CLASS TEST

S.No.: 01SKME_ABCDEFGHI_22525

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ENGINEERING MATHEMATICS

MECHANICAL ENGINEERING

Date of Test : 22/05/2025

ANSWER KEY

1.	(c)	7.	(c)	13.	(d)	19.	(b)	25.	(b)
2.	(c)	8.	(a)	14.	(d)	20.	(a)	26.	(c)
3.	(d)	9.	(a)	15.	(c)	21.	(c)	27.	(b)
4.	(d)	10.	(b)	16.	(a)	22.	(b)	28.	(d)
5.	(c)	11.	(b)	17.	(b)	23.	(b)	29.	(a)
6.	(a)	12.	(a)	18.	(b)	24.	(c)	30.	(d)

DETAILED EXPLANATIONS

1. (c)

A is $m \times n$, B is $p \times q$, $A + B$ is defined if $m = p$, $n = q$

AB is defined if $n = p$

Hence, $m = n = p = q$

2. (c)

$$\text{determinnant of } (3AB) = 27 \text{ determinant of } (AB)$$

[$\because$ Order of A = Order of B = 3]

$$= 27\,|A|\,|B|$$

$$= 27(-1)(3) = -18$$

3. (d)

Here determinant of matrix A,

$$|A| = \begin{vmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{vmatrix}$$

Operating $c_1 \to c_1 + c_2 + c_3$

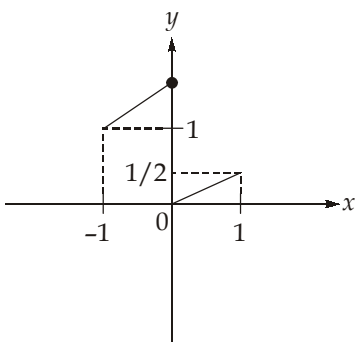
$$= \begin{vmatrix} 1+\omega^2+\omega & \omega^2 & \omega \\ \omega^2+\omega+1 & \omega & 1 \\ \omega+1+\omega^2 & 1 & \omega^2 \end{vmatrix}$$

$$I + \omega + \omega^2 = 0$$

$$|A| = \begin{vmatrix} 0 & \omega^2 & \omega \\ 0 & \omega & 1 \\ 0 & 1 & \omega^2 \end{vmatrix} = 0$$

$\therefore$ A is singular matrix.

4. (d)



The graph of the function is shown above.

Since the function is not continuous at $x = 0$.

$\therefore$ The function has neither maximum nor minimum.

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5. (c)

If function is differentiable then function is continuous also

At $x = 3$

$$f(3) = 2k = f(3^-)$$

$$f(3^+) = 3m + 2$$

$$2k = 3m + 2 \quad \text{...(i)}$$

$$g'(x) = \begin{cases} \dfrac{k}{2\sqrt{x+1}}; & 0 \le x \le 3 \\ m, & 3 < x \le 5 \end{cases}$$

$$g'(3^-) = g(3)$$

$$\frac{k}{4} = m$$

$$k = 4m \quad \text{...(ii)}$$

Put $k = 4m$ in equation (i), we get

$$8m = 3m + 2$$

$$m = \frac{2}{5}$$

Put $m = \dfrac{2}{5}$ in equation (ii)

$$k = \frac{8}{5}$$

$$k + m = \frac{8}{5} + \frac{2}{5} = 2$$

6. (a)

Total of 9 is (3, 6), (4, 5), (5, 4), (6, 3)

$$\text{Probability of getting total 9} = \frac{4}{36} = \frac{1}{9}$$

$$\therefore \quad \text{Required probability} = \frac{1}{9} + \frac{8}{9} \times \frac{8}{9} \times \frac{1}{9} + \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{1}{9} + \ldots\infty$$

$$= \frac{1}{9}\left[1 + \left(\frac{8}{9}\right)^2 + \left(\frac{8}{9}\right)^4 + \ldots\right]$$

$$= \frac{1}{9} \cdot \frac{1}{1 - \frac{64}{81}} = \frac{9}{17}$$

7. (c)

$$\text{Auxiliary equation, } |A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 2-\lambda & -3 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)[(2-\lambda)(3-\lambda)-3]-[(1)(3-\lambda)+6]+1[-1-2(2-\lambda)]=0$$

$$(1-\lambda)\left[6-2\lambda-3\lambda+\lambda^2-3\right]-[3-\lambda+6]+1[-4-1+2\lambda]=0$$

$$-\lambda^3+6\lambda^2-5\lambda-11 = 0$$

$$\lambda^3-6\lambda^2+5\lambda+11 = 0$$

$$x = 6 \text{ and } y = -5$$

$$\therefore \quad x+y = 1$$

8. (a)

Left hand limit at $x = 1$,

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(5x-4) = 5-4 = 1$$

Right hand limit at $x = 1$,

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(4x^2+3bx) = 4+3b$$

Since $f(x)$ is continuous at every point of its domain,

$\therefore$ $f(x)$ is continuous at $x = 1$.

$\therefore$ $\lim_{x\to 1} f(x)$ exists, hence the left and right-hand limits are equal. We get,

$$1 = 4+3b$$

$$3b = -3$$

$$b = -1$$

9. (a)

$$f''(x) = g''(x)$$

Integrating, we get $\quad f'(x) = g'(x) + c$

$$f'(1) = g'(1) + c$$

$$2 = 4 + c$$

$$c = -2$$

$$\therefore \quad f'(x) = g'(x) - 2$$

Integrating above equation

$$f(x) = g(x) - 2x + c'$$

$$f(2) = g(2) - 4 + c'$$

$$3 = 9 - 4 + c'$$

$$c' = -2$$

$$\therefore \quad f(x) = g(x) - 2x - 2$$

$$f(x) - g(x) = -2x - 2$$

At $x = 3$, $\quad f(x) - g(x) = -6 - 2 = -8$

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10. (b)

Using Cauchy's theorem,

$$f(a) = \frac{1}{2\pi i}\oint \frac{f(z)}{z-a}dz$$

$$\therefore \quad \oint_c \frac{f(z)}{z-a}dz = 2\pi i\, f(a)$$

$f(z)$ has a pole, $z = \frac{i}{3}$, which lies inside the unit circle $|z| = 1$. Hence,

$$\oint_c \frac{\left(\frac{z^3}{3}-2\right)}{z-\frac{i}{3}}dz = 2\pi i\left[\frac{z^3}{3}-2\right]_{z=\frac{i}{3}}$$

$$= 2\pi i\left[-\frac{i}{81}-2\right] = \frac{2\pi}{81} - 4\pi i$$

11. (b)

The probability of getting two diamonds from a pack of 52 cards $= \frac{{}^{13}c_2}{{}^{52}c_2} = \frac{3}{51}$

Number of trials, $n = 51$

$$\therefore \quad \lambda = \text{Mean} = np = \frac{3}{51}\times 51 = 3$$

$\therefore$ The poisson distribution is

$$P(x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!} = \frac{e^{-3}\cdot 3^x}{x!}$$

$$P(\text{atleast 3 times}) = P(x \geq 3) = 1 - P(x < 3)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[e^{-3} + 3e^{-3} + e^{-3}\cdot\frac{3^2}{2!}\right]$$

$$= 1 - e^{-3}\left(1+3+\frac{9}{2}\right) = 0.57681$$

12. (a)

f is continuous in $[0, \infty)$ implies that f is continuous at 1 and $\sqrt{2}$ too

So,
$$f(1) = \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$$

$$a = \lim_{x\to 1^-}\frac{x^2}{a} = \lim_{x\to 1^+} a$$

$$a = \frac{1}{a}$$

$$a^2 = 1$$

$$a = \pm 1$$

$$f\left(\sqrt{2}\right) = \lim_{x \to \sqrt{2}^-} f(x) = \lim_{x \to \sqrt{2}^+} f(x)$$

$$\frac{2b^2 - 4b}{2} = \lim_{x \to \sqrt{2}^-} a = \lim_{x \to \sqrt{2}^+} \frac{2b^2 - 4b}{x^2}$$

$$b^2 - 2b = a$$

when $a = 1$

$$b^2 - 2b - 1 = 0$$

$$\therefore \quad b = 1 \pm \sqrt{2}$$

when $a = -1$, then $b^2 - 2b + 1 = 0$

$$\therefore \quad b = 1$$

13. (d)

$$x^2 + y^2 = 1,\ x + y + z = 3 \text{ and } z = 0$$

$$\text{Required volume} = \iiint dx\,dy\,dz$$

$$= \iint (3 - x - y)\,dxdy$$

On putting,

$$x = r\cos\theta,\ y = r\sin\theta$$

$$dxdy = rd\theta dr, \text{ we get}$$

$$= \iint (3 - r\cos\theta - r\sin\theta)\,rd\theta\,dr$$

$$= \int_0^{2\pi} d\theta \left(\frac{3r^2}{2} - \frac{r^3}{3}\cos\theta - \frac{r^3}{3}\sin\theta\right)_0^1$$

$$= \int_0^{2\pi} \left(\frac{3}{2} - \frac{1}{3}\cos\theta - \frac{1}{3}\sin\theta\right) d\theta$$

$$= \left(\frac{3}{2}\theta + \frac{1}{3}\cos\theta - \frac{1}{3}\sin\theta\right)_0^{2\pi}$$

$$= 3\pi - \frac{1}{3}\sin 2\pi + \frac{1}{3}\cos 2\pi - \frac{1}{3} = 3\pi$$

14. (d)

Since $$\frac{\hat{i}\cdot\left(3\hat{i} - 4\hat{j}\right)}{\sqrt{9+16}} = \frac{3}{5} \neq \cos 60^\circ$$

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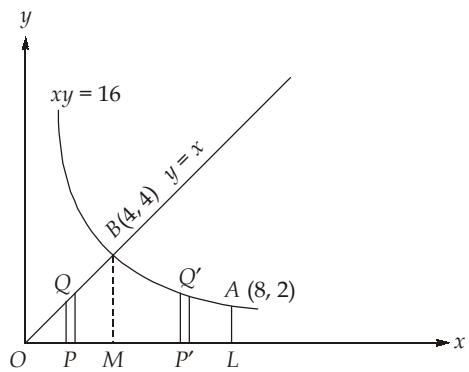
$$\frac{(\hat{i}+\hat{j})(3\hat{i}-4\hat{j})}{\sqrt{9+16}} = \frac{3-4}{10} \neq \cos 60°$$

$$\frac{(\hat{i}-\hat{j})(3\hat{i}-4\hat{j})}{\sqrt{9+16}\times\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}(3+4)}{10} \neq \cos 60°$$

∴ Option (d) is correct.

15. (c)

The line AL (x = 8) intersects the hyperbola xy = 16 at $A(8, 2)$ while the line $y = x$ intersects this hyperbola at $B(4, 4)$. Figure shows the region R of integration which is the area $OLAB$. To evaluate the given integral. We divide this area into two parts OMB and $MLAB$.



$$\therefore \quad \iint_R x^2 dxdy = \int_{x\,\text{at}\,O}^{x\,\text{at}\,M}\int_{y\,\text{at}\,P}^{y\,\text{at}\,Q} x^2 dxdy + \int_{x\,\text{at}\,M}^{x\,\text{at}\,L}\int_{y\,\text{at}\,P'}^{y\,\text{at}\,Q'} x^2 dxdy$$

$$= \int_0^4 x^3 dx + \int_4^8 16x\,dx$$

$$= \left|\frac{x^4}{4}\right|_0^4 + 16\left|\frac{x^2}{2}\right|_4^8$$

$$= 64 + 384 = 448$$

16. (a)

We have:

$$\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x$$

$$\frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$$

Integrating,

$$\log(y+1) = -\log(2+\sin x) + \log c$$

$$y+1 = \frac{c}{2+\sin x}$$

When $x = 0$, $y = 1$

$$\therefore \quad 2 = \frac{c}{2+0}$$

$$c = 4$$

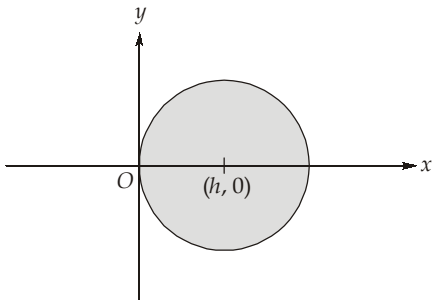
$$\therefore \quad y + 1 = \frac{4}{2+\sin x}$$

$$y = \frac{4}{2+\sin x} - 1 = \frac{2-\sin x}{2+\sin x}$$

Hence, $$y\left(\frac{\pi}{2}\right) = \frac{2-1}{2+1} = \frac{1}{3}$$

17. (b)

Here equation of circle is



$$(x-h)^2 + y^2 = h^2$$

$$x^2 + y^2 - 2hx = 0 \qquad \text{...(i)}$$

Differentiate w.r.t.,

$$2x + 2y\frac{dy}{dx} - 2h = 0$$

$$2h = 2x + 2y\frac{dy}{dx}$$

Putting in equation (i), we get

$$x^2 + y^2 - \left(2x + 2y\frac{dy}{dx}\right)x = 0$$

$$x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$$

$$y^2 = x^2 + 2xy\frac{dy}{dx}$$

18. (b)

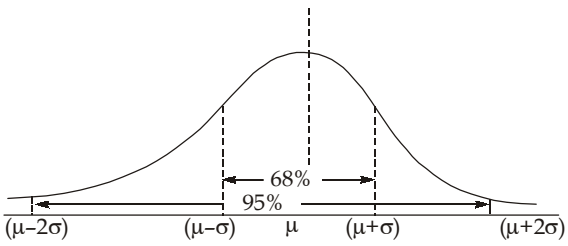
Given,

Mean, $\mu = 1200$

Variance, $\sigma^2 = 9 \times 10^4$

$\Rightarrow$ Standard deviation, $\sigma = \sqrt{9\times10^4} = 300$

Using Standard normal curve,



Probability of finding tigers between

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$$(\mu - 2\sigma) \ \& \ (\mu + 2\sigma) = 0.95$$
$$\mu - 2\sigma = 1200 - 2 \times 300 = 600$$
$$\mu + 2\sigma = 1200 + 2 \times 300 = 1800$$

i.e. $P\ (600 \le X \le 1800) = 0.95$

$\Rightarrow P\ (X \le 600) + P\ (X \ge 1800) = 0.05$

Since normal curve is symmetric wrt mean value,

So, $P\ (X \le 600) = P\ (X \ge 1800)$

$\Rightarrow$ $2P\ (X \ge 1800) = 0.05$

$\Rightarrow$ $P\ (X \ge 1800) = 0.025$

19. (b)

A.E. is $m^2 - 4m + 4 = 0$

$$m = 2, 2$$

$$\text{C.F.} = (C_1 + C_2 x)e^{2x}$$

$$\text{P.I.} = \frac{1}{D^2 - 4D + 4} e^{2x}$$

$$D = 2$$

$$= \frac{1}{4 - 8 + 4} e^{2x}$$

Case fails, $\text{P.I.} = x \cdot \frac{1}{2D - 4 + 0} e^{2x}$

Again case fail, $\text{P.I.} = \frac{x^2}{2 - 0} e^{2x}$

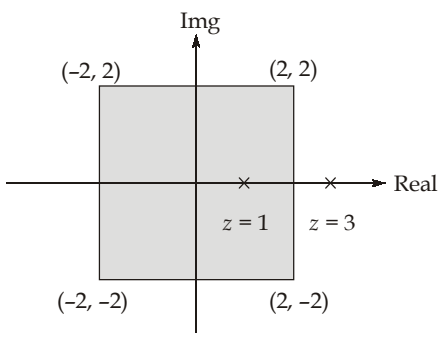
$\therefore$ $\text{P.I.} = x^2 \cdot \frac{1}{2} e^{2x}$

General solution,

$$y = \text{C.F.} + \text{P.I.}$$

$$= (C_1 + C_2 x)e^{2x} + \frac{x^2}{2} e^{2x}$$

20. (a)



Only the pole at z = 1 lies inside the closed curve.

So, $\frac{1}{2\pi i}\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-3)} dz$ = Residue at $(z = 1)$

$$= \left.\frac{\sin \pi z^2 + \cos \pi z^2}{z-3}\right|_{z=1}$$

$$= \frac{\sin \pi + \cos \pi}{-2} = \frac{1}{2} = 0.5$$

21. (c)

Let the events be as below:

G = Original signal is green

E_1 = A receives correct signal

E_2 = B receives correct signal

and E : B receives green signal

$\therefore$ $P(G) = \frac{4}{5}$, $P(E_1) = \frac{3}{4}$ and $P(E_2) = \frac{3}{4}$

Now E can happen in the following ways:

Original signal	A receives	B receives
Red	Red	Green
Red	Green	Green
Green	Green	Green
Green	Red	Green

$$\therefore \quad P(E) = P(\bar{G} \cap E_1 \cap \bar{E}_2) + P(\bar{G} \cap \bar{E}_1 \cap E_2) + P(G \cap E_1 \cap E_2) + A(G \cap \bar{E}_1 \cap \bar{E}_2)$$

$$= \frac{1}{5}\times\frac{3}{4}\times\frac{1}{4} + \frac{1}{5}\times\frac{1}{4}\times\frac{3}{4} + \frac{4}{5}\times\frac{3}{4}\times\frac{3}{4} + \frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}$$

$$= \frac{3}{80} + \frac{3}{80} + \frac{36}{80} + \frac{4}{80}$$

$$= \frac{46}{80} = \frac{23}{40}$$

$$\therefore \quad P\left(\frac{G}{E}\right) = \frac{P(G \cap E)}{P(E)}$$

$$= \frac{P(G \cap E_1 \cap E_2) + P(G \cap \bar{E}_1 \cap \bar{E}_2)}{P(E)}$$

$$= \frac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4} + \frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{23}{40}}$$

$$= \frac{40}{80}\times\frac{40}{23} = \frac{40}{46} = \frac{20}{23}$$

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22. (b)

$$e^{(-2+i5)t} = e^{-2t}\, e^{-5t}$$
$$= e^{-2t}(\cos 5t + i\sin 5t)$$

$$\left[LT \text{ of } \angle\cos at = \frac{s}{s^2+a^2};\ \angle\sin at = \frac{b}{s^2+b^2}\right]$$

$$\angle e^{-ct}\cos at = \frac{s+c}{(s+c)^2+a^2}$$

LT of
$$\angle e^{(-2+i5)t} = \frac{s+2}{(s+2)^2+5^2} + \frac{5i}{(s+2)^2+5^2}$$
$$= \frac{s+2+5i}{(s+2)^2+25}$$

23. (b)

Given: $\oint_C \frac{z^2+1}{z(2z-1)}dz$ is not analytic at the points $z = 0$ and $z = \frac{1}{2}$ both of which lie inside C.

$$\oint_C \frac{z^2+1}{z(2z-1)} = \oint_C \frac{z^2+1}{\left(z-\frac{1}{2}\right)}dz - \oint_C \frac{z^2+1}{z}dz$$

Using Cauchy integral formula:

$$\oint_C \frac{z^2+1}{\left(z-\frac{1}{2}\right)}dz = 2\pi i\left[z^2+1\right]_{z=\frac{1}{2}} = \frac{5\pi i}{2}$$

$$\oint_C \frac{z^2+1}{z} = 2\pi i\left[z^2+1\right]_{z=0} = 2\pi i$$

$$\oint_C \frac{z^2+1}{z(2z-1)} = \frac{5\pi i}{2} - 2\pi i = \frac{\pi i}{2}$$

24. (c)

$$E(x) = 3\times\frac{1}{2} + 6\times\frac{3}{10} + 8\times\frac{1}{5} = 4.9$$

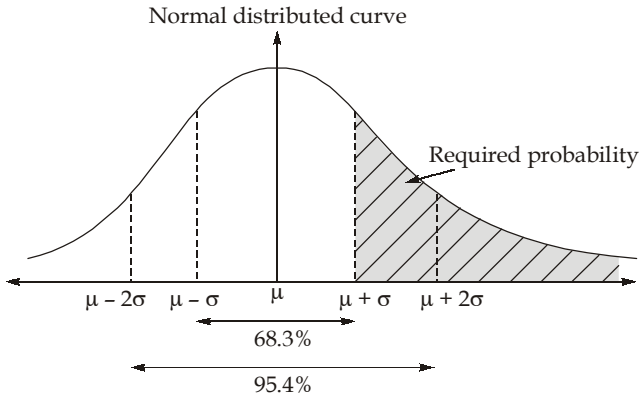
$$E(x^2) = 3^2\times\frac{1}{2} + 6^2\times\frac{3}{10} + 8^2\times\frac{1}{5} = 28.1$$

$$E(2x+6)^2 = E(4x^2+24x+36)$$
$$= 4E(x^2) + 24E(x) + 36$$
$$= 4\times 28.1 + 24\times 4.9 + 36 = 266$$

25. (b)

$$\text{Mean, } \mu = 60$$
$$\text{Variance, } \sigma^2 = 25$$
$$\sigma = 5$$

Normal distributed curve

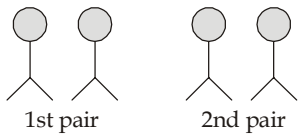


% of students who scored more than 65 marks (μ + σ)

$$= 0.5 - \frac{0.683}{2} = 0.1585 = 15.85\%$$

26. (c)

n person can arrange in round by $(n - 1)!$ way



Consider them as one unit,

Total person $= n - 1 - 1 = (n - 2)$

Total arrangement $= (n - 2 - 1)!$

Both pair arrange itself by 2! way

Probability that two pair of two specified persons do not sit together

$$= 1 - (\text{sit together})$$

$$= 1 - \frac{2 \times 2(n-2-1)!}{(n-1)!}$$

$$= 1 - \frac{4(n-3)!}{(n-1)(n-2)(n-3)!}$$

$$= 1 - \frac{4}{(n-1)(n-2)}$$

$$= \frac{n^2 - 3n + 2 - 4}{n^2 - 3n + 2} = \frac{n^2 - 3n - 2}{n^2 - 3n + 2}$$

27. (b)

$$A = \begin{bmatrix} a & -2 \\ 2.5 & b \end{bmatrix}$$

$$\det(A) = |A| = \begin{bmatrix} a & -2 \\ 2.5 & b \end{bmatrix} = 30$$

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$$\Rightarrow \quad ab + 2 \times 2.5 = 30$$

$$\Rightarrow \quad ab = 25 \quad \text{... (i)}$$

$$\text{trace(A)} = \text{Sum of diagonal element}$$

$$= a + b$$

We know, $\text{AM} \geq \text{GM}$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$\frac{a+b}{2} \geq \sqrt{25}$$

$$\frac{a+b}{2} \geq 5$$

$$\Rightarrow \quad a + b \geq 10$$

Hence, minimum value of trace (A) = 0

28. (d)

Since, $P(A \cup B) = \text{P(A)} + \text{P(B)} - P(A \cap B)$

$$\frac{1}{2} = \frac{1}{4} + \frac{1}{3} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$$

$$= \frac{3+4-6}{12} = \frac{1}{12}$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{1/12}{1/4} = \frac{1}{3}$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/12}{1/3} = \frac{1}{4}$$

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$= \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$$

$$P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{\frac{1}{6}}{1 - P(B)} = \frac{\frac{1}{6}}{1 - \frac{1}{3}} = \frac{1}{4}$$

29. (a)

Let, $du = (u_r)dr + (u_\theta)d\theta = \left(\frac{1}{r}v_\theta\right)dr + (-rv_r)d\theta$

$$\left[\because u_r = \frac{1}{r}v_\theta \text{ and } v_r = -\frac{1}{r}u_\theta\right]$$

So, $du = [(12r^3 \cos 4\theta)\, dr + (-12r^4 \sin(4\theta)\, d\theta)]$

$$u = \int\left[\underbrace{\left(12r^3\cos 4\theta\right)}_{\theta\text{ constant}}dr + \int\underbrace{\left(12r^3\cos 4\theta\right)}_{r=0}dr\right] + k$$

$$\therefore \quad u(r, \theta) = 3r^4\cos(4\theta) + k$$

30. (d)

$$\text{Eigen values of } [A] = 3, 3, 3$$

$$\text{adj}(A) = A^{-1}\,|A|$$

$$\text{Eigen value of } A^{-1} = \frac{1}{3}, \frac{1}{3}, \frac{1}{3}$$

$$\text{Eigen values of adj}[A] = \frac{27}{3}, \frac{27}{3}, \frac{27}{3}$$

$$= 9, 9, 9$$

$$\text{Eigen values of } A + \text{adj}(A) = 12, 12, 12$$

■■■■

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