CLASS TEST

S.No.: 02_SK_ME_J+K_08072025

MADE EASY
Leading Institute for IES, GATE & PSUs
Delhi | Bhopal | Hyderabad | Jaipur | Pune | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

ENGINEERING MATHEMATICS

MECHANICAL ENGINEERING

Date of Test : 08/07/2025

ANSWER KEY

1.	(b)	7.	(c)	13.	(a)	19.	(a)	25.	(b)
2.	(c)	8.	(c)	14.	(c)	20.	(b)	26.	(c)
3.	(b)	9.	(b)	15.	(b)	21.	(c)	27.	(b)
4.	(d)	10.	(a)	16.	(c)	22.	(b)	28.	(a)
5.	(d)	11.	(a)	17.	(b)	23.	(b)	29.	(b)
6.	(a)	12.	(a)	18.	(d)	24.	(a)	30.	(c)

DETAILED EXPLANATIONS

1. (b)

To find the rank of a matrix, we simply transform the matrix to its row Echelon form and count the number of non-zero rows.

$$\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$$

$$R_2 \to R_2 - \frac{1}{2}R_1 \qquad R_3 \to R_3 - \frac{5}{2}R_1$$

$$\begin{bmatrix} 2 & -1 & 3 & 1 \\ 0 & 4.5 & -3.5 & 0.5 \\ 0 & 4.5 & -3.5 & 0.5 \end{bmatrix}$$

$$R_3 \to R_3 - R_2$$

$$\begin{bmatrix} 2 & -1 & 3 & 1 \\ 0 & 4.5 & -3.5 & 0.5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix is in row Echelon form,

Number of non zero rows = 2

So, Rank = 2

2. (c)

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 5 & 6 \\ 0 & 0 & 4 \end{bmatrix}$$

Characteristic equation = $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 0 \\ 0 & 5-\lambda & 6 \\ 0 & 0 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(5-\lambda)(4-\lambda) = 0$$

$$20 + 10\lambda^2 - 29\lambda - \lambda^3 = 0$$

Cayley-Hamilton theorem states that a square matrix satisfies its own characteristic equation.

So, multiply by A^{-1}

$$20 + 10A^2 - 29A - A^3 = 0$$

$$20A^{-1} + 10A - 29I - A^2 = 0$$

$$A^{-1} = \frac{1}{20}\left[A^2 - 10A + 29I\right]$$

3. (b)

Given: $\oint_C \frac{z^2+1}{z(2z-1)}dz$ is not analytic at the points $z = 0$ and $z = \frac{1}{2}$ both of which lie inside C.

$$\oint_C \frac{z^2+1}{z(2z-1)} = \oint_C \frac{z^2+1}{\left(z-\frac{1}{2}\right)}dz - \oint_C \frac{z^2+1}{z}dz$$

Using Cauchy integral formula:

$$\oint_C \frac{z^2+1}{\left(z-\frac{1}{2}\right)}dz = 2\pi i\left[z^2+1\right]_{z=\frac{1}{2}} = \frac{5\pi i}{2}$$

$$\oint_C \frac{z^2+1}{z} = 2\pi i\left[z^2+1\right]_{z=0} = 2\pi i$$

$$\oint_C \frac{z^2+1}{z(2z-1)} = \frac{5\pi i}{2} - 2\pi i = \frac{\pi i}{2}$$

4. (d)

Equation of line is

$$\frac{y}{-1} + \frac{x}{1} = 1$$

$$y = x - 1 \qquad [x = t,\ y = t - 1]$$

$$z = x + iy$$

$$= t + i(t - 1)$$

$$dz = (1 + i)dt$$

Line vary from $t = 0$ $(x = 0)$ to $t = 1$ $(x = 1)$

$$I = \int_C \left(x^3 + ixy\right)dz = \int_{t=0}^{t=1} \left[t^3 + it\cdot(t-1)\right](1+i)dt$$

$$= \int_{t=0}^{t=1} t^3 + i\left(t^2 - t\right) + it^3 - \left(t^2 - t\right)dt$$

$$= \int_{t=0}^{t=1} \left(t^3 - t^2 + t\right) + i\left(t^3 + t^2 - t\right)dt$$

$$= \left[\frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2}\right]_0^1 + i\left[\frac{t^4}{4} + \frac{t^3}{3} - \frac{t^2}{2}\right]_0^1$$

$$= \frac{5}{12} + i\frac{1}{12}$$

5. (d)

We know, $$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$$

$$u = \int \left(3x^2 - 3y^2\right) dx \Big|_{y=C}$$

$$u = x^3 - 3xy^2 + C$$

Alternative:

According to Cauchy-Riemann equation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$f(z) = u + iv$$

$$\frac{\partial f(z)}{\partial x} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$\frac{\partial f(z)}{\partial x} = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$$

$$\frac{\partial f(z)}{\partial x} = (3x^2 - 3y^2) + i6xy$$

Now, put $y = 0$, $x = z$

$$\frac{\partial f(z)}{\partial z} = 3z^2$$

$$\int \partial f(z) = \int 3z^2$$

$$f(z) = 3\frac{z^3}{3} + C$$

$$f(z) = z^3 + C$$

Put $z = x + iy$

$$f(z) = (x + iy)^3 + C$$

$$= x^3 - iy^3 + i3x^2y - 3xy^2 + C$$

$$= \left(x^3 - 3xy^2\right) + i\left(3x^2y - y^3\right) + C$$

Hence, $u = x^3 - 3xy^2 + C$

6. (a)

$$f_1(z) = z^3$$

If $z = x + iy$

$$z^2 = (x + iy)^2 = x^2 - y^2 + 2ixy$$

$$z^3 = (x^2 - y^2 + 2ixy)(x + iy)$$

$$= (x^3 - 3xy^2) + (3x^2y - y^3)i$$

$$u = x^3 - 3xy^2$$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$$

$$\frac{\partial u}{\partial y} = -6\,xy$$

$$v = 3x^2y - y^3$$

$$\frac{\partial v}{\partial y} = 3x^2 - 3y^2$$

$$\frac{\partial v}{\partial x} = 6\,xy$$

$\because \quad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

and $\quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$\therefore f_1(z) = z^3$ is analytic for all z-values

Now, $\quad f_2(z) = \log z = \log(x + iy)$

$$= \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\frac{y}{x}$$

$$u = \frac{1}{2}\log(x^2 + y^2)$$

$$v = \tan^{-1}\frac{y}{x}$$

$$\frac{\partial u}{\partial x} = \frac{x}{x^2 + y^2} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = \frac{y}{x^2 + y^2} = \frac{-\partial v}{\partial x}$$

$\because$ C–R equation are satisfied but the partial derivatives are not continuous at (0, 0)

$\Rightarrow f_2(z)$ is analytic everywhere except $z = 0$

$\Rightarrow$ Option (a) is correct.

7. **(c)**

Comparing the given equation with general form of second order partial differential equation

$$\frac{A\partial^2 P}{\partial x^2} + \frac{B\partial^2 P}{\partial y \partial x} + \frac{C\partial^2 P}{\partial y^2} + \frac{D\partial P}{\partial x} + \frac{E\partial P}{\partial y} + FP = g(x, y)$$

$$A = 1$$

$$B = 3$$

$$C = 1$$

$\Rightarrow \quad B^2 - 4AC = 5 > 0$

$\therefore$ PDE is hyperbolic.

8. **(c)**

$$\frac{dy}{dx} = 0.75y^2 \qquad (y = 1 \text{ at } x = 0)$$

Iterative equation by backward (implicit) Euler's method for above equation would be

$$y_{k+1} = y_k + h_f(x_{k+1}, y_{k+1})$$

$$y_{k+1} = y_k + h \times 0.75\, y^2_{k+1}$$

$\Rightarrow 0.75\, hy^2_{k+1} - y_{k+1} + y_k = 0$

Putting $\quad k = 0$ in above equation

$$0.75hy_1^2 - y_1 + y_0 = 0$$

Since $\quad y_0 = 1$ and $h = 1$

$$0.75y_1^2 - y_1 + 1 = 0$$

$\Rightarrow \quad y_1 = \frac{1 \pm \sqrt{1^2 - 3}}{2 \times 0.75} = \frac{2}{3}(1 \pm i\sqrt{2})$

9. (b)

$$A^{\theta} = \left(\overline{A^T}\right) = \begin{bmatrix} -2i & 3 \\ 3+i & 2-i \end{bmatrix}$$

10. (a)

$$\frac{\partial u}{\partial x} = \frac{\partial (e^x \cos y)}{\partial x} = e^x \cos y$$

11. (a)

We have, Modal matrix $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ and the spectral

$$\text{Matrix } D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

We find that, $P^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

Therefore, $A = \text{PDP}^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ 3 & 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

12. (a)

Substituting, $y = e^{mx}$, we obtain the characteristic equation as

$$4m^2 - 8m + 3 = 0$$

$$m = \frac{1}{2}, \frac{3}{2}$$

Hence, the linearly independent solutions are $e^{x/2}$ and $e^{3x/2}$. The general solution is

$$y(x) = Ae^{3x/2} + Be^{x/2}$$

Substituting the initial conditions, we get

$$y(0) = 1 = \text{A} + \text{B}$$

$$y'(0) = 3 = \frac{3A}{2} + \frac{B}{2}$$

Solving the above equations, we get

$$A = \frac{5}{2} \text{ and } B = -\frac{3}{2}$$

The solution of the differential equation is

$$y(x) = \frac{\left[5e^{3x/2} - 3e^{x/2}\right]}{2}$$

13. (a)

We have, $$\nabla \cdot \left(\frac{\vec{r}}{r^3}\right) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\frac{x}{r^3}\hat{i} + \frac{y}{r^3}\hat{j} + \frac{z}{r^3}\hat{k}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r^3}\right) + \frac{\partial}{\partial z}\frac{z}{r^3}$$

$$= \frac{3}{r^3} - \frac{3}{r^4}\left(x\frac{\partial r}{\partial x} + y\frac{\partial r}{\partial y} + z\frac{\partial r}{\partial z}\right)$$

Since, $r^2 = x^2 + y^2 + z^2$, we obtain

$$\frac{\partial r}{\partial x} = \frac{x}{r}, \frac{\partial r}{\partial y} = \frac{y}{r} \text{ and } \frac{\partial r}{\partial z} = \frac{z}{r}$$

Therefore, $$\nabla \cdot \left(\frac{\vec{r}}{r^3}\right) = \frac{3}{r^3} - \frac{3}{r^5}\left(x^2 + y^2 + z^2\right)$$

$$= 0$$

14. (c)

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

Characteristic equation $= |A - \lambda I| = 0$

$$\begin{bmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{bmatrix} = 0$$

$\Rightarrow$ $(1 - \lambda)(2 - \lambda) - 3 \times 4 = 0$

$\Rightarrow$ $2 - \lambda - 2\lambda + \lambda^2 - 12 = 0$

$\Rightarrow$ $\lambda^2 - 3\lambda - 10 = 0$

$\Rightarrow$ $\lambda = -2, 5 \ (\lambda_1 = -2, \lambda_2 = 5)$

$|A| = 2 \times 1 - 3 \times 4 = -10$

Eigen value of B $\equiv$ (i) $\frac{|A|}{\lambda_1} = \frac{-10}{-2} = 5 = \lambda_1'$

(ii) $\frac{|A|}{\lambda_2} = \frac{-10}{5} = -2 = \lambda_2'$

Now, eigen value of C is

For $\lambda_1' = 5$ $\quad \lambda_1'' = (5)^3 - 4 \times (5)^2 + 5 + 6 = 36$

$\lambda_2' = -2$ $\quad \lambda_2'' = (-2)^3 - 4 \times (-2)^2 - 2 + 6 = -20$

Determinant of C = Product of eigen value of C

$|C| = 36 \times -20 = -720$

15. (b)

p = The probability of hitting a target = $\frac{1}{3}$

q = The probability of not hitting a target

$$= 1-\frac{1}{3}=\frac{2}{3}$$

n = Number of trails = 5

$$\text{P(atleast twice)} = P(x \geq 2)$$
$$= 1 - P(x < 2)$$
$$= 1 - [P(x = 0) + P(x = 1)]$$
$$= 1-\left[{}^5C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^5+{}^5C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^4\right]$$
$$= 1-\left[\left(\frac{2}{3}\right)^5+\frac{5}{3}\left(\frac{2}{3}\right)^4\right]$$
$$= 1-\left(\frac{2}{3}\right)^4\left[\frac{2}{3}+\frac{5}{3}\right]=1-\frac{7}{3}\left(\frac{2}{3}\right)^4 = 0.5391$$

16. (c)

$y^2 = x$, $x^2 = y$ and $z = 12 + y - x^2$

$$V = \int_0^1 dx\int_{x^2}^{\sqrt{x}} dy \int_0^{12+y-x^2} dz = \int_0^1 dx\int_{x^2}^{\sqrt{x}}\left(12+y-x^2\right)dy$$
$$= \int_0^1 dx\left(12y+\frac{y^2}{2}-x^2y\right)_{x^2}^{\sqrt{x}}$$
$$= \left[\frac{2}{3}\times 12x^{3/2}+\frac{x^2}{4}-\frac{2}{7}x^{7/2}-4x^3-\frac{x^5}{10}+\frac{x^5}{5}\right]_0^1$$
$$= 8+\frac{1}{4}-\frac{2}{7}-4-\frac{1}{10}+\frac{1}{5}$$
$$= 4+\frac{1}{4}-\frac{2}{7}-\frac{1}{10}+\frac{1}{5}$$
$$= \frac{560+35-40-14+28}{140}=\frac{569}{140}=4.06$$

17. (b)

$$A = \begin{bmatrix} a & -2 \\ 2.5 & b \end{bmatrix}$$

$$\det(A) = |A| = \begin{bmatrix} a & -2 \\ 2.5 & b \end{bmatrix} = 30$$

$\Rightarrow$ $ab + 2 \times 2.5 = 30$

$\Rightarrow$ $ab = 25$ ... (i)

trace(A) = Sum of diagonal element

$= a + b$

We know, AM $\geq$ GM

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$\frac{a+b}{2} \geq \sqrt{25}$$

$$\frac{a+b}{2} \geq 5$$

$\Rightarrow$ $a + b \geq 10$

Hence, minimum value of trace (A) = 0

18. (d)

Examine the continuity of $f(x)$ at $x = 0$

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} f\left[(0+h)^2 + 1\right] = 1$$

$$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} f\left[(0-h) - 1\right] = -1$$

As $\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$, $f(x)$ is not continuous at $x = 0$.

If the function is not continuous then you can conclude that the function is also non-differentiable.

19. (a)

As $f(x)$ is a polynomial function and every polynomial function is continuous and differentiable every where,

$$f'(x) = (1 - x)^2 + x \times 2(1 - x)(-1)$$
$$= (1 - x)(1 - x - 2x)$$
$$= (1 - x)(1 - 3x)$$

At critical points $f'(x) = 0$

$$(1 - x)(1 - 3x) = 0$$

$$x = 1, \frac{1}{3}$$

Now, $f(0) = 0$

$$f\left(\frac{1}{3}\right) = \frac{1}{3}\left(1 - \frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{4}{9} = \frac{4}{27}$$

$$f(1) = 0$$
$$f(2) = 2 \times (1 - 2)^2$$
$$= 2$$

$\therefore$ The maximum value of $f(x) = 2$

and the minimum value of $f(x) = 0$.

20. **(b)**

There is only one 3rd order minor. For rank 2 determinant of matrix A must be zero.

Now, $$\begin{vmatrix} 3+x & 5 & 2 \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix} = 0 \qquad \text{... (i)}$$

Using, $R_1 \rightarrow R_1 - R_3$

$$\begin{vmatrix} 1+x & 0 & -1-x \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix} = 0$$

Using, $C_3 \rightarrow C_3 + C_1$

$$\begin{vmatrix} 1+x & 0 & 0 \\ 1 & 7+x & 7 \\ 2 & 5 & 5+x \end{vmatrix} = 0$$

$$\Rightarrow \quad (1+x)\begin{vmatrix} 7+x & 7 \\ 5 & 5+x \end{vmatrix} = 0$$

$\Rightarrow (1 + x)\,[(7 + x)\,(5 + x) - 35] = 0$

$\Rightarrow \quad (1 + x)\,(x^2 + 12x) = 0$

$\Rightarrow \quad x(1 + x)\,(x + 12) = 0$

$\therefore$ Equation (i) holds for $x = 0, -1, -12$

when, $x = 0$, the matrix A $= \begin{bmatrix} 3 & 5 & 2 \\ 1 & 7 & 6 \\ 2 & 5 & 3 \end{bmatrix}$

Clearly, a minor $\begin{vmatrix} 3 & 5 \\ 1 & 7 \end{vmatrix} \neq 0$. So the rank = 2.

when, $x = -1$, the matrix $= \begin{bmatrix} 2 & 5 & 2 \\ 1 & 6 & 6 \\ 2 & 5 & 2 \end{bmatrix}$

Clearly, a minor $\begin{vmatrix} 2 & 5 \\ 1 & 6 \end{vmatrix} \neq 0$. So the rank = 2.

when, $x = -12$, the matrix $= \begin{bmatrix} -9 & 5 & 2 \\ 1 & -5 & 6 \\ 2 & 5 & -9 \end{bmatrix}$

Clearly, minor $\begin{vmatrix} -9 & 5 \\ 1 & -5 \end{vmatrix} \neq 0$, so the rank = 2.

$\therefore$ the matrix has the rank 2, if $x = 0, -1, -12$

21. (c)

Let A, B and C produce $2n$, n and n products

Let E_1 = Event of a product being produced at A.

E_2 = Event of a product being produced at B.

E_3 = Event of a product being produced at C.

$$\therefore \quad P(E_1) = \frac{2n}{2n+n+n} = \frac{1}{2}$$

$$P(E_2) = \frac{n}{2n+n+n} = \frac{1}{4}$$

$$P(E_3) = \frac{1}{4}$$

Also, $P\left(\frac{D}{E_1}\right)$ = Probability of a product being defective when produced at A

$$= \frac{2}{100} = \frac{1}{50}$$

Similarly, $$P\left(\frac{D}{E_2}\right) = \frac{4}{100} = \frac{1}{25}$$

$$P\left(\frac{D}{E_3}\right) = \frac{2}{100} = \frac{1}{50}$$

As, $P(E_1) + P(E_2) + P(E_3) = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$

The probability that the product is defective.

$$= P(E_1) \times P\left(\frac{D}{E_1}\right) + P(E_2) \times P\left(\frac{D}{E_2}\right) + P(E_3) \times P\left(\frac{D}{E_3}\right)$$

$$= \frac{1}{2} \times \frac{1}{50} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{50}$$

$$= \frac{1}{100} + \frac{1}{100} + \frac{1}{200} = \frac{1}{40}$$

22. (b)

Here, $$x^2 - 2y - 1 = 2xy\frac{dx}{dy}$$

or $$2xy\frac{dx}{dy} - x^2 = -(1 + 2y)$$

or, $$2x\frac{dx}{dy} - \frac{x^2}{y} = -\left(2 + \frac{1}{y}\right)$$

Put $x^2 = z$

then, $$2x\frac{dx}{dy} = \frac{dz}{dy}$$

$\therefore$ the equation becomes

$$\frac{dz}{dy} - \frac{1}{y} \times z = -\left(2 + \frac{1}{y}\right)$$

$\therefore$ Integrating factor $= e^{\int -\frac{1}{y}dy} = e^{-\log y} = e^{\log\frac{1}{y}} = \frac{1}{y}$

Multiplying by $\frac{1}{y}$

$$\frac{1}{y}\frac{dz}{dy} - \frac{1}{y^2}z = -\frac{1}{y}\left(2+\frac{1}{y}\right)$$

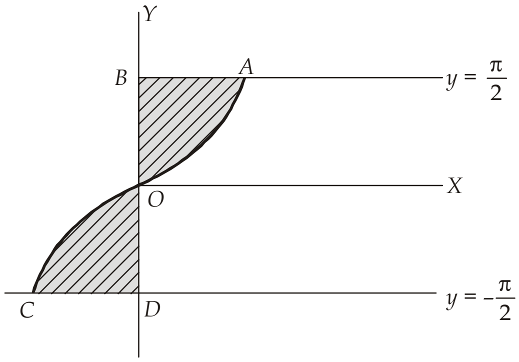
$$\frac{d}{dy}\left(z\frac{1}{y}\right) = -\left(\frac{2}{y}+\frac{1}{y^2}\right)$$

$$\frac{z}{y} = -\int\left(\frac{2}{y}+\frac{1}{y^2}\right)dy$$

$$= -\left(2\log y - \frac{1}{y}\right) + C$$

$$\therefore \quad \frac{x^2}{y} + 2\log y - \frac{1}{y} = C$$

23. (b)



The curve is $y = \sin^{-1}x$

$x = \sin y$

Shaded part is the required area. By symmetry of the curve and the lines.

Area (OABO) = Area (OCDO)

Required area = 2 × Area (OABO)

$$= 2\int_0^{\pi/2} \sin y\,dy = 2\left[-\cos y\right]_0^{\pi/2} = 2$$

24. (a)

Put $\cos\theta = t$

$-\sin\theta d\theta = dt$

$$\int_0^{\pi/2} \sqrt{\cos\theta}\sin^3\theta\, d\theta = \int_0^{\pi/2} \sqrt{\cos\theta}\left(1-\cos^2\theta\right)\sin\theta d\theta$$

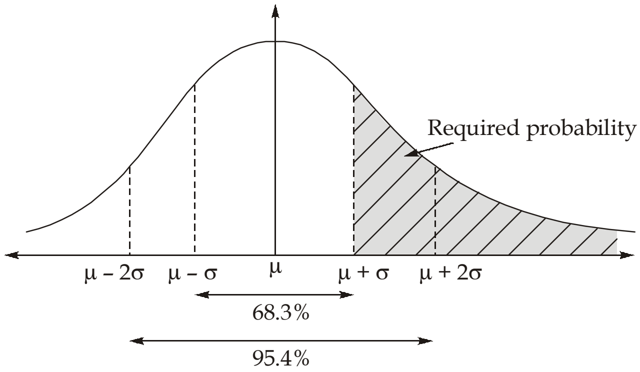
$$= -\int_1^0 \sqrt{t}\left(1-t^2\right)dt = \int_0^1\left(t^{1/2}-t^{5/2}\right)dt$$

$$= \left[\frac{2}{3}t^{3/2} - \frac{2}{7}t^{7/2}\right]_0^1 = \frac{8}{21} = 0.38$$

25. **(b)**

$$\text{Mean, } \mu = 60$$
$$\text{Variance, } \sigma^2 = 25$$
$$\sigma = 5$$

Normal distributed curve

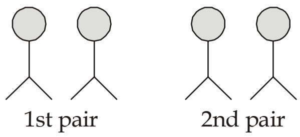


% of students who scored more than 65 marks ($\mu + \sigma$)

$$= 0.5 - \frac{0.683}{2} = 0.1585 = 15.85\%$$

26. **(c)**

n person can arrange in round by $(n - 1)!$ way



Consider them as one unit,

$$\text{Total person} = n - 1 - 1 = (n - 2)$$
$$\text{Total arrangement} = (n - 2 - 1)!$$

Both pair arrange itself by 2! way

Probability that two pair of two specified persons do not sit together

$$= 1 - (\text{sit together})$$
$$= 1 - \frac{2 \times 2(n-2-1)!}{(n-1)!} = 1 - \frac{4(n-3)!}{(n-1)(n-2)(n-3)!}$$
$$= 1 - \frac{4}{(n-1)(n-2)} = \frac{n^2 - 3n + 2 - 4}{n^2 - 3n + 2} = \frac{n^2 - 3n - 2}{n^2 - 3n + 2}$$

27. **(b)**

$$E(x) = 3 \times \frac{1}{2} + 6 \times \frac{3}{10} + 8 \times \frac{1}{5} = 4.9$$
$$E(x^2) = 3^2 \times \frac{1}{2} + 6^2 \times \frac{3}{10} + 8^2 \times \frac{1}{5} = 28.1$$
$$E(2x + 6)^2 = E(4x^2 + 24x + 36)$$
$$= 4E(x^2) + 24E(x) + 36$$
$$= 4 \times 28.1 + 24 \times 4.9 + 36 = 266$$

28. (a)

$(6x^2y + y^2)dx + (4x^3 + 3xy)dy = 0$

Let, $M = 6x^2y + y^2 \qquad N = 4x^3 + 3xy$

$$\frac{\partial M}{\partial y} = 6x^2 + 2y^2 \qquad \frac{\partial N}{\partial x} = 12x^2 + 3y$$

$\because \quad \dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$, equation is not exact

$$f(y) = \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{12x^2 + 3y - 6x^2 - 2y}{6x^2y + y^2}$$

$$= \frac{6x^2 + y}{y(6x^2 + y)} = \frac{1}{y}$$

$$\text{IF} = e^{\int f(y)dy} = e^{\int \frac{1}{y}dy} = e^{\ln y} = y$$

Multiplying throughout by 'y' the equation become

$(6x^2y^2 + y^3)dx + (4x^3y + 3xy^2)dy = 0$

Which is exact.

$\therefore$ The solution is

$$\int_{y\,cont.} Mdx + \int(\text{Terms of N not containing } x)dy = C$$

$$6\frac{x^3}{3}y^2 + xy^3 + 0 = C$$

$$2x^3y^2 + xy^3 = C$$

29. (b)

$$f(z) = (z + 4)\log(z + 2)$$

Let, $z + 1 \rightarrow t$

$$f(t) = (t + 3)\log(t + 1)$$

We know, $\log(1 + t) = t - \dfrac{t^2}{2} + \dfrac{t^3}{3} - \dfrac{t^4}{4}$............

$$f(t) = (t+3)\left[t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4}\text{............}\right]$$

$$= t^2 - \frac{t^3}{2} + \frac{t^4}{3} - \frac{t^5}{4}\text{............} + 3t - \frac{3t^2}{2} + \frac{3t^3}{3} - \frac{3t^4}{4}\text{.......}$$

Put, $t = z + 1$

$$f(z) = (z+1)^2 - \frac{(z+1)^3}{2} + \frac{(z+1)^4}{3}\text{..........} + 3(z+1) - \frac{3}{2}(z+1)^2 + (z+1)^3\text{....}$$

So,Coefficient of $(z + 1)^3$ is $1 - \dfrac{1}{2} = 0.5$

30. (c)

$$\tan^{-1}\frac{x^2y+xy^2}{3\sqrt{x}+3\sqrt{y}} = u$$

$$\frac{x^2y+xy^2}{3\sqrt{x}+3\sqrt{y}} = \tan u = f(x, y)$$

$$f(x, y) = \frac{x^3\left[\frac{y}{x}+\left(\frac{y}{x}\right)^2\right]}{3x^{1/2}\left(1+\sqrt{\frac{y}{x}}\right)} = \frac{1}{3}x^{5/2}\frac{\left[\left(\frac{y}{x}\right)^2+\frac{y}{x}\right]}{\left(1+\sqrt{\frac{y}{x}}\right)}$$

$\therefore$ $f(x, y)$ expressed in the form xn $x^n\phi\left(\frac{y}{x}\right)$ which is called homogeneous function, where n is degree $\left(n=\frac{5}{2}\right)$.

By Euler's theorem,

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = n\frac{f(x,y)}{f'(x,y)}$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \frac{5}{2}\frac{\tan u}{\sec^2 u} = \frac{5}{2}\times\frac{\sin u\cos^2 u}{\cos u\cdot 1}$$

$$= \frac{5}{2}\times\frac{1}{2}\times 2\sin u\cos u$$

$$= \frac{5}{4}\sin 2u$$

■■■■