Production & Industrial Engineering

General Engineering

Vol. II : Applied Mechanics

Comprehensive Theory

with Solved Examples and Practice Questions

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Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016
E-mail: infomep@madeeasy.in
Contact: 011-45124660, 8860378007
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General Engineering

VOLUME II

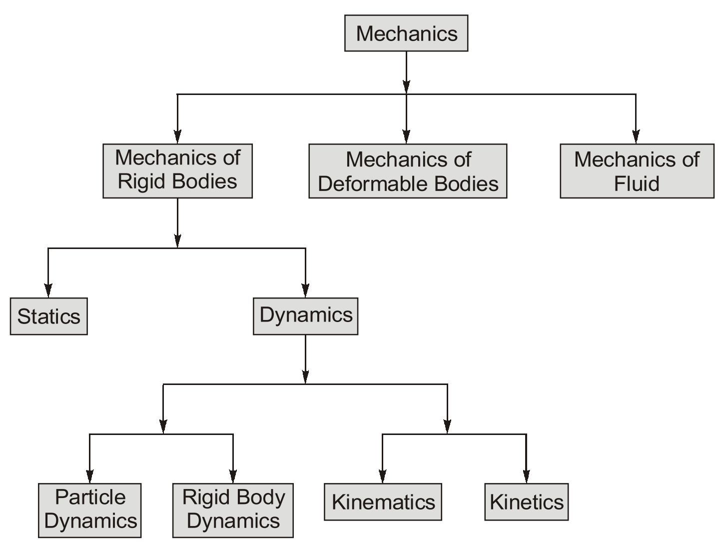
Applied Mechanics

Section I: Engineering Mechanics

INTRODUCTION

Applied mechanics (also **engineering mechanics**) is a branch of the physical sciences and the practical application of mechanics. Pure mechanics describes the response of bodies (solids and fluids) or systems of bodies to external forces. Some examples of mechanical systems include the flow of a liquid under pressure, the fracture of a solid from an applied force, or the vibration of an ear in response to sound. A practitioner of the discipline is known as a **mechanician.**

Applied mechanics describes the behavior of a body, in either a beginning state of rest or of motion, subjected to the action of forces. Applied mechanics, bridges the gap between physical theory and its application to technology. It is used in many fields of engineering, especially mechanical engineering and civil engineering. In this context, it is commonly referred to as **Engineering Mechanics**. Much of modern engineering mechanics is based on Isaac Newton's laws of motion while the modern practice of their application can be traced back to Stephen Timoshenko, who is said to be the father of modern engineering mechanics. Mechanics is classified as follows :



Statics: It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies at rest.

Dynamics: It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies in motion. The subject of dynamics may be further sub-divided into the following two branches :

1. Kinetics, and
2. Kinematics.

Kinetics: It is the branch of dynamics, which deals with the bodies in motion due to the application of forces.

Kinematics: It is that branch of dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

2.1 Force

Force can be defined as an action which changes or tends to change the state of rest or of uniform motion of a body. In order to represent the force acting on a body, the magnitude of the force, its point of action and direction of its action should be known.

There are different types of forces such as gravitational, electrical, magnetic or those caused by mass and acceleration.

According to Newton's second law of motion, force may be expressed as:

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

One Newton force is defined as that which gives an acceleration of 1 m/s^2 when applied to a body of 1 kg in the direction of motion.

2.1.1 Effects of a Force

A force may produce the following effects in a body, on which it acts :

1. It may change the motion of a body. i.e. if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate it.
2. It may retard the motion of a body.
3. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium. We shall study this effect in chapter 5 of this book.
4. It may give rise to the internal stresses in the body, on which it acts. We shall study this effect in the chapters 'Analysis of Perfect Frames' of this book.

2.1.2 Characteristics of Force

In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force :

1. Magnitude of the force (i.e., 100 N, 50 N, 20 kN, 5 kN, etc.)
2. The direction of the line, along which the force acts (i.e., along OX, OY, at 30° North of East etc.). It is also known as line of action of the force.
3. Nature of the force (i.e., whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.
4. The point at which (or through which) the force acts on the body.

2.1.3 Principle of Physical Independence of Force

It states that if a number of forces are simultaneously acting on a particle, then the resultant of these forces will have the same effect as produced by all the forces.

2.1.4 Principle of Transmissibility of Forces

It states that if a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body.

2.1.5 System of Forces

When two or more forces act on a body, they are called to form a system of forces. Following systems of forces are important from the subject point of view :

1. **Coplanar forces.** The forces, whose lines of action lie on the same plane, are known as coplanar forces.
2. **Collinear forces.** The forces, whose lines of action lie on the same line, are known as collinear forces
3. **Concurrent forces.** The forces, which meet at one point, are known as concurrent forces. The concurrent forces may or may not be collinear.
4. **Coplanar concurrent forces.** The forces, which meet at one point and their lines of action also lie on the same plane, are known as coplanar concurrent forces.
5. **Coplanar non-concurrent forces.** The forces, which do not meet at one point, but their lines of action lie on the same plane, are known as coplanar non-concurrent forces.
6. **Non-coplanar concurrent forces.** The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.
7. **Non-coplanar non-concurrent forces.** The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

2.1.6 Resultant Force

If a number of forces, P, Q, R ..... etc. are acting simultaneously on a particle, then it is possible to find out a single force which could replace them *i.e.*, which would produce the same effect as produced by all the given forces. This single force is called *resultant force* and the given forces R ..... etc. are called component forces.

2.1.7 Composition of Forces

The process of finding out the resultant force, of a number of given forces, is called composition of forces or compounding of forces.

2.1.8 Methods for the Resultant Force

Though there are many methods for finding out the resultant force of a number of given forces, yet the following are important from the subject point of view :

1. Analytical method
2. Method of resolution

2.1.9 Analytical Method for Resultant Force

The resultant force, of a given system of forces, may be found out analytically by the following methods :

1. Parallelogram law of forces
2. Method of resolution.

2.1.10 Parallelogram Law of Forces

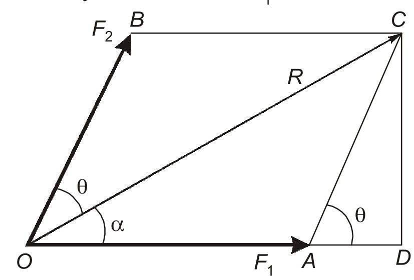
This law is used for finding the resultant of two forces acting at a point.

If two forces F_1 and F_2 are acting at a point and are represented in magnitude and direction by two sides of a parallelogram, then their resultant is represented by the diagonal of the parallelogram both in magnitude and direction.

Consider a parallelogram *OACB* as shown in figure where sides *OA* and *OB* represent the forces F_1 F_2 acting at a point *O*. According to the parallelogram law of forces, the resultant *R* is represented by a diagonal *OC*.

Let θ be the angle between the forces F_1 and F_2 and α be the angle made by *R* with force F_1.

From the figure we can write



$$BC = OA = F_1$$
$$AC = OB = F_2$$
$$\angle BOA = \theta = \angle CAD$$

and ΔODC and ΔADC are right angle triangles.

From triangle *ADC*, we can write

$$AD = AC\cos\theta = F_2\cos\theta$$
$$CD = AC\sin\theta = F_2\sin\theta$$

From triangle *ODC*, we can write

$$OC^2 = OD^2 + CD^2 = (OA + AD)^2 + CD^2$$
$$R^2 = (F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2$$
$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2\cos^2\theta + F_2^2\sin^2\theta$$
$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2(\cos^2\theta + \sin^2\theta)$$
$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2$$
$$R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2} \qquad \text{... (i)}$$

From triangle *ODC*,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta} \qquad \text{... (ii)}$$

Thus
$$R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2}$$

and
$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

Correlation :

1. If $\theta = 0$, i.e., when the forces act along the same line, then

$$R = F_1 + F_2 \qquad \text{...(since } \cos 0° = 1)$$

2. If $\theta = 90°$, i.e., when the forces act at right angle, then

$$R = \sqrt{F_1^2 + F_2^2} \qquad \text{...(since } \cos 90° = 0)$$

3. If $\theta = 180°$, i.e., when the forces act along the same straight line but in opposite directions, then

$$R = F_1 - F_2 \qquad \text{...(since } \cos 180° = -1)$$

In this case, the resultant force will act in the direction of the greater force.

4. If the two forces are equal, i.e., when $F_1 = F_2 = F$, then

$$R = \sqrt{F^2 + F^2 + 2F^2\cos\theta} = \sqrt{2F^2(1+\cos\theta)}$$
$$= \sqrt{2F^2 \times 2\cos^2\left(\frac{\theta}{2}\right)} \qquad \left[\because 1+\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)\right]$$
$$= \sqrt{4F^2\cos^2\left(\frac{\theta}{2}\right)} = 2F\cos\left(\frac{\theta}{2}\right)$$

Example 2.1 Two forces of 100 N and 150 N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°?

Solution:

Given : First force (F_1) = 100 N; Second force (F_2) = 150 N and angle between F_1 and F_2 (θ) = 45°.

We know that the resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

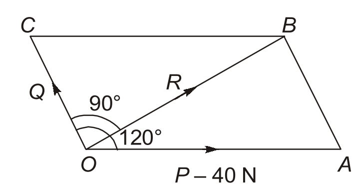
$$= \sqrt{(100)^2 + (150)^2 + 2\times 100\times 150\cos 45°}\ \text{N}$$

$$= \sqrt{10{,}000 + 22{,}500 + (30{,}000\times 0.707)}\ \text{N} = 232\ \text{N}$$

Example 2.2 Two forces act at an angle of 120°. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force.

Solution:

Given : Angle between the forces $\angle AOC$ = 120°. Bigger force (F_1) = 40 N and angle between the resultant and F_2 ($\angle BOC$) = 90°.



Let, F_2 = Smaller force in N

From the geometry of the figure, we find that $\angle AOB$

$$\alpha = 120° - 90° = 30°$$

We know that

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

$$\tan 30° = \frac{F_2\sin 120°}{40 + F_2\cos 120°} = \frac{F_2\sin 60°}{40 + F_2(-\cos 60°)}$$

$$0.577 = \frac{F_2\times 0.866}{40 - F_2\times 0.5} = \frac{0.866F_2}{40 - 0.5F_2}$$

$$40 - 0.5F_2 = \frac{0.866F_2}{0.577} = 1.5F_2$$

$$2F_2 = 40 \text{ or } F_2 = 20$$

Example 2.3 Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they Act at 60°, their resultant is $\sqrt{13}$ N.

Solution :

Given : Two forces = F_1 and F_2

First of all, consider the two forces acting at light angles. We know that when the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{F_1^2 + F_2^2}$$

or $$10 = F_1^2 + F_2^2 \qquad \text{...(Squaring both sides)}$$

Similarly, when the angle between die two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60°}$$

$\therefore$ $$13 = F_1^2 + F_2^2 + 2F_1F_2\times 0.5 \qquad \text{...(Squaring both sides)}$$

or $$F_1F_2 = 13 - 10 = 3 \qquad \text{...(Substituting } F_1^2 + F_2^2 = 10)$$

We know that $$(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2 = 10 + 6 = 16$$

$\therefore \qquad F_1 + F_2 = \sqrt{16} = 4 \qquad$ ...(i)

Similarly $\qquad (F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2 = 10 - 6 = 4$

$\therefore \qquad F_1 - F_2 = \sqrt{4} = 2 \qquad$ ...(ii)

Solving equations (i) and (ii),

$$F_1 = 3 \text{ N} \quad \text{and} \quad F_2 = 1 \text{ N}$$

2.1.11 Resolution of a Force

The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions. In fact, the resolution of a force is the reverse action of the addition of the component vectors.

Principle of Force Resolution

It states, "The algebraic sum of the resolved parts of a number of forces, in a given direction, is equal to the resolved part of their resultant in the same direction."

NOTE : In general, the forces are resolved in the vertical and horizontal directions.

2.1.12 Method of Resolution for Resultant Force

1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (i.e., ΣH).
2. Resolve all the forces vertically and find the algebraic sum of all the vertical components (i.e., ΣV).
3. The resultant R of the given forces will be given by the equation :

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

NOTE

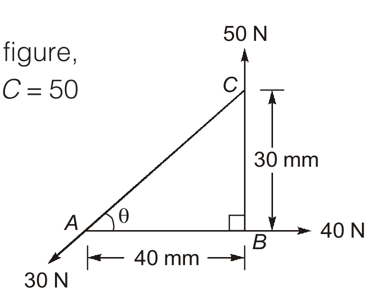
The value of the angle θ will vary depending upon the values of ΣV and ΣH as discussed below :

1. When ΣV is +ve, the resultant makes an angle between 0° and 180°. But when ΣV is –ve, the resultant makes an angle between 180° and 360°.
2. When ΣH is +ve, the resultant makes an angle between 0° to 90° or 270° to 360°. But when ΣH is –ve, the resultant makes an angle between 90° to 270°.

Example 2.4 **A triangle *ABC* has its side *AB* = 40 mm along positive x-axis and side *BC* = 30 mm along positive *y*-axis. Three forces of 40 N, 50 N and 30 N act along the sides *AB*, *BC* and *CA* respectively. Determine magnitude of the resultant of such a system of forces.**

Solution :

The system of given farces is shown in figure. From The geometry of the figure, we find that the triangle *ABC* is a right angled triangle, in which the side *AC* = 50 mm. Therefore



$$\sin\theta = \frac{30}{50} = 0.6$$

and $$\cos\theta = \frac{40}{50} = 0.8$$

Resolving all the farces horizontally (i.e., along AB),

$$\Sigma H = 40 - 30 \cos \theta$$
$$= 40 - (30 \times 0.8) = 16 \text{ N}$$

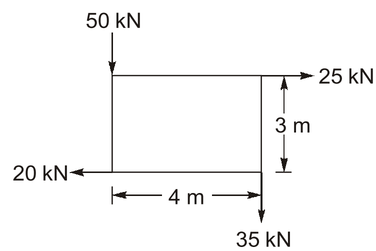
and now resolving all the forces vertically (*i.e.*, along BC)

$$\Sigma V = 50 - 30 \sin \theta = 50 - (30 \times 0.6) = 32 \text{ N}$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(16)^2 + (32)^2} = 35.8 \text{ N}$$

Example 2.5 A system of forces are acting at the corners of a rectangular block as shown in figure, Determine the magnitude and direction of the resultant force.



Solution :

Given : System of forces

Magnitude of the resultant force

Resolving forces horizontally, $\Sigma H = 25 - 20 = 5$ kN

and now resolving the forces vertically,

$$\Sigma V = (-50) + (-35) = -85 \text{ kN}$$

∴ Magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(5)^2 + (-85)^2} = 85.15 \text{ kN}$$

Direction of the resultant force

Let θ = Angle which the resultant force makes with the horizontal.

We know that

$$\tan \theta = \frac{\Sigma V}{\Sigma H} = \frac{-85}{5} = -17 \text{ or } \theta = 86.6°$$

Since ΣH is positive and ΣV is negative, therefore resultant lies between 270° and 360°. Thus actual angle of the resultant force

$$= 360° - 86.6° = 273.4°$$

Example 2.6 The following forces act at a point :

(i) 20 N inclined at 30° towards North of East,

(ii) 25 N towards North,

(iii) 30 N towards North West, and

(iv) 35 N inclined at 40° towards South of West.

Find the magnitude and direction of the resultant force.

Solution:

The system of given forces is shown in figure below.