S.No. : 15 LS_EC_S_031019
Engineering Mathematics

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CLASS TEST 2019-2020

ELECTRONICS ENGINEERING

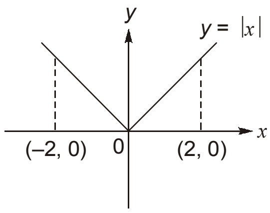
Date of Test : 03/10/2019

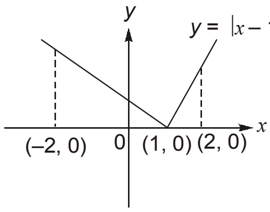
ANSWER KEY > Engineering Mathematics

1.	(c)	7.	(a)	13.	(d)	19.	(d)	25.	(c)
2.	(d)	8.	(d)	14.	(b)	20.	(b)	26.	(b)
3.	(b)	9.	(d)	15.	(b)	21.	(c)	27.	(b)
4.	(d)	10.	(b)	16.	(d)	22.	(c)	28.	(c)
5.	(b)	11.	(c)	17.	(b)	23.	(a)	29.	(d)
6.	(c)	12.	(c)	18.	(a)	24.	(d)	30.	(c)

DETAILED EXPLANATIONS

1. (c)





$$\int_{-2}^{2}(|x|\,dx)+\int_{-2}^{2}(|x-1|\,dx) = \text{Area under the curves}$$

$$= 2\times\frac{1}{2}\times 2\times 2+\frac{1}{2}\times 3\times 3+\frac{1}{2}\times 1\times 1 = 4+\frac{9}{2}+\frac{1}{2} = 9\text{ unit}^2$$

2. (d)

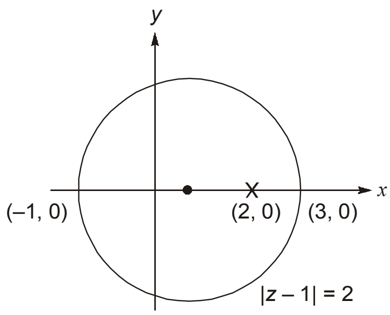
Here,
$$n = 10000$$
$$p = 0.02$$
$$q = 1 - 0.02 = 0.98$$
$$\text{Standard deviation} = \sqrt{npq} = \sqrt{10000\times 0.02\times 0.98} = \sqrt{196} = 14$$

3. (b)

$$\text{div}\,\phi = \nabla\cdot\phi = \left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\cdot(\phi)$$
$$= \frac{\partial}{\partial x}(3xz)+\frac{\partial}{\partial y}(2xy)-\frac{\partial}{\partial z}yz^2 = 3z + 2x - 2yz$$
$$= 3\times 2 + 2\times 2 - 2\times 1\times 2 = 6$$

4. (d)

$$\frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}dz = f(a)$$
$$= 2\times x\big|_{z=2+i0}$$
$$= 2x\big|_{x+iy=2}$$
$$= 2\times 2$$
$$= 4$$



5. (b)

If A is orthogonal, then

$$AA^T = I$$

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x^2 + y^2 + z^2 = 1$$

6. (c)

$$f(0) = \lim_{x\to 0}\left\{\frac{\log(1+x)^{(1+x)} - x}{x^2}\right\} \qquad \left[\frac{0}{0}\text{form}\right]$$

Applying L'Hospital Rule,

$$f(0) = \lim_{x\to 0}\frac{\log(1+x)+1-1}{2x} = \frac{1}{2}\lim_{x\to 0}\frac{\log(1+x)}{x} = \frac{1}{2}$$

7. (a)

$$\begin{bmatrix}3\\b\\c\end{bmatrix} = k_1\begin{bmatrix}1\\3\\2\end{bmatrix} + k_2\begin{bmatrix}2\\6\\4\end{bmatrix} + k_3\begin{bmatrix}-1\\-3\\-2\end{bmatrix}$$

$$= k_1\begin{bmatrix}1\\3\\2\end{bmatrix} + 2k_2\begin{bmatrix}1\\3\\2\end{bmatrix} - k_3\begin{bmatrix}1\\3\\2\end{bmatrix}$$

$$\begin{bmatrix}3\\b\\c\end{bmatrix} = (k_1 + 2k_2 - k_3)\begin{bmatrix}1\\3\\2\end{bmatrix}$$

$$k_1 + 2k_2 - k_3 = 3$$
$$3(k_1 + 2k_2 - k_3) = 3\times 3 = 9$$
$$2(k_1 + 2k_2 - k_3) = 2\times 3 = 6$$

8. (d)

Let x is the thickness of ice at time t

$$V = \frac{4\pi}{3}(x+10)^3$$

Where x is thickness of ice

$$\frac{dV}{dt} = \frac{4\pi}{3}\times 3\times (x+10)^2.\frac{dx}{dt}$$

$$\left.\frac{dx}{dt}\right|_{x=5} = \left.\frac{50}{4\pi(x+10)^2}\right|_{x=5}$$

$$= \frac{50}{4\pi(5+10)^2} = \frac{1}{18\pi}\ \text{cm/min}$$

9. (d)

$$y = \log_e(x+e)$$
$$x + e = e^y$$

$\Rightarrow$ $$x = e^y - e$$

For $x = 0$

$$y = \log_e(0+e) = 1$$

For $y = 0$

$$0 = \log_e(x+e)$$
$$x + e = 1$$

$$x = 1 - e$$

$$\text{Area} = \left|\int_{y_1}^{y_2} x \cdot dy\right| = \left|\int_0^1 (e^y - e)dy\right| = \left|e^y - ey\Big|_0^1\right|$$

$$= |e - 1 - e(1 - 0)|$$

$$= |-1| = 1$$

10. (b)

Given that the partial differential equation is parabolic.

$$\therefore \quad B^2 - 4AC = 0 \qquad \text{Here } A = 3$$

$$\therefore \quad B^2 - 4(3)(3) = 0 \qquad C = 3$$

$$B^2 - 36 = 0$$

$$B^2 = 36$$

11. (c)

$$[A : B]$$

$$\begin{bmatrix} 4 & -2 & 6 & : & 8 \\ 1 & 1 & -3 & : & -1 \\ 5 & -3 & 9 & : & 21 \end{bmatrix}$$

$R_2 \to 4R_2 - R_1$; $R_3 \to 4R_3 - 5R_1$

$$\text{or} \quad \begin{bmatrix} 4 & -2 & 6 & : & 8 \\ 0 & 6 & -18 & : & -12 \\ 0 & -2 & 6 & : & 44 \end{bmatrix}$$

$R_3 \to 3R_3 + R_2$

$$\begin{bmatrix} 4 & -2 & 6 & : & 8 \\ 0 & 6 & -18 & : & 12 \\ 0 & 0 & 0 & : & 120 \end{bmatrix}$$

From here

$$\rho(A) \neq \rho(B) = \rho(A : B) < 3$$

$\Rightarrow$ System is inconsistent with no solution.

12. (c)

Here unit digit of the number is 3.

$$3^1 = \underline{3}$$
$$3^2 = \underline{9}$$
$$3^3 = 2\underline{7}$$
$$3^4 = 8\underline{1}$$
$$3^5 = 34\underline{3}$$

From here, cyclicity of 3 is 4. So, the probability of having 3 in the unit place = $\frac{1}{4}$

13. (d)

Let $$f(x + iy) = u(x, y) + iv(x, y)$$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

Since $f(z)$ is analytic function

So $$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} = 2y = \frac{\partial v}{\partial y}$$

$$u = \int \frac{\partial u}{\partial x}\cdot dx + \int \frac{\partial u}{\partial y}\cdot dy$$

$$= 2xy + Q(x, y)$$

$$f(1+i) = 2$$

$$u(1, 1) = 2$$

$$v(1, 1) = 0$$

$$2 \times 1 \times 1 + Q(1, 1) = 2$$

$$y(x, y) = 0$$

$\Rightarrow$ $$u(x, y) = 2xy$$

$$\frac{\partial u}{\partial y} = 2x = -\frac{\partial v}{\partial x}$$

$$v = \int \frac{\partial v}{\partial x}\cdot dx + \int \frac{\partial v}{\partial y}\cdot dy = \int -2x\cdot dx + \int 2y dy = -x^2 + y^2 + c$$

$$u(1, 1) = 1 - 1 + c = 0$$

$\Rightarrow$ $$c = 0$$

$$v(x, y) = y^2 - x^2$$

14. (b)

The maximum variation is in direction of grad T.

$$T = x^2 + 4xy + y^2$$

$$\nabla T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} = (2x + 4y)\hat{i} + (4x + 2y)\hat{j}$$

$$\nabla T\big|_{(2,2)} = (4+8)\hat{i} + (8+4)\hat{j}$$

$$= 12\hat{i} + 12\hat{j}$$

The direction in which rate is slowest is perpendicular the direction in which variation is maximum.

$$\nabla T\big|_{\min} = 12\hat{i} - 12\hat{j} \text{ or } \hat{i} - \hat{j}$$

15. (b)

By Trapezoidal rule, the area under the curve or $\int_{x_1}^{x_2} y dx$ is

$$= \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3) + y_4\right]$$

where, $$y = f(x)$$

Here, $h = 1$

$$\Rightarrow \quad \text{Area} = \frac{1}{2}\left(10 + 2(50 + 70 + 80) + 100\right) = 255 \text{ sq. unit}$$

16. (d)

Eigen value of A

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & -2 \\ -3 & 2-\lambda \end{vmatrix} = 0$$

$$(1 - \lambda)(2 - \lambda) - 6 = 0$$

$$\lambda^2 - 3\lambda + 2 - 6 = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda = -1, 4$$

Eigen values of A^4 are $(-1)^4$ and $(4)^4$

$= 1$ and 256

17. (b)

$$f(x) = \frac{\log_e(1+ax) - \log_e(1-bx)}{x}$$

For function to be continuous

$$f(0) = \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{\log_e(1+ax) - \log(1-bx)}{x} = \lim_{x \to 0} \frac{\log_e(1+ax) \times a}{ax} + \frac{\log(1-bx) \times b}{-bx}$$

$$= a + b$$

18. (a)

$$A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$$

$$A + I = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$$

$$[A + I]^2 = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2\alpha \\ 0 & 1 \end{bmatrix}$$

$$[A + I]^3 = \begin{bmatrix} 1 & 2\alpha \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3\alpha \\ 0 & 1 \end{bmatrix}$$

$$[A + I]^{50} = \begin{bmatrix} 1 & 50\alpha \\ 0 & 1 \end{bmatrix}$$

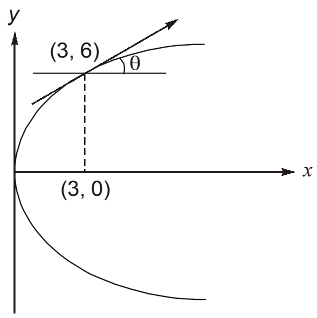
$$50A = \begin{bmatrix} 0 & 50\alpha \\ 0 & 0 \end{bmatrix}$$

$$[A + I]^{50} - 50A = \begin{bmatrix} 1 & 50\alpha \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 50\alpha \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a + b + c + d = 1 + 0 + 0 + 1 = 2$$

19. (d)



Direction of velocity at point (3, 6) is in the direction of tangent at that point.

$\Rightarrow$ Slope of tangent = Slope of velocity

$$\tan\theta = \left.\frac{dy}{dx}\right|_{(3,6)} = \frac{12}{2y} = \frac{12}{2\times 6} = 1$$

$$\Rightarrow \quad \theta = 45^\circ$$

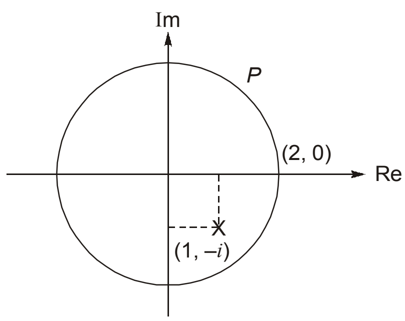
$$v = v_x\hat{i} + v_y\hat{j}$$

$$|v| = \sqrt{v_x^2 + v_y^2} = 10 \text{ m/s}$$

$$v_x = 10\cos 45^\circ = 5\sqrt{2} \text{ m/s}$$

$$v_y = 10\sin 45^\circ = 5\sqrt{2} \text{ m/s}$$

20. (b)



Since (1, $-i$) is inside the circle.

$$\Rightarrow \quad f(\alpha) = \left(\frac{1}{2\pi i}\left.(3z^2 + 7z + 1)\right|_{z=\alpha} \times 2\pi i\right)$$

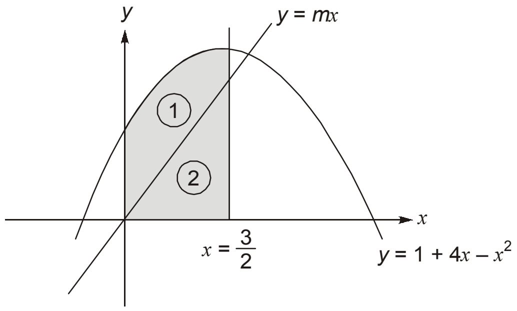
$$f(\alpha) = 3\alpha^2 + 7\alpha + 1$$

$$f'(\alpha) = 6\alpha + 7 = 6(1 - i) + 7 = 13 - 6i$$

$$|f'(\alpha)| = \sqrt{(13)^2 + (6)^2}$$

$$= \sqrt{205} = 14.32$$

21. (c)



$$\text{Area of (1)} = \text{Area of (2)} = \int_0^{3/2} mx\,dx = \frac{1}{2}[\text{Area of (1)} + \text{Area of (2)}]$$

$$\frac{1}{2}\times\int_0^{3/2}\left(1+4x-x^2\right)dx = \frac{1}{2}\left[x+\frac{4x^2}{2}-\frac{x^3}{3}\right]_0^{3/2}$$

$$\int_0^{3/2} mx dx = \frac{1}{2}\left[\left(\frac{3}{2}-0\right)+2\left(\frac{9}{4}-0\right)-\left(\frac{27}{8\times 3}-0\right)\right]$$

$$\left[m\frac{x^2}{2}\right]_0^{3/2} = \frac{1}{2}\left[\frac{3}{2}+\frac{9}{2}-\frac{9}{8}\right]$$

$$m\times\frac{9}{4\times 2} = \frac{1}{2}\times\frac{39}{8}$$

$$m = \frac{13}{6} = 2.17$$

22. (c)

$$N = 5$$

$$h = \frac{b-a}{N} = \frac{5-0}{5} = \frac{5}{5} = 1$$

Therefore

$$\int_0^5 f(x)dx = \frac{1}{2}[0+2.236+2(1+1.414+1.732+2)] = \frac{1}{2}(14.528) = 7.264$$

23. (a)

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 3-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\,[(3-\lambda)^2-1] = 0$$

$$(1-\lambda)\,[\lambda^2+9-6\lambda-1] = 0$$

$$(1-\lambda)\,(\lambda^2-6\lambda+8) = 0$$

$$(1-\lambda)\,(\lambda^2-6\lambda+8) = 0$$

$$(1-\lambda)\,(\lambda-2)\,(\lambda-4) = 0$$

$$\lambda = 1, 2, 4$$

$$[A-\lambda_1 I]\,[x] = 0$$

$$\begin{bmatrix} 1-1 & 0 & 0 \\ 0 & 3-1 & -1 \\ 0 & -1 & 3-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_2 - x_3 = 0$$
$$-x_2 + 2x_3 = 0$$
$$x_1 = k_1, \quad x_2 = 2x_3, \quad 2x_2 = x_3$$

$$\text{eigen vector} = \begin{bmatrix} k_1 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda_2 = 2$

$$[A - \lambda_2 I][x] = 0$$

$$\begin{bmatrix} 1-2 & 0 & 0 \\ 0 & 3-2 & -1 \\ 0 & -1 & 3-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 = 0$$
$$x_2 - x_3 = 0 \qquad x_2 = x_3 \qquad \begin{bmatrix} 0 \\ k \\ k \end{bmatrix}$$
$$-x_2 + x_3 = 0 \qquad x_2 = x_3$$

$\lambda_3 = 4$

$$[A - \lambda_3 I][X] = [0]$$

$$\begin{bmatrix} 1-4 & 0 & 0 \\ 0 & 3-4 & -1 \\ 0 & -1 & 3-4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 = 0$$
$$-x_2 - x_3 = 0 \qquad x_2 = -x_3 \qquad \begin{bmatrix} 0 \\ k \\ -k \end{bmatrix}$$
$$-x_2 - x_3 = 0 \qquad x_2 = -x_3$$

24. (d)

$$P(X = r) = \frac{e^{-m} m^r}{r!}$$

$$P(X = r) = P(X = 0) + P(X = 1) = e^{-m} + \frac{e^{-m} m}{1!}$$

Here is given that $m = 5$

$$\therefore \quad P(X = 5) = e^{-5} + e^{-5} \cdot 5$$

$$= \frac{6}{e^5}$$

25. (c)

$9x^2 + 36x + 16y^2 + 96y + 36 = 0$

$9(x^2 + 4x) + 16(y^2 + 6y) + 36 = 0$

$9(x^2 + 4x + 4) - 36 + 16(y^2 + 6y + 9) - 144 + 36 = 0$

$$9(x+2)^2 + 16(y+3)^2 = 144$$

$$\frac{(x+2)^2}{16} + \frac{(y+3)^2}{9} = 1$$

The given curve is ellipse.

The area of ellipse $= \pi ab$ [a = semi major axis, b = semi minor axis]

$$= \pi \times \sqrt{16} \times \sqrt{9} = 12\pi = 37.69 \text{ sq. unit}$$

26. (b)

By trapezoidal rule,

$$\text{Area} = \int_{7.40}^{7.90} f(x)\,dx$$

$$= \frac{h}{2}\left[y_0 + y_5 + 2(y_1 + y_2 + y_3 + y_4)\right]$$

$$= \frac{0.10}{2}\left[1.93 + 3.10 + 2(1.92 + 2.05 + 2.30 + 2.60)\right] = 1.1385 \approx 1.14$$

27. (b)

$$f(x) = a_0 + a_1x^2 + a_2x^4 \ldots a_nx^{2n}$$

$$\frac{df(x)}{dx} = a_1 \cdot 2x + a_2 4x^3 \ldots a_n \cdot 2nx^{2n-1}$$

For maxima or maxima

$$\frac{df(x)}{dx} = 0$$

$$a_1 \cdot 2x + a_2 \cdot 4x^3 \ldots a_n \cdot 2nx^{2n-1} = 0$$

$$x\left(a_1 \cdot 2 + a_2 4x^2 \ldots a_n \cdot 2nx^{2n-2}\right) = 0$$

$$x = 0, \quad 2a_1 + 4a_2x^2 \ldots\ldots 2n a_n x^{2n-2} \neq 0$$

For $x = 0$

$$f''(x) = 2a_1 > 0$$

Minima at $x = 0$ and exactly one minima.

28. (c)

$$|A| = \begin{vmatrix} {}^xC_0 & {}^xC_1 & {}^xC_2 \\ {}^yC_0 & {}^yC_1 & {}^yC_2 \\ {}^zC_0 & {}^zC_1 & {}^zC_2 \end{vmatrix} = \begin{vmatrix} 1 & \frac{x}{1} & \frac{x(x-1)}{2} \\ 1 & \frac{y}{1} & \frac{y(y-1)}{2} \\ 1 & z & \frac{z(z-1)}{2} \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & x & x^2-x \\ 1 & y & y^2-y \\ 1 & z & z^2-z \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - \frac{1}{2}\begin{vmatrix} 1 & x & x \\ 1 & y & y \\ 1 & z & z \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$R_3 \to R_3 - R_2,\ R_2 \to R_2 - R_1$

$$= \frac{1}{2}\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-y & z^2-y^2 \end{vmatrix}$$

$$= \frac{1}{2}[(y-x)(z^2-y^2)-(y^2-x^2)(z-y)]$$

$$= \frac{1}{2}[(y-x)(z-y)(z+y)-(y-x)(y+x)(z-y)]$$

$$= \frac{1}{2}[(y-x)(z-y)(z+y-y-x)]$$

$$= \frac{1}{2}(y-x)(z-y)(z-x)$$

$$|A| = \frac{1}{2}(x-y)(y-z)(z-x)$$

$$|A| = \frac{1}{2}(x-y)(y-z)(z-x) = 0$$

Then, either $x = y$

or $y = z$

or $z = x$

29. (d)

Let $d \rightarrow$ defective

$y \rightarrow$ supplied by y

$$P\left(\frac{y}{d}\right) = \frac{P(y \cap d)}{P(d)}$$

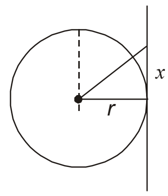
$$P(y \cap d) = 0.3 \times 0.02 = 0.006$$

$$P(d) = 0.6 \times 0.01 + 0.3 \times 0.02 + 0.1 \times 0.03$$

$$= 0.015$$

$$P\left(\frac{y}{d}\right) = \frac{0.006}{0.015} = 0.4$$

30. (c)



$$\tan\theta = \frac{x}{r}$$

$$x = r\tan\theta$$

$$\frac{dx}{dt} = r\sec^2\theta\,\frac{d\theta}{dt}$$

$$\frac{dx}{dt} = r\frac{d\theta}{dt}\sec^2\theta$$

$$= v\sec^2\theta$$

when 1/8 of circle is covered

$$\theta = \frac{1}{8}\times 2\pi = \frac{\pi}{4}$$

$$\frac{dx}{dt} = 20 \times \sec^2\left(\frac{\pi}{4}\right)$$

$$= 20 \times 2 = 40 \text{ km/h}$$

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