CLASS TEST

S.No. : 13 LS1_EC_S_050919

Engineering Mathematics

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CLASS TEST 2019-2020

ELECTRONICS ENGINEERING

Date of Test : 05/09/2019

ANSWER KEY > Engineering Mathematics

1.	(c)	7.	(a)	13.	(c)	19.	(b)	25.	(b)
2.	(b)	8.	(b)	14.	(b)	20.	(b)	26.	(c)
3.	(a)	9.	(d)	15.	(d)	21.	(a)	27.	(a)
4.	(d)	10.	(a)	16.	(a)	22.	(d)	28.	(c)
5.	(a)	11.	(d)	17.	(d)	23.	(a)	29.	(b)
6.	(b)	12.	(a)	18.	(c)	24.	(d)	30.	(d)

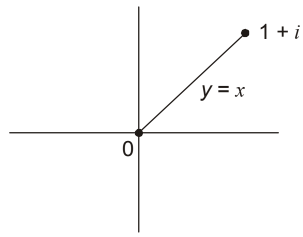
DETAILED EXPLANATIONS

1. (c)

$$dz = dx + i\,dy$$

$$= \int_0^{1+i} (x - y + ix^2)(dx + idy)$$

$$= \int_0^1 (x - x + ix^2)(dx + idx) \qquad (\because \text{ along the given straight line } y = x)$$



$$= \int_0^1 ix^2(1+i)dx$$

$$= \int_0^1 (1+i)\,i\,x^2 dx$$

$$= (i-1)\left.\frac{x^3}{3}\right|_0^1 = \frac{1}{3}(-1+i)$$

2. (b)

Let $e^x = p$

$$e^x dx = dp$$

$$I = \int_0^{\infty} \frac{dx}{e^x + e^{-x}} = \int_0^{\infty} \frac{e^x dx}{e^{2x} + 1}$$

$$= \int_{p=e^0=1}^{p=e^{\infty}=\infty} \frac{dp}{p^2+1} \Rightarrow \left(\tan^{-1} p\right)_1^{\infty}$$

$$= \tan^{-1}\infty - \tan^{-1} 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

3. (a)

Eigen value of A are, $\lambda_1,\ \lambda_2,\ \lambda_3$

$$|A| = \lambda_1 \cdot \lambda_2 \cdot \lambda_3$$

Eigen value of A^{-1} is $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}$

$$\frac{1}{\lambda_1} = 1 \Rightarrow \qquad \lambda_1 = 1$$

$$\frac{1}{\lambda_2} = 2 \Rightarrow \qquad \lambda_2 = \frac{1}{2}$$

$$\frac{1}{\lambda_3} = 5 \Rightarrow \qquad \lambda_3 = \frac{1}{5}$$

$$\lambda_1\,\lambda_2\,\lambda_3 = (1)\left(\frac{1}{2}\right)\left(\frac{1}{5}\right) = \frac{1}{10} = 0.1$$

$$|A| = 0.1$$

4. (d)

$$y = \sec(\tan^{-1} x)$$

$$\frac{dy}{dx} = \sec(\tan^{-1} x)\cdot\tan(\tan^{-1} x)\cdot\frac{1}{1+x^2}$$

$$\left[\frac{d\sec x}{dx} = \sec x\tan x\right]$$

for $x = 1$

$$\tan^{-1}(x) = \frac{\pi}{4}$$

$$\frac{dy}{dx} = \sec\left(\frac{\pi}{4}\right)\cdot\tan\left(\frac{\pi}{4}\right)\cdot\frac{1}{1+1^2}$$

$$= \sqrt{2}\times 1\times\frac{1}{2}$$

$$= \frac{1}{\sqrt{2}} = 0.707$$

5. (a)

Diverge of curl of a vector is always zero.

6. (b)

$$2x + y + 2z = 0$$
$$x + y + 3z = 0$$
$$4x + 3y + z = 0$$

$$[A : B] = \begin{bmatrix} 2 & 1 & 2 & 0 \\ 1 & 1 & 3 & 0 \\ 4 & 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 4 & 3 & 1 & 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 4R_1$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & -11 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$,

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & -7 & 0 \end{bmatrix}$$

Rank of $[A : B] = 3$

Rank of $[A] = 3 =$ Rank of $[A : B]$ = number of unknowns

So, unique soluton exists

7. (a)

$$f_X(x) = \begin{cases} 1; & 0 < x < 1 \\ 0; & \text{otherwise} \end{cases}$$

$$E(X^4) = \int_{-\infty}^{\infty} x^4 f_x(x)\,dx = \int_0^1 x^4.1\,dx = \left(\frac{x^5}{5}\right)_0^1 = \frac{1}{5} = 0.2$$

8. (b)

$$(D^2+4)y = 10\sin x$$

to get PI, put $D^2 = -1$

$$PI = \left.\frac{10\sin x}{D^2+4}\right|_{D^2=-1} = \frac{10\sin x}{3}$$

$$A\sin x = 3.33\sin x$$

$$A = 3.33$$

9. (d)

$$|A-\lambda I| = \begin{bmatrix} \cos\alpha-\lambda & \sin\alpha \\ -\sin\alpha & \cos\alpha-\lambda \end{bmatrix} = 0$$

$$(\cos\alpha-\lambda)^2+\sin^2\alpha = 0$$

$$(\cos\alpha-\lambda) = \pm i\sin\alpha$$

$$\lambda = e^{\pm i\alpha}$$

10. (a)

$$I = \int_0^{\pi/2}\log\left(\frac{\sin x}{\cos x}\right)dx$$

$$= \int_0^{\pi/2}[\log(\sin x)dx - \log(\cos x)dx] \qquad \left[\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

$$= \int_0^{\pi/2}\log\sin\left(\frac{\pi}{2}-x\right)dx - \int_0^{\pi/2}\log(\cos x)dx$$

$$= \int_0^{\pi/2}\log(\cos x)dx - \int_0^{\pi/2}\log(\cos x)dx$$

$$I = 0$$

11. (d)

$$D^2+7D+12 = 0$$

$$(D+3)(D+4) = 0$$

$$D = -3, -4$$

$$y = C_1e^{-3x}+C_2e^{-4x}$$

$$y(0) = C_1+C_2 = 1$$

$$y'(0) = -3C_1-4C_2 = 0$$

$\Rightarrow$

$$-3C_1-4C_2 = 0$$

$$3C_1+3C_2 = 3$$

$$C_2 = -3$$

$$C_1 = 4$$

$$y(x) = 4e^{-3x}-3e^{-4x}$$

12. (a)

$$P(X=k) = \frac{e^{-\lambda}\cdot\lambda^k}{k!} \qquad [\lambda \text{ is mean}]$$

$$P(X=2) = \frac{e^{-\lambda}\cdot\lambda^2}{2!}$$

$$P(X=4) = \frac{e^{-\lambda}\cdot\lambda^4}{4!}$$

$$P(X=6) = \frac{e^{-\lambda}\cdot\lambda^6}{6!}$$

Given that, $P(X=2) = 9P(X=4) + 90P(X=6)$

$$\frac{e^{-\lambda}\cdot\lambda^2}{2} = \left(\frac{9\cdot\lambda^4}{24} + \frac{90\cdot\lambda^6}{30\cdot 24}\right)e^{-\lambda}$$

$\Rightarrow$ $12\lambda^2 = 9\lambda^4 + 3\lambda^6$

$\Rightarrow$ $4\lambda^2 = 3\lambda^4 + \lambda^6$

$\Rightarrow$ $\lambda \neq 0$

$4 = 3\lambda^2 + \lambda^4$

$\lambda^2 = 1$ or $\lambda^2 = -4$ which is not possible

So, $\lambda = \pm 1$

Thus option (a) is correct.

13. (c)

For a diagonal matrix

$$\lambda_1, \lambda_2 = a, b$$

$$\lambda_1 = a$$

$$\lambda_2 = b$$

$$ab = 25$$

we know, $AM \geq GM$

$$\frac{\lambda_1+\lambda_2}{2} \geq \sqrt{\lambda_1\lambda_2} = \sqrt{ab} = 5$$

$$\lambda_1 + \lambda_2 \geq 10$$

$$\min(\lambda_1 + \lambda_2) = 10$$

14. (b)

$$\nabla\times\vec{F} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{vmatrix}$$

$$= -a_x(1) - a_y - a_z$$

$$= -a_x - a_y - a_z$$

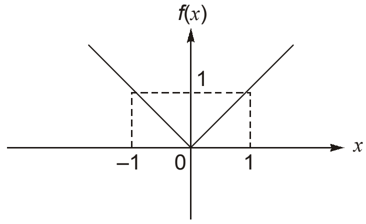
$$\vec{ds} = (dx\,dz)\hat{a}_y$$

$$\int_s (\nabla\times\vec{F})\cdot\vec{ds} = -\iint dx\,dz$$

$$= -\pi r^2\Big|_{r=2}$$

$$= -\pi(4) = -4\pi \approx -12.57$$

15. (d)

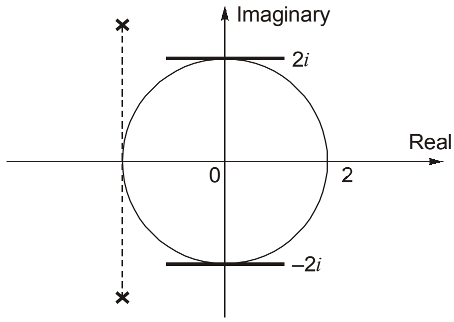


$$f(x) = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 1; & x \geq 0 \\ -1; & x < 0 \end{cases}$$

Thus left hand derivative ≠ Right hand derivative
Hence at $x = 0$, it is not derivable
$f'(x)$ does not exist.

16. (a)

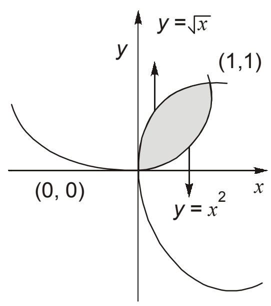


$$z^2 + 4z + 13 = 0$$
$$(z+2)^2 + 9 = 0$$
$$z = -2 \pm 3i$$

Since the poles lie outside the circle/curve given, the integral value is zero.

17. (d)

Here, $M = 2xy - x^2,\ N = -(x^2 + y^2)$



$$\frac{\partial M}{\partial y} = 2x, \quad \frac{\partial N}{\partial x} = -2x$$

$$\Rightarrow \quad \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = -4x$$

∴ By Green's theorem

$$\oint_C M dx + N dy = \iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] dx\, dy$$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} -4x\, dy\, dx$$

$$= \int_0^1 -4x[y]_{x^2}^{\sqrt{x}}\, dx$$

$$= -4\left[\frac{x^{5/2}}{(5/2)} - \frac{x^4}{4}\right]_0^1$$

$$= -4\left[\frac{2}{5} - \frac{1}{4}\right] = -\frac{3}{5} = -0.6$$

18. (c)

$$f'(x) = 2x - 1$$

solve, $f'(x) = 0$

$\Rightarrow$ $x = \dfrac{1}{2}$ point of inflection

$$f''(x) = 2 > 0$$

so, at $x = \dfrac{1}{2}$, $f'(x)$ has minima

$$f\left(\frac{1}{2}\right) = 0.25 - 0.5 - 2 = -2.25$$

$$f(-4) = 16 + 4 - 2 = 18$$

$$f(4) = 16 - 4 - 2 = 10$$

19. (b)

$$u = x^3 + y^3$$

$$= a^3 \cos^3 t + b^3 \sin^3 t$$

$$\frac{du}{dt} = -3a^3 \cos^2 t \sin t + 3b^3 \sin^2 t \cos t$$

$$= 3 \sin t \cos t\,(-a^3 \cos t + b^3 \sin t)$$

$$= \frac{3}{2}\sin 2t(-a^3 \cos t + b^3 \sin t)$$

$$\left.\frac{du}{dt}\right|_{t=\frac{\pi}{4}} = \frac{3}{2} \times \frac{1}{\sqrt{2}}[-a^3 + b^3] = \frac{3}{2\sqrt{2}}(-a^3 + b^3)$$

20. (b)

If $A^T = A^{-1} \Rightarrow A \cdot A^T = I$

$$\Rightarrow \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix} \begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{3}{5}x + \frac{4}{5} \times \frac{3}{5} = 0$$

$$\Rightarrow \quad x = \frac{-4}{5}$$

21. (a)

$$\int_0^{0.4} f(x)dx = \frac{h}{2}[(y_0 + y_4) + 2(y_1 + y_2 + y_3)]$$

$$= \frac{0.1}{2}[(0+160) + 2(10+40+90)] = 22$$

22. (d)

$$\lim_{x\to 0} \frac{\sqrt{3+x} - \sqrt{3-x}}{x}$$

$$= \lim_{x\to 0} \frac{\sqrt{3+x} - \sqrt{3-x}}{x} \times \frac{\sqrt{3+x} + \sqrt{3-x}}{\sqrt{3+x} + \sqrt{3-x}}$$

$$= \lim_{x\to 0} \frac{(3+x) - (3-x)}{x(\sqrt{3+x} + \sqrt{3-x})}$$

$$= \lim_{x\to 0} \frac{2x}{x(\sqrt{3+x} + \sqrt{3-x})}$$

$$= \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

23. (a)

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -8 & 1 & -3/2 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -8 & 1 & -3/2 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -3/2 & -8 \\ 0 & 1 & 0 & 0 & 1/2 & 2 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

$$\therefore \; A^{-1} = \begin{bmatrix} 1 & -3/2 & -8 \\ 0 & 1/2 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

24. (d)

1, –1, 2, –2 are eigens of A

∴ characteristic equation is

$$(\lambda-1)(\lambda+1)(\lambda-2)(\lambda+2) = 0$$
$$(\lambda^2-1)(\lambda^2-4) = 0$$
$$\lambda^4-5\lambda^2+4 = 0$$

By Cayley Hamilton theorem,

$$A^4-5A^2+4I = 0$$
$$A^4-5A^2 = -4I$$
$$B = A^4-5A^2+5I$$
$$\det B = \det(A^4-5A^2+5I)$$
$$= \det(-4I+5I)$$
$$= \det I = 1$$

∴ Option (b) is correct

Since – 1, 1, 2, –2 are eigen values of A

– 1 + 1, 1 + 1, 2 + 1, – 2 + 1 are eigens of A + I

0, 2, 3, –1 are eigens of A + I

$$\text{Hence } \det(A+B) = \det(A+A^4-5A^2+5I)$$
$$= \det(A-4I+5I)$$
$$= \det(A+I)$$
$$=(0)(2)(3)(-1) = 0$$

Hence option (a) is correct

$$\text{trace of } (A+B) = \text{trace of } (A+I) = 0+2+3-1 = 4$$

Hence option (c) is correct

25. (b)

$$\int_0^y \cos t^2\, dt = \int_0^{x^2} \frac{\sin t\, dt}{t}$$

Differentiating both sides w.r.t x

$$\frac{d}{dy}\left(\int_0^y \cos t^2 dt\right)\cdot\frac{dy}{dx} = \frac{d}{dx^2}\left(\int_0^{x^2}\frac{\sin t}{t}dt\right)\cdot\frac{dx^2}{dx}$$

$$\cos y^2\cdot\frac{dy}{dx} = \frac{\sin x^2}{x^2}\cdot 2x$$

$$\frac{dy}{dx} = \frac{2\sin x^2}{x\cdot\cos y^2}$$

26. (c)

$$I = \int_0^{\pi/2} \log\sin x\, dx \qquad \text{...(i)}$$

$$I = \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right) dx$$

$$I = \int_0^{\pi/2} \log\cos x\, dx \qquad \text{...(ii)}$$

Adding (i) and (ii)

$$2I = \int_0^{\pi/2} (\log\sin x + \log\cos x) dx = \int_0^{\pi/2} \log\frac{\sin 2x}{2} dx$$

$$2I = \int_0^{\pi/2} \log\sin 2x \cdot dx - \int_0^{\pi/2} \log 2 dx$$

$$I' = \int_0^{\pi/2} \log\sin 2x\, dx$$

Let $2x = t$, so $dx = \frac{dt}{2}$

$$I' = \int_0^{\pi} \log\sin t \frac{dt}{2}$$

$$I' = \frac{1}{2} \times 2 \int_0^{\pi/2} \log\sin t\, dt$$

$$I' = \int_0^{\pi/2} \log\sin t\, dt = I$$

$$2I = I - \int_0^{\pi/2} \log 2 dt$$

$$I = -\frac{\pi}{2} \log 2$$

27. (a)

$$\frac{dy}{dx} + 2xy = e^{-x^2}$$

$$IF = e^{\int 2x \cdot dx} = e^{x^2}$$

General solution is

$$y \cdot IF = \int Q(x) \cdot IF\, dx + c$$

$$y \cdot e^{x^2} = \int e^{-x^2} \cdot e^{x^2} dx + c$$

$$y \cdot e^{x^2} = x + c$$

$$y = (x + c)e^{-x^2}$$

28. (c)

The equation $x^2 + bx + c = 0$ has roots α and β.

So $x^2 + bx + c = (x - \alpha)(x - \beta)$

$$\lim_{x\to\alpha}\frac{1-\cos(x^2+bx+c)}{(x-\alpha)^2} = \lim_{x\to\alpha}\frac{2\sin^2\left(\frac{x^2+bx+c}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha}\frac{2\sin^2[(x-\alpha)(x-\beta)/2]}{(x-\alpha)^2}$$

$$= 2\lim_{x\to\alpha}\left[\frac{\sin(x-\alpha)(x-\beta)/2]}{\frac{1}{2}(x-\alpha)\cdot(x-\beta)}\right]^2\frac{1}{4}(x-\beta)^2$$

$$= \frac{2}{4}(\alpha-\beta)^2$$

$$= \frac{2}{4}\left[(\alpha+\beta)^2-4\alpha\beta\right]$$

$$\left[\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$= \frac{2}{4}\left[b^2-4c\right]$$

$$= \frac{1}{2}\left[b^2-4c\right]$$

29. (b)

$$h = 10$$

$$\text{Area} = \frac{h}{2}[y_0+2(y_1+y_2+\ldots\ldots+y_{n-1})+y_n]$$

$$= \frac{10}{2}[0+2(4+7+9+12+15+14+8)+3]$$

$$= 705\ \text{m}^2$$

30. (d)

From here,

$$f(x) = x+\sqrt{x}-3$$

$$f'(x) = 1+\frac{1}{2\sqrt{x}}$$

$$x_1 = 2-\frac{f(x)}{f'(x)} = 2-\frac{2+\sqrt{2}-3}{1+\frac{1}{2\sqrt{2}}} = 1.694$$

$$x_2 = 1.694-\frac{f(1.694)}{f'(1.694)}$$

$$= 1.694-\frac{1.694+\sqrt{1.694}-3}{1+\frac{1}{2\sqrt{1.694}}} = 1.697$$

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