OPSC-AEE 2020

Odisha Public Service Commission

Assistant Executive Engineer

Civil Engineering

Open Channel Flow

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY Publications

Open Channel Flow

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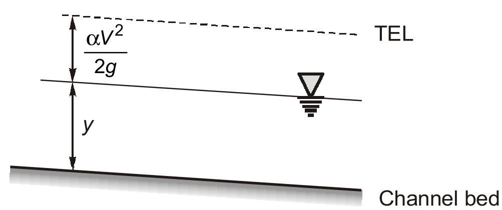
CHAPTER 3

Energy Depth Relationship

3.1 Specific Energy

Specific energy is the total energy at a section w.r.t. the channel bed as datum and is expressed as summation of flow depth and velocity head.

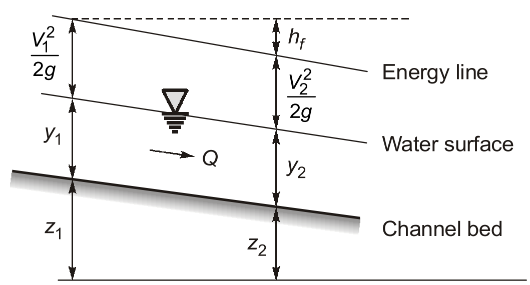


$$E = y + \alpha \cdot \frac{V^2}{2g}$$

where α = Kinetic energy correction factor

Since, channel flow will always be turbulent flow and for turbulent K.E correction factor is approximately unity.

$$\therefore \qquad E = y + \frac{V^2}{2g}$$



- Specific energy at section 1-1,

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

Specific energy at section 2-2,

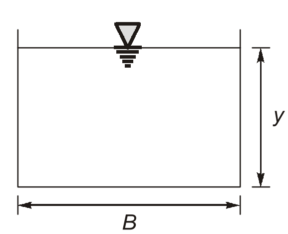
$$E_2 = y_2 + \frac{V_2^2}{2g}$$

- For uniform flow, specific energy will be constant.
- For varied flow specific energy may either increase or decrease in the direction of flow. But total energy will always decrease in the direction of flow.
- For frictionless and horizontal channel specific energy will be constant.

3.1.1 Relationship between Specific Energy and Depth of Flow Specific Energy Curve

It is a plot between the specific energy on abscissa (x-axis) and depth of flow on ordinate (y-axis).

- Consider a rectangular channel having bed width 'B' and depth of flow y.

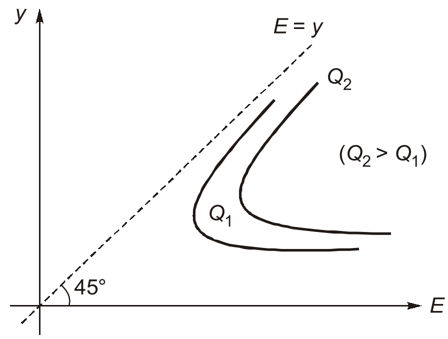


$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$

$$E = y + \frac{Q^2}{2gB^2y^2} \qquad ...(1)$$

If $\quad y \to 0;\ E \to \infty$

$\quad y \to \infty;\ E \to y$



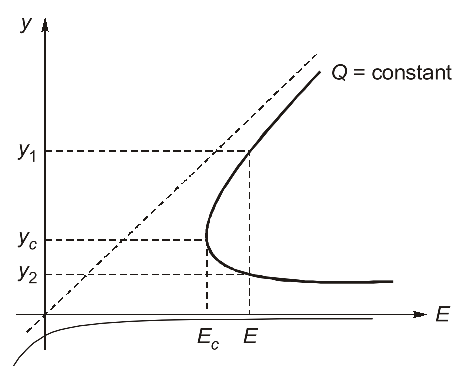
- The curve obtained is valid for one particular discharge as discharge increases the curve shifts to the right.
- The curve would be different for different cross-section however its nature would be same.

From equation (1)

$$E = y + \frac{Q^2}{2g \cdot B^2 y^2}$$

The specific energy depth relationship is cubic in nature hence, we get '3' (three) value of depth for a particular given discharge one of them is negative and other two are positive, these two positive depths of flow y_1 and y_2 are called alternate depth of flow one of that depth (y_1) is corresponding to subcritical flow and other (y_2) is corresponding to supercritical flow.

- The depth of flow obtained at the tip of curve is called critical depth of flow and the corresponding energy is called critical specific energy.



y_1, y_2 – Alternate depth

E_c – Critical specific energy

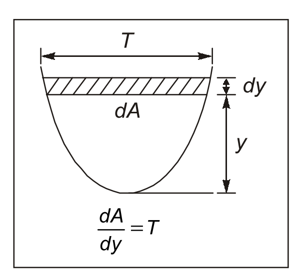
y_c – Critical depth

- Hence, minimum specific energy (E_c) for a particular discharge 'Q' corresponds to the critical state of flow. Hence at the critical state of flow the two alternate depths apparent becomes one, which is known as critical depth (y_c).

As, $$E = y + \frac{Q^2}{2g A^2}$$

For E to be minimum at constant 'Q'

$$\frac{dE}{dy} = 0$$



$$\therefore \quad \frac{dE}{dy} = 1 + \frac{Q^2}{2g}\left(\frac{-2}{A^3}\right) \cdot \frac{dA}{dy} = 0$$

$$\therefore \quad 1 - \frac{Q^2 T}{gA^3} = 0$$

$$\frac{Q^2T}{gA^3} = 1 \qquad \text{For any channel (condition for critical state of flow)}$$

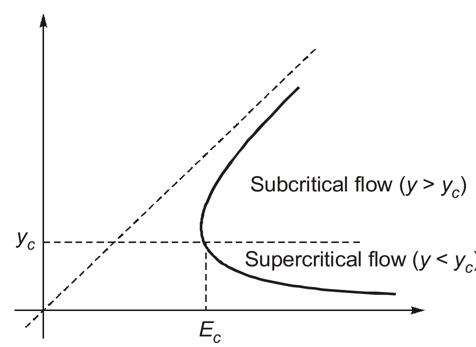
$\Rightarrow$ Now,
$$\frac{\frac{Q^2}{A^2}\cdot T}{gA} = 1$$

$$\frac{V^2T}{gA} = 1$$

$$\frac{V}{\sqrt{g\cdot\frac{A}{T}}} = 1$$

$$F_r = 1$$

- Thus, when the specific energy is minimum for a given discharge flow will be critical flow and depth of flow will be called as critical depth of flow (y_c) and velocity of flow will be called as critical velocity.
- When $y > y_c$; $V < V_c$
  $\Rightarrow$ Subcritical flow
  $y < y_c$; $V > V_c$
  $\Rightarrow$ Supercritical flow



- Also,
$$E = y + \frac{Q^2}{2g\,A^2}$$
$$Q = 2g\,A^2\,(E - y) \qquad ...(1)$$

For a given E, (specific energy) Q is maximum when

$$\frac{dQ}{dy} = 0$$

$\Rightarrow$
$$\frac{dQ^2}{dy} = 0$$

$$-A^2 + 2A\cdot\frac{dA}{dy}(E - y) = 0 \qquad ...(2)$$

From equation (1)

$$(E - y) = \frac{Q^2}{2g\,A^2}$$

Put this value in equation (2)

$$-A^2 + 2A\cdot T\cdot\frac{Q^2}{2g\,A^2} = 0$$

$$\frac{Q^2T}{g\,A^3} = 1 \qquad \text{Condition for critical flow}$$

$\therefore$ Also we can say that for a given specific energy discharge is maximum, flow will be critical flow.

NOTE

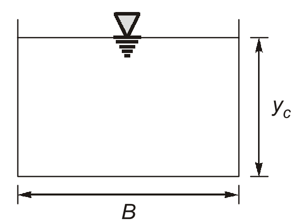
$$\frac{Q^2T}{gA^3} = 1$$ is a condition for critical state of flow such that

(a) for a given discharge, specific energy is minimum.

(b) for a given specific energy, discharge is maximum.

3.1.2 Computation of Critical Depth

1. Rectangular channel:



At $$y = y_c$$

$$\frac{Q^2T}{gA^3} = 1$$

$$\frac{Q^2 \cdot B}{g \cdot (B \times y_c)^3} = 1$$

$$\frac{Q^2 \cdot B}{g \cdot B^3} = y_c^3$$

$$\frac{Q^2}{gB^2} = y_c^3$$

$$\frac{q^2}{g} = y_c^3 \cdot \left(q = \frac{Q}{B}\right)$$

$\therefore$ $$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Critical specific energy:

$$E_c = y_c + \frac{Q^2}{2gA^2} \text{; for critical flow}$$

$$\frac{Q^2T}{gA^3} = 1$$

$\Rightarrow$ $$\frac{Q^2}{gA^2} = \frac{A}{T}$$

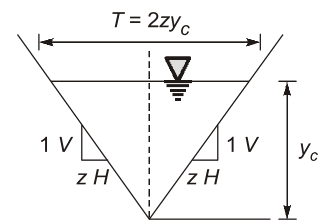
$\therefore$ $$E_c = y_c + \frac{A}{2T} \rightarrow \text{for any channel}$$

For a R.C $$E_c = y_c + \frac{By_c}{2 \cdot B} = y_c + \frac{y_c}{2}$$

$$E_c = \frac{3}{2}y_c$$

$\therefore$ $$V_c = \sqrt{g \cdot y_c}$$

2. Triangular channel:



$$\frac{Q^2 T}{g A^3} = 1$$

$$\frac{Q^2 \cdot (2z y_c)}{g \cdot (z y_c^2)^3} = 1$$

$$y_c^5 = \frac{2Q^2}{gz^2}$$

$$y_c = \left(\frac{2Q^2}{gz^2}\right)^{1/5}$$

$$E_c = y_c + \frac{A}{2T} = y_c + \frac{z y_c^2}{2 \times 2z y_c} = y_c + \frac{y_c}{4}$$

$$E_c = \frac{5}{4} y_c$$

NOTE: Section factor (z): It is a function of depth 'y' for a given channel geometry.

$$z = A \cdot \sqrt{D}$$

Example - 3.1 The term 'alternate depth' in an open channel flow refers to the

(a) Depths having the same specific energy for a given discharge.
(b) Depth before and after the passage of surge.
(c) Depths having same kinetic energy for a given discharge.
(d) Depths on either side of hydraulic jump.

Ans. (a)

Example - 3.2 In a rectangular channel, the ratio of the specific energy at critical depth (E_c) to critical depth (y_c) is

(a) 2.0 (b) 1.0
(c) 1.5 (d) 1.25

Ans. (c)

We know that for a rectangular channel,

$$E_c = \frac{3}{2} y_c$$

$$\therefore \quad \frac{E_c}{y_c} = \frac{3}{2} = 1.5$$

Example - 3.3 Which of the following represents the critical velocity for the discharge per unit width of q(m^3/s/m) from the wide rectangular channel ________.

(a) $\left(\frac{q}{g}\right)^{1/3}$ (b) $(qg)^{1/3}$

(c) $(qg)^{1/2}$ (d) None of these

Ans. (b)

Critical velocity

$$V_c = \sqrt{g y_c}$$

For rectangular channel

We know that

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$\therefore \quad V_c = \sqrt{g \cdot \left(\frac{q^2}{g}\right)^{1/3}} = \sqrt{(gq)^{2/3}}$$

$$= (gq)^{2/3 \times 1/2} = (qg)^{1/3}$$

$$V_c = (qg)^{1/3}$$

Example - 3.4 Which of the following expression represents the critical state of flow in non-rectangular channel?

(a) $y_c = \left(\frac{q^2}{g}\right)^{1/3}$ (b) $\frac{Q^2}{g} = \frac{A^3}{T}$

(c) $\frac{Q^3}{g} = \frac{A^2}{T}$ (d) $\frac{Q^2}{g} = \frac{A}{T^3}$

Ans. (b)

Example - 3.5 What is the specific energy (m-kg/kg) for 1 m depth of flow having velocity of 3 m/s?

(a) 0.54 (b) 1.46

(c) 5 (d) 7.62

Ans. (b)

We know that

Specific energy,

$$E = y + \frac{V^2}{2g} = 1 + \frac{(3)^2}{2 \times 9.81}$$

$$= 1.458 \simeq 1.46 \text{ m}$$

Example - 3.6 If the froude number of flow in a rectangular channel at a depth of flow of y_0 is F_0, then what is y_c/y_0 equal to?

(a) $F_0^{1/3}$ (b) $F_0^{2/3}$

(c) $F_0^{3/2}$ (d) $F_0^{-1/2}$