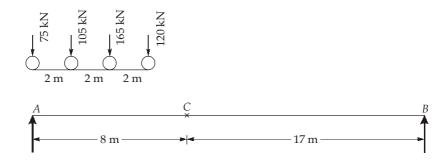
15. (d)



$$M_{BA} = -64 = \frac{6EI\delta}{(1.5L)^2}$$
$$M_{CD} = \frac{3EI\delta}{(3L)^2} = \frac{3EI\delta}{9L^2}$$
$$= -8 \text{ kNm}$$

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16. (c)



Load	Average load on AC (y_1) kN	Average load on <i>CB</i> (y ₂) kN	Remark
All load on span AC	$\frac{75 + 105 + 165 + 120}{8} = 58.125$	$\frac{0}{17} = 0$	y ₁ > y ₂
120 kN load crosses C	$\frac{75 + 105 + 165}{8} = 43.125$	$\frac{120}{17} = 7.06$	y ₁ > y ₂
165 kN load crosses C	$\frac{75 + 105}{8} = 22.5$	$\frac{165 + 120}{17} = 16.76$	y ₁ > y ₂
105 kN load crosses C	$\frac{75}{8} = 9.375$	$\frac{105 + 165 + 120}{17} = 22.94$	y ₂ > y ₁

So, 105 kN load should be placed at the section.

17. (d)

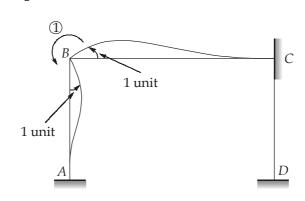
Area under free moment diagram = Area under fixing moment

$$\Rightarrow \frac{1}{2} \times 2 \times 370.02 + \frac{1}{2} \times 2 \times (370.02 + 332.18) + \frac{1}{2} \times 2 \times 332.18 = \frac{1}{2} \times 6 \times (M_A + 228.29)$$

$$\Rightarrow M_A = 239.843 \text{ kNm} \approx 239.84 \text{ kNm}$$

18. (b)

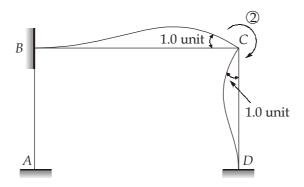
Keeping $D_1 = 1.0$ and $D_2 = 0$, the frame will be as shown below:



$$S_{11} = \frac{4 \times 3(EI)}{6} + \frac{4EI}{4} = 3EI$$

$$S_{21} = \frac{2 \times (3EI)}{6} = -EI$$

Keeping D_1 = 0 and D_2 = 1.0, the frame will be as shown in below:



$$S_{12} = \frac{2 \times 3EI}{6} = -EI$$

 $S_{22} = \frac{4(3EI)}{6} + \frac{4EI}{4} = 3EI$

and

19. (a)

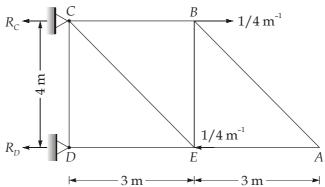
$$\theta_{\rm BE} = \Sigma k \delta$$

 δ is given in the problem on the members *AB*, *AE* and *DE*.

Remaining members have no contraction and no elongation.

For the *k* forces:

Apply unit couple as shown below and find k forces in members of the truss.



Since at joint A, two non-collinear members are meeting and no external load is applied to it.

$$F_{AB} = F_{AE} = 0$$

$$\Sigma M_C = 0$$

$$\frac{1}{4} \times 4 - R_D \times 4 = 0$$

$$R_D = \frac{1}{4}$$

$$\Sigma F_x = 0$$

$$F_{DE} = \frac{1}{4} (\text{Comp.})$$

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$$\theta_{BE} = \Sigma k \delta = k_{DE} \delta_{DE} + k_{AB} \delta_{AB} + k_{AE} \delta_{AE}$$

$$= -\frac{1}{4} \left(+\frac{2}{1000} \right) + 0 + 0 = -\frac{1}{2000} \text{ rad}$$

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20. (b)

$$k_{11} = \Sigma \left(\frac{AE}{L}\right) \cos^2 \theta$$

$$= \left[\frac{AE}{L}(\cos 60^\circ)^2\right]_{OD} + \left[\frac{AE}{L}(\cos 90^\circ)^2\right]_{OC}$$

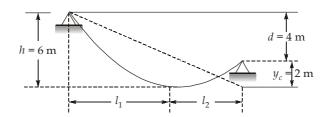
$$+ \left[\frac{AE}{L}(\cos 135^\circ)^2\right]_{OB} + \left[\frac{AE}{L}(\cos 150^\circ)^2\right]_{OA}$$

$$= 1.5 \frac{AE}{L}$$

$$x = 1.5$$

Thus

21. (c)



$$l_1 + l_2 = l = 100 \text{ m}$$
 ...(i)
 $d = 4 \text{ m}, h = 6 \text{ m}, y_c = h - d = 2 \text{ m}$

From eq. (i) and (ii)

$$l_2 = \frac{100}{2.732} = 36.6 \,\mathrm{m}$$

22. (d)

Allow the loads to cross the given section one after another and calculate average loads on AD and BD

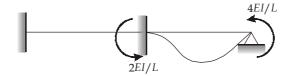
Load crossing section D	Avg. load on AD	Avg. load on BD	Load remarks
50kN	$\frac{650}{4}$	$\frac{50}{6}$	AD > BD
300 kN	$\frac{350}{4}$	$\frac{350}{6}$	AD > BD
150 kN	$\frac{200}{4}$	<u>500</u> 6	BD > AD

After 150 kN passes the section, average load on BD section becomes higher than AD.

Maximum BM at $D = 200 \times 1.92 + 150 \times 2.4 + 300 \times 2 + 50 \times 1.52$ = 1420 kNm

23. (c)

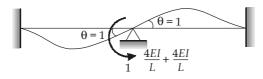
Give unit displacement in the direction (1)



$$k_{11} = \frac{4EI}{L}$$

$$k_{21} = \frac{2EI}{L}$$

Give unit displacement in the direction (2)

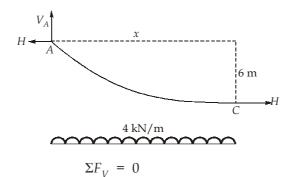


$$k_{12} = \frac{2EI}{L};$$
 $k_{22} = \frac{8EI}{L}$

∴ The stiffness matrix
$$[k] = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{8EI}{L} \end{bmatrix}$$

24. (a)

Let *x* be horizontal distance between A and C consider the equilibrium of the part AC of the cable

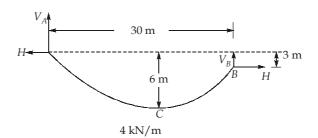


Taking moments about point A, $H \times 6 = 4x \times \frac{x}{2}$

$$\Rightarrow H = \frac{x^2}{3} \qquad \dots \text{ (ii)}$$

Now, consider the equilibrium of whole cable,

Taking moments about B,



$$\Sigma M_{\scriptscriptstyle R} = 0$$

$$V_A \times 30 = H \times 3 + (4 \times 30) \times \frac{30}{2}$$

$$30V_A = 3H + 1800$$
 ...(iii)

From equations (i) and (iii)

$$\Rightarrow \qquad 4x \times 30 = \frac{x^2}{3} \times 3 + 1800$$

$$\Rightarrow$$
 $x^2 - 120x + 1800 = 0$

After solving we get, x = 17.574 m, 102.426 m

Adopt,
$$x = 17.574 \text{ m}$$

$$V_A = 4 \times 17.574 = 70.296 \text{ kN}$$

$$H = \frac{(17.574)^2}{3} = 102.95 \,\text{kN}$$

$$\therefore \qquad \text{Maximum tension, } T_{\text{max}} = \sqrt{H^2 + V_A^2} = \sqrt{(70.296)^2 + (102.95)^2}$$
$$= 124.66 \text{ kN}$$

25. (c)

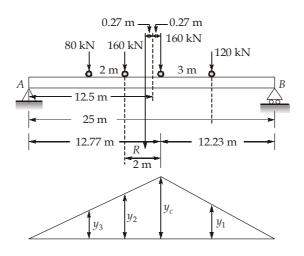
Let CG of loads from leading wheel load (120 kN) be at a distance x,

$$x = \frac{160 \times 3 + 160 \times 5 + 80 \times 7}{120 + 160 + 160 + 80} = 3.54 \text{ m}$$

This is nearer to 160 kN (leading). Hence maximum moment is likely to occur under this load. Its distance from CG,

$$3.54 - 3 = 0.54 \text{ m}$$

Therefore its position from end A = $\frac{25}{2} + \left(\frac{0.54}{2}\right) = 12.77 \text{ m}$



$$y_c = \frac{12.77 \times 12.23}{25}$$

$$y_1 = \frac{12.77}{25} \times 9.23$$

$$y_2 = \frac{12.23}{25} \times 10.77$$

$$y_3 = \frac{12.23}{25} \times 8.77$$

Therefore, absolute maximum bending moment,

$$BM^{max} = 160 \times y_c + 120 \times y_1 + 160 \times y_2 + 80 \times y_3$$

$$= 160 \times \left(\frac{12.77 \times 12.23}{25}\right) + 120 \times \left(\frac{12.77 \times 9.23}{25}\right) + 160 \times \left(\frac{12.23}{25} \times 10.77\right) + 80\left(\frac{12.23}{25} \times 8.77\right)$$

$$= 2751.51 \text{ kNm}$$

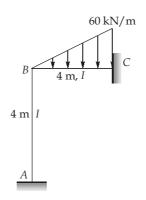
:. Maximum bending stress developed:

$$f_{\text{max}} = \frac{M}{Z} = \frac{2751.51 \times 10^6}{245.4 \times 10^3} = 11212.35 \text{ N/mm}^2$$

= 11.212 × 10³ × 10⁶ N/m² \simeq 11.2 GPa

26. (b)

Here degree of freedom is 3 i.e. the rotations $\theta_B \theta_C$ and θ_D . But due to symmetry, we have $\theta_B = -\theta_D$ and $\theta_C = 0$. For analysis of the frame as shown in figure above, only half as of the given structure is considered and $\theta_C = 0$, therefore only θ_B is the only unknown.



 \dots (α)

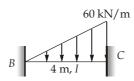
Joint equilibrium equation,

$$M_{BA} + M_{BC} = 0$$

Fixed end moment: Span AB

$$M_{FAB} = M_{FBA} = 0$$

Span BC:



$$M_{FBC} = -\frac{wL^2}{30} = \frac{60 \times 4^2}{30} = 32 \text{ kNm}$$
 $(\theta_A = 0, \Delta = 0)$

$$M_{BC} = M_{FBC} + \frac{2EI}{4} \left(2\theta_A + \theta_B - \frac{3\Delta}{4} \right)$$
$$= 32 + \frac{EI\theta_B}{2}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{4} \left(2\theta_B + \theta_A - \frac{3\Delta}{4} \right)$$

$$M_{BA} = EI\theta_B$$
 ... (β)

$$M_{BC} = M_{FBC} + \frac{2EI}{4} \left(2\theta_B + \theta_C - \frac{3\Delta}{4} \right) \qquad (\theta_C = 0, \Delta = 0)$$

$$= -32 + EI\theta_B \qquad \dots (\gamma)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{4} \left(2\theta_C + \theta_B - \frac{3\Delta}{4} \right)$$

$$= 48 + \frac{EI\theta_B}{2} \qquad \dots (\delta)$$

Now, from equation (i), $M_{BA} + M_{BC} = 0$

$$\Rightarrow EI\theta_B - 32 + EI\theta_B = 0$$

$$\Rightarrow EI\theta_B = 16$$

Now, from equation (δ)

$$M_{CB} = 48 + \frac{16}{2} = 56 \text{ kNm}$$

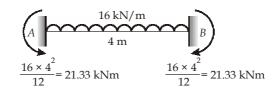
27. (b)

 \Rightarrow

In case of moment distribution method if we have known joint displacements or rotations, the effect of these things are taken in fixed end moment.

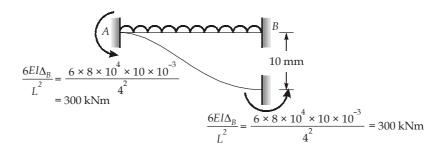
Calculation of fixed end moment.

Span AB



$$\frac{4EI\theta_{A}}{L} = \frac{4 \times 8 \times 10^{4} \times 0.002}{4}$$
= 160 kNm
$$= 160 \text{ kNm}$$

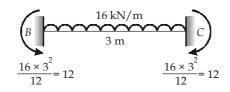
$$\frac{2EI\theta_{A}}{L} = \frac{2 \times 8 \times 10^{4} \times 0.002}{2}$$
= 80 kNm

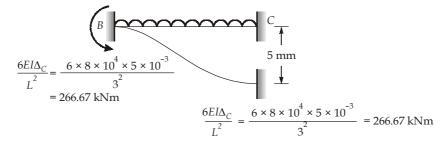


$$M_{FAB} = -21.33 + 160 - 300 = -161.33 \text{ kNm}$$

$$M_{FBA} = 21.33 + 80 - 300 = -198.67 \text{ kNm}$$

Span BC





$$M_{FBC} = -12 - 266.67 = -278.67 \text{ kNm}$$

$$M_{FCB} = 12 - 266.67 = -254.67 \text{ kNm}$$

Span CD:

$$M_{CD} + \frac{16 \times (1)^2}{2} = 0$$

$$M_{CD} = -8 \text{ kNm}$$

 \Rightarrow

Distribution factor (DF):

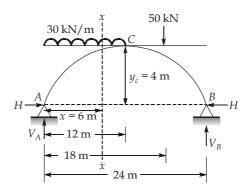
T = : t	N / 1	Member	Member	_D MS	
Joint	Member	Stiffness(MS)	Stiffness(JS)	$D_F = \overline{JS}$	
В	ВА	$\frac{4EI}{4} = EI$		0.5	
			2EI		
	ВС	$\frac{3EI}{3} = EI$		0.5	

Α		$\frac{I}{1/2}$		(2	D
Λ			4	- 8	-8	_
	-161.33	-198.67	-278.67	-254.67		
			127.335	- +254.67		
	-161.34	-198.66	-147.335	8	-8	
		+173	+173			
	86.5			0		
	-74.84	-25.66	25.66	8	-8	



 \therefore Support moment at B = 25.66 kNm (Hogging)

28. (a)



The arch is shown in figure above,

Taking moments about B,

$$\Sigma M_B = 0$$

$$V_A \times 24 - 30 \times 12 \times 18 - 50 \times 6 = 0$$

$$V_A = 28$$

$$V_A = 282.5 \text{ kN}$$

 $V_B = 30 \times 12 + 50 - 282.5 = 127.5 \text{ kN}$

$$\Sigma M_C = 0$$
 (From right)

$$V_B \times 12 - H \times 4 - 50 \times 6 = 0$$

$$\Rightarrow H = \frac{127.5 \times 12 - 50 \times 6}{4} = 307.5 \text{ kN}$$

Now, at 6 m from left support,
$$y = \frac{4y_c}{L^2}x(L-x) = \frac{4 \times 4}{24^2} \times 6(24-6) = 3 \text{ m}$$

Also,
$$\frac{dy}{dx} = \frac{4y_c}{L^2}(L - 2x)$$

At
$$x = 6$$
 m, $\frac{dy}{dx} = \tan \theta = \frac{4 \times 4}{24^2} (24 - 2 \times 6) = \frac{1}{3}$

$$\cos\theta = \frac{3}{\sqrt{10}}; \quad \sin\theta = \frac{1}{\sqrt{10}}$$

Radial shear at a section 6 m from left support,

$$Q = V\cos\theta - H\sin\theta$$

$$V = \text{Vertical shear at } x-x$$

$$= V_A - wx = 282.5 - 30 \times 6$$

$$= 102.5 \text{ kN}$$

$$Q = 102.5 \times \frac{3}{\sqrt{10}} - 307.5 \times \frac{1}{\sqrt{10}} = 0$$

29. (b)

$$M_x = \frac{wR^2}{2} (\sin\theta - \sin^2\theta)$$

For maximum bending moment,

$$\frac{dM_x}{d\alpha} = 0$$

$$\frac{wR^2}{2}(\cos\theta - 2\sin\theta\cos\theta) = 0$$

$$\Rightarrow \cos\theta(1 - 2\sin\theta) = 0$$

$$\Rightarrow$$
 $\cos\theta = 0 \Rightarrow \theta = 90^{\circ} \text{ and } \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^{\circ}$

$$\theta = 90^{\circ}$$
 (Rejected as BM = 0)

$$\therefore \qquad \qquad \theta = 30^{\circ}$$

$$M_x = \frac{-wR^2}{2} (\sin 30^\circ - \sin^2 30^\circ)$$
$$= \frac{-wR^2}{8} \text{ or } \frac{wR^2}{8} \text{ (Hogging)}$$

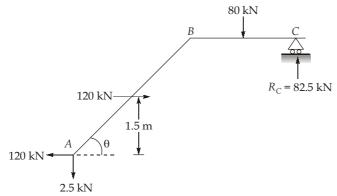
30. (c)
$$\Sigma F_y = 0 \qquad \Rightarrow \qquad R_A + R_C = 80 \text{ kN}$$

$$\Sigma M_A = 0 \qquad \Rightarrow \qquad R_C \times 8 = 80 \times 6 + 40 \times 3 \times 1.5$$

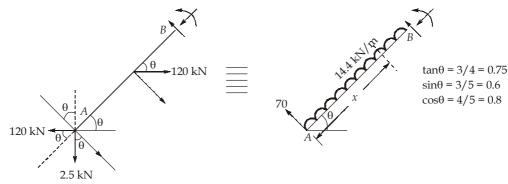
$$\Rightarrow \qquad \qquad R_C = 82.5 \text{ kN } (\uparrow)$$

$$\therefore \qquad \qquad R_A = -2.5 \text{ kN} = 2.5 \text{ kN} (\downarrow)$$





Now, resolving forces perpendicular to AB



:.Force perpendicular to
$$AB = 120 \sin\theta - 2.5 \cos\theta$$

= $120 \times 0.6 - 2.5 \times 0.8$

$$= 70 \text{ kN}$$

Also, transformed UDL =
$$\frac{120 \times \sin \theta}{5} = \frac{120 \times 0.6}{5} = 14.4 \text{ kN/m}$$

For maximum BM, SF = 0

$$\Rightarrow 70 - 14.4x = 0$$

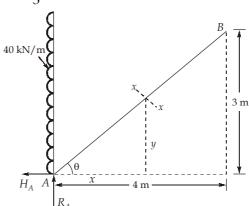
$$x = \frac{70}{14.4} = 4.86 \text{ m} \text{ (4.86 m is along AB)}$$

Distance vertically above A, $d = 4.86 \sin\theta = 4.86 \times 0.6 = 2.916 \text{ m} \simeq 2.92 \text{ m}$

 ${\bf Alternatively:}$

$$\tan\theta = \frac{y}{x} = \frac{3}{4}$$

$$x = \frac{4}{3}$$



Now, bending moment at section X-X,

$$M_{xx} = R_A x - 40 \frac{y^2}{2} + H_A y$$

$$\Rightarrow M_{xx} = -2.5 \left(\frac{4}{3}y\right) - 40 \frac{y^2}{2} + 120y$$

$$\Rightarrow M_{xx} = \left(120 - \frac{2.5 \times 4}{3}\right) y - 20y^2 = 116.67y - 20y^2$$

For maximum bending moment, $\frac{dM_x}{dy} = 0$

$$\Rightarrow 116.67 - 20 \times 2y = 0$$

$$\Rightarrow \qquad \qquad y = 2.917 \text{ m} \simeq 2.92 \text{ m}$$

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