CLASS TEST

S.No.: 01_JP_CE_03072025

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STRENGTH OF MATERIALS

CIVIL ENGINEERING

Date of Test : 03/07/2025

ANSWER KEY

1.	(c)	7.	(b)	13.	(b)	19.	(c)	25.	(b)
2.	(d)	8.	(c)	14.	(d)	20.	(a)	26.	(d)
3.	(d)	9.	(c)	15.	(a)	21.	(b)	27.	(b)
4.	(b)	10.	(b)	16.	(a)	22.	(b)	28.	(c)
5.	(d)	11.	(d)	17.	(c)	23.	(b)	29.	(a)
6.	(a)	12.	(a)	18.	(b)	24.	(b)	30.	(c)

DETAILED EXPLANATIONS

1. (c)

The stress induced in metal-1 due to restriction = $E_1\alpha_1\Delta T$

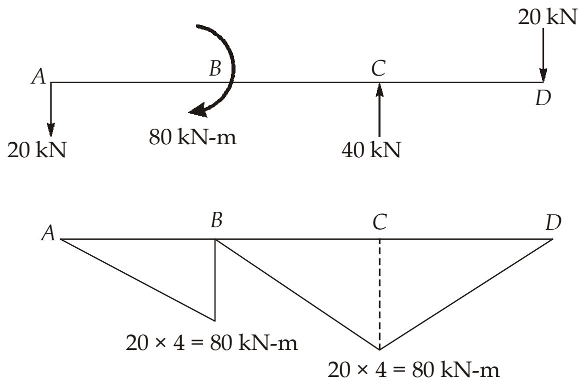
So, force required in metal-1 = $E_1\alpha_1\Delta T \times A_1$

Similarly for metal-2, force required = $E_2\alpha_2\Delta T \times A_2$

So, Total force required $= E_1\alpha_1\Delta T A_1 + E_2\alpha_2\Delta T A_2$

$= (E_1\alpha_1 A_1 + E_2\alpha_2 A_2)\,\Delta T$

2. (d)



BMD

3. (d)

For the given stress condition,

$\sigma_x = +50\text{ N/mm}^2,\ \sigma_y = 0,\ \tau_{xy} = \pm 20\text{ N/mm}^2$

Now, principal stresses,

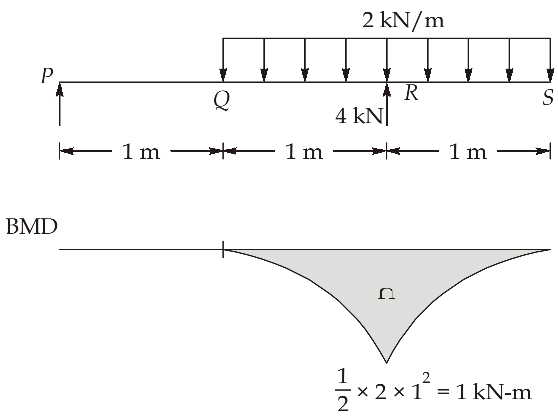
$$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{50+0}{2} \pm \sqrt{\left(\frac{50-0}{2}\right)^2 + 20^2}$$

$$= 57, -7\text{ N/mm}^2$$

So, $\sigma_1 = 57\text{ N/mm}^2$ (T)

$\sigma_2 = 7\text{ N/mm}^2$ (C)

4. (b)



5. **(d)**

As we know, $\dfrac{dV}{dx} = w$

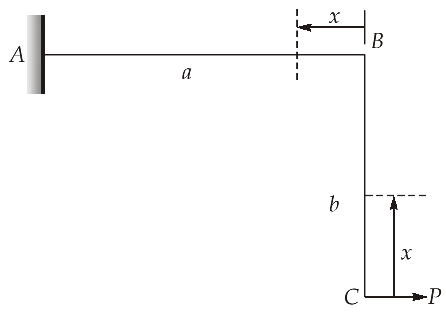
$\Rightarrow \quad w = \dfrac{d}{dx}\left(5x^2\right) = 10x$

For midspan, $x = 1$ m

So, load intensity, $w = 10$ N/m

6. **(a)**

Moment,

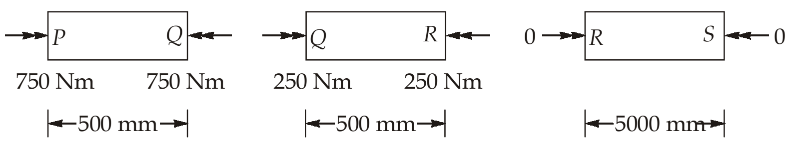


$$M_x = Px \qquad 0 < x < b$$
$$= Pb \qquad 0 < x < a$$

$$\text{Strain energy} = \int \frac{M_x^2 dx}{2EI} = \int_0^b \frac{(Px)^2\,dx}{2EI} + \int_0^a \frac{(Pb)^2\,dx}{2EI}$$

$$= \frac{P^2b^3}{6EI} + \frac{P^2b^2a}{2EI} = \frac{P^2b^2}{2EI}\left(\frac{b}{3} + a\right)$$

7. **(b)**



$$\text{Angle of twist } \phi = \frac{Tl}{GJ}$$

$$\phi_{PS} = \phi_{PQ} + \phi_{QR} + \phi_{RS}$$

$$= \frac{750 \times 10^3 \times 500}{80 \times 10^3 \times \frac{\pi}{32} \times 50^4} + \frac{250 \times 10^3 \times 500}{80 \times 10^3 \times \frac{\pi}{32} \times 50^4} + 0$$

$$= 10 \times 10^{-4} \times \frac{32}{\pi} \text{ rad} = 0.58°$$

8. **(c)**

$$\text{Deflection due to load} = \frac{wl^4}{8EI} = \frac{10 \times (3000)^4}{8 \times 5 \times 10^{11}} = 202.5 \text{ mm}$$

Since gap is only 3 mm.

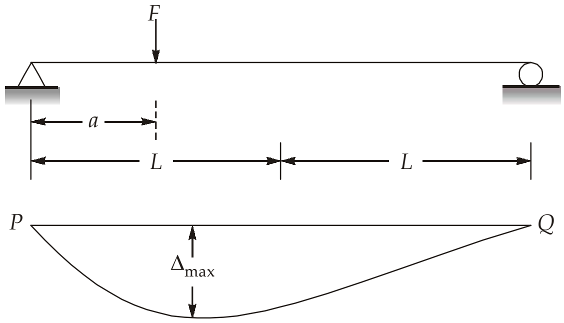
$$\therefore \quad 202.5 - 3 = \frac{Rl^3}{3EI}$$

$$\Rightarrow \quad \frac{(202.5 - 3) \times 3 \times 5 \times 10^{11}}{(3000)^3} = R$$

$$\Rightarrow \quad R = 11.083 \text{ kN} \simeq 11.08 \text{ kN}$$

9. (c)

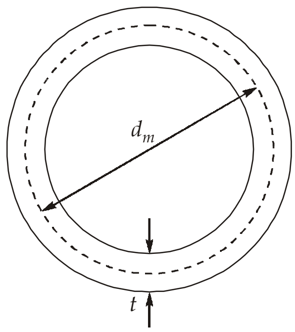
The tentative deflection for the loading is shown.



So, option (c) is possible.

10. (b)

For a thin hollow tube,



$$\tau_{max} = \frac{T}{2A_m t}$$

For an allowable maximum shear,

$$T_{max} = \tau_{allow} \times 2A_m t$$

Since, thickness and material is same in both cases,

$$T_{max} = [2\tau_{allow} \times t] \times A_m$$

Also, Weight of section = $\gamma \times$ Volume

$$\frac{T_{max}}{\text{Weight}} = \left[\frac{2\tau_{allow} \times t}{\gamma}\right] \times \frac{A_m}{\text{Volume}}$$

$$\therefore \quad \frac{(T_{max}/\text{Weight})_1}{(T_{max}/\text{Weight})_2} = \frac{(A_m/\text{Vol.})_1}{(A_m/\text{Vol.})_2} = \frac{\dfrac{\frac{\pi}{4} \times d_1^2}{\pi d_1 t}}{\dfrac{\frac{\pi}{4} \times d_2^2}{\pi d_2 t}}$$

$$\Rightarrow \quad \frac{(T_{max}/\text{Weight})_1}{(T_{max}/\text{Weight})_2} = \frac{d_1}{d_2} = \frac{1}{2}$$

So, maximum allowable torque to weight becomes nearly double.

11. (d)

For the given conditions, stress in x and z will be zero as the block in free to expand in x and z directions. Only y-direction will have stress.

On increasing temperature, free elongation

$$= l\alpha\Delta T = 0.1 \times 20 \times 10^{-6} \times 150 \text{ m}$$

$$= 3 \times 10^{-4} \text{ m}$$

$$= 0.3 \text{ mm} > 0.2 \text{ mm}$$

Thus stress induced will be only due to (0.3 - 0.2) = 0.1 mm

$$\text{Stress, } \sigma_{yy} = \left(\frac{\Delta l}{l}\right)E = \frac{0.1}{0.1 \times 10^3} \times E = \frac{70 \times 10^3}{10^3} \text{ N/m}^2 = 70 \text{ MPa} \quad \text{(Compression)}$$

12. **(a)**

As there is a jump at A, there will be an upward load at A.

From A to D, slope of SFD = 0, so no load intensity acts from A to D.

At D, there is the fall so there will be a downward load at D.

$$P_D = -4 - 14 = -18 \text{ kN}$$

So, only option (a) is possible.

13. **(b)**

Axial stress, $\sigma_a = -\dfrac{F}{\pi r^2}$

In radial direction, strain is zero.

So, $\varepsilon_r = \dfrac{\sigma_r}{E} - \mu\dfrac{\sigma_a}{E} - \mu\dfrac{\sigma_r}{E}$

$\Rightarrow$ $0 = \dfrac{\sigma_r(1-\mu)}{E} - \mu\dfrac{\sigma_a}{E}$

$\Rightarrow$ $\sigma_r = -\dfrac{\mu}{1-\mu}\left(\dfrac{F}{\pi r^2}\right)$

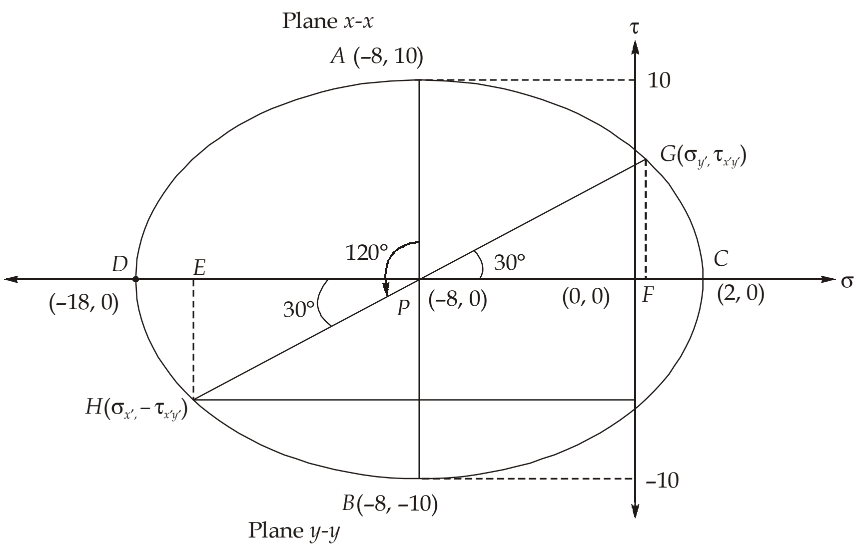
Strain in axial direction, $\varepsilon_a = \dfrac{\sigma_a}{E} - \mu\dfrac{\sigma_r}{E} - \mu\dfrac{\sigma_r}{E}$

$\Rightarrow$ $\dfrac{\Delta h}{h} = -\dfrac{F}{E\pi r^2}\left[1 - \dfrac{2\mu^2}{1-\mu}\right]$

$\Rightarrow$ $\Delta h = -\dfrac{Fh}{\pi r^2 E}\left[1 - \dfrac{2\mu^2}{1-\mu}\right]$

14. **(d)**

Mohr's circle for this stress condition is shown below:



For $\sigma_{y'}$, in ΔPGF

$$\cos 30° = \frac{OF}{OG}$$

$$OF = \frac{\sqrt{3}}{2} \times 10$$

$$OF = 8.66 \text{ MPa}$$

$$\sigma_{y'} = OF - 8 = 0.66 \text{ MPa}$$

For σ_x', In ΔPEH $\quad PE = 8.66$ MPa

$$\sigma_x' = -8 - 8.66 \text{ MPa} = -16.66 \text{ MPa}$$

For $\tau_{x'y'}$, in ΔPEH,

$$\tan 30° = \frac{\tau_{x'y'}}{8.66}$$

$\Rightarrow$ $\quad \tau_{x'y'} = 5$ MPa

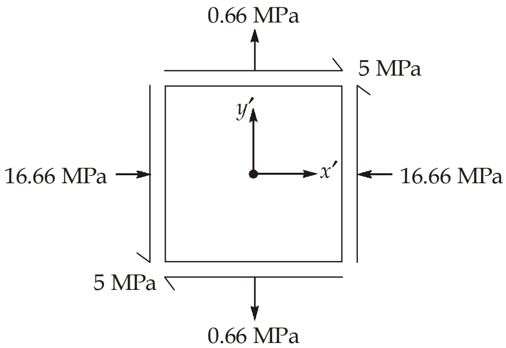
So, at plane x'-x', $\quad \sigma_{y'} = 0.66$ MPa

and $\quad \tau_{x'y'} = 5$ MPa

And at plane y'-y', $\quad \sigma_{x'} = -16.66$ MPa

And $\quad \tau_{x'y'} = -5$ MPa

Hence, option (d) is correct.

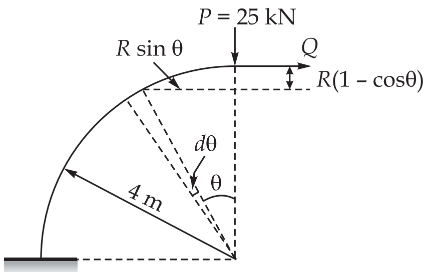


15. (a)

Apply a horizontal load (Q) at tip.

$$M_x = PR \sin\theta + QR(1 - \cos\theta)$$

$$\therefore \quad \frac{\partial M_x}{\partial Q} = R(1 - \cos\theta)$$



$$\therefore \quad \Delta_H = \left.\frac{\partial U}{\partial Q}\right|_{Q=0}$$

$$= \frac{1}{EI}\int M_x \frac{dM_x}{dQ}dx\Big|_{Q=0}$$

$$= \frac{1}{EI}\int\{(PR\sin\theta + QR(1-\cos\theta))R(1-\cos\theta)dx\}\Big|_{Q=0}$$

$$= \frac{1}{EI}\int_0^{\pi/2} PR\sin\theta R(1-\cos\theta)Rd\theta$$

$$\therefore \quad \Delta_H = \frac{PR^3}{EI}\int_0^{\pi/2}\sin\theta(1-\cos\theta)d\theta$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2}\left(\sin\theta - \frac{\sin 2\theta}{2}\right)d\theta$$

$$= \frac{PR^3}{EI}\left(-\cos\theta + \frac{\cos 2\theta}{4}\right)_0^{\pi/2}$$

$$= \frac{PR^3}{EI}\left(-0 - \frac{1}{4} - \left(-1 + \frac{1}{4}\right)\right)$$

$$= \frac{PR^3}{2EI}$$

Given $P = 25$ kN, $R = 4$ m, $E = 200 \times 10^3$ N/mm^2

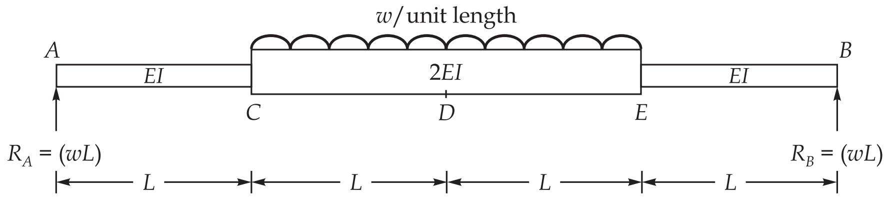
Dia. of cross-section, $d = 150$ mm

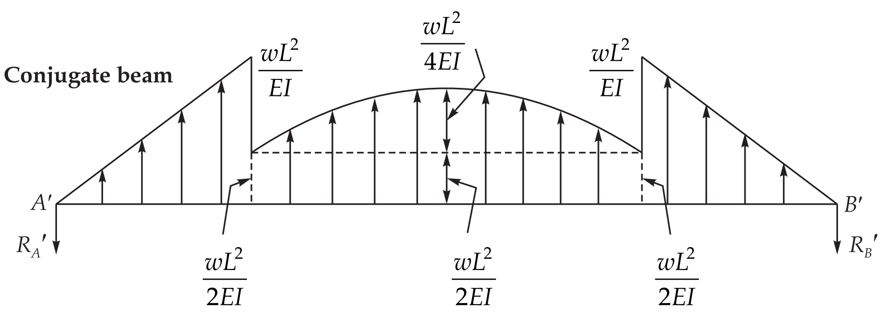
$$\therefore \quad \Delta_H = \frac{25\times10^3\times(4000)^3}{2\times200\times10^3\times\frac{\pi}{64}\times(150)^4}$$

$$= 160.963 \text{ mm}$$

16. (a)

Using conjugate beam method.





The reaction at A' and B' are equal due to symmetry.

$$R_A' = \frac{1}{2}\times L\times\frac{wL^2}{EI}+\left(\frac{wL^2}{2EI}\right)\times L+\frac{2}{3}\times\frac{wL^2}{4EI}\times L$$

$$= \frac{7wL^3}{6EI}$$

From conjugate beam 'Theorem-1':

The slope at any point in a real beam will be equal to shear force at the corresponding point in conjugate beam.

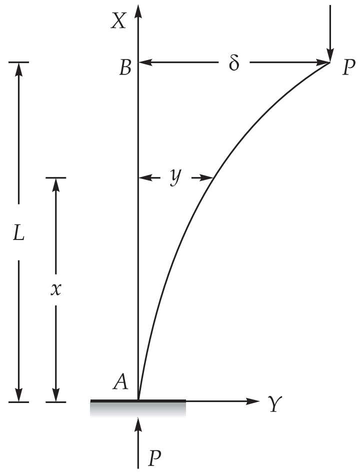
$\therefore$ θ_A = SF at A' in conjugate beam

$$= -R_A' = -\frac{7wL^3}{6EI}$$

$$= \frac{7wL^3}{6EI}(\text{CW})$$

17. (c)

Given differential equation is:



$$EI\frac{d^2y}{dx^2} = P(\delta-y)$$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} = P\delta-Py$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}+\frac{P}{EI}\times y = \frac{P\delta}{EI}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}+\alpha^2 y = \frac{P\delta}{EI}\left(\text{where } \alpha^2=\frac{P}{EI}\right)$$

The solution of the above differential equation

$$y = A\sin\alpha x+B\cos\alpha x+\frac{P\delta}{EI\alpha^2}$$

$$= A\sin\alpha x+B\cos\alpha x+\delta$$

Applying boundary condition.

At $x = 0$; $y = 0$

$\therefore$ $B = -\delta$

Also, $\frac{dy}{dx} = A\alpha\cos\alpha x-B\alpha\sin\alpha x$

At $x = 0$, $\frac{dy}{dx} = 0$

$\Rightarrow$ $0 = A\alpha - 0$

$\Rightarrow$ $A = 0$

$\therefore$ $y = -\delta\cos\alpha x+\delta=\delta(1-\cos\alpha x)$

Also, at $x = L$, $y = \delta$

$\therefore$ $\delta = \delta(1-\cos\alpha L)$

$\Rightarrow$ $\cos\alpha L = 0$

$\Rightarrow$ $\cos\alpha L = \cos\frac{\pi}{2}$ or $\cos\frac{3\pi}{2}$ or $\cos\frac{5\pi}{2}$ or , ...

$$\Rightarrow \qquad \alpha L = \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \text{ or } \frac{5\pi}{2} \text{ or }, \ldots$$

For second critical load, $\alpha L = \frac{3\pi}{2}$

$$\Rightarrow \qquad \sqrt{\frac{P}{EI}} \times L = \frac{3\pi}{2}$$

Squaring both sides, $\frac{P}{EI} \times L^2 = \frac{9\pi^2}{4}$

$$\Rightarrow \qquad P = \frac{9\pi^2 EI}{4L^2}$$

18. (b)

For no tension to occur,

Direct stress ≥ Bending tensile stress

In limiting case,

Direct stress = Maximum bending tensile stress

$$\Rightarrow \qquad \frac{P}{D^2} = \frac{Pe_y}{\left(D^3/6\right)} + \frac{Pe_x}{D^3/6} \quad \ldots(i)$$

Since column is square and point being on diagonal

$$\therefore \qquad e_x = e_y$$

From eq. (i) $\qquad 2e_x = \frac{D}{6}$

$$\Rightarrow \qquad e_x = \frac{D}{12}$$

$$\therefore \qquad e = \sqrt{e_x^2 + e_y^2}$$

$$= \sqrt{\left(\frac{D}{12}\right)^2 + \left(\frac{D}{12}\right)^2} = \sqrt{2 \times \left(\frac{D}{12}\right)^2}$$

$$= \frac{\sqrt{2}\,D}{12}$$

19. (c)

For the 60° rosette:

$$\varepsilon_x = \varepsilon_a = 60 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{3}\left(2\varepsilon_b + 2\varepsilon_c - \varepsilon_a\right)$$

$$= \frac{1}{3}\left(2 \times 135 \times 10^{-6} + 2 \times 264 \times 10^{-6} - 60 \times 10^{-6}\right)$$

$$= 246 \times 10^{-6}$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}\left(\varepsilon_b - \varepsilon_c\right) = \frac{2}{\sqrt{3}}\left(135 \times 10^{-6} - 264 \times 10^{-6}\right)$$

$$\simeq -149 \times 10^{-6}$$

∴ In plane principal strain

$$\varepsilon_{1/2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\Rightarrow \quad \varepsilon_{1/2} = \left[\frac{60+246}{2} \pm \sqrt{\left(\frac{60-246}{2}\right)^2 + \left(-\frac{149}{2}\right)^2}\right] \times 10^{-6}$$

$$= [153 \pm 119.2] \times 10^{-6}$$

$$\therefore \quad \varepsilon_1 = 272.2 \times 10^{-6}$$

$$\varepsilon_2 = 33.8 \times 10^{-6}$$

Principal stress,
$$\sigma_1 = \frac{E}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2)$$

$$= \frac{200 \times 10^3}{1-(0.3)^2}\left(272.2 \times 10^{-6} + 0.3 \times 33.8 \times 10^{-6}\right)$$

$$= 62.05 \text{ MPa} \simeq 62 \text{ MPa}$$

$$\sigma_2 = \frac{E}{1-\mu^2}(\varepsilon_2 + \mu\varepsilon_1)$$

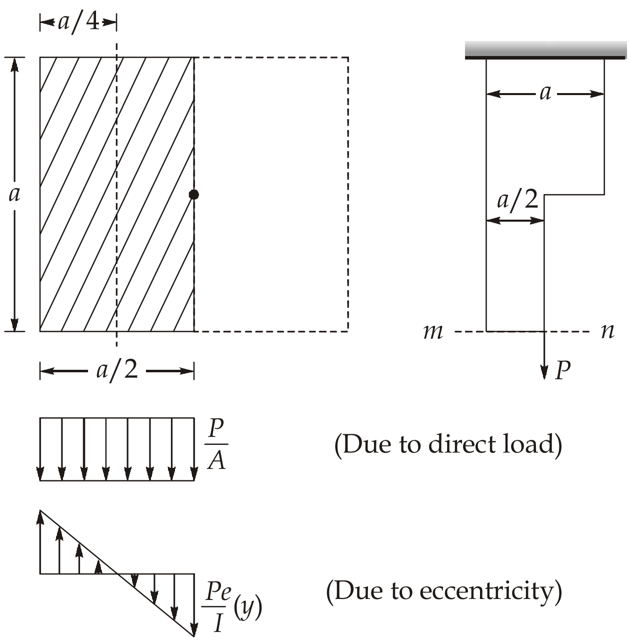
$$= \frac{200 \times 10^3}{1-(0.3)^2}\left(33.8 \times 10^{-6} + 0.3 \times 272.2 \times 10^{-6}\right)$$

$$= 25.4 \text{ MPa}$$

$\therefore$ Largest normal stress = 62 MPa

Largest shearing stress $= \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{62 - 25.4}{2} = 18.3$ MPa

20. **(a)**



(Due to direct load)

(Due to eccentricity)

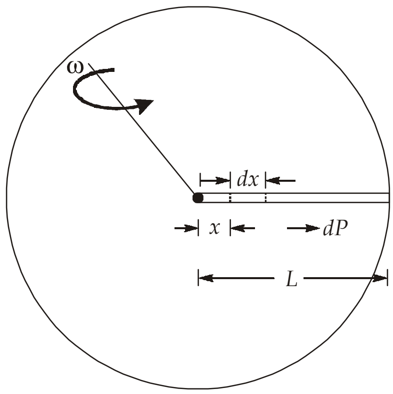
Maximum compressive stress at section *mn* is given by

$$\sigma_c = \frac{P}{A} - \frac{Pe}{I} \times y$$

$$= \frac{P}{\left(a \times \frac{a}{2}\right)} - \frac{P \times \frac{a}{4}}{\left[\frac{a}{12} \times \left(\frac{a}{2}\right)^3\right]} \times \frac{a}{4} = \frac{2P}{a^2} - \frac{96Pa^2}{16a^4}$$

$$= \frac{2P}{a^2} - \frac{6P}{a^2} = -\frac{4P}{a^2} = \frac{4P}{a^2}(\text{Compressive})$$

21. (b)



$$\delta = \frac{PL}{AE}$$

From the figure,

$$d\delta = \frac{dPx}{AE}$$

Centrifugal force on differential mass dM,

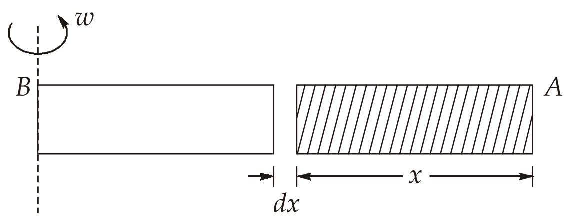
$$dP = dM \cdot \omega^2 x = (\rho A dx)\omega^2 x$$

$$\therefore \quad d\delta = \frac{(\rho A\omega^2 x dx)x}{AE}$$

$$\therefore \quad \delta = \frac{\rho\omega^2}{E}\int_0^L x^2.dx = \frac{\rho\omega^2}{E}\left[\frac{x^3}{3}\right]_0^L$$

$$\Rightarrow \quad \delta = \frac{\rho\omega^2}{3E}\left[L^3 - 0^3\right] = \frac{\rho\omega^2 L^3}{3E}$$

Alternate Solution:



$$m_x = \rho A x$$

Distance between the C.G. of mass to the centre of rotation,

$$r = L - \frac{x}{2}$$

Centrifugal force, $$F = m_x w^2 r = \rho A x \left(w^2\right)\left(L - \frac{x}{2}\right)$$

Element elongation, $$d\delta = \frac{F dx}{AE} = \frac{Ax\rho\left(L - \frac{x}{2}\right)w^2 dx}{AE}$$

$$d\delta = \frac{\rho w^2\left(L - \frac{x}{2}\right)}{E}x.dx$$

Total elongation, $$\delta = \int_0^L \frac{\rho w^2}{E}\left(L - \frac{x}{2}\right)x dx = \frac{\rho w^2}{E}\left[L\left(\frac{L^2}{2}\right) - \left(\frac{x^3}{6}\right)\right]_0^L = \frac{\rho w^2}{E}\left[\frac{L^2}{2} - \frac{L^3}{6}\right]$$

$$\delta = \frac{\rho w^2 L^2}{3E}$$

22. (b)

$$\text{Total load} = 2\left[\frac{1}{2}\left(\frac{L}{2}\right)\times w_0\right] = \frac{w_0 L}{2}$$

By symmetry, $$R_1 = R_2 = \frac{1}{2}\times\text{Total load}$$

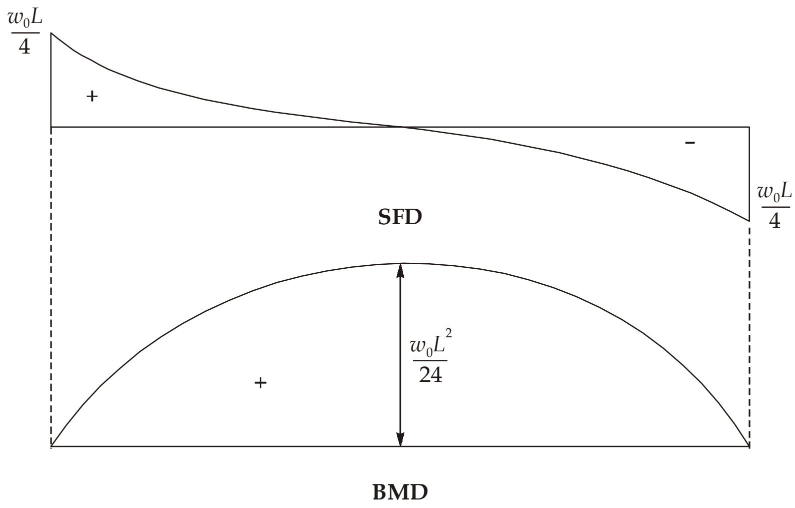
$\Rightarrow$ $$R_1 = R_2 = \frac{w_0 L}{4}$$

Bending moment at B,

$$(M_B) = R_1 \times \frac{L}{2} - \frac{1}{2}w_0 \times \frac{L}{2}\times\frac{2}{3}\left(\frac{L}{2}\right)$$

$$= \frac{w_0 L}{4}\times\frac{L}{2} - \frac{w_0}{2}\times\frac{L}{2}\times\frac{2}{3}\left(\frac{L}{2}\right)$$

$$= \frac{w_0 L^2}{8} - \frac{w_0 L^2}{12} = \frac{w_0 L^2}{24}$$



23. **(b)**

Since section is symmetric about x-x and y-y, therefore centre of section will lie on the geometrical centroid of section.

The semi-circular grooves may be assumed together and consider one circle of diameter 60 mm.

So, $$I_{xx} = \frac{80\times(100)^3}{12} - \frac{\pi}{64}(60)^4 = 6.03 \times 10^6 \text{ mm}^4$$

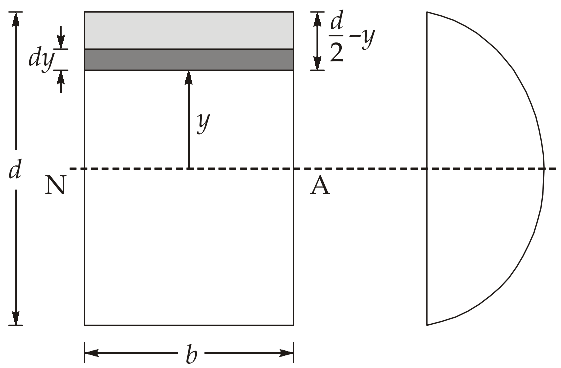
Now for shear stress at neutral axis, consider the area above the neutral axis,

$$A\bar{y} = [80\times50\times25] - \frac{\pi}{2}(30)^2\times\frac{4\times30}{3\pi} = 100000 - 18000 = 82000 \text{ mm}^3$$

$$b = 20 \text{ mm}$$

So, $$\tau = \frac{VA\bar{y}}{Ib} = \frac{20\times10^3\times82000}{6.03\times10^6\times20} = 13.60 \text{ MPa}$$

24. **(b)**



Shear stress at 'y' distance from neutral axis.

$$\tau = \frac{VQ}{It}$$

Where $$Q = A\bar{y} = \left(\frac{d}{2}-y\right)b\times\left(y+\frac{\frac{d}{2}-y}{2}\right)$$

$\Rightarrow$ $$Q = \left(\frac{d}{2}-y\right)b\left(\frac{\frac{d}{2}+y}{2}\right)$$

$\Rightarrow$ $$Q = \left(\frac{d^2}{4}-y^2\right)\frac{b}{2}$$

$$I = \frac{bd^3}{12}$$

So, $$\tau = \frac{V\left(\frac{d^2}{4}-y^2\right)\frac{b}{2}}{\frac{bd^3}{12}\times b} = \frac{6V}{d^3b}\left(\frac{d^2}{4}-y^2\right)$$

Now shear force carried by elementary portion

$$dF = \tau dA$$
$$= \tau bdy$$
$$dF = \frac{6V}{d^3}\left(\frac{d^2}{4} - y^2\right)dy$$

So, shear force carried by upper 1/3rd portion:

$$F = \int_{d/6}^{d/2} \frac{6V}{d^3}\left(\frac{d^2}{4} - y^2\right)dy$$
$$= \frac{6V}{d^3}\left[\frac{d^2}{4}y - \frac{y^3}{3}\right]_{d/6}^{d/2}$$
$$= \frac{6V}{d^3}\left[\frac{d^2}{4}\frac{d}{2} - \frac{d^3}{24} - \frac{d^2}{4}\frac{d}{6} + \frac{d^3}{216 \times 3}\right]$$
$$= \frac{6V}{d^3} \times \frac{7d^3}{162}$$
$$\therefore \quad F = \frac{7V}{27}$$

25. (b)

Let, P_s = Load shared by steel rod

P_c = Load shared by copper rod

Taking moments about A,

$$P_s \times 1 + P_c \times 3 = 20 \times 4$$
$$\Rightarrow \quad P_s + 3P_c = 80 \quad \text{...(i)}$$

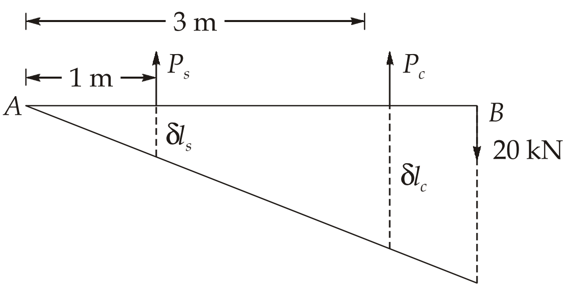
Now deformation of steel road due to load P_s is,

$$\delta l_s = \frac{P_s l_s}{A_s E_s} = \frac{P_s \times 1 \times 10^3}{200 \times 200 \times 10^3} = 0.025 \times 10^{-3} P_s$$

And deformation of copper rod due to load P_c is,

$$\delta l_c = \frac{P_c l_c}{A_c E_c} = \frac{P_c \times 2 \times 10^3}{400 \times 100 \times 10^3} = 0.05 \times 10^{-3} P_c$$

From the geometry of elongation of the steel rod and copper rod,



$$\frac{\delta l_c}{3} = \delta l_s$$

$\Rightarrow \quad \delta l_c = 3\delta l_s$

$\Rightarrow \quad 0.05 \times 10^{-3}\, P_c = 3 \times 0.025 \times 10^{-3}\, P_s$

$\Rightarrow \quad P_c = 1.5\, P_s$

Substituting this in eq. (i)

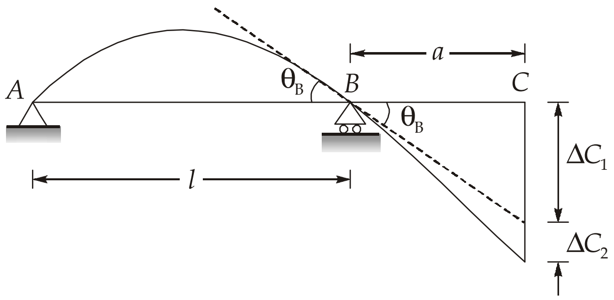
$P_s + 3\,(1.5\, P_s) = 80$

$\Rightarrow \quad P_s = 14.5 \times 10^3$ N

So, stress in steel rod, $\sigma_s = \dfrac{P_s}{A_s} = \dfrac{14.5 \times 10^3}{200} = 72.5 \text{ N/mm}^2$

26. (d)

The deformation of the beam will be as shown below.



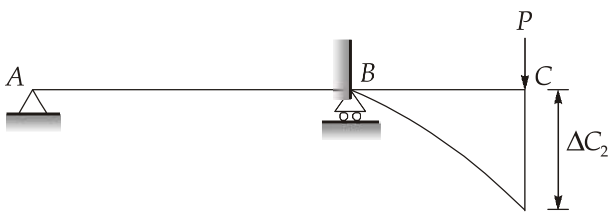
Now ΔC_1 is produced due to deflection of C as caused due to deformation of AB,

$$\Delta C_1 = \theta_B\,(BC) = \theta_B a$$

$$\theta_B = \frac{M_{BA}l}{3EI} = \frac{Pal}{3EI}$$

$$\therefore \quad \Delta C_1 = \frac{Pala}{3EI} = \frac{Pa^2 l}{3EI}$$

ΔC_2 is produced due to deformation of BC



$$\Delta C_2 = \frac{Pa^3}{3EI}$$

So total deflection at $C, \Delta C = \Delta C_1 + \Delta C_2$

$$= \frac{Pa^2 l}{3EI} + \frac{Pa^3}{3EI}$$

27. (b)

Stress developed in the bar, $\sigma = \dfrac{P}{A}\left[1 + \sqrt{1 + \dfrac{2AEh}{Pl}}\right]$

$$= \frac{15\times10^3}{2500}\left[1+\sqrt{1+\frac{2\times2500\times200\times10^3\times10}{15\times10^3\times3\times10^3}}\right]$$

$$= 6[1+14.9] = 95.4 \text{ N/mm}^2$$

We know that volume of bar, $V = l.A$

$$= 3 \times 10^3 \times 2500$$

$$= 7.5 \times 10^6 \text{ mm}^3$$

∴ Strain energy stored in the bar, $U = \frac{\sigma^2}{2E}V = \frac{(95.4)^2}{2\times200\times10^3}\times7.5\times10^6$ Nmm

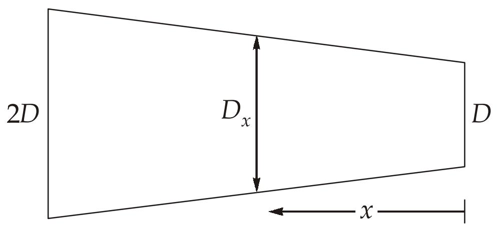
$$= 170.6 \text{ Nm}$$

28. (c)

As the shaft is subjected to pair of equal and opposite torques T applied at its ends.

The diameter of shaft at distance x from smaller end is

$$D_x = D+\frac{(2D-D)x}{L} = \frac{D}{L}(L+x)$$



Corresponding polar moment of inertia,

$$J_x = \frac{\pi D_x^4}{32} = \frac{\pi}{32}\frac{D^4}{L^4}(L+x)^4$$

The angle of twist of the element, $d\theta = \frac{Tdx}{GJ_x}$

$$\Rightarrow \quad d\theta = \frac{T}{G}\left[\frac{32L^4}{\pi D^4(L+x)^4}\right]dx$$

The total angle of twist θ over the entire length is,

$$\theta = \frac{32TL^4}{\pi GD^4}\int_0^L\frac{dx}{(L+x)^4} = \frac{32TL^4}{\pi GD^4}\left[-\frac{1}{3(L+x)^3}\right]_0^L$$

$$= \frac{32TL^4}{\pi GD^4}\times\frac{7}{24L^3} = \frac{28}{3\pi}\frac{TL}{GD^4}$$

29. (a)

Shear strain, $\gamma_{xy} = \frac{\partial U_x}{\partial y}+\frac{\partial U_y}{\partial x}$

$$= (6y+0)\times10^{-2}\Big| = (6y\times10^{-2})\Big|_{(1,2,0)} = 0.12$$

$$\gamma_{yz} = \frac{\partial U_y}{\partial z} + \frac{\partial U_z}{\partial y} = (6y+0)\times 10^{-2} = 6y \times 10^{-2}\Big|_{(1,2,0)} = 0.12$$

$$\gamma_{xz} = \frac{\partial U_x}{\partial z} + \frac{\partial U_z}{\partial x}$$

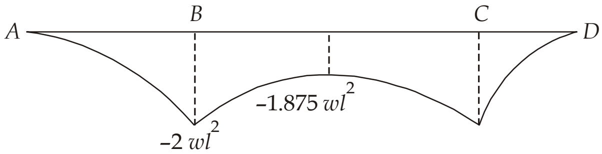
$$= (0+16x)\times 10^{-2} = 16x \times 10^{-2}\Big|_{(1,2,0)} = 0.16$$

30. **(c)**

BM at B $= -2wl^2$

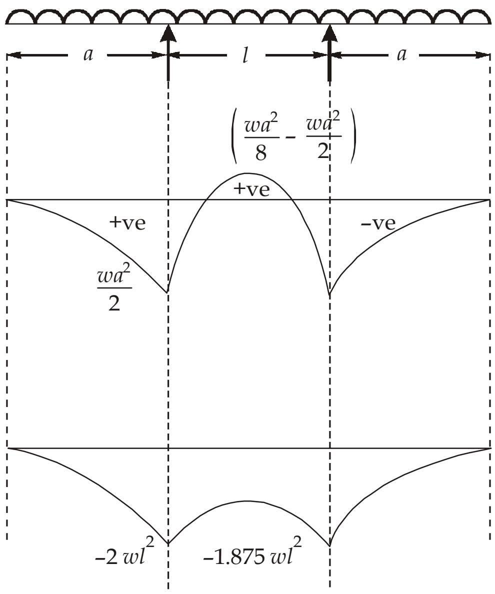
BM at mid of BC $= \dfrac{-w\times(2.5l)^2}{2} + (2.5wl)\dfrac{l}{2} = -1.875wl^2$

So, BM doesn't change sign throughout the beam.



BMD

Alternative:



So, for given condition,

$a = 2l$

and $l = l$

B.M. at support

$\Rightarrow \quad \dfrac{-w(2l)^2}{2} = -2wl^2$

Both value is –ve one.

Hence no point of contraflexure in the beam.

■■■■