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Engineering Mechanics

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# CLASS TEST 2019-2020

## MECHANICAL ENGINEERING

**Date of Test : 26/09/2019**

## ANSWER KEY > Engineering Mechanics

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (d) | 7. | (a) | 13. | (d) | 19. | (c) | 25. | (d) |
| 2. | (a) | 8. | (b) | 14. | (b) | 20. | (b) | 26. | (d) |
| 3. | (b) | 9. | (a) | 15. | (c) | 21. | (c) | 27. | (d) |
| 4. | (b) | 10. | (a) | 16. | (b) | 22. | (a) | 28. | (b) |
| 5. | (b) | 11. | (a) | 17. | (c) | 23. | (a) | 29. | (a) |
| 6. | (b) | 12. | (a) | 18. | (b) | 24. | (a) | 30. | (a) |

## DETAILED EXPLANATIONS

**1. (d)**

Given: Mass of elevator = 500 kg

Mass of operator = 100 kg

Upward acceleration = 3 m/s$^2$

Total tension in the cable of the elevator = $(m_1 + m_2)(g + a)$

$= (500 + 100)(10 + 3) = 600 \times 13$

Total tension in the cable of the elevator = 7800 N = 7.8 kN

**2. (a)**

Given:Velocity of first particle, $u_1$ = 10 m/s

Angle of projection for first particle, $\alpha_1$ = 60°

Angle of projection for second particle, $\alpha_2$ = 30°

Velocity of second particle, $u_2$ = ?

Given, Time of flight is same.

$$t_1 = t_2$$

$$\frac{2u_1 \sin\alpha_1}{g} = \left(\frac{2u_2 \sin\alpha_2}{g}\right)$$

$$u_2 = \frac{10\times\sin 60°}{(\sin 30°)} = \frac{10\times\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 10\times\sqrt{3}$$

$$u_2 = 17.32 \text{ m/s}$$

**3. (b)**

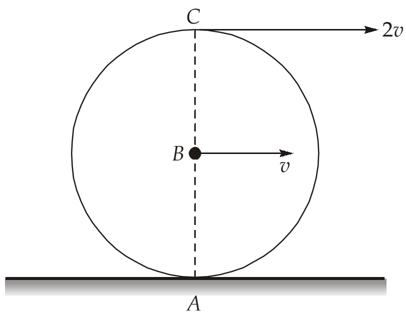
Given: Velocity, $v$ = 54 kmph = $(54)\times\frac{5}{18}$ = 15 m/s

Diameter, $d$ = 1 m

Radius, $r$ = 0.5 m

(i) Velocity of the top of the wheel relative to the person sitting in the carriage:

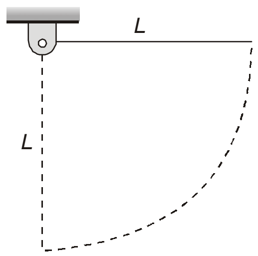
We know that the velocity of the top of the wheel ($C$) = $2v$ = 2 × 15 = 30 m/s




Velocity of the person sitting in the carriage, $v$ = 15 m/s

Velocity of the top of the wheel relative to the person sitting in the carriage = 30 – 15 = 15 m/s

**4.** **(b)**




Applying conservation of energy,

$$mgL = \frac{mgL}{2} + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad I\omega^2 = mgL$$

$$\Rightarrow \quad \frac{mL^2}{3}\omega^2 = mgL \quad \text{[The moment of inertia about the end of the rod is } \frac{mL^2}{3}\text{]}$$

$$\therefore \quad \omega = \sqrt{\frac{3g}{L}}$$

**5.** **(b)**

Using conservation of energy,

$$mgh = \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2 \times 0.04 \times 9.81 \times 4.9}{400}}$$

$$\therefore \quad x = 0.098 \text{ m} = 9.8 \text{ cm}$$

**6.** **(b)**

Work done, $dW = F \cdot dx = (10 + 0.5 \ln x)dx$

Thus, $$\int_0^W dW = \int_2^4 (10 + 0.5 \ln x)dx$$

$$W = 10(4-2) + 0.5\int_2^4 \ln x\, dx$$

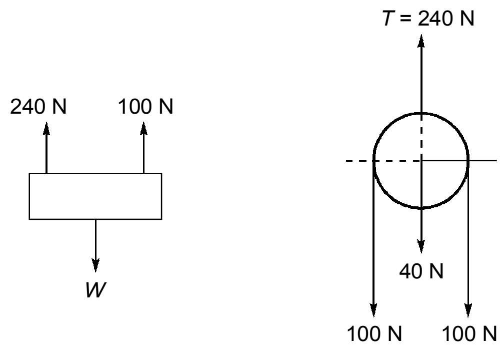
$$W = 20 + 0.5\,(x \ln x - x)_2^4$$

$$W = 20 + 0.5\,(4 \ln 4 - 4 - 2 \ln 2 + 2)$$

$$W = 21.079 \text{ J}$$

**7.** **(a)**

The FBD of the weight $W$ is

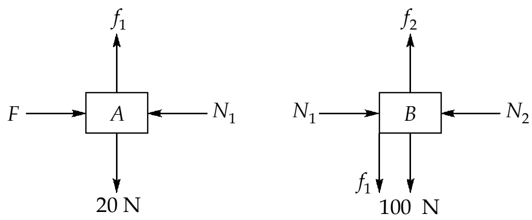


So, $240 + 100 = W$ (240 N includes weight of pulley and tension carried by rope)

$\therefore$ $W = 340$ N

**8. (b)**

The *FBD* of blocks *A* and *B* are,




So, $f_1 = 20$ N

$f_2$(friction on *B* due to wall) $= 100 + f_1 = 120$ N

**9. (a)**

Given: $\vec{F} = 10\hat{i} + 5\hat{j} + \hat{k}$ (N)

$x = \sqrt{106}$ m

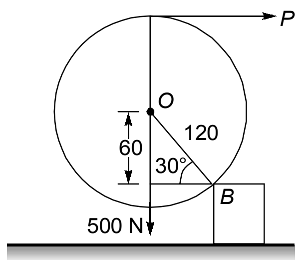
Now, $\vec{A} \times \vec{B} = (3\hat{i} + 4\hat{j}) \times (3\hat{j} + \hat{k}) = 4\hat{i} - 3\hat{j} + 9\hat{k}$

Now, $$W = \left[(10\hat{i} + 5\hat{j} + \hat{k}) \times \sqrt{106}\right] \cdot \frac{(4\hat{i} - 3\hat{j} + 9\hat{k})}{\sqrt{4^2 + 3^2 + 9^2}}$$

or $W = 40 - 15 + 9$

$\therefore$ $W = 34$ Nm

**10. (a)**




Taking moment about *B*,

$P \times (60 + 120) = 500 \times 120 \cos 30°$

$\therefore$ $P = 288.68$ N

**11. (a)**

Given: Mass of first ball $= m$ kg

Mass of second ball $= 2m$ kg

Initial velocity of first mass $= u$ m/s

Initial velocity of second mass $= \frac{u}{7}$ m/s

Coefficient of restitution, $e = 0.75$

Let, Velocity of the first ball after impact $= v_1$ m/s

Velocity of the second ball after impact $= v_2$ m/s

By momentum conservation,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$mu + 2m \times \frac{u}{7} = mv_1 + 2mv_2$$

$$\frac{9u}{7} = (v_1 + 2v_2) \quad \text{...(i)}$$

We also know that, $(v_2 - v_1) = e(u_1 - u_2)$

$$(v_2 - v_1) = 0.75\left(u - \frac{u}{7}\right)$$

$$v_2 - v_1 = \frac{9u}{14} \quad \text{...(ii)}$$

Multiplying equation (ii) by 2,

$$2v_2 - 2v_1 = \frac{9u}{7} \quad \text{...(iii)}$$

$$v_1 + 2v_2 = \frac{9u}{7} \quad \text{from eq.(i)}$$

By equation (iii) -(i), $v_1 = 0$ m/s

**12. (a)**

Given:Speed, $u$ = 25 m/s; diameter, $d$ = 50 cm = 0.5 m; Radius, $r$ = 0.25 m

We know that, $v^2 = u^2 + 2as$

$$0 = (25)^2 + 2 \times a \times (25)$$

$$a = \frac{-625}{50} = -12.5 \text{ m/s}^2$$

[Here minus sign represents retardation]

Angular retardation of the wheel,

$$\alpha = \frac{a}{r} = \frac{-12.5}{0.25} = -50 \text{ rad/s}^2$$

[Minus sign indicates retardation]

$$\alpha = 50 \text{ rad/s}^2 \text{ (Retardation)}$$

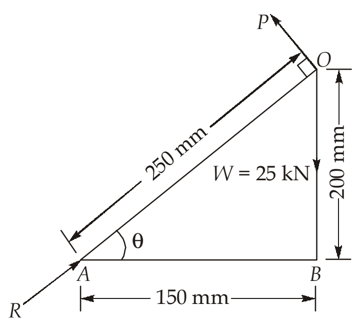
**13. (d)**

Given, Diameter of wheel = 500 mm, weight of wheel = 25 kN

For the least pull, force $P$ must be normal to the line $AO$.

$$AB = \sqrt{(OA)^2 - (OB)^2} = \sqrt{(250)^2 - (200)^2}$$

$$AB = 150 \text{ mm}$$




Taking moment about $A$,

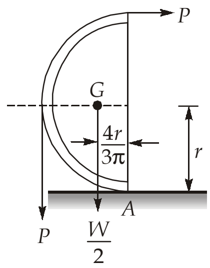
$$P \times 250 = W \times 150$$

$$P = \frac{25 \times 150}{250}$$

$$P = 15 \text{ kN}$$

**14. (b)**

Taking one halve of cylinder. Centre of gravity of a semicircle is at a distance of $\frac{4r}{3\pi}$ from centre.




Taking moment about $A$,

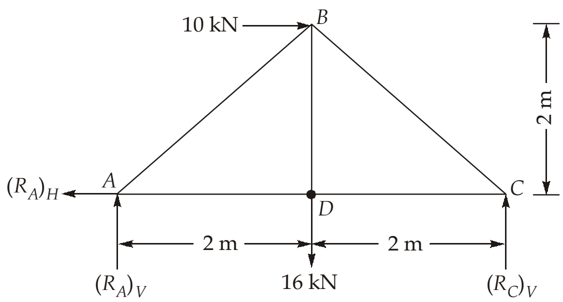
$$P \times 2r = P \times r + \left(\frac{W}{2}\right) \times \left(\frac{4r}{3\pi}\right)$$

$$P \times r = W\left(\frac{2r}{3\pi}\right)$$

$$P = \frac{2W}{3\pi}$$

**15. (c)**

Let, $R_A$ and $R_C$ are the support reactions at $A$ and $C$ respectively. Support $C$ is a roller support so there will be only vertical reaction at $C$.




Now, $\Sigma F_x = 0$

$$(R_A)_H = 10 \text{ kN } (\leftarrow)$$

Now, Taking moment about $A$,

$$(R_C)_V \times 4 = (16 \times 2) + (10 \times 2) = 52$$

$\Rightarrow$ $(R_C)_V = 13 \text{ kN}$

Now, $\Sigma F_y = 0$

$\Rightarrow$ $(R_A)_V + (R_C)_V = 16$

$\Rightarrow$ $(R_A)_V = 16 - 13 = 3 \text{ kN}$

Reaction at $A$, $R_A = \sqrt{(R_A)_H^2 + (R_A)_V^2} = \sqrt{10^2 + 3^2} = 10.44 \text{ kN}$

**16. (b)**

Assume,Initial angular velocity = $\omega_0$

Angular acceleration = $\alpha$

**Condition I:**

Angular velocity after 4 sec = $\omega$

$$\omega = \omega_0 + (\alpha t)$$

$$\omega = \omega_0 + 4\alpha \quad \text{...(i)}$$

We know that, $$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$100 = (4\omega_0) + \frac{1}{2}\times\alpha\times 4^2$$

$$100 = 4\omega_0 + 8\alpha \quad \text{...(ii)}$$

**Condition II:**

$$\theta = \omega \times t \qquad \{\because \alpha = 0 \text{ in } 2^{nd} \text{ case}\}$$

$$80 = (\omega_0 + 4\alpha) \times 4$$

$$20 = \omega_0 + 4\alpha \quad \text{...(iii)}$$

Multiply equation (iii) by (2), $40 = 2\omega_0 + 8\alpha$ ...(iv)

By equation (ii) – equation (iv),

$$(100 - 40) = (4\omega_0 + 8\alpha) - (2\omega_0 + 8\alpha)$$

$$60 = 2\omega_0$$

Initial angular velocity, $\omega_0$ = 30 rad/s

**17. (c)**

Given: Mass, $m$ = 80000 kg,

Resistance = 2% of (80000 × 10)N

$$= \frac{2\times 80000\times 10}{100} = 16000 \text{ N} = 16 \text{ kN}$$

Available force = Tractive force – Resistance

= (26 – 16) = 10 kN

$$\text{Acceleration of train} = \frac{\text{Available force}}{\text{mass}} = \frac{10\times 10^3}{80\times 10^3} = \frac{1}{8} \text{ m/s}^2$$

Final velocity of the train, $v$ = 10 m/s

$\therefore$ $$v = u + at$$

$$10 = 0 + \left(\frac{1}{8}\times t\right)$$

$$t = 80 \text{ s}$$

**18. (b)**

Given: $a = \frac{5}{v+3}$, where '$v$' is velocity and 's' is distance.

We know that,

$$v\frac{dv}{ds} = a$$

$$\frac{vdv}{ds} = \left(\frac{5}{v+3}\right)$$

$$v(v+3)dv = 5ds$$

Integrating on both sides,

$$\left(\frac{v^3}{3}+\frac{3v^2}{2}\right) = 5s + c_1$$

$\therefore$ at, $t = 0$, $s = 0$ and $v = 0$

$\therefore$ $$0 + 0 = 0 + c_1$$

$\therefore$ $$c_1 = 0$$

Now, $$\frac{v^3}{3}+\frac{3v^2}{2} = 5s$$

at, $v = 30$ m/s

$$\frac{(30)^3}{3}+\frac{3(30)^2}{2} = 5s$$

$$\frac{(30)^3}{3}+\frac{3\times 30^2}{2} = 5s$$

$$9000 + 1350 = 5s$$

$$s = \frac{10350}{5}$$

$$s = 2070 \text{ m}$$

**19.** **(c)**

Given: $m_A = 15$ kg, $m_B = 10$ kg

For mass $B$, $$m_B g - T = m_B a$$

$$10g - T = 10\,a \quad \text{...(i)}$$

For mass $A$, $$T = m_A a$$

$$T = 15\,a \quad \text{...(ii)}$$

Addition equation (i) and (ii)

$$(10g - T) + (T) = (15 + 10)a$$

$$a = \frac{10g}{25} = \frac{10\times 10}{25} = 4 \text{ m/s}^2$$

$$\text{Acceleration, } a = 4 \text{ m/s}^2$$

**20.** **(b)**

Given: Angle of inclination, $\alpha = 30°$; Deceleration, $a = 1$ m/s$^2$; Weight of block, $W = 5$ kN

Coefficient of friction, $\mu = 0.3$

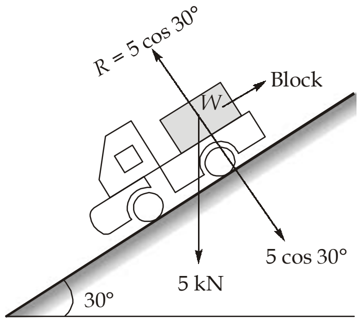
$\because$ As truck is decelerated, the load will tend to slip forward (i.e. downward)

Force due to deceleration, $F_1 = m \cdot a$

$$= \left(\frac{5\times 10^3}{10}\right)\times 1 = 500 \text{ N}$$

Component of the load along the plane,

$$F_2 = W\sin\theta = (5 \text{ kN}) \sin 30° = 2.5 \text{ kN}$$




∴ Total force that will cause slipping:

$$F_{net} = F_1 + F_2 = 0.5 + 2.5 = 3.0 \text{ kN}$$

$$\text{Force of friction} = \mu W\cos\theta = 0.3 \times 5 \times \cos 30°$$

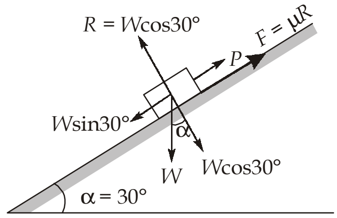
$$= 1.5 \times 0.866 \text{ kN} = 1.30 \text{ kN}$$

$$\text{Factor of safety} = \frac{\text{Force of friction}}{\text{Force causing slipping}} = \frac{1.30}{3.00} = 0.433$$

**21. (c)**

Given: Weight of body, $W$ = 1000 N

Angle of plane of inclination, $\alpha$ = 30°, Angle of friction, $\phi$ = 15°




For minimum value of $P$, the body will be at the point of sliding downwards. In this condition, friction force will act in upward direction parallel to the plane.

Let, $F$ and $R$ are friction force and normal reactions respectively.

In equilibrium condition, $W\sin 30° = P + \mu R$

$$P = W\sin 30° - \mu W\cos 30°$$

$$[\because \mu = \tan\phi = \tan 15°]$$

$$= W\left[0.5 - 0.268 \times \frac{\sqrt{3}}{2}\right] = W[0.5 - 0.268 \times 0.866]$$

$$= 1000 \times 0.268$$

Minimum force required for equilibrium, $P$ = 268 N

**Alternate:**

$$P_{min} = W\frac{\sin(\alpha - \phi)}{\cos\phi}$$

$$= 1000 \times \frac{\sin(30 - 15)°}{\cos 15°} = 1000 \times \tan 15°$$

$$P_{min} = 268 \text{ N}$$

**22. (a)**

Given: $P$ = 250 N; $BF_1$ = 25 mm; $F_1A$ = 325 mm; $CD$ = 360 mm; $DF_2$ = 40 mm

Leverage of the upper lever, $AB = \dfrac{AF_1}{BF_1} = \dfrac{325}{25} = 13$

Leverage of the lower lever, $CF_2 = \dfrac{CF_2}{DF_2} = \dfrac{360+40}{40} = 10$

Total leverage of the compound lever = 13 × 10 = 130

We know that,Total leverage = $\dfrac{W}{P} = \dfrac{W}{250}$

$$130 = \frac{W}{250}$$

$$W = 130 \times 250 = 32500 \text{ N} = 32.5 \text{ kN}$$

**23. (a)**

MOI of triangle about base $AB$,

$$I_1 = \frac{1}{12}\times(2r)\times(2r)^3 = \left(\frac{16}{12}\right)r^4$$

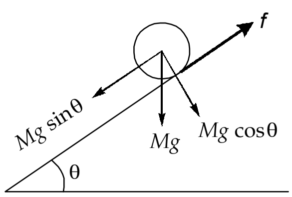
MOI of semi-circle about diameter, $I_2 = \left(\dfrac{1}{2}\right)\times\left(\dfrac{\pi}{64}\right)\times(2r)^4 = \left(\dfrac{\pi}{8}\right)r^4$

MOI of smaller circle about diameter, $I_3 = \left(\dfrac{\pi}{64}\right)r^4$

MOI of whole section about $AB$ axis, $I = I_1 + I_2 - I_3$

$$= \left(\frac{4}{3}+\frac{\pi}{8}-\frac{\pi}{64}\right)r^4 = \left(\frac{4}{3}+\frac{7\pi}{64}\right)r^4 = \left(\frac{4}{3}+\frac{22}{64}\right)r^4 = 1.677\, r^4$$

**24. (a)**




Let the friction force be '$f$'

So, $$Mg\sin\theta - f = Ma$$

$$f = Mg\sin\theta - Ma$$

and $$fR = I\alpha = I\frac{a}{R}$$

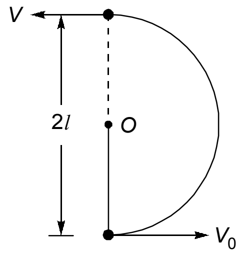
$$f = \frac{Ia}{R^2}$$

$$\frac{Ia}{R^2} = Mg\sin\theta - Ma$$

$$\therefore \qquad a = \frac{g\sin\theta}{\left(1+\dfrac{I}{MR^2}\right)}$$

**25. (d)**

Let the bob is given horizontal speed $V_0$ at the bottom.




By energy conservation,

So, $$\frac{1}{2}mV_0^2 = \frac{1}{2}mV^2 + mg\times(2l)$$

or, $$mV^2 = mV_0^2 - 4\,mgl \qquad \text{...(1)}$$

Also, at the top most point, force balance is,

$$mg + T = \frac{mV^2}{l}$$

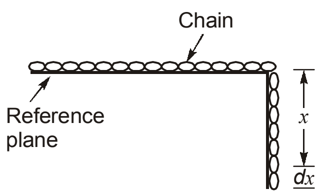
$$mV^2 = mgl + Tl$$

or $$mV_0^2 = 5\,mgl + Tl \qquad \text{(using (1))}$$

For minimum $V_0$, $T$ should be zero

$$\therefore \qquad V_0 = \sqrt{5gl}$$

**26. (d)**




The potential energy of $\frac{l}{3}$ of the chain that overhangs is

$$u_1 = \int_0^{l/3} -\frac{mgx}{l}dx = \frac{-mgl}{18}$$

The potential energy of the full chain when it completely slips off the table is

$$u_2 = \int_0^{l} -\frac{mgx}{l}dx = \frac{-mgl}{2}$$

$$\text{The loss in } PE = \frac{-mgl}{18} - \left(\frac{-mgl}{2}\right) = \frac{4mgl}{9}$$

This should be equal to gain in kinetic energy, but the initial kE is zero. Hence this is the kE when the chain completely falls off the table.

**27.** **(d)**

Using conservation of angular momentum,

$$MR^2\omega = \left(MR^2 \times \frac{8\omega}{9}\right) + \left(\frac{M}{8} \times \frac{9R^2}{25} \times \frac{8\omega}{9}\right) + \left(\frac{M}{8} \times x^2 \times \frac{8\omega}{9}\right)$$

$$\therefore \quad x = \frac{4R}{5}$$

**28.** **(b)**

Now, $\Sigma F_x = 0$

$R_{B2} = P$

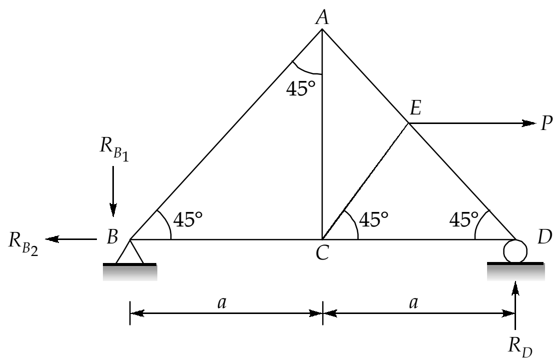
and, $\Sigma F_y = 0$

$R_D = R_{B1}$

Also, $\Sigma M_B = 0$

$$R_D \times 2a = P \times \frac{a}{2}$$

$$R_D = R_{B1} = \frac{P}{4}$$




Analysis of joint $B$,

So, $F_{AB} \sin 45° = \dfrac{P}{4}$

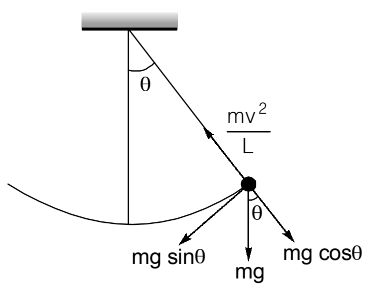
$$\Rightarrow \quad F_{AB} = \frac{\sqrt{2}P}{4}$$

Also, $P = F_{BC} + F_{AB}\cos 45°$

$$\Rightarrow \quad F_{BC} = P - F_{AB}\cos 45° = P - \frac{\sqrt{2}P}{4} \times \frac{1}{\sqrt{2}} = \frac{3P}{4}$$

Hence, $F_{BC} = 0.75P$

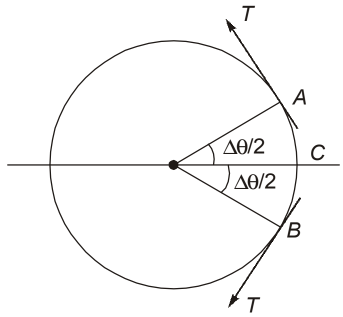
**29.** **(a)**



$$\Rightarrow \qquad T - mg\cos\theta = \frac{mv^2}{L}$$

$$\Rightarrow \qquad T = m\left(g\cos\theta + \frac{v^2}{L}\right)$$

**30. (a)**




Consider a small part ACB of the ring that subtends an angle $\Delta\theta$ at the centre. Let $T$ be tension in the ring.
$\Delta m$ be the mass of small element.

$$\Rightarrow \qquad 2T\sin\frac{\Delta\theta}{2} = \Delta m\left(\frac{v^2}{r}\right) \qquad ...(1)$$

Length of arc ACB is $R\Delta\theta$.

Now, $$\Delta m = \frac{M}{2\pi R}\times R\Delta\theta = \frac{M\Delta\theta}{2\pi}$$

$$\Rightarrow \qquad 2T\sin\frac{\Delta\theta}{2} = \frac{M\Delta\theta}{2\pi}\times\frac{v^2}{R}$$

$$\Rightarrow \qquad T = M\times\frac{v^2}{2\pi R}\times\frac{\Delta\theta/2}{\sin(\Delta\theta/2)} = \frac{Mv^2}{2\pi R} \qquad \left[\because \text{for small angle, } \sin\frac{\theta}{2} = \frac{\theta}{2}\right]$$

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