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ENGINEERING MECHANICS

CIVIL ENGINEERING

Date of Test : 14/03/2024

ANSWER KEY >

1. (d)	7. (d)	13. (b)	19. (b)	25. (d)
2. (b)	8. (b)	14. (a)	20. (c)	26. (b)
3. (c)	9. (a)	15. (c)	21. (b)	27. (c)
4. (a)	10. (c)	16. (d)	22. (c)	28. (a)
5. (b)	11. (a)	17. (c)	23. (b)	29. (b)
6. (b)	12. (d)	18. (a)	24. (a)	30. (b)

DETAILED EXPLANATIONS

1. (d)

Without slipping, maximum acceleration provided by friction is given as

$$a = \mu g = 0.75 \times 9.81 = 7.36 \text{ m/s}^2$$

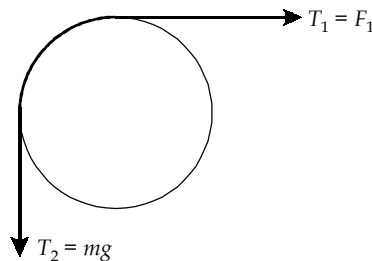
Using second equation of kinematics as the car is starting from rest,

$$s = ut + \frac{1}{2}at^2$$

$$\therefore s = \frac{1}{2}at^2 \quad (u = 0)$$

$$\begin{aligned} \therefore t &= \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 700}{7.36}} = \sqrt{100.217} \\ &= 13.79 \text{ seconds} \end{aligned}$$

2. (b)



The angle of contact between the cable and the round support is $\theta = \frac{\pi}{2}$ radians.

$$\begin{aligned} \therefore T_2 &= T_1 e^{\mu_s \theta} \\ T_2 &= T_1 e^{0.4 \times \frac{\pi}{2}} \\ T_2 &= T_1 e^{0.628} = 1.874 T_1 \\ T_1 &= \frac{T_2}{1.874} = \frac{50 \times 9.81}{1.874} = 261.74 \text{ N} \end{aligned}$$

3. (c)

The work done in a small displacement dx is given as,

$$dw = \vec{F} \cdot \vec{dx} = F dx$$

$$w = \int dw = \int_0^3 (10 + x) dx$$

$$\begin{aligned} w &= 10[x]_0^3 + \frac{1}{2}[x^2]_0^3 = 10(3-0) + 0.5(9-0) \\ &= 30 + 4.5 = 34.5 \text{ Joules} \end{aligned}$$

4. (a)

Using impulse-momentum theorem,

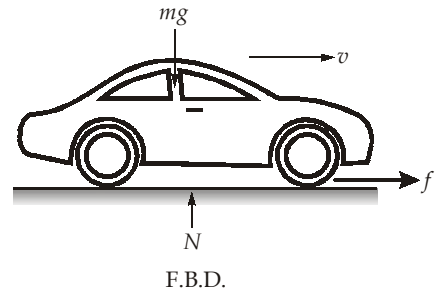
$$mu + \int F dt = mv$$

$$mu + \int_0^6 600t^2 dt = mv$$

$$2500 \times 20 + 600 \left(\frac{t^3}{3} \right)_0^6 = 2500 \times v$$

$$50000 + 600 \times \frac{216}{3} = 2500 \times v$$

$$v = \frac{93200}{2500} = 37.28 \text{ m/s}$$



5. (b)

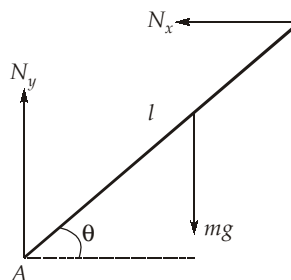
Given $u = 8 \text{ m/s}$, $a = 2.4 \text{ m/s}^2$, $t = 10 \text{ s}$

Using, $S = ut + \frac{1}{2}at^2$

$$S = 8 \times 10 + \frac{1}{2} \times 2.4 \times 10^2$$

$$S = 80 + 120 = 200 \text{ m}$$

6. (b)



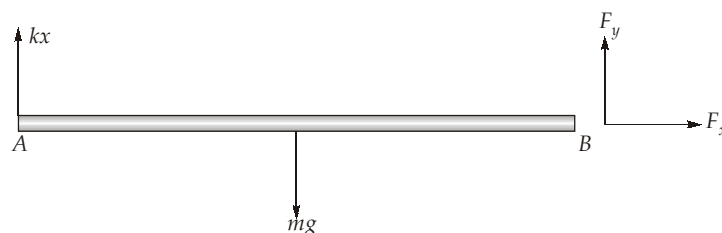
The free body diagram of the rod is shown. From the equilibrium of the rod and taking moment about the end A, we get,

$$mg \times \frac{l}{2} \cos \theta = N_x \times l \sin \theta$$

$$N_x = \frac{mg}{2 \tan \theta}$$

7. (d)

Free body diagram of rod is given as



Taking moment about B.

$$mg \times \frac{l}{2} = kx \times l$$

$$\therefore x = \frac{mg}{2k}$$

Since, there is no external horizontal force on the rod, so $F_x = 0$ and $F_y + kx = mg$

$$F_y + \frac{mg}{2k} \times k = mg$$

$$F_y = \frac{mg}{2}$$

8. (b)

As the block is at rest, the net horizontal force acting on it should be zero. Therefore, friction force is equal to 20 N.

9. (a)

$$\text{Acceleration of particle, } a = \frac{F}{m} = \frac{7.5}{30 \times 10^{-3}} = 250 \text{ m/s}^2$$

Time taken to cover 2.5 meters distance is

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 2.5}{250}} = 0.141 \text{ seconds}$$

Velocity after this displacement,

$$v = \sqrt{2as} = \sqrt{2 \times 250 \times 2.5} = 35.35 \text{ m/s}$$

$$P_{av} = \frac{\text{Total work done}}{\text{Time}} = \frac{\text{Change in KE}}{\text{Time}}$$

$$= \frac{\frac{1}{2}mv^2}{t} = \frac{\frac{1}{2} \times 30 \times 10^{-3} \times 35.35^2}{0.141}$$

$$= 133 \text{ Watts}$$

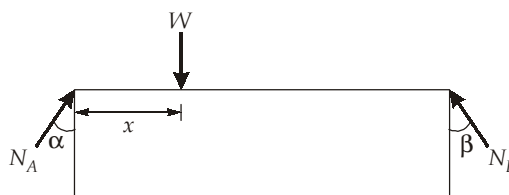
10. (c)

Acceleration is defined as the rate of change of velocity with respect to time. So, a change in either the speed or the direction of motion or both results into acceleration. Statement I is correct.

For a particle moving in circular motion with a constant speed, the direction of velocity is changing at every instant. Therefore, the particle is having an acceleration. So, statement II is false.

11. (a)

Free-body diagram of the beam is drawn as,



From equilibrium equation,

$$N_A \cos \alpha + N_B \cos \beta = W \quad \dots(i)$$

$$N_A \sin\alpha = N_B \sin\beta \quad \dots(\text{ii})$$

Moment about A is given by,

$$W \times x = N_B \cos\beta \times l \quad \dots(\text{iii})$$

Solving these equations,

$$N_B = \frac{Wx}{l \cos\beta}$$

Putting this value of N_B in equation (ii),

$$N_A = \frac{Wx \sin\beta}{l \cos\beta \sin\alpha} = \frac{Wx \tan\beta}{l \sin\alpha}$$

Now, putting the values N_A and N_B in equation (i)

$$\frac{Wx \tan\beta}{l \sin\alpha} \times \cos\alpha + \frac{Wx}{l \cos\beta} \times \cos\beta = W$$

$$\frac{Wx \tan\beta}{l \tan\alpha} + \frac{Wx}{l} = W$$

$$\frac{x}{l} \left(\frac{\tan\beta}{\tan\alpha} + 1 \right) = 1$$

$$x = \frac{l}{1 + \frac{\tan\beta}{\tan\alpha}}$$

12. (d)

The angles between the pillar ED and three cables are

$$\alpha_A = \tan^{-1}\left(\frac{4}{6}\right) = 33.7^\circ$$

$$\alpha_B = \tan^{-1}\left(\frac{8}{6}\right) = 53.1^\circ$$

$$\alpha_C = \tan^{-1}\left(\frac{12}{6}\right) = 63.4^\circ$$

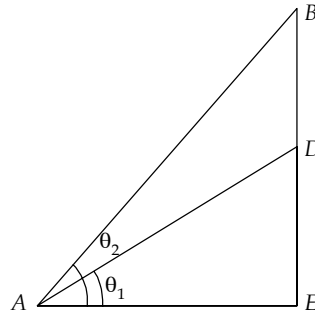
The vertical components of each force at point D exert no moment about E . Noting that $F_A = F_B = F_C$ the magnitude of the moment about E due to the horizontal components is

$$\Sigma M_E = F_A (\sin\alpha_A + \sin\alpha_B + \sin\alpha_C) \times 6 = 2700$$

$$F_A = \frac{2700}{6 \times (\sin\alpha_A + \sin\alpha_B + \sin\alpha_C)} = \frac{2700}{6 \times (0.55 + 0.8 + 0.89)}$$

$$F_A = 200.89 \text{ kN}$$

13. (b)



$$\tan\theta_1 = \frac{DE}{AE} = \frac{\frac{1}{4}a}{a} = \frac{1}{4} = 0.25$$

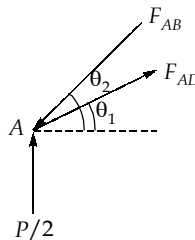
$$\theta_1 = \tan^{-1}(0.25) = 14.04^\circ$$

$$\tan\theta_2 = \frac{BE}{AE} = \frac{a}{a} = 1$$

$$\theta_2 = \tan^{-1}(1) = 45^\circ$$

Given: $F_{AB} = 800 \text{ N (C)}$

Joint A:



$$\Sigma F_x = 0;$$

$$F_{AD} \cos\theta_1 = 800 \cos\theta_2$$

$$F_{AD} = 800 \frac{\cos 45^\circ}{\cos 14.04^\circ}$$

$$F_{AD} = 583.01 \text{ N} < 2000 \text{ N}$$

$$\Sigma F_y = 0;$$

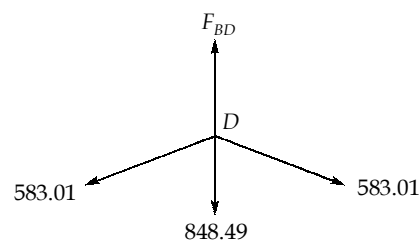
$$\frac{P}{2} + F_{AD} \sin\theta_1 = F_{AB} \sin\theta_2$$

$$P = (2) (800 \sin 45^\circ - 583.01 \sin 14.04^\circ)$$

$$P = 2 (565.68 - 141.44)$$

$$P = 848.49 \text{ N}$$

Joint D,



$$\Sigma F_y = 0;$$

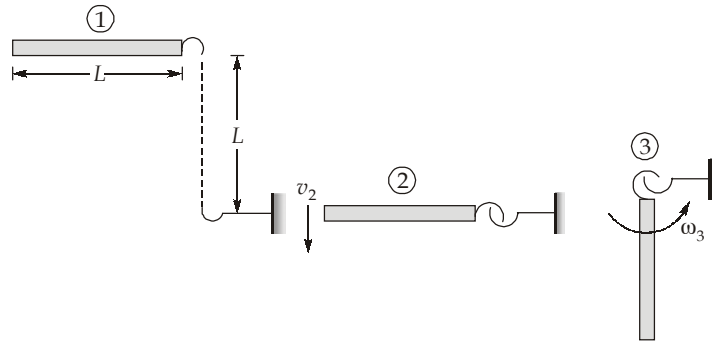
$$F_{DB} = 848.49 + 2 \times 583.01 \times \sin 14.04$$

$$= 1131.29 \text{ N} < 2000 \text{ N}$$

Therefore, $P_{\max} = 848.49 \text{ N}$

14. (a)

The three different situations of motion of rod is shown as :



Using energy conservation between (1) and (2),

$$U_1 + K_1 = U_2 + K_2$$

$$\Rightarrow mgL + 0 = 0 + \frac{1}{2}mv_2^2$$

$$\Rightarrow v_2 = \sqrt{2gL}$$

From momentum conservation before and after striking the hook

$$\therefore P_1 = P_2$$

$$\Rightarrow mv_2r = I\omega_2$$

$$\Rightarrow m\sqrt{2gL} \times \frac{L}{2} = \left(\frac{mL^2}{3}\right)\omega_2$$

$$\Rightarrow \omega_2 = \frac{3}{2}\sqrt{\frac{2g}{L}}$$

Energy conservation between (2) and (3),

$$U_2 + K_2 = U_3 + K_3$$

$$0 + \frac{1}{2}\left(\frac{1}{3}ML^2\right) \times \frac{9}{4} \times \frac{2g}{L} = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega_3^2 - Mg\frac{L}{2}$$

$$\frac{3}{4}gL = \frac{1}{6}L^2\omega_3^2 - g\left(\frac{L}{2}\right)$$

$$\omega_3 = \sqrt{\frac{7.5g}{L}} = \sqrt{\frac{7.5 \times 9.81}{1}} = \sqrt{73.575}$$

$$\omega_3 = 8.57 \text{ rad/sec}$$

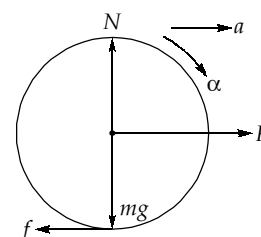
15. (c)

Free body diagram of the disc is given as

$$F - f = ma$$

$$fR = I\alpha = \frac{1}{2}mR^2\alpha = \frac{1}{2}mR^2 \frac{a}{R}$$

$$fR = \frac{mRa}{2}$$



...(i)

$$f = \frac{ma}{2} \quad \dots(ii)$$

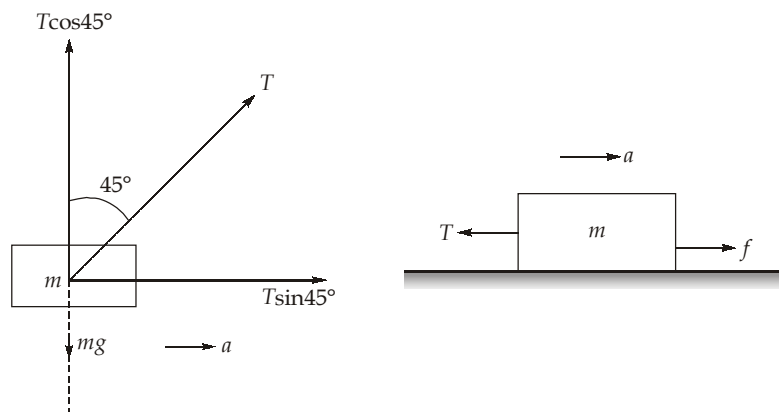
From equation (i) and (ii),

$$F = \frac{3ma}{2}$$

$$\Rightarrow a = \frac{2F}{3m} = \frac{2 \times 10}{3 \times 12} = 0.55 \text{ m/s}^2$$

16. (d)

Free body diagram of the blocks are:



$$T \sin 45^\circ = ma \quad \dots (i)$$

$$T \cos 45^\circ = mg \quad \dots (ii)$$

Dividing equation (i) by (ii),

$$\frac{T \sin 45^\circ}{T \cos 45^\circ} = \frac{ma}{mg}$$

$$\Rightarrow a = g$$

$$\text{From equation (ii), } T = \frac{mg}{\cos 45^\circ} = \sqrt{2} mg.$$

Applying Newton's law equation for the block placed on the cart.

$$f - T = ma$$

$$\mu mg - T = ma$$

$$\mu mg = T + ma = \sqrt{2} mg + mg$$

$$\mu mg = mg(\sqrt{2} + 1)$$

$$\mu = \sqrt{2} + 1$$

17. (c)

$$v \frac{dv}{ds} = a$$

$$-6s^{-3} = v \frac{dv}{ds}$$

$$\int_{\infty}^6 -6s^{-3} ds = \int_0^v v dv$$

$$\left[-\frac{6}{-2} s^{-2} \right]_{\infty}^6 = \frac{v^2}{2}$$

$$\left[\frac{3}{s^2} \right]_{\infty}^6 = \frac{v^2}{2}$$

$$v^2 = \left[\frac{6}{s^2} \right]_{\infty}^6$$

$$\Rightarrow v^2 = \frac{6}{6 \times 6} = \frac{1}{6}$$

$$v = 0.408 \text{ m/s}$$

$$\left[\begin{aligned} a &= \frac{dv}{dt}, dt = \frac{ds}{v} \\ a &= \frac{dv}{\left(\frac{ds}{v}\right)} = v \left(\frac{dv}{ds}\right) \end{aligned} \right]$$

18. (a)

Given data: $m = 8.4 \text{ kg}$, $\omega = 6.9 \text{ rad/s}$, $F = 6.6 \text{ N}$, $M = 59 \text{ Nm}$, $L = 4 \text{ m}$, $\omega_0 = 90^\circ = ?$

Moment of Inertia of rod about hinge O ,

$$I_O = \frac{mL^2}{12} + m \times \left(\frac{L}{2}\right)^2 = \frac{mL^2}{3} = \frac{8.4 \times 4 \times 4}{3} = 44.8 \text{ kg m}^2.$$

By conservation of energy:

$$mgh_{cm} + (M + F \times L)\Delta\theta = \frac{1}{2}I_0(\omega_1^2 - \omega_0^2)$$

$$\text{For } \theta = 90^\circ, h_{cm} = 2 \text{ m}$$

$$(8.4 \times 9.81 \times 2) + (59 + 6.6 \times 4) \times \frac{\pi}{2} = \frac{1}{2} \times (44.8) [\omega_1^2 - 6.9^2]$$

$$\frac{298.954 \times 2}{44.8} = \omega_1^2 - 6.9^2$$

$$\omega_1^2 = 60.9562$$

$$\omega_1 = 7.807 \text{ rad/s}$$

19. (b)

Given: Pitch (P) = 12 mm, Mean radius (r) = $\frac{80}{2} = 40 \text{ mm}$, Coefficient of static friction (μ_s) = 0.15,

Coefficient of kinetic friction (μ_k) = 0.10, Lever length (a) = 600 mm, Weight to be lifted (W) = 25 kN.

Since, the screw is single threaded, lead (C) = Pitch(P) = 12 mm.

Determination of helix angle,

$$\tan\theta = \frac{L}{2\pi r} = \frac{12}{2\pi \times 40} = 0.0477$$

$$\theta = \tan^{-1}(0.0477) = 2.733^\circ$$

Force required to just lift a weight of 25 kN.

$$\begin{aligned} \tan\phi_s &= \mu_s \\ \phi_s &= \tan^{-1}(\mu_s) = \tan^{-1}(0.15) \\ \phi_s &= 8.53^\circ \\ \phi_s + \theta &= 8.53^\circ + 2.733^\circ = 11.263^\circ \\ \tan(\phi_s + \theta) &= \tan(11.263) = 0.199 \end{aligned}$$

Therefore, the force required to just raise the load is given as:

$$\begin{aligned} P &= \frac{Wr}{a} \tan(\phi_s + \theta) \\ &= \frac{25000 \times 0.04}{0.6} \times 0.199 = 331.67 \text{ N} \end{aligned}$$

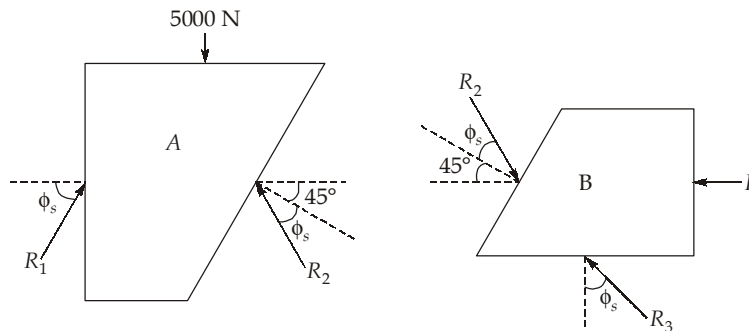
20. (c)

Coefficient of friction, $\mu_s = 0.2$.

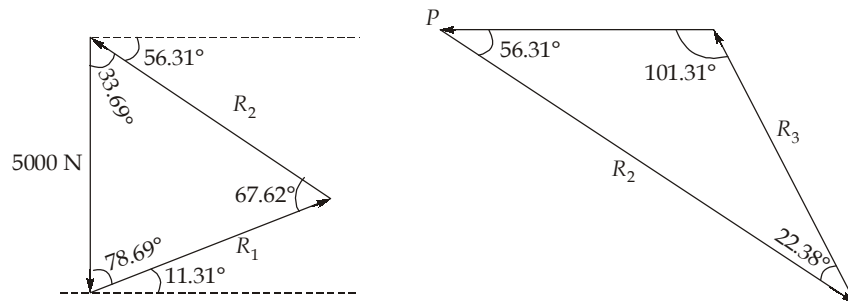
Here the force P is required to maintain the equilibrium. The direction of impending motion of the block A is downwards and that of block B is rightwards.

The free body-diagrams of the block are:

[Angle of friction: $\phi_s = \tan^{-1}\mu, \phi_s = \tan^{-1}(0.2), \phi_s = 11.31^\circ$]



Making force triangles for A and B



Applying Lami's theorem for block A

$$\frac{5000}{\sin(67.62^\circ)} = \frac{R_1}{\sin(33.69^\circ)} = \frac{R_2}{\sin(78.69^\circ)}$$

$$\therefore R_2 = 5000 \times \frac{\sin(78.69^\circ)}{\sin(67.62^\circ)} = 5302.27 \text{ N}$$

From Lami's theorem for block B

$$\frac{P}{\sin(22.38^\circ)} = \frac{R_2}{\sin(101.31^\circ)} = \frac{R_3}{\sin(56.31^\circ)}$$

$$\therefore P = R_2 \times \frac{\sin(22.38^\circ)}{\sin(101.31^\circ)}$$

$$P = 5302.27 \times \frac{\sin(22.38^\circ)}{\sin(101.31^\circ)} = 2058.81 \text{ N}$$

21. (b) Beginning by analyzing the equilibrium of joint D.

$$\Sigma F_x = 0$$

$$F_{DE} \cos\theta - 500 = 0$$

$$F_{DE} = \frac{500}{\cos\theta} = \frac{500}{\left(\frac{2.5}{3.90}\right)} = 780 \text{ N}$$

F_{DE} is compressive in nature.

$$\Sigma F_y = 0$$

$$F_{DC} = F_{DE} \sin\theta$$

$$F_{DC} = 780 \times \frac{3}{3.90} = 600 \text{ N}$$

F_{DC} is tensile in nature.

Free-body diagram of the joint C,

$$\Sigma F_x = 0$$

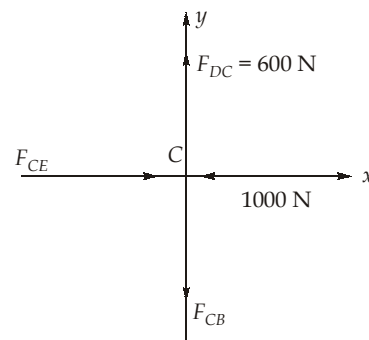
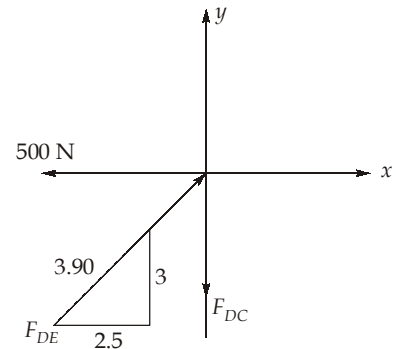
$$F_{CE} = 1000 \text{ N}$$

F_{CE} is compressive in nature,

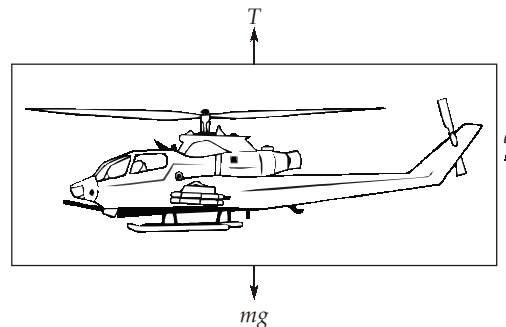
$$\Sigma F_y = 0, \quad 600 - F_{CB} = 0$$

$$F_{CB} = 600 \text{ N}$$

F_{CB} is tensile in nature.



22. (c) Free-body diagram of the helicopter is given by:



Net force on the helicopter is given as,

$$F_{net} = T - mg = (200 + 2t^3 - 100) \text{ kN}$$

Impulse of the net force is given as

$$\begin{aligned} I &= \int_0^4 F_{net} dt = \int_0^4 (200 + 2t^3 - 100) dt \\ &= \int_0^4 (2t^3 + 100) dt = 2 \left[\frac{t^4}{4} \right]_0^4 + 100[t]_0^4 \\ &= \frac{2}{4} [4^4 - 0] + 100[4 - 0] \\ &= 128 + 400 = 528 \text{ kN-s} \end{aligned}$$

23. (b)

The area under the force displacement curve will give the net work done by the force on the particle.

$$W_{\text{net}} = 10 \times 2 - \frac{1}{2} \times 10 \times 2 = 20 - 10 = 10 \text{ J}$$

Using work energy theorem,

$$W_{\text{net}} = \text{Change in kinetic energy}$$

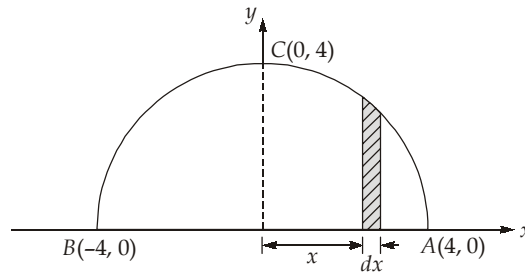
$$10 = (\text{KE})_f - (\text{KE})_i$$

$$10 = \frac{1}{2} Mv^2 - 0$$

$$v = \sqrt{\frac{20}{M}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2 \text{ m/s}$$

24. (a)

The co-ordinates of the plate on the axis are:



Let ρ be the area density of the plate,

$$\rho = \frac{m}{A} = \frac{W}{gA}$$

The mass of an element ydx at a distance x from the y -axis is,

$$dm = \frac{W}{gA} y dx$$

Using the formula for moment of inertia,

$$\begin{aligned} I_{y\text{-axis}} &= \int r^2 dm = \int x^2 \frac{W}{gA} y dx \\ &= \frac{W}{gA} \int_{-4}^4 x^2 \left(4 - \frac{x^2}{4} \right) dx = \frac{W}{gA} \int_{-4}^4 \left(4x^2 - \frac{x^4}{4} \right) dx \\ &= \frac{W}{gA} \left[\frac{4x^3}{3} - \frac{x^5}{20} \right]_{-4}^4 = \frac{W}{gA} (68.26) \quad \dots (i) \end{aligned}$$

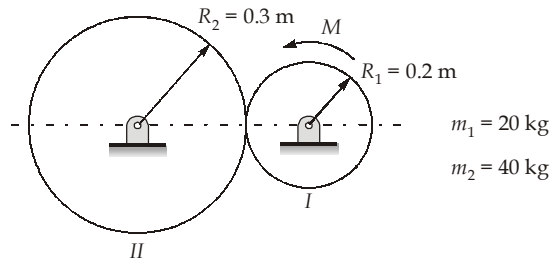
$$\text{Area of the plate is } A = \int \left(4 - \frac{x^2}{4} \right) dx = \left[4x - \frac{x^3}{12} \right]_{-4}^4$$

$$A = 21.33 \text{ m}^2$$

Putting this value of area in equation (i)

$$I_{y\text{-axis}} = \frac{W}{g \times 21.33} \times (68.26) = \frac{20}{(9.81 \times 21.33)} \times (68.26) = 6.5458 \text{ kg-m}^2 \approx 6.55 \text{ kg-m}^2$$

25. (d)



$$\text{Moment of inertia, } I_1 = \frac{m_1 R_1^2}{2} = \frac{20 \times 0.2^2}{2} = 0.4 \text{ kgm}^2$$

$$I_2 = \frac{m_2 R_2^2}{2} = \frac{40 \times 0.3^2}{2} = 1.8 \text{ kgm}^2$$

A force of friction F acts between disc I and II which drives disc II .

$$\begin{aligned} F \times R_2 &= I_2 \alpha_2 & \dots(1) \\ R_1 \alpha_1 &= R_2 \alpha_2 \\ \Rightarrow 0.2 \times 8.33 &= 0.3 \times \alpha_2 \\ \alpha_2 &= 5.55 \text{ m/s}^2 \end{aligned}$$

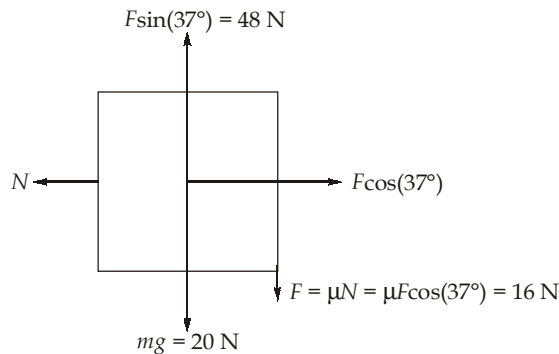
Put α_2 value in equation (1),

We get $F = 33.32 \text{ N}$

$$\begin{aligned} M - FR_1 &= I_1 \alpha_1 \\ \Rightarrow M - 33.32 \times 0.2 &= 0.4 \times 8.33 \\ M &= 9.996 \approx 10 \text{ Nm} \end{aligned}$$

26. (b)

Free-body diagram of the block is given as:



As the upward force [$F \sin(37^\circ) = 48 \text{ N}$] is greater than the total downward force ($20 + 16 = 36 \text{ N}$) hence, it has an upward acceleration,

$$\begin{aligned} F_{\text{net}, y} &= ma \\ [48 - (20 + 16)] &= 2a \\ 48 - 36 &= 2a \\ a &= \frac{12}{2} = 6 \text{ m/s}^2 \end{aligned}$$

27. (c)

Given: $N_1 = 100 \text{ rev/min}$, $N_2 = 200 \text{ rev/min}$, $N = 130 \text{ rev/min}$, $I_1 = 1 \text{ kg-m}^2$.

Since the external torque acting on the two wheels system is zero, the angular momentum will be conserved.

$$\begin{aligned}
 L_i &= L_f \\
 I_1 N_1 + I_2 N_2 &= (I_1 + I_2) N \\
 1 \times 100 + I_2 \times 200 &= (I_1 + I_2) \times 130 \\
 100 + 200 I_2 &= 130 + 130 I_2 \\
 70 I_2 &= 30 \\
 I_2 &= 0.4286 \text{ kg-m}^2 \simeq 0.43 \text{ kg-m}^2
 \end{aligned}$$

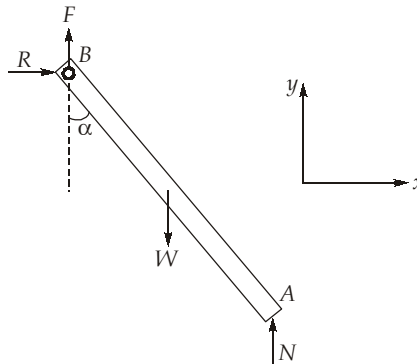
28. (a)

Mass of bar, $m = 4 \text{ kg}$ Length of bar, $L = 6 \text{ m}$

The elongation in the spring,

$$x = L - L \cos \alpha \quad \dots (i)$$

Free-body diagram of the bar is given as:



From equilibrium equations,

$$\Sigma F_x = 0, \quad R = 0$$

$$\Sigma F_y = 0, \quad F + N = W$$

$$\Sigma M_A = 0, \quad W \left(\frac{L}{2} \sin \alpha \right) - R(L \cos \alpha) - F(L \sin \alpha) = 0$$

as $R = 0$

$$W \left(\frac{L}{2} \sin \alpha \right) = F(L \sin \alpha)$$

$$\therefore F = \frac{W}{2} = \frac{4 \times 10}{2} = 20 \text{ N}$$

 \therefore Putting this value of F in equation (i),

$$F = k(L) (1 - \cos \alpha)$$

$$k = \frac{F}{L(1 - \cos \alpha)} = \frac{20}{6(1 - \cos 30^\circ)}$$

$$k = 24.88 \text{ N/m}$$

29. (b)

The initial extension of the spring,

$$x_0 = \frac{mg}{k}$$

Using conservation of linear momentum to find the combined speed of A and B.

$$\begin{aligned}
 P_i &= P_t \\
 2m \times u + 0 &= (3m) \times v
 \end{aligned}$$

$$\Rightarrow v = \frac{2mu}{3m} = \frac{2u}{3}$$

For the spring to just attain its natural length, the combined blocks must rise by $\frac{mg}{k}$. Therefore, using conservation of mechanical energy:

$$E_i = E_f$$

$$\Rightarrow \frac{1}{2} \times 3m \left(\frac{2u}{3}\right)^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = (3mg)\left(\frac{mg}{k}\right)$$

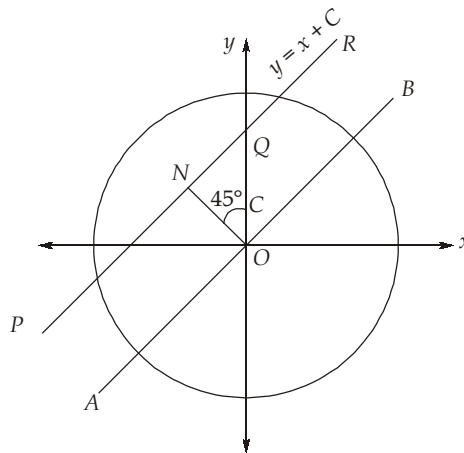
$$\Rightarrow \frac{2mu^2}{3} + \frac{m^2g^2}{2k} = \frac{3m^2g^2}{k}$$

$$\Rightarrow \frac{2mu^2}{3} = \left(\frac{m^2g^2}{k}\right)\left(\frac{5}{2}\right)$$

$$\Rightarrow u^2 = \frac{15mg^2}{4k}$$

$$\Rightarrow u = \sqrt{\frac{15mg^2}{4k}}$$

30. (b)



$$I_{PQR} = I_{AOB} + m(ON)^2$$

$$I_{PQR} = \frac{mR^2}{4} + m\left(\frac{C}{\sqrt{2}}\right)^2$$

$$[ON : C \cos(45^\circ) = \frac{C}{\sqrt{2}}]$$

$$= \frac{mR^2}{4} + \frac{mC^2}{2}$$

$$I_z = I_{PQR}$$

$$\frac{mR^2}{2} = \frac{mR^2}{4} + \frac{mC^2}{2}$$

$$\frac{mR^2}{4} = \frac{mC^2}{2}$$

$$C = \pm \frac{R}{\sqrt{2}}$$

