CLASS TEST

S.No.:01SK_ABCDEFGHIJK_10825

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# DISCRETE MATHEMATICS

## COMPUTER SCIENCE & IT

**Date of Test : 10/08/2025**

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (d) | 7. | (a) | 13. | (a) | 19. | (d) | 25. | (c) |
| 2. | (c) | 8. | (c) | 14. | (c) | 20. | (b) | 26. | (b) |
| 3. | (d) | 9. | (a) | 15. | (d) | 21. | (d) | 27. | (c) |
| 4. | (b) | 10. | (b) | 16. | (c) | 22. | (b) | 28. | (a) |
| 5. | (c) | 11. | (c) | 17. | (c) | 23. | (d) | 29. | (c) |
| 6. | (b) | 12. | (d) | 18. | (a) | 24. | (d) | 30. | (b) |

## DETAILED EXPLANATIONS

**1.** **(d)**

$\sim\forall x\ \exists y\ P(x, y) \equiv \exists x\ \forall y\ (\sim P(x, y))$

$\sim\forall x\ P(x) \equiv \exists x\ [\sim P(x)]$

$\sim\exists x\ \forall y\ [P(x, y) \vee Q(x, y) \equiv \forall x\ \exists y\ [\sim P(x,y)\ \wedge \sim Q(x, y)]$

$\therefore$ All logical equivalents are correct.

**2.** **(c)**

For all $a \in A_i$, put $a$ in $A_i$, where $i$ is the length of the longest chain ending at $a$ and for all $b \leq a$, with $b \neq a$, we have $b \in A_1 \cup A_2 \cup A_{i-1}$.

We prove this by contradiction. Assume there is some $i$, $a \in A_i$ and $b \leq a$ with $b \neq a$ and $b \notin A_1 \cup A_2 \cup \ldots\ldots \cup A_{i-1}$. By the way we defined $A_i$, this implies there is a chain of length at least $i + 1$, ending at $a$. But then a could not be in $A_i$. This is a contradiction.

A poset has no directed cycles other than self-loops is true statement.

**3.** **(d)**

To make a connected graph atleast $(n - 1)$ edges required. To make it disconnected graph should contain at most $(n - 2)$ edges. The graph has $m$ edges, to be make it disconnected at most $n - 2$ edge must be deleted. So, $m - (n - 2) = m - n + 2$ edges.

$\therefore$ $(m - n + 2)$ edges deletion always guarantee that any graph will become-disconnected.

**4.** **(b)**

$f : A \rightarrow B$

$g : B \rightarrow C$ is injection: $\forall b \in B, g(b) = c$ distinct images in C.

$g \circ f : A \rightarrow C$ is surjection

$$g(f(a)) = c$$

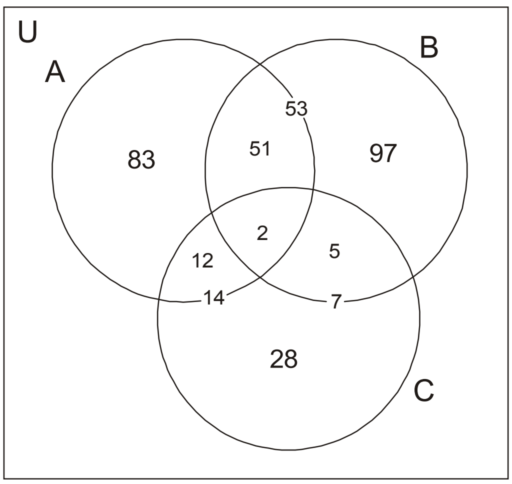
$$\Rightarrow \quad g(f(a)) = g(b)$$

$$\exists a \in A$$

$$\therefore \quad f(a) = b$$

So, $f : A \rightarrow B$ is surjection.

**5.** **(c)**



$A \rightarrow$ The set of students owning cars

$B \rightarrow$ The set of students owning bikes

$C \rightarrow$ The set of students owning motorcycles.

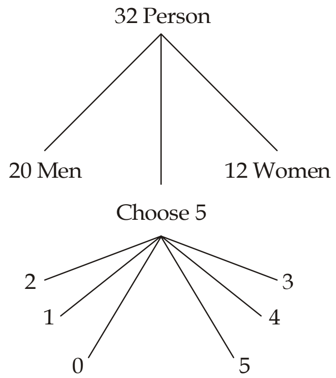
$$|A| = 83, |B| = 97, |C| = 28, |A \cap B| = 53$$
$$|A \cap C| = 14, |B \cap C| = 7, |A \cap B \cap C| = 2$$
$$|A \cup B \cup C| = 150$$

"$B - (A \cup C)$" denotes the set of students who own a bike and nothing else.

$$|B - (A \cup C)| = 97 - (51 + 5 + 2)$$
$$= 39$$

**6.** **(b)**

Given,




We must choose at least 3 women, so, we calculate 3 women, 4 women and 5 women and by addition rule add the results:

$$= {}^{12}C_3 \times {}^{20}C_2 + {}^{12}C_4 \times {}^{20}C_1 + {}^{12}C_5 \times {}^{20}C_0$$
$$= 220 \times 190 + 495 \times 20 + 792 \times 1 = 52492$$

**7.** **(a)**

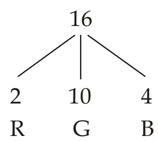
$A \cup B \subseteq A \cap B$ holds true when $A = B$. It is true for empty as well as nonempty sets.

$\Rightarrow$ $|A| = |B|$ is true $|A| \geq 0$ ***eg.*** $A = B \{a, b\}$

Hence $A = \{\ \}, B = \{\ \}$ "always" is false.

**8.** **(c)**

This problem reduces to selecting 10 offices from 16, then 4 from remaining 6.




i.e., number of ways = $\dfrac{16!}{10!\,4!\,2!} = 120120$

**9. (a)**

Clearly, $a_n = n + 1$

$\Rightarrow \quad a_{n-1} = n$

$\Rightarrow \quad a_{n-2} = n - 1$

$\Rightarrow \quad a_n = 2a_{n-1} - a_{n-2} \qquad [\because\ 2(n) - (n-1) = n + 1]$

**10. (b)**

The candidate is unsuccessful if he fails in 9 or 8 or 7or 6 or 5 papers.

∴ The number of ways to be unsuccessful

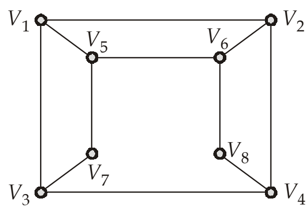
$= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5 = 256$

**11. (c)**

A **bipartite graph** is a graph whose vertices can be divided into two disjoint sets $U$ and $V$ such that every edge connects a vertex in $U$ to one in $V$. Vertex sets $U$ and $V$ are usually called the parts of the graph. Equivalently, a bipartite graph is a graph that does not contain any odd length cycles. It is also 2-colorable and the degree sum formula for a bipartite graph states that

$$\sum_{v \in V} \deg(V) = \sum_{u \in U} \deg(U) = |E|$$

Given graph:




(a) $(3 + 3 + 3) \neq (3 + 3 + 3 + 2 + 2)$

(b) $(3 + 2 + 2) \neq (3 + 3 + 3 + 3)$

(c) $(3 + 3 + 3 + 2) = (3+3+3+2) = 11 = |E|$

(d) $(3 + 3 + 3 + 2 + 2) \neq (3 + 3 + 3)$

**12. (d)**

$h^c$ = '$g$' or '$i$' or '$f$' or '$d$' or '$b$'

$x$ and $y$ are complement to each other

iff $\quad x \vee y$ = Greatest element = $j$

and $\quad x \wedge y$ = Least element = $a$

**13. (a)**

***R* is reflexive:** Since $(a, b)\ R\ (a, b)$ for all elements $(a, b)$ because $a = a$ and $b = b$ are always true.

***R* is symmetric:** Since $(a, b)\ R\ (c, d)$ and $a = c$ or $b = d$ which can be written as $c = a$ or $d = b$.

So, $(a, b)\ R\ (a, b)$ is true.

***R* is not antisymmetric:** Since $(1, 2)\ R\ (1, 3)$ and $1 = 1$ or $2 = 3$ true b/c $1 = 1$.

So $(1, 3)\ R\ (1, 2)$ but here $2 \neq 3$ so $(1, 2) \neq (1, 3)$.

So, only statement 1 and 2 are correct.

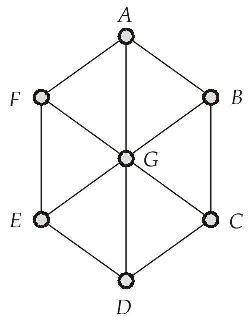
**14. (c)**

Consider each options:

(a) Null graph of 6 vertices is 1-chromatic so it is correct.

(b) It is correct because tree with 2 or more vertices is always bichromatic.

(c) It is incorrect. Consider a wheel graph of 7 vertices.




The chromatic number of graph is 3.

- Color 1 for $G$
- Color 2 for $A$, $E$, $C$
- Color 3 for $F$, $B$, $D$
- A wheel graph is 3-chromatic when $n$-vertices are odd and 4-chromatic when $n$-vertices is even.
- So here $n = 7$, $\left(\left\lfloor \frac{n}{2} \right\rfloor + 1\right) = \left(\left\lfloor \frac{7}{2} \right\rfloor + 1\right) = 4$ which is incorrect because only 3 colors are required to color the above wheel graph.

(d) This statement is correct because graph without odd length cycle having atleast 1 edge is bichromatic.

All other statements are true except option (c).

**15. (d)**

"Not every satisfiable logic is Valid"

$= $ Not (every satisfiable logic is Valid)

$= $ Not $(\forall x(\text{satisfiable }(x) \Rightarrow \text{Valid }(x))$    option (a)

$= $ Not $(\forall x(\neg\text{satisfiable} \vee \text{Valid }(x))$    option (c)

$= \exists x(\text{satisfiable }(x) \wedge \neg\, \text{Valid }(x))$    option (b)

Statement (d) says every satisfiable logic is invalid.

So option (d) is not represent given statement.

**16. (c)**

Set $S = \{1, 2\}$

Total number of binary relation $= 2^{n^2} = 2^{2^2} = 2^4 = 16$

We need to count relations which are neither reflexive ($R$) nor irreflexive ($IR$) which is $n(R \cup IR)^c$

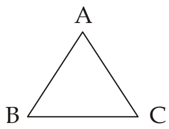
Now, $n(R \cup IR)^c = 2^{n^2} - [n(R \cup IR)]$

$= 2^{n^2} - [n(R) + n(IR) - n(R \cap IR)]$

$= 2^{n^2} - \left(2^{n^2 - n} + 2^{n^2 - n} - 0\right) = 16 - (4 + 4) = 8$

**17.** **(c)**

Considering each statements:

- If a graph is bipartite, then its two colourable. Because a bipartite graph can be represented as two groups of vertices such that vertices in same graph are not adjacent.
  Similarly, statement 2 is equivalent to statement 1.
- If a bipartite graph has a cycle, then it has to be of even length. Graph G is bipartite iff no odd length cycle.
- 
  
  This graph has a Hamiltonian circuit, but the cycle is of odd length and not bipartite.

$\therefore$ $3 \not\equiv 4$

- 
  
  This graph is bipartite and 2 colorable but does not have Hamiltonian circuit.

So 1, 2 and 4 are equivalent statements.

**18.** **(a)**

LUB = $g$, GLB = $c$

$b \vee h = g$ and $b \wedge h = c$

$\therefore$ $b$ has only 1 complement [complement of $a = h$]

[**Note:** $x$ and $y$ are complement to each other iff $x \vee y = 1$ (Greatest element) and $x \wedge y = 0$ (least element)]

**19.** **(d)**

A number is relatively prime to 15 iff it is not divisible by 3 and not divisible by 5.

Set of integer from 1 to 1000 divisible by $3 = \left\lfloor \frac{1000}{3} \right\rfloor = 333.$

Set of integer from 1 to 1000 divisible by $5 = \left\lfloor \frac{1000}{5} \right\rfloor = 200.$

So, number of integer not relatively prime to 15 are

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$= \left\lfloor \frac{1000}{3} \right\rfloor + \left\lfloor \frac{1000}{5} \right\rfloor - \left\lfloor \frac{1000}{15} \right\rfloor$$

$$= 333 + 200 - 66 = 467$$

So, number of integer relatively prime to 15 are

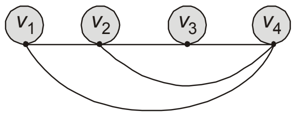
$$|\overline{A \cup B}| = 1000 - 467 = 533$$

**20. (b)**

Let (a) $n = 2$;


$\#$ edge $= 1$

(b) $n = 4$




$\#$ edges $= 5$

So option (b) is correct.

**21. (d)**

$R^1$ is nothing but R itself.

Now, $R^2 = R \cdot R$ i.e. composite of $R$ with $R$.

If $(a, b) \in R$, then $(a, c) \in R^2$ iff $(b, c) \in R$.

This composite of relations

$$R^2 = \{(1, 1), (2, 1), (3, 1), (4, 2)\}$$
$$R^3 = \{(1, 1), (2, 1), (3, 1)\ (4, 1)\}$$
$$P = \{(1, 1)\ (2, 1), (3, 2), (4, 1), (4, 2), (4, 3), (3, 1)\}$$

$\therefore$ Cardinality of $P = 7$.

**22. (b)**

Let $p$ : GATE rank is needed

$q$ : I will write the GATE exam

$r$ : I will join in MADEEASY.

Given arguments:

**$P_1$:** If GATE rank is needed, i will not write GATE exam, if i do not join MADEEASY.

$$p \to (\sim r \to \sim q) = (p \wedge \sim r) \to \sim q$$

**$P_2$:** GATE rank is needed : $p$

**$P_3$:** I will join MADEEASY : $r$

**Q:** I will write the GATE exam : $q$

Inference is: $(p \wedge \neg r) \to \neg q$

$$\begin{array}{c} p \\ r \\ \hline q \\ \hline \end{array}$$

We can also write the above inference as following: $(p \wedge \neg r)$

$$\left([(p \wedge \neg r) \to \neg q] \wedge p \wedge r\right) \to q$$

If above proposition is tautology then given inference is valid.

$((pr')' + q')' + p' + r' + q$

$= pr'q + p' + r' + q$

$= p' + r' + q$ which is consistency hence invalid.

**23.** **(d)**

Given problem is dearrangement problem

$$D_n = \sum_{r=2}^{n} (-1)^r \frac{n!}{r!}$$

$$D_5 = \sum_{r=2}^{5} (-1)^r \frac{5!}{r!}$$

$$= \left(\frac{5!}{2!} - \frac{5!}{3!} + \frac{5!}{4!} - \frac{5!}{5!}\right)$$

$$= (60 - 20 + 5 - 1)$$

$$= 44$$

**24.** **(d)**

The operation is not commutative as since upper and lower triangle is not same.

$q * p = p$ and $p * q = r$

The operation is not associative as $p * (q * r) \neq (p * q) * r$

LHS $p * r = s$

RHS $r * r = p$

**25.** **(c)**

The problem corresponds to the number of non negative integral solutions to

$$x_1 + x_2 + x_3 = 10 \text{ with the conditions,}$$

$$0 \leq x_1 \leq 10$$

$$0 \leq x_2 \leq 5$$

$$0 \leq x_3 \leq 3$$

Generating functions are required, since the variables have an upper constraint

The generating function is

$(1 + x + x^2 ...)(1 + x + x^2 + x^3 ... + x^5)(1 + x + ... x^3)$

$$= \left(\frac{1}{1-x}\right)\left(\frac{1-x^6}{1-x}\right)\left(\frac{1-x^4}{1-x}\right)$$

$$= \frac{(1-x^6)(1-x^4)}{(1-x)^3}$$

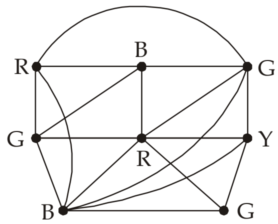
$$= (1 - x^4 - x^6 + x^{10}) \sum_{r=0}^{\infty} 3 - 1 + rC_r x^r$$

$$= (1 - x^4 - x^6 + x^{10}) \sum_{r=0}^{\infty} r + 2C_r x^r$$

The coefficient of $x^{10}$ in above generating function is ${}^{12}C_{10} - {}^{8}C_6 - {}^{6}C_4 + {}^{2}C_0 = 24$.

**26.** **(b)**

$d(i)$ = Degree of node $i$. $d(A) = 4$, $d(B) = 4$, $d(C) = 5$ $d(D) = 4$, $d(E) = 3$, $d(F) = 6$, $d(G) = 4$, $d(H) = 6$ using Welsh-powell's algorithm.




Chromatic number = 4

**27.** **(c)**

Maximum and minimum number of component given by:

$$n - K \le e \le \frac{(n-K+1)(n-K)}{2}$$

**1.**

$$n - K \le e$$
$$n - e \le K$$
$$10 - 6 \le K \qquad (\because \text{ Minimum number of component})$$

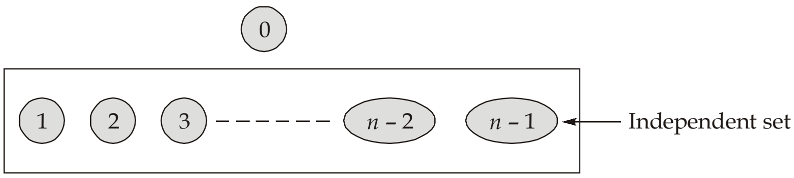
**2.**

$$e \le \frac{(n-K+1)(n-K)}{2}$$
$$6 \le \frac{(10-K+1)(10-K)}{2}$$
$$2 \times 6 \le (11 - K)(10 - K)$$
$$12 \le (10 - K)(11 - K)$$
$$12 \le K^2 + 110 - 21K$$
$$0 \le K^2 + 98 - 21K$$
$$K^2 + 98 - 21K = 0$$
$$K = 14, 7$$

Maximum value of $K$ is 7 because number of components never be larger than nodes.

**28.** **(a)**

Let $|V|$ = n, nodes named as 0, 1, 2, ..... $n - 1$, let independent set consists nodes named 1, 2, ..... $n - 1$, and node 0 is not in this set. Consider following representation




Let given independent set is $S$.

For any $V_i$ and $V_j$ node from $S$, there is no edge. And $S$ is maximal independent set, so you cannot add node 0 in $S$.

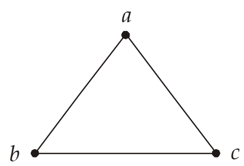
So, there is atleast one edge from node 0 to any node of $S$.

So minimum number of edge is 1. And for maximum, every node of $S$ has edge with node 0.

So, $|V| - 1$ maximum edges possible.

**29.** **(c)**

Let $k_3$ is




Cases:

(i) One vertex in subgraph, 3 vertices so 3 possibilities, and there is no edge possible here.

(ii) Two vertices in subgraph,

$\Rightarrow$ $^3C_2$ = 3 ways to select the vertices

$\Rightarrow$ $^2C_2$ = 1 edge possible in a subgraph

$\Rightarrow$ 2 possibilities for an edge (select or not)

So, Subgraph possible = $^3C_2 \times 2^1 = 6$

(iii) Three vertices in subgraph,

$\Rightarrow$ $^3C_3$ = 1 way to select vertices

$\Rightarrow$ $^3C_2$ = 3 edges possible in a subgraph

$\Rightarrow$ 2 possibilities for an edge.

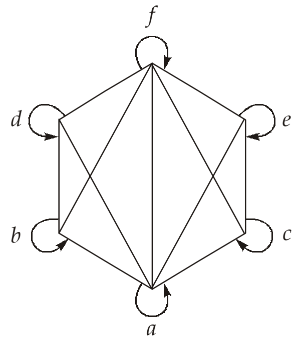
So, Possible subgraphs = $1 \times 2^3 = 8$

From (i), (ii), (iii)

Total number of subgraphs = $3 + 6 + 8 = 17$

**30.** **(b)**

(a) Full graph for given Hasse diagram




Total number of undirected edges (without loops) = (Comparable edge) + (Non-comparable edge)

$$\frac{6(6-1)}{2} = 11 + x$$

$$x = 15 - 11 = 4$$

Here $(b, e)$, $(b, c)$, $(d, e)$, $(d, c)$ are incomparable pairs.

(b) $(b, d) \in R$, so $(d, b) \notin R$

(c) $(a, c), (c, e) \in R$, so $(a, e) \in R$

■■■■