CLASS TEST

S.No.: 01SKCE_EFGHI_21072025

MADE EASY

Leading Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

STRENGTH OF MATERIALS

CIVIL ENGINEERING

Date of Test : 21/07/2025

ANSWER KEY

1.	(a)	7.	(c)	13.	(b)	19.	(b)	25.	(d)
2.	(b)	8.	(a)	14.	(b)	20.	(a)	26.	(b)
3.	(a)	9.	(c)	15.	(c)	21.	(d)	27.	(c)
4.	(c)	10.	(a)	16.	(b)	22.	(b)	28.	(b)
5.	(b)	11.	(a)	17.	(b)	23.	(a)	29.	(d)
6.	(c)	12.	(c)	18.	(a)	24.	(a)	30.	(c)

DETAILED EXPLANATIONS

1. (a)

$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_{CI} = \delta_{steel} = 0.8 \text{ mm}$$

$$\delta_{CI} = \frac{P_{CI} \times 2000 \times 1000}{\frac{\pi}{4}\left(60^2 - 50^2\right) \times 10^5} = 0.8 \text{ mm}$$

$\Rightarrow$ $$P_{CI} = 11\,\pi \text{ kN}$$

$$\delta_{steel} = \frac{P_{steel} \times 2000 \times 1000}{\frac{\pi}{4} \times 50^2 \times 2 \times 10^5} = 0.8 \text{ mm}$$

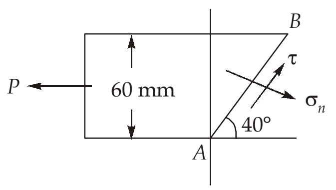
$\Rightarrow$ $$P_{steel} = 50\,\pi \text{ kN}$$

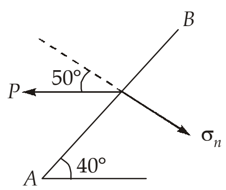
$\therefore$ $$P = P_{CI} + P_{steel} = (11\pi + 50\pi) \text{ kN}$$

$$= 191.64 \text{ kN}$$

2. (b)

$$P_n = P\cos 50°$$





$$\sigma_n = \frac{P\cos 50°}{AB \times 120} \qquad \sin 40° = \frac{60}{AB}$$

Given, $\sigma_n \le 100 \text{ N/mm}^2$ $\quad\Rightarrow\quad$ $AB = \dfrac{60}{\sin 40°}$

$\Rightarrow$ $$\frac{P\cos 50°}{\left(\frac{60}{\sin 40°}\right) \times 120} \le 100$$

$\Rightarrow$ $$P = \frac{100 \times \frac{60}{\sin 40°} \times 120}{\cos 50°}$$

$$P = 1742599.17 \text{ N} \simeq 1743 \text{ kN (say)}$$

3. (a)

Consider corner A,

Shear strain, $\gamma = \theta_{AC} + \theta_{AB}$

Rotation angle of AC + Rotation angle of AB

$$= \frac{4.8 \times 10^{-3}}{6} + \frac{6.4 \times 10^{-3}}{8} = 1.6 \times 10^{-3}$$

4. (c)

$$\frac{\tau_{max}}{\tau_{avg}} = \frac{3}{2}$$

$$\therefore \quad P = \frac{\tau_{max} - \tau_{avg}}{\tau_{avg}} \times 100$$

$$\Rightarrow \quad P = \frac{\frac{3}{2}\tau_{avg} - \tau_{avg}}{\tau_{avg}} \times 100$$

$$\Rightarrow \quad P = 50\%$$

5. (b)

Shear flow, $q = \dfrac{T}{2A_0}$

where, T = Torsion

A_0 = Area enclosed by the median line

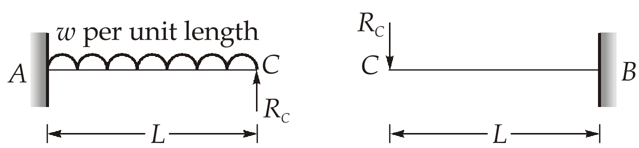
$$\therefore \quad q = \frac{8.75 \times 10^6}{2 \times 22500}$$

$$\Rightarrow \quad q = 194.44 \text{ N/mm} \simeq 194.4 \text{ N/mm}$$

6. (c)

We know that at internal hinge deflection will be same at just left and right of hinge.

So,



$$(\delta_c)_{left}\downarrow = \frac{wL^4}{8EI} - \frac{R_cL^3}{3EI} \quad \ldots(i)$$

$$(\delta_c)_{right}\downarrow = \frac{R_cL^3}{3EI} \quad \ldots(ii)$$

So from eq. (i) and (ii)

$$\frac{wL^4}{8EI} - \frac{R_cL^3}{3EI} = \frac{R_cL^3}{3EI}$$

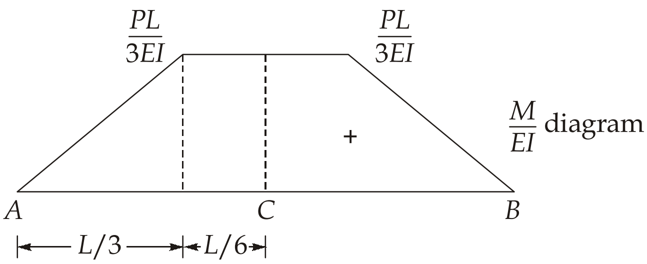
$$\Rightarrow \quad \frac{2R_cL^3}{3EI} = \frac{wL^4}{8EI}$$

$$\Rightarrow \quad R_c = \frac{3}{16}wL$$

7. (c)

For rotation at A, we will solve it by moment area method.

So $\dfrac{M}{EI}$ diagram of given beam AB is as shown below.



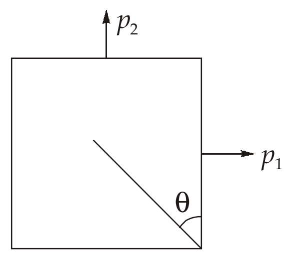
So, $\theta_{CA} = \theta_C - \theta_A = -\theta_A$ = Area of $\frac{M}{EI}$ diagram between A and C

$$-\theta_A = \frac{1}{2}\times\frac{PL}{3EI}\times\frac{L}{3}+\frac{L}{6}\times\frac{PL}{3EI}$$

$$-\theta_A = \frac{PL^2}{18EI}+\frac{PL^2}{18EI} = \frac{PL^2}{9EI}$$

$$\Rightarrow \quad \theta_A = \frac{PL^2}{9EI}(\text{CW})$$

8. (a)



The maximum shear stress will occur at 45° plane from principle plane

So, $$\sigma_{@\,45} = \sigma_1\cos^2\theta+\sigma_2\sin^2\theta$$

$$= \frac{p_1}{2}+\frac{p_2}{2} = \frac{p_1+p_2}{2}$$

$$\tau_{@\,45°} = \frac{p_1-p_2}{2}$$

So, resultant stress, $$p_r = \sqrt{\sigma^2+\tau^2}$$

$$= \sqrt{\frac{p_1^2+p_2^2+2p_1p_2+p_1^2+p_2^2-2p_1p_2}{4}}$$

$$= \sqrt{\frac{p_1^2+p_2^2}{2}}$$

9. **(c)**

Torque at distance x from free end

$$T_x = \int_0^x \tau_x dx = \int_0^x \frac{kx^2}{2L^2} dx = \frac{kx^3}{6L^2}$$

Now take small element at x i.e., dx

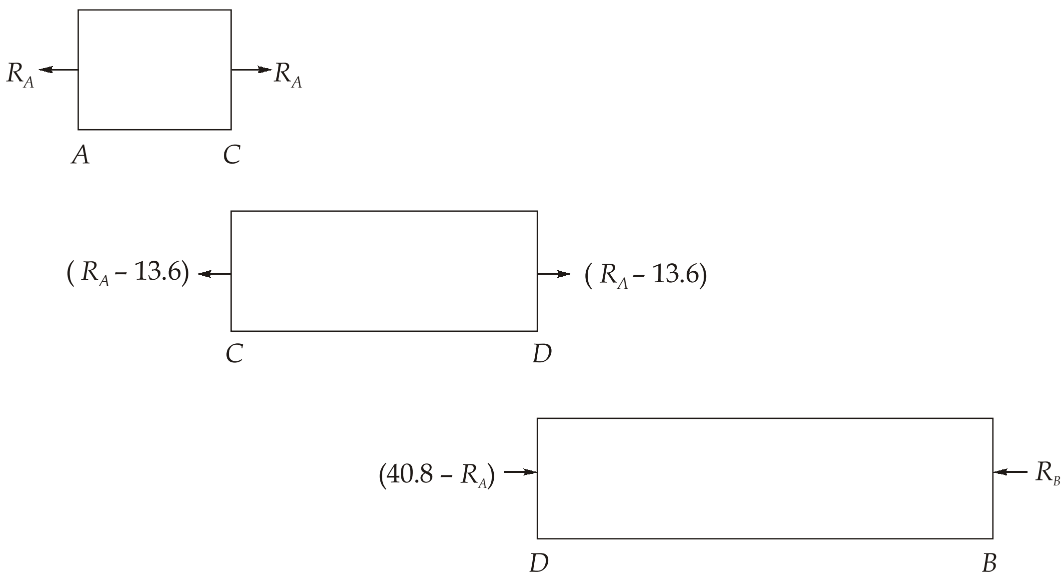
So, $$d\theta_x = \frac{T_x dx}{GJ}$$

On integrating, $$\theta_{AB} = \int d\theta_x = \int_0^L \frac{kx^3}{6L^2}\frac{dx}{GJ} = \left[\frac{kx^4}{24L^2 GJ}\right]_0^L$$

$\Rightarrow$ $$\theta_{AB} = \frac{kL^2}{24GJ}$$

10. **(a)**

Free body diagram is shown below,



$$R_A + R_B = 40.8 \quad \text{...(i)}$$

Total change in length = 0

So, $$\delta_{AC} + \delta_{CD} + \delta_{DB} = 0$$

$$\frac{R_A \times 100}{AE} + \frac{(R_A - 13.6)\times 200}{AE} - \frac{(-R_A + 40.8)\times 300}{AE} = 0$$

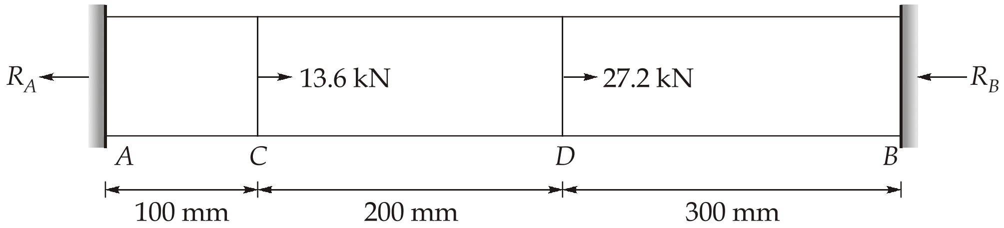
$\Rightarrow$ $$R_A + 2R_A - 27.2 + 3R_A - 122..4 = 0$$

$\Rightarrow$ $$R_A = 24.93 \text{ kN}$$

$$R_B = (40.8 - R_A) = -15.87 \text{ kN}$$

So ratio of magnitude of reactions i.e., $$\frac{R_A}{R_B} = \frac{24.93}{15.87} = 1.57$$

Alternatively:



$$R_A = \frac{13.6\times500}{600}+\frac{27.2\times300}{600}=24.93\text{kN}$$

$$R_B = \frac{27.2\times300}{600}+\frac{13.6\times100}{600}=15.87\text{kN}$$

$\therefore$ Ratio of $\left(\frac{R_A}{R_B}\right) = \frac{24.93}{15.87}=1.57$

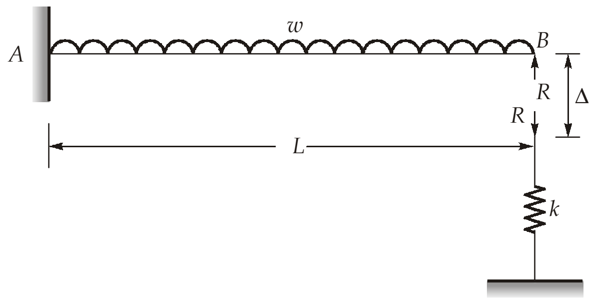
11. **(a)**

$$\Delta_B = \frac{R}{k}=\frac{wl^4}{8EI}-\frac{Rl^3}{3EI}$$

Since, $k = \frac{EI}{l^3}$

$\therefore$ $\frac{Rl^3}{EI}+\frac{Rl^3}{3EI} = \frac{wl^4}{8EI}$

$\Rightarrow$ $R = \frac{3wL}{32}$



12. **(c)**

As it is given that, $\varepsilon = \frac{\sigma}{E}=\frac{y}{R}=3.0\times10^{-5}$

So, $\frac{1}{R} = \frac{3.0\times10^{-5}}{30}\text{mm}^{-1}=10^{-6}\text{mm}^{-1}$

Also, in pure bending, $\frac{\sigma}{y} = \frac{M}{I}=\frac{E}{R}=\text{constant}$

For σ_{max}, y_{max} has to be used

So, $\sigma_{max} = \frac{E}{R}y_{max}=\frac{200\times10^3}{R}\text{MPa}\times y_{max}$

$\Rightarrow$ $\sigma_{max} = 200\times10^3\times10^{-6}\times50$ MPa

$\Rightarrow$ $\sigma_{max} = 10$ MPa

13. **(b)**

Principal strains, $\varepsilon_{1/2} = \frac{\varepsilon_x+\varepsilon_y}{2}\pm\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$

$$= \left[\frac{800+200}{2} \pm \sqrt{\left(\frac{800-200}{2}\right)^2 + \left(\frac{-600}{2}\right)^2}\right] \times 10^{-6}$$

$$\varepsilon_1 = 924.264 \times 10^{-6}$$

$$\varepsilon_2 = 75.74 \times 10^{-6}$$

Thus major principal stress is,

$$\sigma_1 = \frac{E}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2) = \frac{200 \times 10^3}{1-0.3^2}(924.264 + 0.3 \times 75.74) \times 10^{-6}$$

$$= 208.13 \text{ MPa} \simeq 208 \text{ MPa}$$

14. (b)

Let the forces induced in aluminium and steel rods be P_a and P_s respectively.

Now, $P_a + P_s = W$

Balancing moment about the point where aluminum rod is hinged we have,

$$W \times 1 = P_s \times 3$$

$$\Rightarrow \quad P_s = \frac{W}{3}$$

But $P_a + P_s = W$

$$\Rightarrow \quad P_a = W - \frac{W}{3} = \frac{2W}{3}$$

For the beam to remain horizontal, elongation in both the rods must be equal in the vertical direction.

$$\frac{P_a l_a}{A_a E_a} = \frac{P_s l_s}{A_s E_s}$$

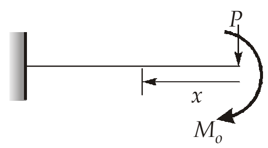
$$\Rightarrow \quad \frac{A_a}{A_s} = \frac{P_a l_a E_s}{P_s l_s E_a}$$

$$\Rightarrow \quad \frac{A_a}{A_s} = \frac{\left(\frac{2W}{3}\right)}{(W/3)} \times \frac{2000}{3000} \times \frac{200}{70}$$

$$\Rightarrow \quad \frac{A_a}{A_s} = 3.8095 \simeq 3.8$$

15. (c)

Strain energy due to bending $= \int \frac{M_x^2 dx}{2EI}$



Since it varies with square, we can't apply principle of superposition.

$$\int_o^L \frac{(-Px - M_0)^2\, dx}{2EI} = \int_o^L \frac{P^2x^2 + M_0^2 + 2PxM_0}{2EI} dx$$

$$= \frac{1}{2EI}\left[\frac{P^2x^3}{3} + M_0^2 x + PM_0 x^2\right]_0^L$$

$$= \frac{P^2L^3}{6EI} + \frac{M_o^2L}{2EI} + \frac{PM_oL^2}{2EI}$$

16. (b)

$$\sigma_x = 50 \text{ unit}, \sigma_y = 40 \text{ unit}, \tau_{xy} = 20 \text{ unit}$$

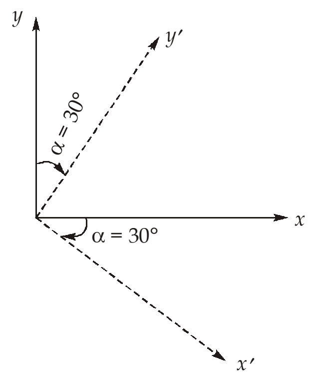
$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \quad \sigma_2 = \frac{50+40}{2} - \sqrt{\left(\frac{50-40}{2}\right)^2 + 20^2}$$

$$= 45 - 20.615$$

$$= 24.384 \text{ unit}$$

17. (b)



$$\theta = -30°$$

Considering 2nd loading,

$$\sigma_{x'} = 0 \text{ MPa}$$

$$\sigma_{y'} = 0 \text{ MPa}$$

$$\tau_{x'y'} = 4 \text{ MPa}$$

$$\tau_{xy} = -\left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\sin 2\theta + \tau_{x'y'}\cos 2\theta$$

$$\Rightarrow \quad \tau_{xy} = -\left(\frac{0-0}{2}\right)\sin(90° - 30°) + 4\cos(90° - 30°)$$

$$\Rightarrow \quad \tau_{xy} = 0 + 2 = 2 \text{ MPa}$$

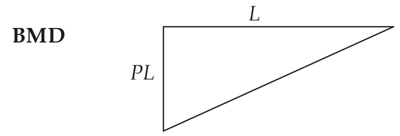
When both loads act simultaneously then,

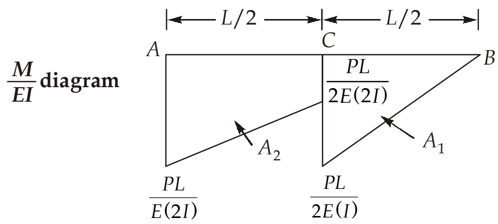
$$(\tau_{xy})_{net} = (\tau_{xy})_{1st\ loading} + (\tau_{xy})_{2nd\ loading}$$

$$= 3 \text{ MPa} + 2 \text{ MPa} = 5 \text{ MPa}$$

18. (a)

Using moment area method,





Deflection at free end $B, \delta_B = \dfrac{A_1 x_1}{EI} + \dfrac{A_2 x_2}{EI}$

$$\frac{A_1 x_1}{EI} = \left(\frac{1}{2} \times \frac{L}{2} \times \frac{PL}{2EI}\right) \times \frac{2}{3}\left(\frac{L}{2}\right) = \frac{PL^3}{24EI}$$

$$\frac{A_2 x_2}{EI} = \frac{1}{2} \times \frac{L}{2}\left[\frac{PL}{4EI} + \frac{PL}{2EI}\right] \times x_2 = \frac{3PL^2}{16EI} \times 2$$

$$x_2 = \frac{L}{2} + \frac{L/2}{3}\left[\frac{2 \times \frac{PL}{2EI} + \frac{PL}{4EI}}{\frac{PL}{2EI} + \frac{PL}{4EI}}\right]$$

$$\Rightarrow \quad x_2 = \frac{L}{2} + \frac{L}{6}\left[\frac{5}{3}\right]$$

$$= \frac{L}{2} + \frac{5L}{18} = \frac{14L}{18} = \frac{7L}{9}$$

$$\therefore \quad \frac{A_2 x_2}{EI} = \frac{3PL^2}{16EI} \times \frac{7L}{9} = \frac{7PL^3}{48EI}$$

$$\therefore \quad \delta_B = \frac{PL^3}{24EI} + \frac{7PL^3}{48EI} = \frac{9}{48}\frac{PL^3}{EI} = \frac{3}{16}\frac{PL^3}{EI}$$

19. (b)

$$\tau = \frac{VA\bar{y}}{Ib}$$

Area above x-x axis, $A = 100 \times 4 \text{ mm}^2 = 400 \text{ mm}^2$

$$\bar{y} = 35 - \frac{4}{2} = 33 \text{ mm}$$

$$I = 1600000 \text{ mm}^4, \; b = 100 \text{ mm}$$

$$\therefore \quad \tau = \frac{10 \times 10^3 \times 400 \times 33}{1600000 \times 100} = 0.825 \text{ MPa}$$

20. (a)

$$\varepsilon_0 = \varepsilon_x = -120\ \mu m/m$$
$$\varepsilon_{90^\circ} = \varepsilon_y = 1120\ \mu m/m$$
$$\varepsilon_{45^\circ} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\gamma_{xy}}{2}$$
$$\Rightarrow \quad 400 = \frac{-120 + 1120}{2} + \frac{\gamma_{xy}}{2}$$
$$\Rightarrow \quad \gamma_{xy} = -200\ \mu m/m$$

Now,

$$\varepsilon_{135^\circ} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\gamma_{xy}}{2}$$
$$\Rightarrow \quad 600 = \frac{-120 + 1120}{2} - \frac{\gamma_{xy}}{2}$$
$$\Rightarrow \quad \gamma_{xy} = -200\ \mu m/m$$

Since γ_{xy} is same as obtained from both the cases, so given data is consistent.

21. (d)

Strongest beam is the one for which section modulus Z is maximum.
Let rectangular section is of width b and depth d,
Diameter of cylindrical log, D = 300 mm

$$\therefore \quad b^2 + d^2 = D^2$$
$$\therefore \quad Z = \frac{bd^2}{6} = \frac{b(D^2 - b^2)}{6} = \frac{bD^2 - b^3}{6}$$

For maximum Z,

$$\frac{\partial Z}{\partial b} = 0$$
$$\Rightarrow \quad \frac{D^2 - 3b^2}{6} = 0$$
$$\Rightarrow \quad b = \frac{D}{\sqrt{3}}$$
$$\Rightarrow \quad b = \frac{300}{\sqrt{3}} = 173.2\ \text{mm}$$

22. (b)

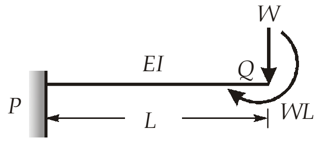
$$\text{Strain energy, } U = \frac{P^2 L}{2AE} \text{ for axially loaded bar}$$
$$\therefore \quad U_A = \frac{P^2 L_1}{2A_1 E} + \frac{P^2 L_2}{2A_2 E}$$
$$U_B = \frac{P^2 L_2}{2A_1 E} + \frac{P^2 L_1}{2A_2 E}$$
$$\Rightarrow \quad \frac{U_B}{U_A} = \frac{\dfrac{L_2}{A_1} + \dfrac{L_1}{A_2}}{\dfrac{L_1}{A_1} + \dfrac{L_2}{A_2}} = \frac{L_1 d_1^2 + L_2 d_2^2}{L_1 d_2^2 + L_2 d_1^2}$$

$$= \frac{10 \times 2^2 + 20 \times 4^2}{10 \times 4^2 + 20 \times 2^2}$$

$$= \frac{36}{24} = 1.5$$

23. (a)

The given beam can be modified as,



$$\text{Deflection at } Q = \frac{WL^3}{3EI} + \frac{WL \times L^2}{2EI} = \frac{5WL^3}{6EI}$$

$$\text{Slope at } Q = \frac{WL^2}{2EI} + \frac{WL \times L}{EI} = \frac{3}{2}\frac{WL^2}{EI}$$

24. (a)

Let D = Diameter of shaft (in mm)

We know that power transmitted by shaft (P)

$$= \frac{2\pi NT}{60}$$

$$\Rightarrow \quad 100 = \frac{2\pi NT}{60} = \frac{2\pi \times 160 \times T}{60}$$

$$\Rightarrow \quad T = 5.968 \text{ kNm}$$

$\therefore$ Maximum torque, $T_{max} = 1.2\ T$

$= 1.2 \times 5.968$

$= 7.162$ kNm

$$T_{max} = f_{\max}\frac{\pi}{16}D^3$$

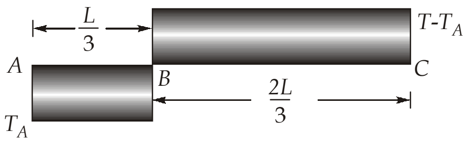
$$\Rightarrow \quad 7.162 \times 10^6 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 70 \times D^3$$

$$\Rightarrow \quad D^3 = \frac{7.162 \times 10^6 \times 16}{\pi \times 70}$$

$$\Rightarrow \quad D = (521082.378)^{1/3} = 80.57 \text{ mm}$$

25. (d)

$$T_A + T_C = T \quad \text{...(i)}$$



The angle of twist at B for AB and BC is same

$$\theta_{BA} = \theta_{BC}$$

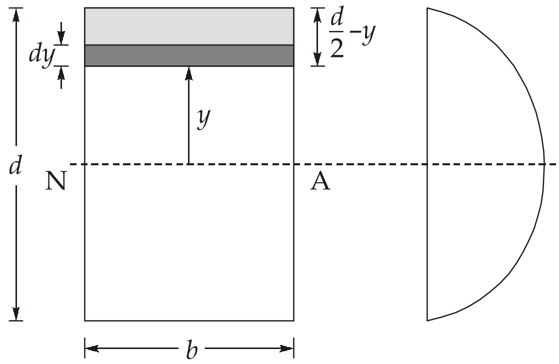
$$\Rightarrow \quad \frac{T_A L/3}{GJ} = \frac{(T-T_A)2L/3}{GJ}$$

$$\Rightarrow \quad T_A = \frac{2T}{3}$$

$\therefore$ Twisting moment in portion $AB = \frac{2T}{3} = kT$

$$k = \frac{2}{3} = 0.67$$

26. (b)



Shear stress at 'y' distance from neutral axis.

$$\tau = \frac{VQ}{It}$$

Where

$$Q = A\bar{y} = \left(\frac{d}{2} - y\right)b \times \left(y + \frac{\frac{d}{2} - y}{2}\right)$$

$$\Rightarrow \quad Q = \left(\frac{d}{2} - y\right)b\left(\frac{\frac{d}{2} + y}{2}\right)$$

$$\Rightarrow \quad Q = \left(\frac{d^2}{4} - y^2\right)\frac{b}{2}$$

$$I = \frac{bd^3}{12}$$

So,

$$\tau = \frac{V\left(\frac{d^2}{4} - y^2\right)\frac{b}{2}}{\frac{bd^3}{12} \times b} = \frac{6V}{d^3 b}\left(\frac{d^2}{4} - y^2\right)$$

Now shear force carried by elementary portion

$$dF = \tau dA$$

$$= \tau b dy$$

$$dF = \frac{6V}{d^3}\left(\frac{d^2}{4} - y^2\right)dy$$

So, shear force carried by upper 1/3rd portion:

$$F = \int_{d/6}^{d/2} \frac{6V}{d^3}\left(\frac{d^2}{4} - y^2\right)dy$$

$$= \frac{6V}{d^3}\left[\frac{d^2}{4}y - \frac{y^3}{3}\right]_{d/6}^{d/2}$$

$$= \frac{6V}{d^3}\left[\frac{d^2}{4}\frac{d}{2} - \frac{d^3}{24} - \frac{d^2}{4}\frac{d}{6} + \frac{d^3}{216\times 3}\right]$$

$$= \frac{6V}{d^3}\times\frac{7d^3}{162}$$

$$\therefore \quad F = \frac{7V}{27}$$

27. (c)

$$\text{Maximum principle stress} \le \frac{f_y}{\text{FOS}}$$

Equivalent BM,
$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$

$$= \frac{1}{2}\left[20 + \sqrt{20^2 + 40^2}\right]$$

$$= 32.36 \text{ kNm}$$

$$\text{Maximum principle stress} = \frac{32M_e}{\pi D^3} \le \frac{250}{2}$$

$$\Rightarrow \quad \frac{32\times 32.36\times 10^6}{\pi D^3} \le 125$$

$$\Rightarrow \quad D \ge 138.15 \text{ mm}$$

28. (b)

Let, P_s = Load shared by steel rod

P_c = Load shared by copper rod

Taking moments about A,

$$P_s \times 1 + P_c \times 3 = 20 \times 4$$

$$\Rightarrow \quad P_s + 3P_c = 80 \quad \text{...(i)}$$

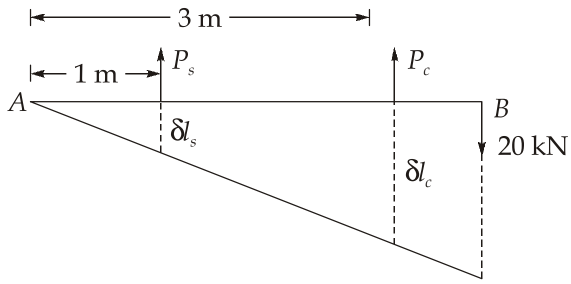
Now deformation of steel road due to load P_s is,

$$\delta l_s = \frac{P_s l_s}{A_s E_s} = \frac{P_s \times 1\times 10^3}{200\times 200\times 10^3} = 0.025\times 10^{-3} P_s$$

And deformation of copper rod due to load P_c is,

$$\delta l_c = \frac{P_c l_c}{A_c E_c} = \frac{P_c \times 2\times 10^3}{400\times 100\times 10^3} = 0.05\times 10^{-3} P_c$$

From the geometry of elongation of the steel rod and copper rod,



$$\frac{\delta l_c}{3} = \delta l_s$$

$\Rightarrow \quad \delta l_c = 3\delta l_s$

$\Rightarrow \quad 0.05 \times 10^{-3}\, P_c = 3 \times 0.025 \times 10^{-3}\, P_s$

$\Rightarrow \quad P_c = 1.5\, P_s$

Substituting this in eq. (i)

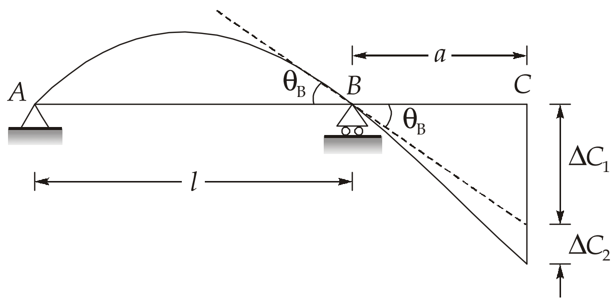
$P_s + 3\,(1.5\, P_s) = 80$

$\Rightarrow \quad P_s = 14.5 \times 10^3$ N

So, stress in steel rod, $\sigma_s = \dfrac{P_s}{A_s} = \dfrac{14.5 \times 10^3}{200} = 72.5\ \text{N/mm}^2$

29. (d)

The deformation of the beam will be as shown below.



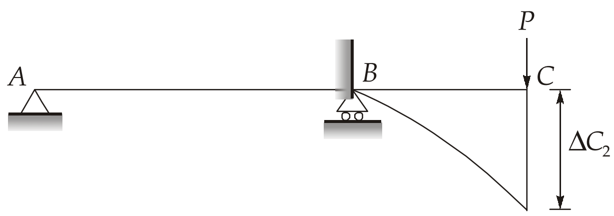
Now ΔC_1 is produced due to deflection of C as caused due to deformation of AB,

$$\Delta C_1 = \theta_B\,(BC) = \theta_B a$$

$$\theta_B = \frac{M_{BA} l}{3EI} = \frac{Pal}{3EI}$$

$$\therefore \quad \Delta C_1 = \frac{Pala}{3EI} = \frac{Pa^2 l}{3EI}$$

ΔC_2 is produced due to deformation of BC



$$\Delta C_2 = \frac{Pa^3}{3EI}$$

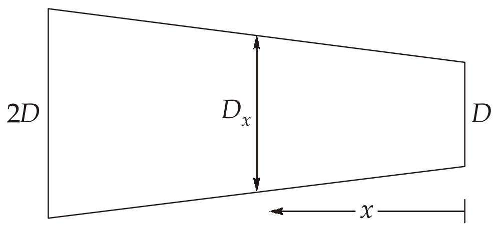
So total deflection at $C, \Delta C = \Delta C_1 + \Delta C_2$

$$= \frac{Pa^2 l}{3EI} + \frac{Pa^3}{3EI}$$

30. (c)

As the shaft is subjected to pair of equal and opposite torques T applied at its ends.
The diameter of shaft at distance x from smaller end is

$$D_x = D + \frac{(2D - D)x}{L} = \frac{D}{L}(L + x)$$



Corresponding polar moment of inertia,

$$J_x = \frac{\pi D_x^4}{32} = \frac{\pi}{32}\frac{D^4}{L^4}(L+x)^4$$

The angle of twist of the element, $d\theta = \frac{Tdx}{GJ_x}$

$$\Rightarrow \quad d\theta = \frac{T}{G}\left[\frac{32L^4}{\pi D^4 (L+x)^4}\right]dx$$

The total angle of twist θ over the entire length is,

$$\theta = \frac{32TL^4}{\pi G D^4}\int_0^L \frac{dx}{(L+x)^4}$$

$$= \frac{32TL^4}{\pi G D^4}\left[-\frac{1}{3(L+x)^3}\right]_0^L$$

$$= \frac{32TL^4}{\pi G D^4} \times \frac{7}{24L^3} = \frac{28}{3\pi}\frac{TL}{GD^4}$$

■■■■