CLASS TEST

S.No.: 02SK_ECEE_JKLMN_8825

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CONTROL SYSTEM

EC-EE

Date of Test : 08/08/2025

ANSWER KEY

1.	(b)	7.	(d)	13.	(a)	19.	(a)	25.	(b)
2.	(d)	8.	(d)	14.	(a)	20.	(c)	26.	(d)
3.	(b)	9.	(d)	15.	(d)	21.	(b)	27.	(c)
4.	(b)	10.	(a)	16.	(c)	22.	(b)	28.	(c)
5.	(c)	11.	(c)	17.	(b)	23.	(d)	29.	(a)
6.	(a)	12.	(d)	18.	(b)	24.	(b)	30.	(a)

DETAILED EXPLANATIONS

1. (b)

$$G(s)H(s) = \frac{K}{s(s+3-j\sqrt{3})(s+3+j\sqrt{3})} = \frac{K}{s(s^2+6s+12)}$$

2. (d)

The roots of the system in s-plane coincides with the poles if the gain of the system is reduced to a value zero.

3. (b)

From the given equation,

We can get open loop transfer function of the system

$$G(s) = \frac{K(s+1)}{s(s^2+5s+2)}$$

$$P = 3 \quad ; \quad Z = 1$$

$\therefore$ Number of asymptotes, $P - Z = 2$

$\therefore$ Angle of asymptotes $= \frac{(2q+1)\,180°}{P-Z}$

$\because$ $P - Z = 2 \quad \Rightarrow q = 0, 1$

$\therefore$ Angle of asymptotes are 90° and 270°.

4. (b)

Poles location, $s = 0, s = -2, s = -4$

Zero's location, $s = -3$

$$G(s) = \frac{k(s+3)}{s(s+2)(s+4)}$$

$$G(s)|_{s=1} = \frac{k(1+3)}{1\times(1+2)\times(1+4)} = 4$$

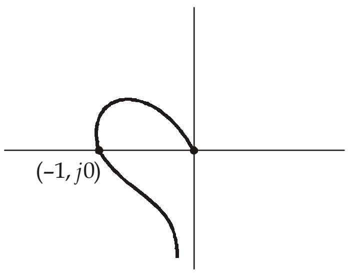
$$\Rightarrow \quad k = \frac{4\times 5\times 3}{4} = 15$$

Hence, $G(s) = \frac{15(s+3)}{s(s+2)(s+4)}$

5. (c)

6. (a)

$$PM = 180° + \phi_{\omega gc}$$
$$= 0$$



7. **(d)**

The output of the system is given by,

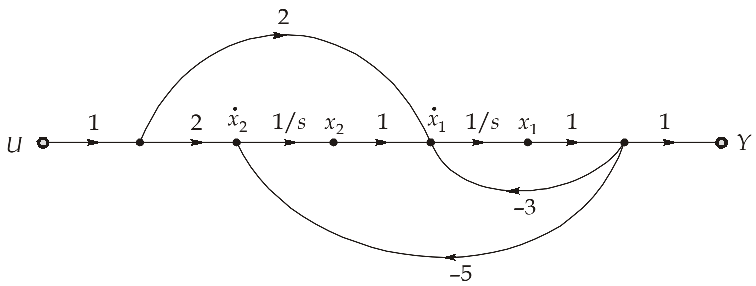
$$y(t) = C\phi(t)x(0) + \int_0^t C\phi(t-\tau)Bu(\tau)d\tau$$

$\because$ Initial conditions are zero, the first part of output is zero. The impulse input exists only at $t = 0$ with an area of 1 which converts the integral in the second part as $C\phi(t)B$.

We know that, $\phi(t) = \exp(At) \Rightarrow$ State transition matrix

$\therefore$ Impulse response $= C\exp(At)B$

8. **(d)**



$$\dot{x}_1 = 2u + x_2 - 3x_1$$

$$\dot{x}_2 = 2u - 5x_1$$

$$y = x_1$$

9. **(d)**

$s(s + 4)(s + 7) + K(s + 1)(s + 5) = 0$

$$K = \frac{-s(s+4)(s+7)}{(s+1)(s+5)}$$

$\because K \to \infty, \therefore$ $(s + 1)(s + 5) = 0$

$$s = -1, -5$$

$\therefore$ Roots are –1, –5 and ∞.

10. **(a)**

As we know,

$$\phi(t) = e^{At} = \begin{bmatrix} e^{2t}\cos t & e^{2t}\sin t \\ -e^{2t}\sin t & e^{2t}\cos t \end{bmatrix}$$

$$\frac{d\phi(t)}{dt} = Ae^{At} = \begin{bmatrix} 2e^{2t}\cos t - e^{2t}\sin t & 2e^{2t}\sin t + e^{2t}\cos t \\ -2e^{2t}\sin t - e^{2t}\cos t & 2e^{2t}\cos t - e^{2t}\sin t \end{bmatrix}$$

Now, $$\left.\frac{d\phi(t)}{dt}\right|_{t=0} = A = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$$

Also, $$[sI - A] = \begin{bmatrix} s-2 & -1 \\ 1 & s-2 \end{bmatrix}$$

$$|sI - A| = (s - 2)^2 + 1 = s^2 - 4s + 5$$

For eigen values,

$$s^2 - 4s + 5 = 0 \;\Rightarrow\; s = 2 \pm j \;\Rightarrow \text{Eigen values of system}$$

11. (c)

Here, $$\dot{X}_1 = -X_1 \Rightarrow \dot{X}_1(0) = -1$$

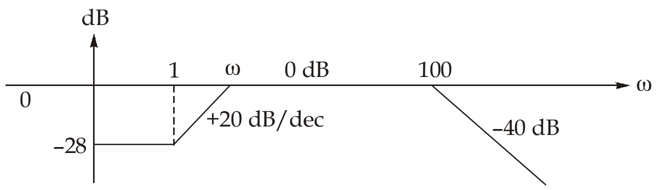
$$\dot{X}_2 = -2X_2 + U \Rightarrow \dot{X}_2(0) = 2$$

$\because$ $$Y = X_1 + 2X_2 \Rightarrow \frac{dy}{dt} = \dot{X}_1 + 2\dot{X}_2$$

at $t = 0$

$$\left.\frac{dy}{dt}\right|_{t=0} = \dot{X}_1(0) + 2\dot{X}_2(0) = -1 + 4 = 3$$

12. (d)



$$\text{slope of the line} = \frac{\text{Amount of rise}}{\text{duration}}$$

$$+20 \text{ dB} = \frac{-28 - 0}{\log(1) - \log\omega}$$

$$\log\left(\frac{1}{\omega}\right) = -\frac{28}{20} = -1.4$$

$$\omega = 25.118$$

$\therefore$ $$G(s)H(s) = \frac{K(1+s)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)^2}$$

$$= \frac{K(1+s)}{\left(1+\frac{s}{25.118}\right)\left(1+\frac{s}{100}\right)^2} = \frac{K(1+s)}{(1+0.0398s)(1+0.01s)^2}$$

13. (a)

The characteristic equation is

$$1 + G(s) = 0$$

$$s^3 + Ks^2 + 8s + 2 + 5s + 50 = 0$$

$$s^3 + Ks^2 + 13s + 52 = 0$$

For system response to be oscillatory

$$13 \times K = 52$$

$$K = \frac{52}{13} = 4$$

14. (a)

For gain margin of 10 dB,

$$GM = 20\log\frac{1}{|G|_{\omega=\omega_{pc1}}}$$

$$\therefore \quad \frac{1}{|G|_{\omega pc1}} = 10^{\left(\frac{10}{20}\right)} = 3.167$$

$$|G|_{\omega pc_1} = 0.315$$

For new gain margin of 18 dB, the gain at ω_{pc2} is

$$18 = 20\log\frac{1}{|G|_{\omega pc2}}$$

$$\log\frac{1}{|G|_{\omega pc2}} = \frac{18}{20} = 0.9$$

$$\frac{1}{|G|_{\omega pc2}} = 7.943$$

$$\therefore \quad |G|_{\omega pc_2} = 0.126$$

$\therefore$ The gain must be multiplied by a factor of $\frac{0.126}{0.315} = 0.4$

15. (d)

The open loop transfer function of the system is

$$G_c(s) \times G(s) = \left(\frac{1+s\tau}{1+\alpha s\tau}\right)\left(\frac{K}{s(s+1)(s+3)}\right)$$

$\therefore$ Velocity error constant,

$$K_v = \lim_{s\to 0} sG_c(s)G(s)$$

$$= \lim_{s\to 0}\frac{K(1+s\tau)}{(s+1)(s+3)(1+\alpha s\tau)}$$

$$K_v = \frac{K}{3}$$

$$\because \quad e_{ss} = \frac{1}{K_v} = \frac{3}{K}$$

Also

$$e_{ss} \le 1.5\%$$

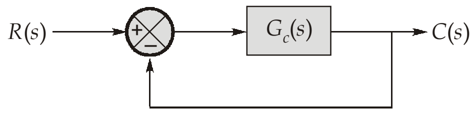
$$e_{ss} \le 0.015$$

$$\frac{3}{K} \le 0.015$$

$$K \ge \frac{3}{0.015} = 200$$

16. (c)

The closed loop transfer function of the given system can be obtained as



where, $$G_c(s) = \frac{5s}{s^2(s+2)+5K}$$

and $$G(s) = \frac{C(s)}{R(s)} = \frac{5s}{s^3+2s^2+5K+5s}$$

$$= \frac{5s}{s^3+2s^2+5s+5K}$$

Using Routh's tabulation method, we have

s^3	1	5
s^2	2	$5K$
s^1	$\frac{10-5K}{2}$	0
s^0	$5K$	

$$5K > 0$$

or $$K > 0$$

and $$\frac{10-5K}{2} > 0$$

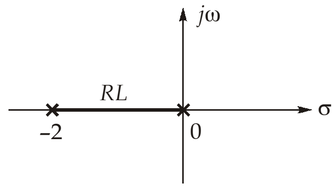
or $$-5K > -10$$

or $$K < 2$$

$\therefore$ $$0 < K < 2$$

17. (b)

The root locus can be plotted as



$$G(s)H(s) = \frac{K}{s(s+2)(s+1-j)(s+1+j)}$$

$\therefore$ $$n = 4 \text{ and } m = 0$$

$$\text{Asymptotes} = \theta_q = \frac{(2q+1)\pi}{P-Z} = \frac{(2q+1)\pi}{n-m}$$

$$= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \text{ and } \frac{7\pi}{4}$$

$$\text{Centroid} = \sigma = \frac{\Sigma P - \Sigma Z}{n-m} = \frac{-2-1-1-0}{4} = -1$$

$$\text{break point} = \frac{dK}{ds} = 0$$

$$K = -(s)(s+2)(s^2+2s+2)$$

$$\frac{dK}{ds} = \frac{d}{ds}\left[s^4+4s^3+6s^2+4s\right] = 0$$

or $$s^3+3s^2+3s+1 = 0$$

$$(s+1)^3 = 0$$

$\therefore$ actual break points are –1, –1 and –1

$\therefore$ break angle at $s = -1$

is $$\pm\frac{\pi}{r} = \pm\frac{180°}{4} = \pm 45°$$

The intersection with imaginary axis can be obtained by RH criteria.

as $s^4 + 4s^3 + 6s^2 + 4s + K = 0$

s^4	1	6	K
s^3	4	4	0
s^2	5	K	
s^1	$\frac{20-4K}{5}$	0	
s^0	K		

For intersection with $j\omega$ axis,

$$K \text{ mar} = 5$$

$$\therefore \quad \text{auxiliary equation} = 5s^2 + K = 0$$

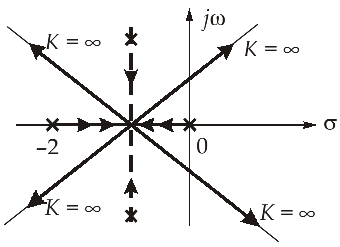
$$5s^2 + 5 = 0$$

$$s = \pm j1$$

Angle of departure for complex pole $(-1 + j)$ is

$$\theta_d = 180° - (90° + 180° + \phi - \phi)$$
$$= -90°$$

∴ Complete root locus can be plotted as



18. (b)

For GM to be 30 dB

$$30 \text{ dB} = -20\log|G(j\omega)|_{\omega=\omega_{pc}}$$

$$\therefore \quad -180° = -90° - \tan^{-1}\frac{\omega}{3} - \tan^{-1}\frac{\omega}{8}$$

$$-90° = -\tan^{-1}\frac{\left(\frac{\omega}{3}+\frac{\omega}{8}\right)}{\left(1-\frac{\omega^2}{24}\right)}$$

or $$1 - \frac{\omega^2}{24} = 0$$

$$\therefore \quad \frac{\omega^2}{24} = 1 \Rightarrow \omega = \sqrt{24} = \omega_{pc}$$

$$|G(j\omega)|_{\omega=\omega_{pc}} = \frac{K}{\sqrt{24}\times\sqrt{(24)+9}\sqrt{24+64}} = \frac{K}{264}$$

$\therefore$ $$30 = -20\log\left(\frac{K}{264}\right)$$

$$\log\frac{K}{264} = \left(-\frac{3}{2}\right)$$

$$\frac{K}{264} = 0.0316$$

or $$K = 8.348$$

19. **(a)**

From the given pole-zero map, we can write the open-loop transfer function as,

$$G(s)H(s) = \frac{K'}{s(s+4)(s+8)}$$

The gain at $s = j\omega = 4j$ is given to be 202.38. We know this gain is calculated as,

$$K = 202.38 = \frac{K' \times (\Pi \text{ distance of all poles from } s = 4j)}{(\Pi \text{ distance of all zeros from } s = 4j)}$$

$\therefore$ $$202.38 = K'(4)\sqrt{4^2+4^2}\sqrt{8^2+4^2}$$

$$202.38 = K' \times 202.38$$

$\therefore$ $$K' = 1$$

Hence, the gain at $s = -2 + 2j$

$$K = 1\times\left(\sqrt{(-2)^2+2^2}\right)\left(\sqrt{(4-2)^2+2^2}\right)\left(\sqrt{(8-2)^2+2^2}\right)$$

$$= 50.596$$

$$\simeq 50.6$$

20. **(c)**

Given open loop transfer function,

$$G(s) = \frac{1}{s(2s+1)(s+1)}, \quad H(s) = 1$$

So, $$G(s)H(s) = \frac{1}{s(2s+1)(s+1)}$$

By putting, $$s = j\omega$$

$$G(j\omega)\ H(j\omega) = \frac{1}{(j\omega)(2j\omega+1)(j\omega+1)}$$

The intersection with real axis occurs in negative half of the $j\omega$ plane when $\text{Im}\{G(j\omega)H(j\omega)\} = 0$

$\therefore$ $$\text{Im}\left\{\frac{(+1-2j\omega)(+1-j\omega)(-j\omega)}{\omega^2(4\omega^2+1)(1+\omega^2)}\right\} = 0$$

$$\text{Im}\{(\omega - 2\omega^3)j - (\omega^2 + 2\omega^2)\} = 0$$

So, $$\omega(1 - 2\omega^2) = 0$$

$$\omega = 0, \text{ and } \omega = \frac{1}{\sqrt{2}}$$

$\therefore$ $$\text{Value } G(j\omega)H(j\omega)\Big|_{\omega=\frac{1}{\sqrt{2}}} = \frac{1}{\left(j\frac{1}{\sqrt{2}}\right)\left(2j\frac{1}{\sqrt{2}}+1\right)\left(j\frac{1}{\sqrt{2}}+1\right)} = \frac{1}{\frac{j^2}{2}-1} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$$

21. (b)

Poles : –1, –2

Zeros : $-1 + j, -1 - j$

∴ Transfer function is given by,

$$G(s) = \frac{K((s+1)^2+1)}{(s+1)(s+2)}, \text{ where gain is assumed to be } K$$

$$G(s) = \frac{K(s^2+2s+2)}{(s+1)(s+2)}$$

at $s = 2$, $\quad G(s) = 15$

$$G(s) = 15 = \frac{K(2^2+2(2)+2)}{(2+1)(2+2)}$$

∴ $$K = \frac{15\times 3\times 4}{10} = 18$$

Therefore, transfer function of the system is given as,

$$G(s) = \frac{18(s^2+2s+2)}{(s+1)(s+2)}$$

22. (b)

Routh Table :

s^6	1	–1	–2	8
s^5	–2	0	8	
s^4	–1	2	8	
s^3	–4	–8	0	
s^2	$\in$	8		
s^1	$\frac{-8\in+32}{\in}$	0		
s^0	8			

There are two sign changes in first column of Routh table, hence 2 roots are present in R.H.S. of s-plane.

23. (d)

Let us consider an OLTF,

$$G(s)H(s) = \frac{K(s+2)}{s(s+1)}$$

Open loop zeros, $\quad s = -2$

Open loop poles, $\quad s = 0, -1$

Now, characteristic equation of roots of the system

$$1 + G(s)H(s) = s(s+1) + K(s+2) = 0$$

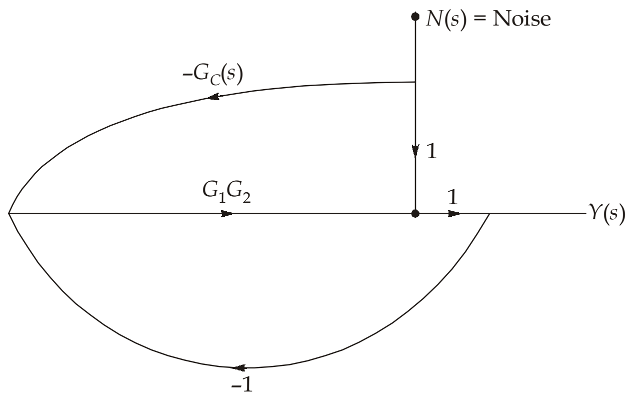
Now, set $K = 0$

Roots of the system, $\quad s = 0, -1$

Hence, at $K = 0$, roots of the system is poles itself.

24. (b)

Let the output due to $N(s)$ is $Y(s)$. When $N(s)$ is under consideration, set $X(s) = 0$.



Forward paths, $P_1 = 1$

$P_2 = -G_c(s)G_1G_2$

Loop gain, $L_1 = -G_1G_2$

Transfer function

$$\frac{Y(s)}{N(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{1 - G_c(s)G_1G_2}{1 + G_1G_2}$$

Therefore, $$Y(s) = \left[\frac{1 - G_c(s)G_1G_2}{1 + G_1G_2}\right]N(s) = 0$$

$$1 - G_c(s)G_1G_2 = 0$$

$$G_c(s) = \frac{1}{G_1G_2}$$

Hence, $G_c(s)$ should be equal to $\frac{1}{G_1G_2}$, in order to nullify the output due to noise $N(s)$.

25. (b)

Checking angle criterion of root locus

$$\angle G(s)H(s)\big|_{s=s_1} = -4\tan^{-1}(-2) \neq 180°$$

$$\angle G(s)H(s)\big|_{s=s_2} = -4\tan^{-1}(1) = -180°$$

$\angle G(s)H(s)\big|_{s=s_2}$ satisfies angle criterion.

So it lies on root locus.

26. (d)

The output is given as a function of time.

The final value of the output is,

$$\lim_{t\to\infty} c(t) = 1$$

Hence, t_d (at 50% of final value) is the solution, i.e. $0.5 = 1 - e^{-t_d}$

$$e^{-t_d} = 0.5$$

$$t_d = \ln 2 \quad \text{(or)} \quad 0.693 \text{ sec}$$

27. (c)

Given, $h(t) = e^{-t} + e^{-2t}$

$$H(s) = \frac{1}{s+1} + \frac{1}{s+2}$$

But $H(s) = \frac{C(s)}{R(s)} = \frac{1}{s+1} + \frac{1}{s+2}$

Given, $R(s) = \frac{1}{s}$

$$C(s) = R(s) \cdot H(s) = \frac{1}{s}\left[\frac{1}{s+1} + \frac{1}{s+2}\right]$$

$$C(s) = \frac{1}{s(s+1)} + \frac{1}{s(s+2)} = \left(\frac{1}{s} - \frac{1}{s+1}\right) + \frac{1}{2}\left(\frac{1}{s} - \frac{1}{s+2}\right)$$

$\therefore$ $c(t) = L^{-1}\,[C(s)]$

$$c(t) = (1.5 - e^{-t} - 0.5e^{-2t})\, u(t)$$

28. (c)

Given, $K_i = \frac{0.1}{s}$

$$I_o = 40\%$$

$$e = \text{Error (constant)} = 20\%$$

For an integral controller

$$(I_{\text{out}} - I_o) = \int_0^t K_i\, e dt$$

When the error does not vary with time, the equation becomes

$$(I_{\text{out}} - I_o) = K_i e t$$

$\therefore$ $I_{\text{out}} = K_i e t + I_o$

At $t = 2$ sec, $I_{\text{out}} = 0.1 \times 20 \times 2 + 40 = 44\%$

29. (a)

The percentage peak overshoot is

$$\% \ M_p = \frac{C(t_p) - C(\infty)}{C(\infty)} = 100 \times e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$e^{-\pi\xi/\sqrt{1-\xi^2}} = \frac{0.5}{3} = 0.1666$$

$$e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.1666$$

$$\frac{\xi}{\sqrt{1-\xi^2}} = 0.5703$$

$\therefore$ $\xi = 0.495 \approx 0.5$

30. **(a)**

Given, $c(t) = 1 + 0.5e^{-30t} - 1.5e^{-10t}$

Impulse response, $\frac{dc(t)}{dt} = 0 - 15e^{-30t} + 15e^{-10t}$

Then the transfer function,

$$\text{TF} = L\ [\text{Impulse response}]$$

$$\text{TF} = \frac{-15}{s+30} + \frac{15}{s+10}$$

$$\text{TF} = \frac{-15}{s+30} + \frac{15}{s+10} = \frac{-15s - 150 + 15s + 450}{(s+10)(s+30)}$$

Therefore natural frequency, $\omega_n = \sqrt{300} = 17.32 \text{ rad}$

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