Q.25 The forward path transfer function of a unity feedback closed-loop control system is given by $G(s) = \dfrac{5s+9}{s(5s^2+4s+3K_t)}$. Then the value of K_t which makes the ramp input error coefficient K_r = 2 and the steady state error due to step input for this K_t respectively are

(a) 1.5, ∞ (b) 1.5, 0
(c) 0, 1.5 (d) ∞, 1.5

Q.26 The response $h(t)$ of a linear time invariant system to an impulse $\delta(t)$, under initially relaxed condition, $h(t) = e^{-t} + e^{-2t}$. Then the response of this system for a unit step input $u(t)$ is

(a) $u(t) + e^{-t} + e^{-2t}$
(b) $(e^{-t} + e^{-2t})\, u(t)$
(c) $(1.5 - e^{-t} - 0.5e^{-2t})\, u(t)$
(d) $e^{-t}\,\delta(t) + e^{-2t}\, u(t)$

Q.27 Consider an integral controller with a value of $K_i = \dfrac{0.1}{s}$ and an output of 40% at the set point. Then the output after 2 sec, if there is a sudden change to a constant error of 20% is

(a) 40% (b) 42%
(c) 44% (d) 100%

Q.28 The step response of a second order system is

$$c(t) = 1 + 0.5e^{-30t} - 1.5e^{-10t}$$

Then the natural frequency of the system is

(a) 17.32 rad (b) 19.99 rad
(c) 29.88 rad (d) 12.88 rad

Q.29 A system is described by the state equation $\dot{X} = AX + BU$. The output is given by $Y = CX$, where $A = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = [1 \quad 0]$.

Then the damping ratio of the above system is ________.

(a) 0.65 (b) 0.72
(c) 0.94 (d) 1.23

Q.30 The state model of a system is

$$\dot{X}(t) = \begin{bmatrix} 0 & 0 & -3 \\ 1 & 0 & -4 \\ 0 & 1 & 5 \end{bmatrix} X(t) + \begin{bmatrix} 6 \\ 4 \\ 5 \end{bmatrix} u(t)$$

$Y(t) = [0 \quad 0 \quad 1]\, X(t)$

The final value of the system to a unit step input is _________.

(a) 1 (b) 2
(c) 3 (d) 4

■■■■

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CONTROL SYSTEM

EC-EE

Date of Test : 14/06/2025

ANSWER KEY

1.	(b)	7.	(d)	13.	(a)	19.	(a)	25.	(b)
2.	(a)	8.	(b)	14.	(a)	20.	(b)	26.	(c)
3.	(a)	9.	(b)	15.	(d)	21.	(a)	27.	(c)
4.	(b)	10.	(d)	16.	(a)	22.	(b)	28.	(a)
5.	(c)	11.	(c)	17.	(b)	23.	(b)	29.	(c)
6.	(a)	12.	(d)	18.	(d)	24.	(d)	30.	(b)

DETAILED EXPLANATIONS

1. (b)

$$N = P - 2$$

Where,
N = 2 (Anti clock wise)
P = Open loop poles on right hand side of s-plane.
Z = Closed loop poles on right hand side of s-plane.

$$2 = 2 - Z$$
$$Z = 0$$

As closed loop poles on right hand side of s-plane are zero.
System is stable.

2. (a)

Open loop transfer function

$$= \frac{Ks+b}{s^2+as+b-Ks-b} = \frac{Ks+b}{s^2+(a-K)s}$$

$$K_v = \lim_{s\to 0} s\cdot\frac{(Ks+b)}{s(s+a-K)} = \frac{b}{a-K}$$

$$\therefore \quad e_{ss} = \frac{1}{\left(\frac{b}{a-K}\right)} = \frac{a-K}{b}$$

3. (a)

$$c(t) = 12.5\, e^{-6t} \sin 8t$$

$$c(s) = \frac{12.5(8)}{(s+6)^2+8^2} = \frac{100}{s^2+12s+100}$$

$$\omega_n^2 = 100$$
$$\omega_n = 10 \text{ r/s}$$
$$2\xi\omega_n = 12$$
$$\xi = \frac{12}{2(10)} = 0.6$$

4. (b)

$$G(s)H(s) = \frac{K}{s(s+3-j\sqrt{3})(s+3+j\sqrt{3})} = \frac{K}{s(s^2+6s+12)}$$

5. (c)

$$G(s) = \frac{2(1-sT)}{s(s+2)}$$

Characteristic equation for the system is

$$1 + G(s) = 0$$
$$\Rightarrow \quad s^2 + 2s + 2 - 2sT = 0$$
$$\Rightarrow \quad s^2 + (2 - 2T)s + 2 = 0$$

By Routh array analysis,

$$\begin{array}{c|cc} s^2 & 1 & 2 \\ s^1 & 2-2T & 0 \\ s^0 & 2 & \end{array}$$

For system to be stable, if $2 - 2T > 0$; i.e. $T < 1$.

6. (a)

Characteristic equation is

$$1+\frac{K(s+2)}{s(s+10)(s-1)} = 0$$

System has roots farther to the left of –1.
Substitute $(s - 1)$ in place of 's' we get

$$1+\frac{K(s-1+2)}{s(s-1)(s-1+10)(s-2)} = 0$$

$$1+\frac{K(s+1)}{(s-1)(s+9)(s-2)} = 0$$

$$(s-1)(s+9)(s-2) + K(s+1) = 0$$

$$(s-1)(s^2+7s-18) - s^2 - 7s + 18 + Ks + K = 0$$

$$s^3 + 7s^2 - 18s - s^2 - 7s + 18 + Ks + K = 0$$

$$s^3 + 6s^2 - 25s + 18 + Ks + K = 0$$

$$s^3 + 6s^2 + (K-25)s + (18+K) = 0$$

From RH criterion

$$\begin{array}{c|cc} s^3 & 1 & K-25 \\ s^2 & 6 & 18+K \\ s^1 & \dfrac{6(K-25)-(18+K)}{6} & \\ s^0 & 18+K & \end{array}$$

$$18 + K > 0$$

$$K > -18$$

and $$6(K-25) - 18 - K > 0$$

$$5K > 168$$

$$K > \frac{168}{5}$$

$\therefore$ range of K is $\quad K > 33.6$

7. (d)

The gain margin is the inverse of the intersection of the root loci plot to the imaginary axis and if it does not intersect then the gain margin will be infinite.

8. (b)

From the given equation,
We can get open loop transfer function of the system

$$G(s) = \frac{K(s+1)}{s(s^2+5s+2)}$$

$$P = 3 \quad ; \quad Z = 1$$

∴ Number of asymptotes, $P - Z = 2$

∴ Angle of asymptotes $= \frac{(2q+1)\,180°}{P-Z}$

∵ $P - Z = 2 \quad \Rightarrow q = 0, 1$

∴ Angle of asymptotes are 90° and 270°.

9. (b)

$$\text{Transfer function} = \frac{-R_2}{R_1}\left[\frac{R_1 C_1 s+1}{R_2 C_2 s+1}\right]$$

10. (d)

$$\frac{C(s)}{R(s)} = \frac{5\times4\times2\times10(1+1)+10\times10\times2\times(1+4)}{1+4+4+1+4+4}$$

$$\frac{C(s)}{R(s)} = \frac{400\times2+200\times5}{18} = 100$$

11. (c)

Forward paths:

$$F_1 = G_1G_2G_3G_4 \quad ; \quad \Delta_1 = 1$$

$$F_2 = G_1G_6G_4 \quad ; \quad \Delta_2 = 1$$

$$F_3 = G_1G_7 \quad ; \quad \Delta_3 = 1 - G_5$$

Individual loops:

$$L_1 = G_2H_1 \quad ; \quad L_2 = G_5$$

Two non-touching loops,

$$L_{12} = G_2H_1(G_5)$$

$$\Delta = 1 - G_2H_1 - G_5 + G_2H_1G_5$$

∴ $$\frac{X_5}{X_1} = \frac{G_1G_2G_3G_4 + G_1G_4G_6 + G_1G_7(1-G_5)}{1-G_2H_1+G_2G_5H_1-G_5}$$

12. (d)

Characteristics equation is

$$s^5 - 2s^4 - 2s^3 + 4s^2 + s - 2 = 0$$

s^5	1	–2	1
s^4	–2	4	–2
s^3	0(–1)	0(1)	0
s^2	2	–2	0
s^1	0(4)	0	0
s^0	–2		

Auxiliary equation-1 is,

$$-2s^4 + 4s^2 - 2 = 0$$

$$\frac{d}{ds}(-2s^4 + 4s^2 - 2) = 0$$
$$-8s^3 + 8s = 0$$
$$-s^3 + s = 0$$

Auxiliary equation-2 is,

$$2s^2 - 2 = 0$$
$$\frac{d}{ds}(2s^2 - 2) = 0$$
$$4s = 0$$

3 sign changes in the first column.

Therefore 3 poles are in RHP out of which 2 are symmetric.

13. (a)

$$1 + KG = 0$$
$$s^3 + 6s^2 + 12s + Ks + 8 - 4K = 0$$
$$(s^3 + 6s^2 + 12s + 8) + K(s - 4) = 0$$
$$1 + \frac{K(s-4)}{s^3 + 6s^2 + 12s + 8} = 0$$

∴ Open loop transfer function,

$$G(s) = \frac{(s-4)}{s^3 + 6s^2 + 12s + 8}$$

s^3	1	12
s^2	6	8
s^1	$\frac{72-8}{6} = 10.66$	
s^0	8	

There is no sign change in first column of RH table.

∴ Open loop system is stable.

14. (a)

$$\text{Transfer function} = C[sI - A]^{-1} B + D$$

$$= \frac{\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}s+3 & 1 \\ 0 & s\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix}}{s(s+3)} = \frac{\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}s+3 \\ 0\end{bmatrix}}{s(s+3)} = \frac{1}{s}$$

15. (d)

$$\frac{s+3}{s+6} = \frac{3}{6}\left[\frac{1+\frac{1}{3}s}{1+\frac{1}{6}s}\right] = \frac{\alpha(1+Ts)}{(1+\alpha Ts)}$$

$$\alpha = \frac{1}{2}; \qquad T = \frac{1}{3}$$

$$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right) = \sin^{-1}\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right) = \sin^{-1}\left(\frac{1}{3}\right)$$

$$\phi_m = 19.47°$$

$$\omega_m = \frac{1}{T\sqrt{\alpha}} = \frac{1}{\frac{1}{3}\sqrt{\frac{1}{2}}} = 3\sqrt{2}$$

$$\omega_m = 3\sqrt{2}\ \text{rad/sec}$$

16. (a)

Characteristics equation,

$$1 + G(s) = 0$$

$$1+\frac{(K_1 s + K_2 s^2 + K_3)}{s^2(s-1)} = 0$$

$$s^3 + (K_2 - 1)s^2 + K_1 s + K_3 = 0$$

From the given closed loop poles,

$$(s + 1)\ (s + 1)\ (s + 2) = 0$$

$$(s^2 + 2s + 1)\ (s + 2) = 0$$

$$s^3 + 2s^2 + 2s^2 + 4s + s + 2 = 0$$

$$s^3 + 4s^2 + 5s + 2 = 0$$

$$K_2 - 1 = 4$$

$$K_2 = 5\ ; \qquad K_1 = 5\ ; \qquad K_3 = 2$$

$\therefore$ K_1, K_2, K_3 are 5, 5, 2

$$\therefore \quad K_1 + K_2 + K_3 = 5 + 2 + 2 = 12$$

17. (b)

$$G(s) = \frac{K}{s(s+2)(s+10)}$$

$$\angle G(j\omega) = -90 - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

$$-180 = -90 - \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) - \tan^{-1}\left(\frac{\omega_{pc}}{10}\right)$$

$$\tan^{-1}\frac{\omega_{pc}}{2} + \tan^{-1}\left(\frac{\omega_{pc}}{10}\right) = 90$$

$$\frac{\frac{\omega_{pc}}{2}+\frac{\omega_{pc}}{10}}{1-\frac{\omega_{pc}^2}{20}} = \infty$$

$$\omega_{pc}^2 = 20$$

$$\omega_{pc} = \sqrt{20}$$

$$|G(j\omega)| = \frac{K}{\omega\sqrt{\omega^2+2^2}\sqrt{\omega^2+10^2}}$$

$$\text{Gain margin} = 20\log\left|\frac{1}{G(j\omega)}\right|$$

$$G = 20\log_{10}\frac{\sqrt{20}\sqrt{20+4}\sqrt{20+100}}{K}$$

$$20\log_{10}2 = 20\log_{10}\frac{\sqrt{20}\sqrt{20+4}\sqrt{20+100}}{K}$$

$$K = 120$$

18. (d)

$$K_v = \lim_{s\to 0} s\cdot G(s)$$

$$K_v = \lim_{s\to 0} s\cdot\frac{K(s+10)(s+15)}{s(s+3)(s+7)(s+20)}$$

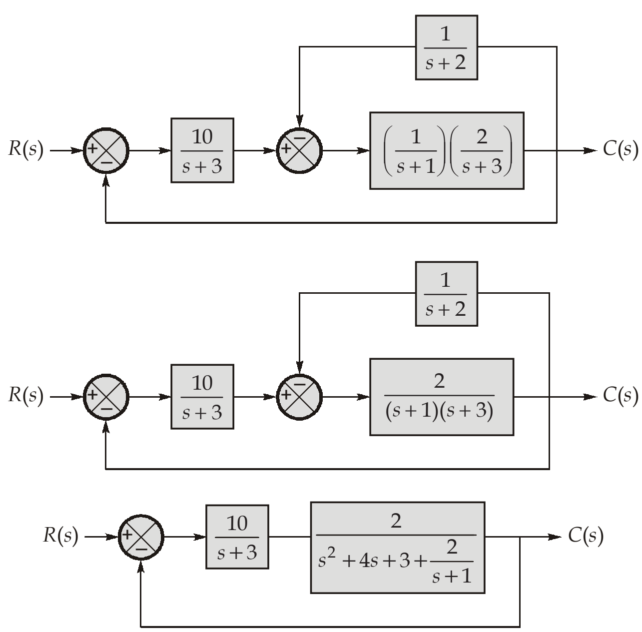
$$K_v = \frac{K(10)(15)}{(3)(7)(20)} = \frac{K(5)}{14}$$

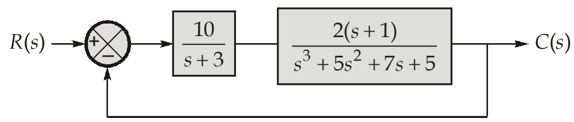
$$e_{ss} = \frac{25}{K_v} = \frac{25}{K(5)}\times 14$$

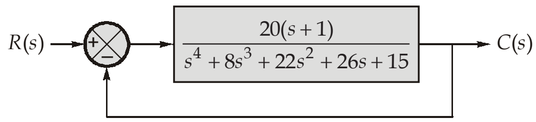
$$0.1 = \frac{5\times 14}{K}$$

$$K = \frac{5\times 14}{0.1} = 700$$

19. (a)







No. of poles at $s = 0$ in open loop transfer function is zero,

$\therefore$ Type = 0

20. (b)

$$|G(j\omega)| = \frac{2}{\omega^2 + 1}$$

$$\frac{2}{1+\omega_{gc}^2} = 1$$

$$2 = 1 + \omega_{gc}^2$$

$$\omega_{gc} = 1 \text{ rad/sec}$$

$$\angle G(j\omega)\big|_{\omega=\omega_{gc}} = -2\tan^{-1}\omega_{gc} = -90°$$

$$\text{PM} = 180 - 90° = 90°$$

21. (a)

$$\%M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$$

$$= e^{-0.5\pi/\sqrt{1-0.5^2}} \times 100$$

$$\%M_p = 16.3\%$$

$$\%M_p = \frac{P-3}{3} \times 100 = 16.3$$

$$\frac{P-3}{3} = 0.163$$

$$P = 3(1.163)$$

$$P = 3.489$$

22. (b)

Characteristic equation of the closed loop system,

$$s^3 + as^2 + 2s + 1 + Ks + K = 0$$

$$s^3 + as^2 + (K+2)s + (K+1) = 0$$

By using Routh Hurwitz table,

s^3	1	$K+2$
s^2	a	$K+1$
s^1	$\frac{a(K+2)-(K+1)}{a}$	
s^0	$K+1$	

If 's^1' term is equal to zero, then row of zeros occurs

$$\therefore \quad \frac{a(K+2)-(K+1)}{a} = 0$$

$$\therefore \quad a = \frac{1+K}{2+K}$$

and also, $K > -1$

From the given options, if $K = 2$ then

$$a = \frac{1+2}{2+2} = \frac{3}{4} = 0.75$$

Option (b) satisfy the condition.

23. **(b)**

Checking angle criterion of root locus

$$\angle G(s)H(s)\big|_{s=s_1} = -4\tan^{-1}(-2) \neq 180°$$

$$\angle G(s)H(s)\big|_{s=s_2} = -4\tan^{-1}(1) = -180°$$

$\angle G(s)H(s)\big|_{s=s_2}$ satisfies angle criterion.

So it lies on root locus.

24. **(d)**

The output is given as a function of time.

The final value of the output is,

$$\lim_{t\to\infty} c(t) = 1$$

Hence, t_d (at 50% of final value) is the solution,

i.e. $$0.5 = 1 - e^{-t_d}$$

$$e^{-t_d} = 0.5$$

$$t_d = \ln 2 \quad \text{(or)} \quad 0.693 \text{ sec}$$

25. **(b)**

Given, $$K_r = 2 \quad \Rightarrow \lim_{s\to 0} sG(s) = \lim_{s\to 0} s\frac{5s+9}{s(5s^2+4s+3K_t)}$$

$$2 = \frac{9}{3K_t}$$

$$\therefore \quad K_t = 1.5$$

$$\therefore \quad G(s) = \frac{5s+9}{s(5s^2+4s+4.5)}$$

Also step error coefficient, $K_s = \lim_{s\to 0} \frac{5s+9}{s(5s^2+4s+4.5)} = \infty$

$\therefore$ Steady state error, $e_{ss} = \frac{1}{1+K_s} = \frac{1}{\infty} = 0$

26. **(c)**

Given, $$h(t) = e^{-t} + e^{-2t}$$

$$H(s) = \frac{1}{s+1} + \frac{1}{s+2}$$

But $H(s) = \frac{C(s)}{R(s)} = \frac{1}{s+1} + \frac{1}{s+2}$

Given, $R(s) = \frac{1}{s}$

$$C(s) = R(s) \cdot H(s) = \frac{1}{s}\left[\frac{1}{s+1} + \frac{1}{s+2}\right]$$

$$C(s) = \frac{1}{s(s+1)} + \frac{1}{s(s+2)} = \left(\frac{1}{s} - \frac{1}{s+1}\right) + \frac{1}{2}\left(\frac{1}{s} - \frac{1}{s+2}\right)$$

$\therefore$ $c(t) = L^{-1}\,[C(s)]$

$$c(t) = (1.5 - e^{-t} - 0.5e^{-2t})\,u(t)$$

27. **(c)**

Given, $K_i = \frac{0.1}{s}$

$I_o = 40\%$

e = Error (constant) = 20%

For an integral controller

$$(I_{out} - I_o) = \int_0^t K_i\, edt$$

When the error does not vary with time, the equation becomes

$$(I_{out} - I_o) = K_i\, et$$

$\therefore$ $I_{out} = K_i\, et + I_o$

At $t = 2$ sec, $I_{out} = 0.1 \times 20 \times 2 + 40 = 44\%$

28. **(a)**

Given, $c(t) = 1 + 0.5e^{-30t} - 1.5e^{-10t}$

Impulse response, $\frac{dc(t)}{dt} = 0 - 15e^{-30t} + 15e^{-10t}$

Then the transfer function,

TF = L [Impulse response]

$$\text{TF} = \frac{-15}{s+30} + \frac{15}{s+10}$$

$$\text{TF} = \frac{-15}{s+30} + \frac{15}{s+10} = \frac{-15s - 150 + 15s + 450}{(s+10)\,(s+30)}$$

Therefore natural frequency, $\omega_n = \sqrt{300} = 17.32$ rad

29. **(c)**

$$sI - A = \begin{bmatrix} s+4 & 1 \\ -3 & s+1 \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} s+1 & -1 \\ 3 & s+4 \end{bmatrix} \frac{1}{(s+4)\,(s+1)+3}$$

Transfer function, $G(s) = C(sI - A)^{-1} B = [1 \;\; 0]\dfrac{\begin{bmatrix} s+1 & -1 \\ 3 & s+4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{(s+4)(s+1)+3}$

$$G(s) = \frac{s}{s^2+5s+7}$$

$$2\xi\sqrt{7} = 5 \qquad \{\because 2\xi\omega_n = 5\}$$

$$\Rightarrow \qquad \xi = \frac{5}{2\sqrt{7}} = 0.944$$

30. (b)

The transfer function, $TF = C[sI - A]^{-1} B + D$

$$C = [0 \;\; 0 \;\; 1] \quad ; \quad B = \begin{bmatrix} 6 \\ 4 \\ 5 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 0 & -3 \\ 1 & 0 & -4 \\ 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} s & 0 & 3 \\ -1 & s & 4 \\ 0 & -1 & s-5 \end{bmatrix}$$

$$TF = [0 \;\; 0 \;\; 1]\begin{bmatrix} s & 0 & 3 \\ -1 & s & 4 \\ 0 & -1 & s-5 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ 4 \\ 5 \end{bmatrix}$$

On solving, $TF = \dfrac{5s^2+4s+6}{s^3+5s^2+4s+3}$

$$C(\infty) = \lim_{s \to 0} s[TF]\frac{1}{s} = \frac{6}{3} = 2$$

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