CLASS TEST

S.No.: 02SK_ECEE_JKL_07072025

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ENGINEERING MATHEMATICS

EE+EC

Date of Test : 07/07/2025

ANSWER KEY

1.	(b)	7.	(b)	13.	(b)	19.	(c)	25.	(d)
2.	(d)	8.	(a)	14.	(c)	20.	(a)	26.	(d)
3.	(c)	9.	(b)	15.	(a)	21.	(b)	27.	(c)
4.	(c)	10.	(b)	16.	(a)	22.	(a)	28.	(c)
5.	(b)	11.	(c)	17.	(d)	23.	(a)	29.	(b)
6.	(b)	12.	(c)	18.	(d)	24.	(b)	30.	(d)

DETAILED EXPLANATIONS

1. (b)

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x-2) = 0$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x^3-8) = 0$$

Also $f(2) = 0$

Thus $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\therefore$ f is continuous at $x = 2$

and $Lf'(2) = 1$ and $Rf'(2) = 12$

$\therefore$ f is not differentiable at $x = 2$.

2. (d)

$D = -96$ for the given matrix

$$|A| = \begin{vmatrix} 4 & 12 & 0 \\ 8 & 24 & 16 \\ -4 & 0 & 8 \end{vmatrix} = 2^3 \begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix}$$

(Taking 2 common from each row)

$\therefore$ $\text{Det}(A) = (2)^3 \times D$

$= 8 \times (-96) = -768$

3. (c)

Given,

$$\text{Trace } A = 9$$

$$|A| = 24$$

$$\lambda_1 = 3$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 9$$

$\Rightarrow$ $3 + \lambda_2 + \lambda_3 = 9$

$\Rightarrow$ $\lambda_2 + \lambda_3 = 6$

4. (c)

Consider a symmetric matrix $A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$

Given $a + d = -8$

$|A| = ad - b^2$

Now since b^2 is always non-negative, maximum determinant will come when $b^2 = 0$.

So we need to maximize

$|A| = ad$

$= ad = a \times [-(8 + a)] = -a^2 - 8a$

$$\frac{d|A|}{da} = -2a - 8 = 0$$

$\Rightarrow$ $a = -4$ is the only stationary point

Since, $\left[\dfrac{d^2|A|}{da^2}\right]_{a=-4} = -2 < 0,$ we have a maximum at $a = -4$

Since, $a + d = -8$, $d = -4$. Now maximum value of determinant is $|A| = 16$

5. (b)

$$\left(\frac{1+i}{1-i}\right)^n = 1$$

$$\Rightarrow \quad \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n = 1$$

$$\Rightarrow \quad \left(\frac{(1+i)^2}{1-i^2}\right)^n = 1$$

$$\Rightarrow \quad \left(\frac{1+i^2+2i}{1+1}\right)^n = 1$$

$$\Rightarrow \quad \left(\frac{1-1+2i}{2}\right)^n = 1$$

$$\Rightarrow \quad i^n = 1$$

$$\Rightarrow \quad n = 4$$

6. (b)

$$(3AB + \lambda BA)' = 3(AB)' + \lambda(BA)'$$

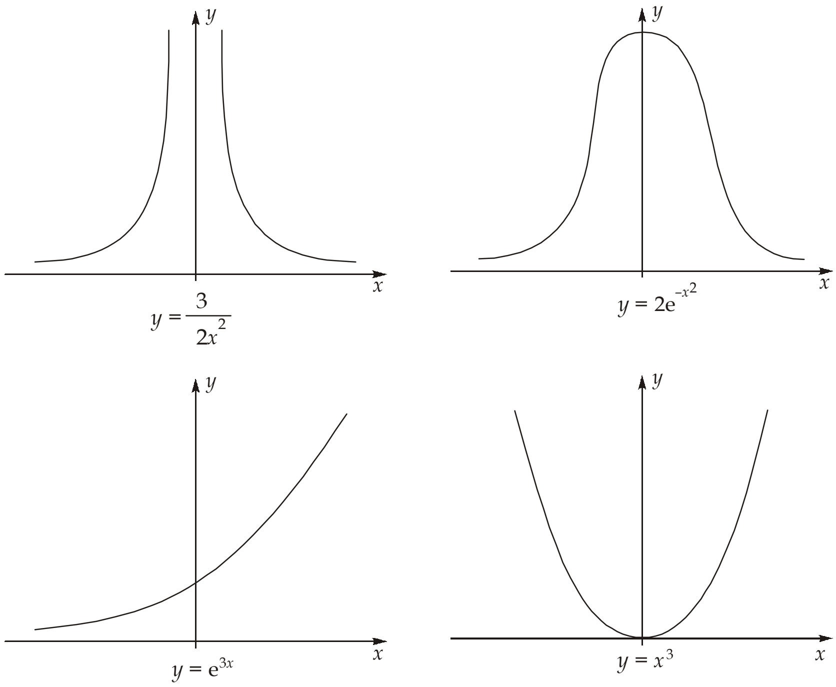
$$= 3B'A' + \lambda A'B'$$

$$= -3BA - \lambda AB = -(\lambda AB + 3BA)$$

$$\lambda = 3$$

7. (b)

From the graphs below, we can see that only $2e^{-x^2}$ is strictly bounded.



8. (a)

$$z = \frac{6+8i}{1-2i} = \frac{(6+8i)(1+2i)}{(1-2i)(1+2i)}$$

$$= \frac{-10+20i}{5} = -2 + 4i$$

$$|z| = \sqrt{(-2)^2 + (4)^2} = 2\sqrt{5}$$

9. (b)

$$P[X > 1] = \int_1^{\infty} f(x)dx = \int_1^{\infty} e^{-2x} dx = \left.\frac{-e^{-2x}}{2}\right|_1^{\infty} = -\left(\frac{e^{-2\infty}}{2} - \frac{e^{-2}}{2}\right) = \frac{e^{-2}}{2} = 0.067$$

10. (b)

$P(E)$ = Probability of head appearing in odd number of tosses

$= P(H) + P(TTH) + \ldots\ldots\ldots$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + \ldots\ldots.$$

$$= \frac{1/2}{1-\frac{1}{4}} = \frac{2}{3} = 0.67$$

11. (c)

$$f(\theta) = a\sin^3\left(\frac{\theta}{3}\right)$$

$$f'(\theta) = a\sin^2\left(\frac{\theta}{3}\right)\cos\left(\frac{\theta}{3}\right)$$

$$S = \int_0^{3\pi} \sqrt{\left[a\sin^3\left(\frac{\theta}{3}\right)\right]^2 + \left[a\sin^2\left(\frac{\theta}{3}\right)\cos\left(\frac{\theta}{3}\right)\right]^2}\, d\theta$$

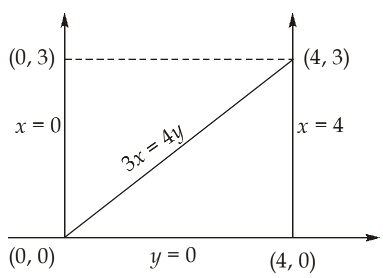
$$= \int_0^{3\pi} a\sin^2\left(\frac{\theta}{3}\right) d\theta = \frac{a}{2}\int_0^{3\pi} 1 - \cos\left(\frac{2\theta}{3}\right) d\theta$$

$$= \frac{a}{2}\left[\theta\Big|_0^{3\pi} - \frac{3}{2}\sin\left(\frac{2\theta}{3}\right)\Big|_0^{3\pi}\right] = \frac{a}{2}(3\pi)$$

$= ab\pi$

$\Rightarrow$ $b = 1.5$

12. (c)



$$\text{Volume} = \iiint dz\,dx\,dy = \iint z\,dy\,dx$$

$$= \int_0^4 \int_0^{\frac{3x}{4}} (8 - x - y)\,dy\,dx = \int_0^4 \left(8y - xy - \frac{y^2}{2}\right)\Bigg|_0^{\frac{3x}{4}} dx$$

$$= \int_0^4 \left(6x - \frac{33x^2}{32}\right) dx = \left(\frac{6x^2}{2} - \frac{33}{32} \times \frac{x^3}{3}\right)\Bigg|_0^4$$

$$= \left(3x^2 - \frac{11x^3}{32}\right)\Bigg|_0^4 = 48 - 22 = 26$$

13. (b)

Characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 2 & -2 \\ -12 & -22-\lambda & 12 \\ -12 & -22 & 10-\lambda \end{vmatrix} = 0$$

$\Rightarrow -\lambda[-220 + 22\lambda - 10\lambda + \lambda^2 + 264] - 2[-120 + 12\lambda + 144] - 2\,[264 - 264 - 12\lambda] = 0$

$\Rightarrow -\lambda(\lambda^2 + 12\lambda + 44) - 2(12\lambda + 24) + 24\lambda = 0$

$\Rightarrow -\lambda^3 - 12\lambda^2 - 44\lambda - 24\lambda - 48 + 24\lambda = 0$

$\Rightarrow -\lambda^3 - 12\lambda^2 - 44\lambda - 48 = 0$

$\Rightarrow \quad \lambda^3 + 12\lambda^2 + 44\lambda + 48 = 0$

$\Rightarrow \quad (\lambda + 2)(\lambda + 4)(\lambda + 6) = 0$

$\Rightarrow \quad \lambda = -2, -4, -6$

$$\text{Ratio} = \frac{6}{2} = 3$$

14. (c)

$$I = \int \sec\theta(\sec^2\theta)\,d\theta$$

$$= \sec\theta \int \sec^2\theta\,d\theta - \int \tan\theta(\sec\theta\tan\theta)\,d\theta$$

$$= \sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta$$

$$= \sec\theta\tan\theta - \int (\sec^2\theta - 1)\sec\theta\,d\theta$$

$$= \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$$

$$= \sec\theta\tan\theta - I + \ln|\sec\theta + \tan\theta| + C_1$$

$$I = \frac{1}{2}\left[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right] + C$$

On comparing, $a = 2$

15. (a)

Order of a differential equation is order of highest order derivative occurring in it.

Degree of a differential equation is degree of highest ordered derivative occurring in it when the derivatives are free from fractional power.

16. (a)

$$\int_0^{\pi/2} \sin^m \cos^n x\,dx = K\frac{(m-1)(m-3)(m-5)......(n-1)(n-3)...}{(m+n)(m+n-2)(m+n-4)...}$$

Where, $$K = \begin{cases} \pi/2 & m,n \in \text{even} \\ 1 & \text{else} \end{cases}$$

Here, m, $n \in$ even

$$\Rightarrow \quad K = \frac{\pi}{2}$$

Substituting values,

$$\int_0^{\pi/2} \sin^4 x \cos^2 x\,dx = \frac{(3)\cdot(1)\cdot(1)}{6\cdot 4\cdot 2}\cdot\frac{\pi}{2}$$

$$\Rightarrow \quad \int_0^{\pi/2} \sin^4 x \cos^2 x\,dx = \frac{3\pi}{96}$$

17. (d)

$$\frac{(\cos\theta + i\sin\theta)^7}{i^4(\cos\theta + \frac{\sin\theta}{i})^4} = \frac{(\cos\theta + i\sin\theta)^7}{(\cos\theta - i\sin\theta)^4} = \frac{(\cos\theta + i\sin\theta)^7}{[\cos(-\theta) + i\sin(-\theta)]^4}$$

$$= \frac{(\cos\theta + i\sin\theta)^7}{[\cos\theta + i\sin\theta]^{-4}} = (\cos 11\theta + i\sin 11\theta)$$

Comparing with $x + iy$, we get

$$x = \cos 11\theta$$
$$y = \sin 11\theta$$

18. (d)

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 3-\lambda & 1 \\ -2 & -\lambda \end{vmatrix} = 0$$

$$-3\lambda + \lambda^2 + 2 = 0$$
$$\lambda^2 - 3\lambda + 2 = 0$$
$$A^2 - 3A + 2 = 0$$
$$A - 3I + 2A^{-1} = 0$$

19. (c)

$$AX = \lambda X$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -6 \end{bmatrix} = (-6)\begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

$$3a - 6b = -18 \quad \text{... (i)}$$
$$3c - 6d = 36 \quad \text{... (ii)}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -3 \end{bmatrix} = (-3)\begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$3a - 3b = -9 \quad \text{... (iii)}$$

$$3c - 3d = 9 \quad \text{... (iv)}$$

From equation (i) and (ii), $a = 0$ and $b = 3$.

From equation (ii) and (iv), $c = -6$ and $d = -9$.

$$\therefore \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -6 & -9 \end{bmatrix}$$

20. (a)

$$\int_0^{\pi} \frac{dx}{c\left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right) + d\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)} = \int_0^{\pi} \frac{dx}{(c+d)\cos^2\frac{x}{2} + (c-d)\sin^2\frac{x}{2}}$$

$$= \int_0^{\pi} \frac{\sec^2\frac{x}{2}dx}{(c+d) + (c-d)\tan^2\frac{x}{2}} = \frac{1}{(c-d)}\int_0^{\pi} \frac{\sec^2\frac{x}{2}dx}{\frac{(c+d)}{(c-d)} + \tan^2\frac{x}{2}}$$

$$= \frac{2}{c-d}\sqrt{\frac{c-d}{c+d}}\left[\tan^{-1}\left\{\tan\frac{x}{2}\sqrt{\frac{c-d}{c+d}}\right\}\right]_0^{\pi}$$

$$= \frac{2}{c-d}\sqrt{\frac{c-d}{c+d}}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$

$$= \frac{2}{\sqrt{(c-d)(c+d)}} \times \frac{\pi}{2} = \frac{\pi}{\sqrt{c^2 - d^2}}$$

21. (b)

The Taylor series expansion for

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots -\infty < x < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots -\infty < x < \infty$$

$$\therefore \quad \frac{3}{2}\sin x + \cos x = 1 + \frac{3}{2}x - \frac{x^2}{2!} - \frac{3}{2}\frac{x^3}{3!} + \frac{x^4}{4!} + \frac{3}{2}\frac{x^5}{5!}\ldots\ldots\ldots$$

$$= 1 + \frac{3}{2}x - \frac{x^2}{2} - \frac{x^3}{4} + \ldots\ldots$$

22. (a)

Since

$$\sum_{x=0}^{4} P(x) = 1$$

$$c + 2c + 2c + c^2 + 5c^2 = 1$$

$$6c^2 + 5c - 1 = 0$$

$$c = \frac{1}{6}, -1$$

x	0	1	2	3	4
$P(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{36}$	$\frac{5}{36}$
$xP(x)$	0	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{3}{36}$	$\frac{20}{36}$

Since $P(x) \geq 0$, the possible value of

$$c = \frac{1}{6}$$

$$\text{Mean} = \sum_{x=0}^{4} xP(x) = 0 + \frac{2}{6} + \frac{4}{6} + \frac{3}{36} + \frac{20}{36}$$

$$= \frac{59}{36} = 1.638$$

$$\text{Variance} = \sigma^2 = E(x^2) - [E(x)]^2$$

$$= \left[0\left(\frac{1}{6}\right) + 1\left(\frac{2}{6}\right) + 4\left(\frac{2}{6}\right) + 9\left(\frac{1}{36}\right) + 16\left(\frac{5}{36}\right) - \left(\frac{59}{36}\right)^2\right]$$

$$= 1.45$$

23. (a)

Given, $\frac{dy}{dx} + 3y = 0$ and $y(1) = 4$

$$\frac{dy}{dx} = -3y$$

$$\int \frac{dy}{y} = \int -3dx$$

$\Rightarrow$ $\ln y = -3x + c$

$\Rightarrow$ $y = e^{-3x} . e^{c} = c_1 e^{-3x}$

$$y(1) = c_1 e^{-3} = 4$$

$\Rightarrow$ $c_1 = \frac{4}{e^{-3}}$

So, $y = \frac{4}{e^{-3}} e^{-3x} = 4e^{3} e^{-3x} = 80.34 e^{-3x}$

24. (b)

We know that,

$$A \,.\, \text{Adj}\,(A) = |A|\, I$$

and, $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 7 & 0 \\ 4 & 6 & 1 \end{vmatrix}$

Expanding along c_3,

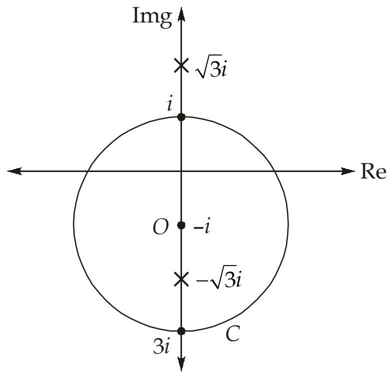
$\Rightarrow$ $|A| = 1 \times \begin{vmatrix} 3 & 7 \\ 4 & 6 \end{vmatrix} + 0 + 1 \times \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}$

$\Rightarrow$ $|A| = (18 - 28) + (7 - 6)$

$\Rightarrow$ $|A| = -9$

$\Rightarrow$ $A \,.\, \text{Adj}\,(A) = -9 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} -9 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & -9 \end{vmatrix}$

25. (d)



Singularity in $f(z) = \dfrac{z}{z^2+3}$ is

at $z = \pm\sqrt{3}i$ out of which $\sqrt{3}i$ lies outside c and $-\sqrt{3}i$ lies inside c.

Using Cauchy's Residue theorem,

$$\int_c f(z)\,dz = 2\pi i\left[\Sigma \text{ Residues of } f(z) \text{ inside } c\right]$$

Now,

$$\operatorname*{Res}_{z\to-\sqrt{3}i} = \lim_{z\to\sqrt{3}i}\left(z+\sqrt{3}i\right)\frac{z}{z^2+3}$$

$$= \lim_{z\to\sqrt{3}i}\frac{\left(z+\sqrt{3}i\right)z}{\left(z+\sqrt{3}i\right)\left(z-\sqrt{3}i\right)} = \frac{-\sqrt{3}i}{-2\sqrt{3}i} = \frac{1}{2}$$

$$\Rightarrow \quad \int_c \frac{z}{z^2+3}dz = 2\pi i\left(\frac{1}{2}\right) = \pi i$$

26. (d)

In matrix A, column-1 elements are proportional to elements of column-3 so,

Det [A] = 0

Note: In a square matrix if any two rows (column) are identical (proportional) then value of determinant of that matrix is zero.

27. (c)

A function $f(x)$ can be represented by a Fourier series if it satisfies Dirichlet's condition i.e.:

- $f(x)$ is defined, single valued and periodic with period 2π.
- $f(x)$ is piecewise continuous.
- $f(x)$ has finite number of discontinuities in any one period,

$$f(x) = \frac{1}{x+2} \text{ has infinite discontinuity at } x = -2.$$

28. (c)

$f(z)$ has two simple poles at,

$$z = 6$$

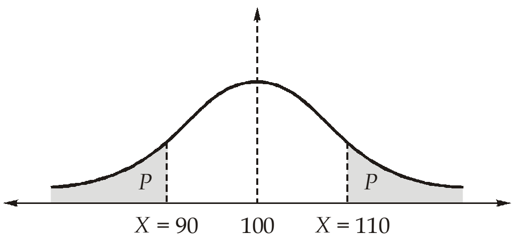
and $$z = -1$$

So, $$\underset{z=6}{\text{Res}}\, f(z) = \lim_{z\to 6}(z-6)f(z) = \frac{(6+4)(6-1)}{6+1} = \frac{50}{7}$$

$$\underset{z=-1}{\text{Res}}\, f(z) = \lim_{z\to -1}(z+1)f(z) = \frac{(-1+4)(-1-1)}{(-1-6)} = \frac{6}{7}$$

$\Rightarrow$ $$\text{sum of residues} = \frac{50}{7} + \frac{6}{7} = 8$$

29. (b)



Given, $P(90 < X < 110) = 0.4$

Also, $P(-\infty < X < \infty) = 1$

and $P(X \geq 110) = P(X \leq 90) = P$ (due to symmetry)

$\Rightarrow$ $1 = 0.4 + 2P$

$\Rightarrow$ $P = 0.3$

$\Rightarrow$ $P(X \leq 90) = P(X \geq 110) = 0.3$

$\Rightarrow$ $P(X \leq 110) = 0.7$

30. (d)

$$\frac{\partial z}{\partial x} = 2x \cdot f'(x^2 + y^2) \qquad \text{...(i)}$$

and $$\frac{\partial z}{\partial y} = 2y \cdot f'(x^2 + y^2) \qquad \text{...(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{\partial z}{\partial x}}{\frac{\partial z}{\partial y}} = \frac{2x \cdot f'(x^2 + y^2)}{2y \cdot f'(x^2 + y^2)}$$

$\Rightarrow$ $$\frac{\partial z}{\partial x} = \frac{x}{y}\frac{\partial z}{\partial y}$$

$\Rightarrow$ $$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$$

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