CLASS TEST

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ENGINEERING MECHANICS

CIVIL ENGINEERING

Date of Test : 10/05/2024

ANSWER KEY

1.	(d)	6.	(a)	11.	(c)	16.	(c)	21.	(c)
2.	(a)	7.	(d)	12.	(b)	17.	(c)	22.	(c)
3.	(b)	8.	(b)	13.	(c)	18.	(b)	23.	(d)
4.	(b)	9.	(a)	14.	(a)	19.	(b)	24.	(a)
5.	(a)	10.	(c)	15.	(d)	20.	(d)	25.	(b)

DETAILED EXPLANATIONS

1. (d)

Given: Mass of elevator = 500 kg

Mass of operator = 100 kg

Upward acceleration = 3 m/s^2

Total tension in the cable of the elevator = $(m_1 + m_2)(g + a)$

$= (500 + 100)(10 + 3) = 600 \times 13$

Total tension in the cable of the elevator = 7800 N = 7.8 kN

2. (a)

Given:Velocity of first particle, u_1 = 10 m/s

Angle of projection for first particle, α_1 = 60°

Angle of projection for second particle, α_2 = 30°

Velocity of second particle, u_2 = ?

Given, Time of flight is same.

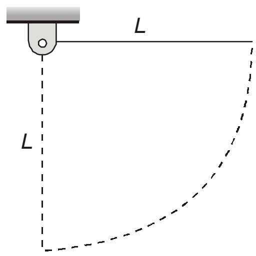
$$t_1 = t_2$$

$$\frac{2u_1 \sin\alpha_1}{g} = \left(\frac{2u_2 \sin\alpha_2}{g}\right)$$

$$u_2 = \frac{10 \times \sin 60°}{(\sin 30°)} = \frac{10 \times \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 10 \times \sqrt{3}$$

$$u_2 = 17.32 \text{ m/s}$$

3. (b)



Applying conservation of energy,

$$mgL = \frac{mgL}{2} + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad I\omega^2 = mgL$$

$$\Rightarrow \quad \frac{mL^2}{3}\omega^2 = mgL$$ [The moment of inertia about the end of the rod is $\frac{mL^2}{3}$]

$$\therefore \quad \omega = \sqrt{\frac{3g}{L}}$$

4. (b)

Using conservation of energy,

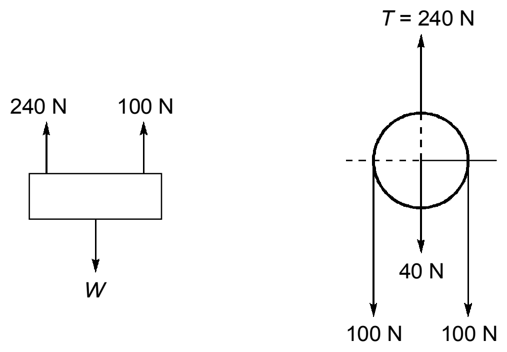
$$mgh = \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2 \times 0.04 \times 9.81 \times 4.9}{400}}$$

$$\therefore \quad x = 0.098 \text{ m} = 9.8 \text{ cm}$$

5. (a)

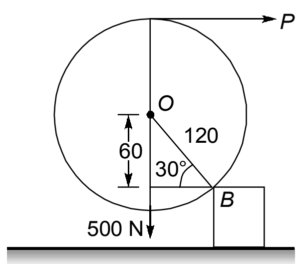
The FBD of the weight W is



So, $240 + 100 = W$ (240 N includes weight of pulley and tension carried by rope)

$$\therefore \quad W = 340 \text{ N}$$

6. (a)

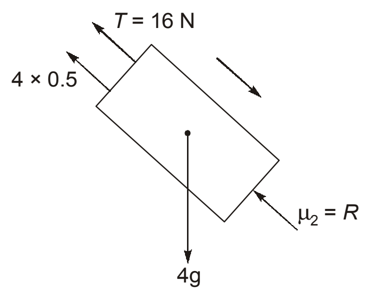


Taking moment about B,

$$P \times (60 + 120) = 500 \times 120\cos 30°$$

$$\therefore \quad P = 288.68 \text{ N}$$

7. (d)



$$\Rightarrow \quad \mu_2 R + 4 \times 0.5 + 16 - 4g\sin 30° = 0$$

$$\Rightarrow \quad \mu_2 20\sqrt{3} + 2 + 16 - 20 = 0$$

$$\Rightarrow \quad \mu_2 = \frac{2}{20\sqrt{3}} = 0.0577$$

8. (b)

$$\text{Speed of flow} = 7 - 5 = 2 \text{ km/h}$$

$$\text{Speed of swimmer with flow} = 7 + 2 = 9 \text{ km/hr}$$

$$\text{Time required} = \frac{90}{9} = 10 \text{ hour}$$

9. (a)

Change in the stored energy of rubber band = $F\,dx$

$$\Rightarrow \quad dE = 300x^2 dx$$

Integrating,
$$\int_0^E dE = \int_0^{0.1} 300x^2\,dx$$

$$\Rightarrow \quad E = 300 \times \left.\frac{x^3}{3}\right|_0^{0.1} = 0.1 \text{ Joule}$$

10. (c)

Given: $m_A = 15$ kg, $m_B = 10$ kg

For mass B,
$$m_B g - T = m_B a$$
$$10g - T = 10\,a \quad \text{...(i)}$$

For mass A,
$$T = m_A a$$
$$T = 15\,a \quad \text{...(ii)}$$

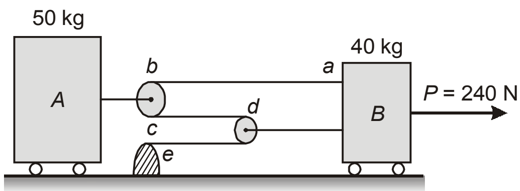
Addition equation (i) and (ii)

$$(10g - T) + (T) = (15 + 10)a$$

$$a = \frac{10g}{25} = \frac{10 \times 10}{25} = 4 \text{ m/s}^2$$

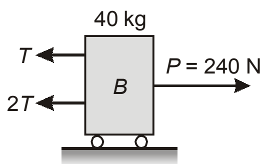
$$\text{Acceleration, } a = 4 \text{ m/s}^2$$

11. (c)



As given, acceleration $a_A = 1.5\,a_B$

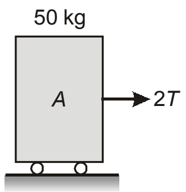
For block *B*:



$$\Sigma F = \text{Mass} \times \text{Acceleration}$$

$$240 - 3T = 40\,a_B \quad \text{...(i)}$$

For block *A*:



$\Rightarrow \quad \Sigma F = \text{Mass} \times \text{Acceleration}$

$\Rightarrow \quad 2T = 50\, a_A$ ...(ii)

$\Rightarrow \quad 2T = 50 \times 1.5\, a_B$

$\Rightarrow \quad 2T = 75\, a_B$ ...(iii)

Using equation (i) and (iii), we get

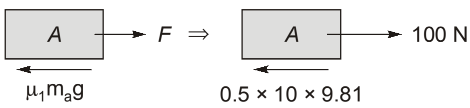
$\Rightarrow \quad 240 - 1.5 \times 75\, a_B = 40\, a_B$

$\Rightarrow \quad 152.5\, a_B = 240$

$\therefore \quad a_B = 1.57 \text{ m/s}^2$

12. (b)

Free body diagram of A:

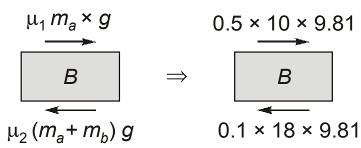


Writing equation of motion for A.

$100 - 0.5 \times 10 \times 9.81 = 10a$

$\Rightarrow \quad a = 5.095 \text{ m/s}^2$

Free body diagram of B:



Writing equation of motion for B.

$49.05 - 17.658 = 8\, a$

$\Rightarrow \quad a = 3.924 \text{ m/s}^2$

After 0.1s, $V_A = U_a + a_a t.$

$V_A = 0 + 5.095 \times 0.1$

$V_A = 0.5095 \text{ m/s}$

Similarly, $V_B = 0 + 3.924 \times 0.1$

$V_B = 0.3924 \text{ m/s}$

$\therefore$ Relative velocity of A w.r.t. B $= V_A - V_B$

$= 0.5095 - 0.3924 \simeq 0.12 \text{ m/s}$

13. (c)

$\omega = 12 + 9t - 3t^2$

$$\frac{d\omega}{dt} = 9 - 6t = 0$$

$\Rightarrow \quad t = 1.5\text{s}$

$$\frac{d^2\omega}{dt^2} = -6 < 0$$

Hence, at t = 1.5 sec maximum value of angular velocity will occur

$$\therefore \quad \omega_{max} = 12 + 9 \times 1.5 - 3 \times 1.5^2$$
$$= 12 + 13.5 - 6.75$$
$$= 18.75 \text{ rad/s}$$

14. (a)

$$x = 10 \sin 2t + 15 \cos 2t + 100$$
$$v = \frac{dx}{dt} = 20 \cos 2t - 30 \sin 2t$$
$$a = \frac{dv}{dt} = -40 \sin 2t - 60 \cos 2t \quad \text{...(i)}$$

For a_{max}, $\frac{da}{dt} = 0$

$\Rightarrow$ $-80 \cos 2t + 120 \sin 2t = 0$

$$\tan 2t = \frac{2}{3}$$

$\Rightarrow$ $2t = 33.69$

Now using equation (i), we get

$$a_{max} = -40 \sin (33.69) - 60 \times \cos (33.69) = -72.11 \text{ mm/s}^2$$

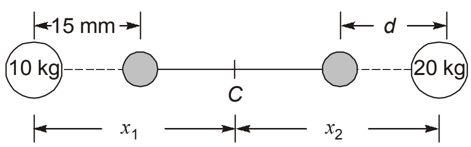
15. (d)

$$I = -2\hat{i} - \hat{j} + \hat{k}$$
$$r = 2\hat{i} - 3\hat{j} + 2\hat{k}$$

Angular momentum $= H = r \times I$

$$= \left(2\hat{i} - 3\hat{j} + 2\hat{k}\right) \times \left(-2\hat{i} - \hat{j} + \hat{k}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ -2 & -1 & 1 \end{vmatrix}$$
$$= \hat{i}(-3+2) - \hat{j}(2+4) + \hat{k}(-2-6)$$
$$= \hat{i}(-1) - 6\hat{j} - 8\hat{k} = -\hat{i} - 6\hat{j} - 8\hat{k}$$
$$|H| = \sqrt{1^2 + 6^2 + 8^2} = 10.01 \text{ kg m}^2/\text{s} \simeq 10 \text{ kg m}^2/\text{s}$$

16. (c)



To keep centre of mass at C

$$m_1 x_1 = m_2 x_2 \quad \rightarrow \quad (\text{Let } 10 \text{ kg} = m_1, 20 \text{ kg} = m_2)$$

and $m_1(x_1 - 15) = m_2(x_2 - d)$

$$15\,m_1 = m_2 d$$

$$d = \frac{15\times10}{20} = 7.5\text{ mm}$$

17. **(c)**

$$|\vec{V}| = \frac{ds}{dt} = 3t^2$$

Now, $$a_r = \frac{v^2}{R} = \frac{\left(3\times(2)^2\right)^2}{20} = 7.2\text{ m/s}^2$$

and $$a_t = \frac{dv}{dt} = 6t = 12\text{ m/s}^2$$

∴ $$a = \sqrt{a_r^2 + a_t^2} = 14\text{ m/s}^2$$

18. **(b)**

Using conservation of angular momentum,

$$2\,mvr = I\omega,$$

where, $$I = \frac{MR^2}{2}$$

⇒ $$2\times0.05\times9\times0.25 = \frac{1}{2}\times0.45\times0.5^2\times\omega$$

∴ $$\omega = 4\text{ rad/s}$$

19. **(b)**

Now, $$\Sigma F_x = 0 - R_{B2} = -P$$

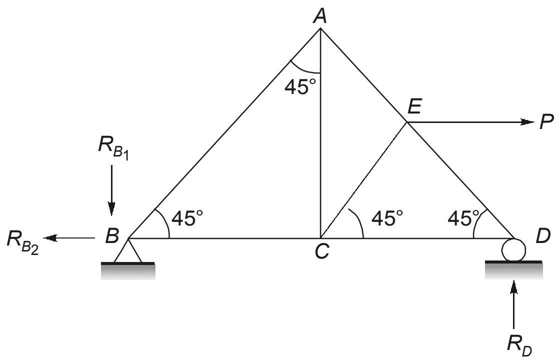
⇒ $$R_{B2} = P$$

and, $$\Sigma F_y = 0 - R_D = -R_{B1}$$

⇒ $$R_D = R_{B1}$$

Also, $$\Sigma M_B = R_D\times 2a = P\times\frac{a}{2}$$

⇒ $$R_D = R_{B1} = \frac{P}{4}$$



Analysis of joint B,

So, $$F_{AB}\sin45 = \frac{P}{4}$$

⇒ $$F_{AB} = \frac{\sqrt{2}P}{4}$$

Also, $$P = F_{BC} + F_{AB}\cos45°$$

⇒ $$F_{BC} = P - F_{AB}\cos45° = P - \frac{\sqrt{2}P}{4}\times\frac{1}{\sqrt{2}} = \frac{3P}{4}$$

Hence, $$F_{BC} = 0.75P$$

20. (d)

$$5 = \frac{1}{2}\times(10)t^2$$

$\Rightarrow$ $t = 1$ sec

Now, $V_{ball} = 20$ m/s

$V_{bullet} = 100$ m/s

Also, by conservation of momentum, we have

$$0.01\,V = 0.2 \times 20 + 0.01 \times 100$$

$$\therefore \quad V = \frac{4+1}{0.01} = \frac{5}{0.01} = 500 \text{ m/s}$$

21. (c)

Coefficient of restitution,

$$e = -\frac{\Delta V}{\Delta u} = -\frac{v_2 - v_1}{u_2 - u_1}$$

here, $u_2 = 0$,

$v_2 = 0$

$$e = \frac{v_1}{u_1}$$

$$v^2 - u^2 = 2\,ah$$

when ball is dropped from height,

$$u = 0$$

Let final velocity is u_1

$$u_1^2 = 2ah_1$$

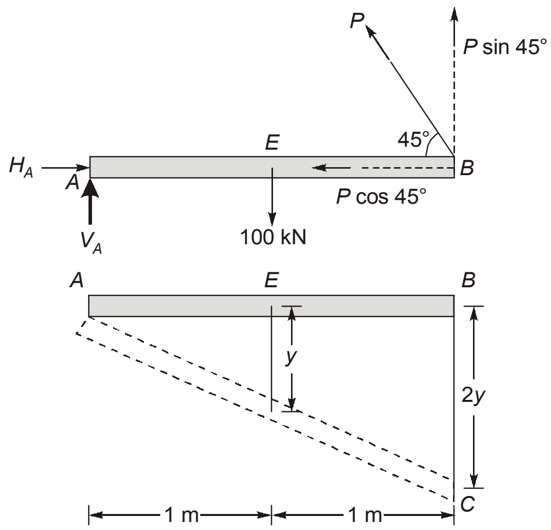
$$v_1^2 = 2ah_2$$

$$e^2 = \left(\frac{v_1}{u_1}\right)^2 = \frac{h_2}{h_1}$$

$$\therefore \quad h_2 = h_1 \times e^2 = 0.36 \text{ m}$$

22. (c)

Free body diagram of beam *AB*,

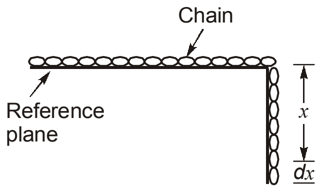


Now using the principle of virtual work done, if C.G. of beam AB shifts by an amount 'y' then end B must shift by '$2y$' (using similar triangles).

$\therefore \quad 100 \times y - P \sin 45° \times 2y = 0$

$\Rightarrow \quad P = 70.71$ kN

23. (d)



The potential energy of $\frac{l}{3}$ of the chain that overhangs is

$$u_1 = \int_0^{l/3} -\frac{mgx}{l}dx = \frac{-mgl}{18}$$

The potential energy of the full chain when it completely slips off the table is

$$u_2 = \int_0^{l} -\frac{mgx}{l}dx = \frac{-mgl}{2}$$

$$\text{The loss in } PE = \frac{-mgl}{18} - \left(\frac{-mgl}{2}\right) = \frac{4mgl}{9}$$

This should be equal to gain in kinetic energy, but the initial kE is zero. Hence this is the kE when the chain completely falls off the table.

24. (a)

Given: $P = 250$ N; $BF_1 = 25$ mm; $F_1A = 325$ mm; $CD = 360$ mm; $DF_2 = 40$ mm

Leverage of the upper lever, $AB = \frac{AF_1}{BF_1} = \frac{325}{25} = 13$

Leverage of the lower lever, $CF_2 = \frac{CF_2}{DF_2} = \frac{360+40}{40} = 10$

Total leverage of the compound lever = 13 × 10 = 130

We know that,Total leverage = $\frac{W}{P} = \frac{W}{250}$

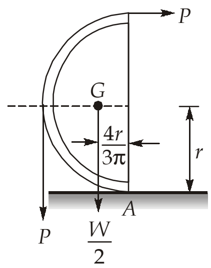
$$130 = \frac{W}{250}$$

$$W = 130 \times 250 = 32500 \text{ N} = 32.5 \text{ kN}$$

25. (b)

Taking one halve of cylinder. Centre of gravity of a semicircle is at a distance of $\frac{4r}{3\pi}$ from centre.

Taking moment about A,



$$P \times 2r = P \times r + \left(\frac{W}{2}\right) \times \left(\frac{4r}{3\pi}\right)$$

$$P \times r = W\left(\frac{2r}{3\pi}\right)$$

$$P = \frac{2W}{3\pi}$$

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