CLASS TEST

S.No.: 01SKME_ABCDEFGHI_050725

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STRENGTH OF MATERIAL

MECHANICAL ENGINEERING

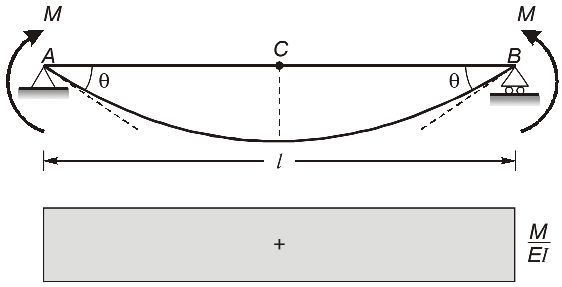
Date of Test : 05/07/2025

ANSWER KEY

1.	(a)	7.	(c)	13.	(a)	19.	(a)	25.	(b)
2.	(c)	8.	(a)	14.	(b)	20.	(d)	26.	(c)
3.	(c)	9.	(a)	15.	(b)	21.	(b)	27.	(a)
4.	(c)	10.	(b)	16.	(c)	22.	(a)	28.	(d)
5.	(b)	11.	(d)	17.	(a)	23.	(b)	29.	(a)
6.	(a)	12.	(d)	18.	(c)	24.	(b)	30.	(b)

DETAILED EXPLANATIONS

1. (a)



$$\theta_C - \theta_A = \text{Area of } \frac{M}{EI} \text{ diagram}$$

$$0 - \theta_A = +\frac{M}{EI}\times\frac{l}{2} = \frac{Ml}{2EI}$$

$$\theta_A = -\frac{Ml}{2EI}$$

$$\theta_A = \frac{Ml}{2EI} \text{ (Anticlockwise)}$$

$$\therefore \quad \frac{M}{EI} = 2\theta_A = 2\theta$$

2. (c)

$$\text{Dilation, } \varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \frac{1-2\mu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

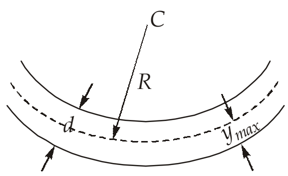
For uniaxial loading, $\sigma_y = \sigma_z = 0$

So,
$$\varepsilon_v = \frac{1-2\times0.3}{200\times10^3}\left(\frac{30\times10^3}{\frac{\pi}{4}\times20^2}\right) = 1.9098\times10^{-4}$$

3. (c)

Diameter of wire, d = 20 mm

So,
$$y_{max} = \frac{d}{2} = \frac{20}{2} = 10 \text{ mm}$$



Radius of curvature, R = 10 m

Bending equation

$$\frac{\sigma_{max}}{y_{max}} = \frac{E}{R}$$

$$\Rightarrow \quad \frac{\sigma_{max}}{10\times10^{-3}} = \frac{200\times10^3}{10}$$

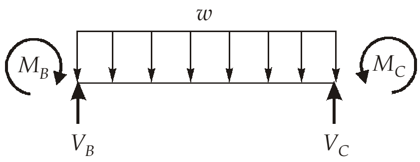
$$\Rightarrow \quad \sigma_{max} = 200 \text{ MPa}$$

4. (c)

For sections *AB* and *CD*, the beam may be modeled as

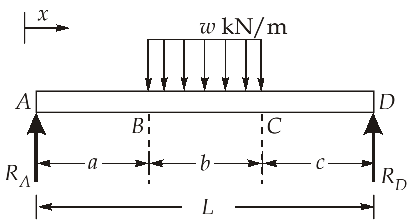
$M(x)$ is linear with respect to x.

For section *BC*, the beam is modeled as



$M(x)$ is parabolic, reaching a maximum near or at the center.

Alternate solution:



$$\Sigma M_A = 0;\ R_D \times L = wb\left(a + \frac{b}{2}\right)$$

$$R_D = \frac{wb}{L}\left(a + \frac{b}{2}\right)$$

$$\Sigma F_V = 0;\ R_A + R_D = w \cdot b$$

$$R_A = \frac{wb}{L}\left(c + \frac{b}{2}\right)$$

Now, for $0 \le x \le a$

$$M_x = \frac{wb}{L}\left(c + \frac{b}{2}\right)x$$

For $a \le x \le a + b$ $$M_x = \frac{wx}{2l}(a^2 + L^2 - c^2) - \frac{wx^2}{2} - \frac{wa^2}{2}$$

For $a + b \le x \le L$ $$M_x = wb\left(a + \frac{b}{2}\right)x - \frac{wb}{L}\left(a + \frac{b}{2}\right)x$$

5. (b)

For a thin-walled cylinder of diameter d and internal pressure p,

$$\sigma_{long} \le (0.2)\sigma_{yield}$$

$$\frac{Pd}{4t} \le (0.2)\sigma_{yield}$$

$$t \ge \frac{pd}{0.8\sigma_{yield}} = \frac{\left(7\times10^6\,\frac{N}{m^2}\right)(1\,m)}{(0.8)\left(200\times10^6\,\frac{N}{m^2}\right)}$$

$$\ge 0.04375 \text{ m} = 43.75 \text{ mm} \approx 44 \text{ mm}$$

6. (a)

$$\Delta L_s = \Delta L_A$$

$$\Rightarrow \quad \left(\frac{PL}{AE}\right)_S = \left(\frac{PL}{AE}\right)_A$$

$$\frac{P_s}{P_A} = \frac{A_S E_S / L_S}{A_A E_A / L_A} = \frac{0.5\times200/2}{2\times100/1} = 0.25$$

7. (c)

$$d_1 = 50 \text{ mm}; G = 27 \text{ GPa}$$

$$T = 4.0 \text{ kNm}, \varepsilon_{allow} = 900\times10^{-6}$$

Allowable shear stress based on normal strain,

$$\varepsilon_{max} = \frac{\gamma}{2} = \frac{\tau}{2G}$$

$$\tau = 2G\varepsilon_{max}$$

$$(\tau_{allow}) = 2G\varepsilon_{allow} = 2(27\times10^{-3})(900\times10^{-6})$$

$$= 48.6 \text{ MPa}$$

8. (a)

Rankine's buckling load, $P_R = 4000$ N

Crushing load, $P_C = 5000$ N

As we know, $$\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_E}$$

$$\frac{1}{4000} = \frac{1}{5000} + \frac{1}{P_E}$$

$$P_E = \frac{1}{\frac{1}{4000} - \frac{1}{5000}} = 20000 \text{ N}$$

$$P_E = 20 \text{ kN}$$

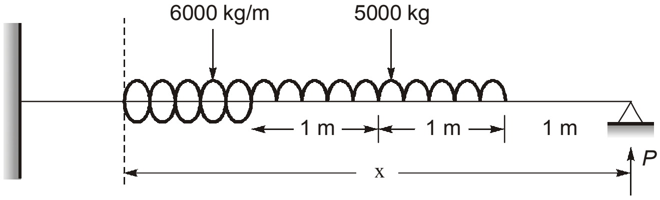
9. (a)

10. (b)

High carbon spring steel has maximum value of yield point strength.

11. **(d)**

Writing Macaulay's theorem at a distance 'x' from the right hand support



$$EI\frac{d^2y}{dx^2} = M$$

$$EI\frac{d^2y}{dx^2} = P[x]-5000[x-2]-\frac{6000}{2}[x-1]^2+\frac{6000}{2}[x-3]^2 \quad ...(1)$$

$$EI\frac{dy}{dx} = \frac{P}{2}[x]^2-2500[x-2]^2-1000[x-1]^3+1000[x-3]^3+A \quad ...(2)$$

$$EIY = \frac{P}{6}[x]^3-\frac{2500}{3}[x-2]^3-250[x-1]^4+250[x-3]^4+Ax+B \quad ...(3)$$

At $x = 4$ m; $\frac{dy}{dx} = 0 \Rightarrow 8P - 36000 + A = 0$ ...(4)

At $x = 4$ m; $y = 0 \Rightarrow \frac{32}{3}P-\frac{80000}{3}+4A=0$ ...(5)

From equation (4), $\frac{32}{3}P-\frac{80000}{3}+4(36000-8P)=0$

$\Rightarrow$ $P = 5500$ kg

12. **(d)**

Bending moment at section x-x:

$$M_x = Fr\sin\theta$$

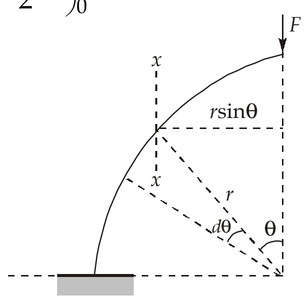
Bending strain energy,

$$U = \int_0^{\pi/2}\frac{M_x^2}{2EI}dS \qquad (dS = rd\theta)$$

$$= \int_0^{\pi/2}\frac{F^2r^2\sin^2\theta rd\theta}{2E\frac{\pi}{64}d^4} = \frac{32F^2r^3}{\pi d^4E}\int_0^{\pi/2}\sin^2\theta d\theta$$

$$= \frac{16F^2r^3}{\pi d^4E}\int_0^{\pi/2}(1-\cos 2\theta)d\theta = \frac{16F^2r^3}{\pi d^4E}\left(\theta-\frac{\sin 2\theta}{2}\right)_0^{\pi/2}$$

$$U = \frac{8F^2r^3}{d^4E}$$



Strain energy stored = Work done by external load

$$\frac{1}{2}F\Delta = \frac{8F^2r^3}{d^4E}$$

$$\text{Deflection, } \Delta = \frac{16Fr^3}{d^4E}$$

13. (a)

Given, a biaxial stress system,

$$\sigma_x = 100 \text{ MPa}, \sigma_y = 60 \text{ MPa}$$

For maximum obliquity of the resultant with the normal to a plane is given by

$$\tan\theta = \sqrt{\frac{\sigma_x}{\sigma_y}} = \sqrt{\frac{100}{60}} = 1.29$$

or $\theta = 52.24°$

Direct stress, $\sigma_\theta = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta$

$= 100\cos^2 52.24° + 60\sin^2 52.24°$

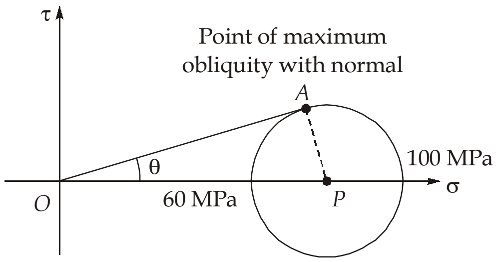
$= 100 \times 0.375 + 60 \times 0.625 = 37.5 + 37.5 = 75$ MPa

Shear stress, $\tau_\theta = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$

$= -\frac{1}{2}(100-60)\sin 104.48° = -19.365$ MPa

Resultant stress, $\sigma_r = \sqrt{\sigma_\theta^2 + \tau_\theta^2} = \sqrt{75^2 + 19.365^2} = 77.46$ MPa

Alternate Solution:



$\sigma_1 = \sigma_x = 100$ MPa

$\sigma_2 = \sigma_y = 60$ MPa

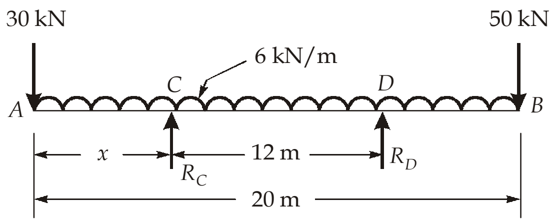
$$AP = \tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{100-60}{2} = 20 \text{ MPa}$$

$$OP = \frac{\sigma_1 + \sigma_2}{2} = \frac{100+60}{2} = 80 \text{ MPa}$$

$$\sigma_\sigma = \sqrt{OP^2 - AP^2} = \sqrt{80^2 - 20^2} = 77.46 \text{ MPa}$$

14. (b)

Let the left support C be at a distance x meters from A.



Now, $R_C = R_D$ (Given)

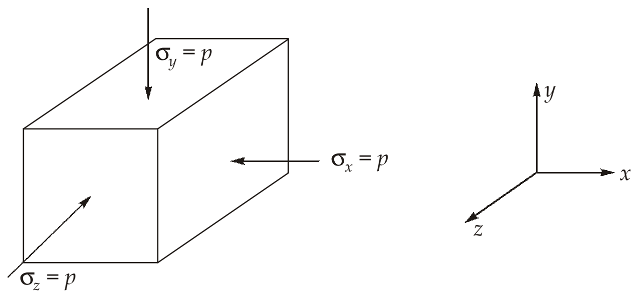
$\Sigma V = 0$

$$R_C + R_D - 30 - 6 \times 20 - 50 = 0$$
$$\Rightarrow \quad 2R_C = 30 + 120 + 50$$
$$\Rightarrow \quad R_C = 100 \text{ kN}$$
$$\therefore \quad R_D = 100 \text{ kN}$$
$$\Sigma M_A = 0$$
$$100x + 100(12 + x) - 6 \times 20 \times 10 - 50 \times 20 = 0$$
$$200x = 1000$$
$$x = 5 \text{ m}$$

15. (b)

Under water, the solid will be subjected to hydrostatic pressure (compressive) of equal magnitude on all sides as shown in figure. In the three principal directions, the strains will be

$$\varepsilon_x = \frac{\sigma}{E}(1-2\mu) = \varepsilon_y = \varepsilon_z$$



Therefore,

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{3p}{E}(1-2\mu)$$

$$\text{Change in volume, } \varepsilon_v = \frac{0.05}{100} = \frac{3p}{E}(1-2\mu)$$

$$p = \frac{0.05}{100} \times \frac{E}{3(1-2\mu)} = \frac{0.05 \times 200{,}000}{100 \times 3(1 - 2\times 0.3)} = 83.33 \text{ N/mm}^2 \quad \ldots(1)$$

$$\text{Pressure at any depth, } p = wh = 10{,}080h \text{ N/m}^2 \quad \ldots(2)$$

From eq. (1) and (2) we get,

$$10{,}080h = 83.33 \times 10^6$$
$$\Rightarrow \quad h = 8267 \text{ m}$$

16. (c)

Impact loading

$$\delta = \text{Max. instantaneous extension} = 1.25 \text{ mm}$$
$$W = Mg = 60 \times 9.81 = 588.6 \text{ N}$$
$$\therefore \quad \sigma = E\frac{\delta}{L} = \frac{2.05\times 10^5 \times 1.25}{2.5\times 10^3} = 102.5 \text{ N/mm}^2$$

Now potential energy lost by weight = Strain energy stored in bar.

$$\therefore \quad W(h+\delta) = \frac{\sigma^2}{2E} \times V$$

$$\text{or} \quad 588.6(h + 1.25) = \frac{(102.5)^2}{2\times 2.05\times 10^5}\left[\frac{\pi}{4}(40)^2 \times 2500\right]$$

or $h + 1.25 = 136.78$

$\Rightarrow$ $h = 136.78 - 1.25 = 135.53$ mm

17. (a)

Length of the rod, l = 6000 mm

Fall of temperature, T = 120 – 40 = 80°C

When the ends yield by 1.1 mm.

$$\text{Thermal strain} = \frac{\text{Contraction prevented}}{\text{Original length}} = \frac{\alpha Tl - \delta}{l} = \frac{12\times10^{-6}\times80\times6000-1.1}{6000}$$

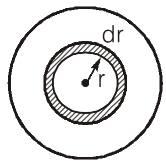
$$= \frac{4.66}{6000} = 7.7667 \times 10^{-4}$$

Thermal stress = Thermal strain × Young's modulus

$$= \frac{4.66}{6000}\times210\times10^3\ \text{N/mm}^2 = 163.1\ \text{N/mm}^2 = 163.1\ \text{MPa}$$

18. (c)

Considering a ring element of thickness dr at a distance r from the centre of the shaft.



$$T_1 = \int_0^{R/3} \tau(2\pi r dr)r = \int_0^{R/3} \frac{\tau_{max}}{R}.r(2\pi r dr)r$$

$$T_1 = \frac{2\pi\tau_{max}}{R}\int_0^{R/3} r^3 dr = \frac{2\pi\tau_{max}}{R}\left(\frac{r^4}{4}\right)_0^{R/3}$$

$$T_1 = \left(\frac{\pi\tau_{max}R^3}{2\times3^4}\right) \quad .....(i)$$

$$T = \frac{\tau_{max}}{R}\times\frac{\pi R^4}{2}$$

$$T = \left(\frac{\pi\tau_{max}R^3}{2}\right) \quad .....(ii)$$

From equations (i) and (ii), we get

$$\frac{T_1}{T} = \frac{1}{3^4} = \frac{1}{81} = 0.012$$

Alternate:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{L}$$

$\therefore$ $T \propto J$

$\therefore$ $$\frac{T_1}{T} \propto \frac{J_1}{J} \propto \left(\frac{D_1}{D}\right)^4$$

$\therefore$ $$\frac{T_1}{T} \propto \left[\left(\frac{R/3}{R}\right)\right]^4 \propto \left(\frac{1}{3}\right)^4 = 0.012$$

19. (a)

For a rectangular strain rosette

$$\varepsilon_x = \varepsilon_{0°} = 400 \times 10^{-6},\ \varepsilon_y = \varepsilon_{90°} = -100 \times 10^{-6}$$

and $\gamma_{xy} = 2\varepsilon_{45°} - \varepsilon_x - \varepsilon_y$

$$\gamma_{xy} = 2 \times 200 \times 10^{-6} - 400 \times 10^{-6} + 100 \times 10^{-6} = 100 \times 10^{-6}$$

$$\text{Principal strains, } \varepsilon_1, \varepsilon_2 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) \pm \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

$$= \frac{10^{-6}}{2}\left[(400 - 100) \pm \sqrt{(400+100)^2 + 100^2}\right]$$

$$= 404.95 \times 10^{-6} \text{ and } -104.95 \times 10^{-6}$$

Principal stresses

$$\sigma_1 = \frac{E(\mu\varepsilon_2 + \varepsilon_1)}{1-\mu^2} = \frac{210000(-0.3 \times 104.95 + 404.95) \times 10^{-6}}{1-0.3^2} = 86.2 \text{ MPa}$$

$$\sigma_2 = \frac{E(\mu\varepsilon_1 + \varepsilon_2)}{1-\mu^2} = \frac{210000(0.3 \times 404.95 - 104.95) \times 10^{-6}}{1-0.3^2} = 3.82 \text{ MPa}$$

20. (d)

Total length, L = 2000 mm

Uniform stress, σ = 0.3 MPa

Weight density, w = 7.644×10^{-5} N/mm^3

Diameter at bottom, d_1 = 60 mm

$$\text{Area at bottom, } A_1 = \frac{\pi}{4} \times d_1^2 = \frac{\pi}{4} \times 60^2 = 2827.433 \text{ mm}^2$$

As we know,

$$\text{Area at top, } A_2 = A_1 e^{wL/\sigma} = 2827.433 e^{\frac{7.644 \times 10^{-5} \times 2000}{0.3}}$$

$$A_2 = 4706.62 \text{ mm}^2$$

21. (b)

$\Sigma F_x = 0$,

$\Rightarrow \quad H_A = P$

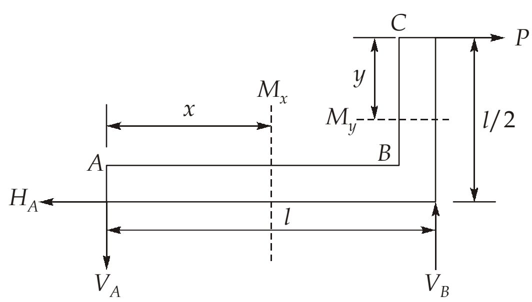
$\Sigma M_A = 0$

$$\Rightarrow \quad V_B \times l = P \times \frac{l}{2}$$

$$V_B = \frac{P}{2}$$

$\Sigma F_y = 0$

$$\Rightarrow \quad V_A = V_B = \frac{P}{2}$$



Strain energy stored by the bracket,

$$U = U_{AB} + U_{BC}$$

$$= \int_0^l \frac{M_x^2 dx}{2EI} + \int_0^{l/2} \frac{M_y^2 dy}{2EI} = \int_0^l \frac{\left(-\frac{Px}{2}\right)^2 dx}{2EI} + \int_0^{l/2} \frac{(Py)^2 dy}{2EI}$$

$$= \frac{P^2}{8EI}\left[\frac{x^3}{3}\right]_0^l + \frac{P^2}{2EI}\left[\frac{y^3}{3}\right]_0^{l/2}$$

$$= \frac{P^2}{8EI} \times \frac{l^3}{3} + \frac{P^2 (l/2)^3}{6EI} = \frac{P^2 l^3}{24EI} + \frac{P^2 l^3}{48EI}$$

$$U = \frac{P^2 l^3}{16EI}$$

Horizontal deflection at C,

$$\delta_C = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P}\left(\frac{P^2 l^3}{16EI}\right)$$

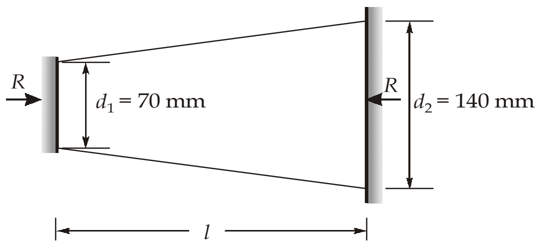
$$\delta_C = \frac{2Pl^3}{16EI}$$

$$\delta_C = \frac{Pl^3}{8EI}$$

22. (a)

Let R be the reaction developed at the ends.

Decrease in the length due to support reaction $= \dfrac{4Rl}{\pi d_1 d_2 E}$ ...(1)



Due to free expansion,

Increase in length $= \alpha t l$ ... (ii)

Since there is no change in length,

From (i) and (ii):

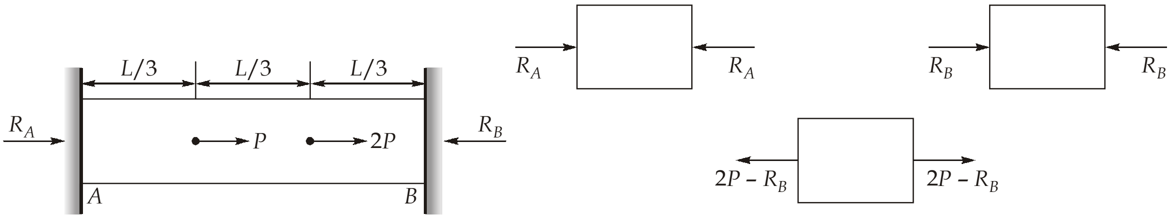
$$\frac{4Rl}{\pi d_1 d_2 E} = \alpha t l$$

$$R = \frac{\pi}{4} d_1 d_2 E \alpha t$$

$$\text{Maximum stress, } \sigma_{max} = \frac{R}{\frac{\pi}{4}d_1^2} = \frac{\frac{\pi}{4}d_1 d_2 E\alpha t}{\frac{\pi}{4}d_1^2} = \frac{d_2 E\alpha t}{d_1}$$

$$= \frac{140 \times 200 \times 10^3 \times 12 \times 10^{-6} \times 50}{70} = 240 \text{ MPa}$$

23. (b)



$$\Sigma F_x = 0$$

$$R_A + P + 2P - R_B = 0 \quad \text{... (i)}$$

$$\text{Total deformation, } \Delta = 0$$

$$\Delta = \Sigma\frac{PL}{AE} = 0$$

$$-R_A\frac{L}{3} + (2P - R_B)\frac{L}{3} - R_B\frac{L}{3} = 0$$

$$-R_A + 2P - 2R_B = 0$$

$$R_A = 2P - 2R_B \quad \text{... (ii)}$$

From (i) and (ii)

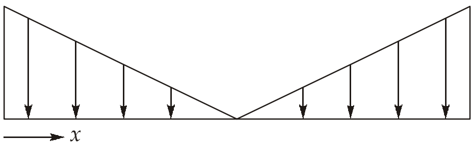
$$R_B = \frac{5P}{3} \qquad R_A = -\frac{4P}{3}$$

$$\left|\frac{R_A}{R_B}\right| = \left|\frac{-(4P/3)}{(5P/3)}\right| = 0.8$$

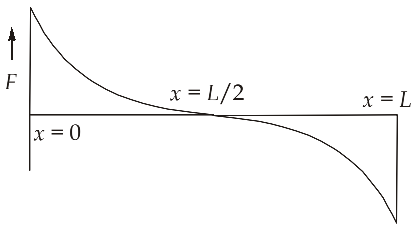
24. (b)

$$\frac{dF}{dx} = -w$$

Where, $\frac{dF}{dx}$ = Slope of shear force diagram



As load intensity, w decreases from w_o at $x = 0$ to 0 at at $x = L/2$, slope at $x = 0$ will be maximum and zero at $x = L/2$.



25. (b)

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\left(\frac{\gamma_{max,1}}{2}\right)}{\left(\frac{\gamma_{max,2}}{2}\right)} = \frac{\left(\frac{\tau_{max,1}}{G}\right)}{\left(\frac{\tau_{max,2}}{G}\right)} = \frac{\tau_{max,1}}{\tau_{max,2}}$$

Torsion equation: $$\frac{T}{J} = \frac{\tau_{max}}{r}$$

$\Rightarrow$ $$\tau_{max} = \frac{T}{J} \times r$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\frac{T}{J_1} \times r_1}{\frac{T}{J_2} \times r_2} = \frac{J_2}{J_1} \times \frac{r_1}{r_2} = \frac{\frac{\pi}{32} \times d_2^4}{\frac{\pi}{32} \times d_1^4} \times \frac{d_1 / 2}{d_2 / 2}$$

$$= \left(\frac{d_2}{d_1}\right)^3 = (2)^3 = 8$$

26. (c)

Maximum shear stress theory states that

$$\left|\frac{\sigma_1 - \sigma_2}{2}\right| = \frac{\sigma_y}{2} \quad \text{... (i)}$$

σ_1 and σ_2 are the principal stresses

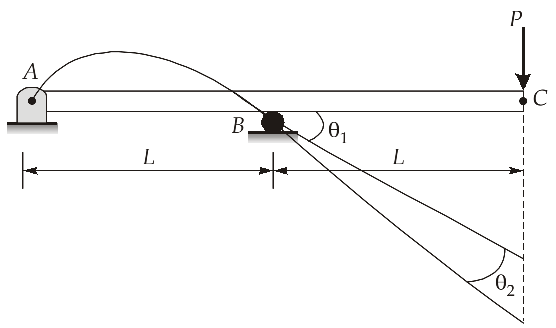
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\left|\frac{\sigma_1 - \sigma_2}{2}\right| = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\therefore$ Substituting in (i) and squaring

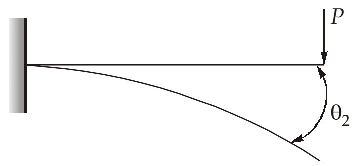
$$(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 = \sigma_y^2$$

27. (a)



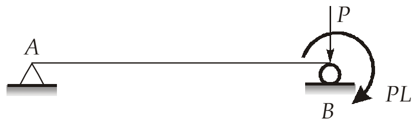
$$\theta_C = \theta_1 + \theta_2 \quad \text{... (i)}$$

For θ_2, assume BC as a cantilever beam:

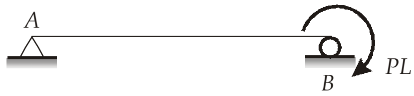


$$\theta_2 = \frac{PL^2}{2EI}$$

For θ_1, AB is given as:



This beam will bend only due to moment PL

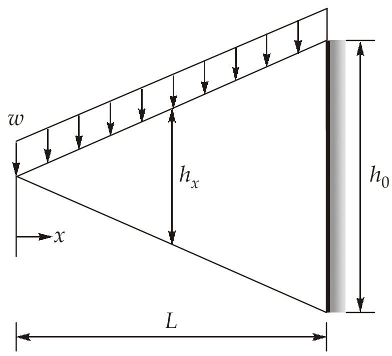


$$\theta_B = \theta_1 = \frac{ML}{3EI} = \frac{PL^2}{3EI}$$

Equation (i),
$$\theta_C = \theta_1 + \theta_2 = \frac{PL^2}{3EI} + \frac{PL^2}{2EI} = \frac{5PL^2}{6EI}$$

28. (d)

BM at distance x



$$M = \frac{wx^2}{2}$$

$$\sigma_{max} = \frac{wx^2}{2 \times \frac{h_x^2 b}{6}} = \frac{3wx^2}{h_x^2 b} \quad \text{... (i)}$$

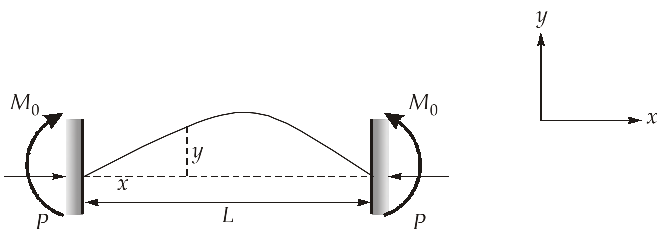
From figure,
$$h_x = \frac{h_0 x}{L} \quad \text{... (ii)}$$

Substitute (ii) in (i)

$$\sigma_{max} = \frac{3wx^2}{\frac{h_0 x^2}{L^2} b} = \frac{3wL^2}{h_0 b}$$

σ_{max} is independent of x. Thus it is a beam of uniform strength.

29. (a)



The bending equation is

$$EI\frac{d^2y}{dx^2} = M$$

M is the bending moment at any cross-section,

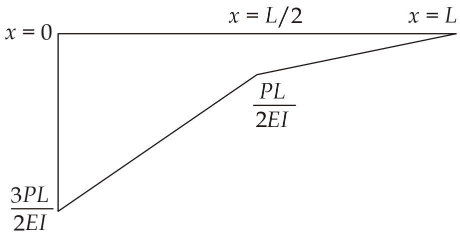
$$M = M_0 - Py$$

$$\therefore \quad EI\frac{d^2y}{dx^2} = M_0 - Py$$

or

$$\frac{d^2y}{dx^2} = -\frac{Py}{EI} + \frac{M_0}{EI}$$

30. (b)



$$\text{Slope, } \theta_{x=L} = \text{Area of } \frac{M}{EI} \text{ diagram}$$

$$= \frac{1}{2}\left[\frac{3PL}{2EI} + \frac{PL}{2EI}\right]\left(\frac{L}{2}\right) + \frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) = \frac{5PL^2}{8EI}$$

■■■■