RIB-R | T13

Session 2019-20 | S.No. : 210919_GH1A

ANSWER KEY ➤ Strength of Material

1.	(a)	7.	(a)	13.	(b)	19.	(d)	25.	(a)
2.	(b)	8.	(c)	14.	(c)	20.	(b)	26.	(b)
3.	(a)	9.	(a)	15.	(d)	21.	(c)	27.	(a)
4.	(b)	10.	(d)	16.	(b)	22.	(b)	28.	(b)
5.	(c)	11.	(b)	17.	(c)	23.	(d)	29.	(a)
6.	(c)	12.	(b)	18.	(d)	24.	(b)	30.	(d)

DETAILED EXPLANATIONS

1. (a)

$$D = 20 \text{ mm (Diameter)}$$
$$L = 210 \text{ mm (Length)}$$
$$\Delta L = 0.30 \text{ mm (Expansion)}$$
$$P = 50 \text{ kN (Load)}$$
$$\text{Strain} = \frac{\Delta L}{L} = \frac{0.30}{210} = 1.428 \times 10^{-3}$$

2. (b)

$$E = 1.25 \times 10^5 \text{ N/mm}^2$$
$$\mu = 0.25$$
$$E = 2G(1 + \mu)$$
$$1.25 \times 10^5 = 2G(1 + 0.25)$$
$$G = \frac{1.25 \times 10^5}{2 \times 1.25} = 0.5 \times 10^5 \text{ N/mm}^2$$

3. (a)

Since fluid element will be subjected to hydrostatic loading therefore Mohr's circle will reduce into a point on σ-axis.

$\therefore$ Radius of Mohr circle = 0 unit

4. (b)

Strain energy due to torsion per unit volume,

$$U = \frac{\tau^2}{4G}(1+K^2)$$

where,

$$K = \frac{d}{D}$$

d = inner diameter of hollow shaft

D = outer diameter of hollow shaft

Given,

$$U = \frac{\tau^2}{3G}$$

$$\therefore \quad \frac{\tau^2}{3G} = \frac{\tau^2}{4G}(1+K^2)$$

$$\Rightarrow \quad K^2 = \frac{1}{3}$$

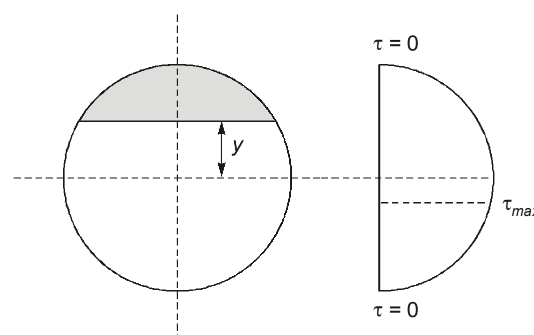
$$\Rightarrow \quad K = \frac{d}{D} = \sqrt{\frac{1}{3}}$$

$$\therefore \quad \frac{D}{d} = \sqrt{3}$$

5. (c)

Shear stress in circular cross-section

$$\tau = \frac{F}{3I}(r^2 - y^2)$$



We observe

1. Variation of τ versus y is a parabolic curve.
2. τ increases as y decreases.
3. $\tau = 0$ $(y = r)$
4. at $y = 0$, τ is maximum

$$\tau = \frac{F}{3I}(r^2 - y^2)$$

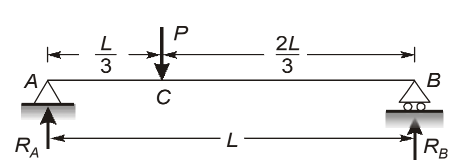
$$\tau_{y=0} = \tau_{max} = \frac{F \times (d/2)^2}{3 \times \frac{\pi}{64} d^4} = \frac{F}{3\left(\frac{\pi}{16} d^2\right)}$$

$$\Rightarrow \quad \tau_{max} = \frac{4}{3}\frac{F}{A} = \frac{4}{3}\tau_{avg.}$$

$$\Rightarrow \quad \frac{\tau_{max}}{\tau_{avg}} = \frac{4}{3} = 1.33$$

6. (c)

$$R_A + R_B = P$$



Taking moment about A

$$R_B \times L = P \times \frac{L}{3}$$

$$R_B = \frac{P}{3}$$

$$\text{Bending moment at } C = R_B \times \frac{2L}{3} = \frac{P}{3} \times \frac{2L}{3} = \frac{2PL}{9}$$

$$\frac{2 \times 50 \times 3}{9} = 33.33 \text{ Nm}$$

7. (a)

$$P = T\omega$$

So, $T = \frac{P}{N}$

Now, $\tau = \frac{16T}{\pi d^3} = \frac{16P}{\pi N d^3}$

So, $d \propto \left(\frac{P}{N}\right)^{1/3}$

9. (a)

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

$$\sigma_{max} = \frac{E}{R} \times y_{max} = \frac{2 \times 10^5}{10 \times 1000} \times \frac{20}{2} = 200 \text{ N/mm}^2 = 0.2 \text{ kN/mm}^2$$

10. (d)

$$\text{Area of the bar} = A = \frac{\pi}{4} \times 15^2 = 176.7145 \text{ mm}^2$$

$$\text{Stress in the bar} = \sigma = \frac{10 \times 10^3}{176.7145} = 56.59 \text{ N/mm}^2$$

Strain energy stored per unit volume

$$= \frac{\sigma^2}{2E} = \frac{56.59^2}{2 \times 2 \times 10^5} = 8.006 \times 10^{-3} \text{ N/mm}^2$$

11. (b)

$$D = 60 \text{ mm}$$
$$P = 180 \text{ kW}$$
$$f = 25 \text{ Hertz}$$

$$\text{thickness (t)} = \frac{D - d}{2}$$

$D \rightarrow$ outer diameter

$d \rightarrow$ Inner diameter

$$P = \frac{2\pi NT}{60} \text{ or } \frac{2\pi fT}{1000}$$

$$T = \frac{180 \times 1000}{2 \times \pi \times 25}$$

$$T = 1145.92 \text{ Nm}$$

By torsional rigidity:

$$\frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{J}{R} = \frac{T}{\tau}$$

$\therefore$

$$\frac{J}{R} = \text{Polar section modulus}$$

$$Z_P = \frac{1145.92 \times 10^3}{60} = 19098.67 \text{ mm}^3$$

$$Z_P = \frac{\pi}{16D}\left(D^4 - d^4\right)$$

$$d = 51.66 \text{ mm}$$

$$\text{thickness} = \frac{D-d}{2} = \frac{60-51.66}{2} = 4.17 \text{ mm}$$

12. (b)

$$\frac{\tau}{R} = \frac{G\theta}{L}$$

$$\theta = 2.5° = \frac{\pi}{180} \times 2.5° \text{ radian}$$

$$L = 450 \text{ mm}$$

$\therefore$

$$G = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$\Rightarrow$

$$R = \frac{\tau}{G} \times \frac{L}{\theta}$$

$$= \frac{0.0006 \times 450 \times 180}{\pi \times 2.5} \qquad \left[\text{strain} = \frac{\tau}{G} = 0.0006\right]$$

$$= 6.1879 \text{ mm}$$

$$D = 12.3758 \text{ mm}$$

13. (b)

$$L = 1 \text{ m}$$

$$P = 4500 \text{ N/m}$$

$$\text{Maximum deflection} = \frac{wL^4}{384EI} = \frac{4.5 \times 1000^4}{384 \times 200000 \times 5 \times 10^7} = 1.1718 \times 10^{-3} \text{ mm}$$

14. (c)

$$\in_{\theta 1} = \in_x \cos^2\theta_1 + \in_y \sin^2\theta_1 + \gamma \sin\theta_1 \cos\theta_1$$
$$\in_{\theta 2} = \in_x \cos^2\theta_2 + \in_y \sin^2\theta_2 + \gamma \sin\theta_2 \cos\theta_2$$
$$\in_{\theta 3} = \in_x \cos^2\theta_3 + \in_y \sin^2\theta_3 + \gamma \sin\theta_3 \cos\theta_3$$

$$\in_{0^\circ} = \in_x + \in_{45^\circ} = \frac{1}{2}\left(\in_x + \in_y\right) + \frac{\gamma}{2} \in_{90^\circ} = \in_y$$

$$\in_x = 400 \times 10^{-6},\ \in_y = -100 \times 10^{-6},\ \gamma = 2\in_{45^\circ} - \in_x - \in_y$$
$$\gamma = 2 \times 200 \times 10^{-6} - (400 - 100) \times 10^{-6} = 100 \times 10^{-6}$$

$$\in_{1,2} = \frac{\in_x + \in_y}{2} \pm \frac{1}{2}\sqrt{\left(\in_x - \in_y\right)^2 + \gamma^2}$$

$$\in_{1,2} = \frac{10^{-6}}{2}\left[(400 - 100) \pm \sqrt{(500)^2 + (100)^2}\right]$$

$$\in_{1,2} = 404.95 \times 10^{-6} \text{ and } -104.95 \times 10^{-6}$$

$$\sigma_1 = \frac{E\left(\nu \in_2 + \in_1\right)}{1 - \nu^2} = \frac{210 \times 10^3 (-0.3 \times 104.95 + 404.95) \times 10^{-6}}{1 - (0.3)^2}$$

$$\sigma_1 = 86.2 \text{ MPa}$$

$$\sigma_2 = \frac{E\left(\nu \in_1 + \in_2\right)}{1 - \nu^2} = \frac{210 \times 10^3 (0.3 \times 404.95 - 104.95) \times 10^{-6}}{1 - (0.3)^2}$$

$$\sigma_2 = 3.82 \text{ MPa}$$

Major principal stress = 86.2 MPa

15. (d)

Specific weight, $w = \rho g = 10.2 \text{ kN/m}^3$

Pressure at the given depth, $P = \rho g h$

$$P = 10.2 \times 600 = 6120 \text{ kN/m}^2$$
$$P = 6.12 \text{ N/mm}^2$$

Bulk modulus, $K = 160 \times 10^3 \text{ N/mm}^2$

Volumetric strain, $\in_v = \frac{P}{K} = \frac{6.12}{160 \times 10^3} = 3.825 \times 10^{-5}$

Initial volume, $V = \frac{4}{3}\pi R^3 = \frac{\pi D^3}{6} = \frac{3.14 \times 160^3}{6} = 2.144 \times 10^6 \text{ mm}^3$

$$\frac{\Delta V}{V} = \in_v$$

or $\Delta V = 2.144 \times 10^6 \times 3.825 \times 10^{-5}$

Change in volume, $\Delta V = 82 \text{ mm}^3$

16. (b)

$$\Delta L_{AC} = \Delta L_{CB}$$

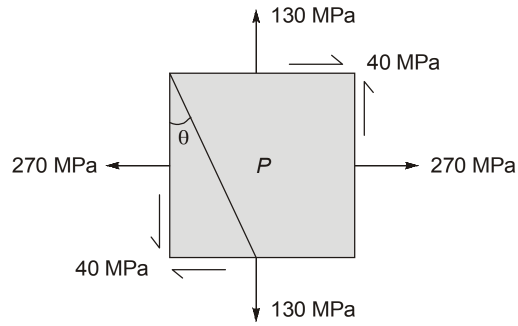
$$\frac{R_A 1}{AE} = \frac{2R_B}{AE}$$

$\Rightarrow$ $R_A = 2R_B$

Also $R_A + R_B = 10$

$\Rightarrow$ $R_A = \frac{20}{3} \text{ kN} = 6.67 \text{ kN}$

17. (c)



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2}$$

$$= \frac{270 + 130}{2} \pm \frac{1}{2}\sqrt{(270 - 130)^2 + 4 \times 40^2}$$

$$= 200 \pm \frac{1}{2}\sqrt{140^2 + 4 \times 1600}$$

$$= 200 \pm 80.62 \text{ MPa}$$

$$\sigma_1 = 280.62 \text{ MPa}$$

$$\sigma_2 = 119.38 \text{ MPa}$$

Maximum shearing stress $= \frac{1}{2}\left(\sigma_1 - \sigma_2\right) = \frac{1}{2}\left(280.62 - 119.38\right) = 80.62$ MPa

18. (d)

Thermal Stress $= \alpha E \Delta t = (12 \times 10^{-6})\,(200 \times 10^3)\,(120 - 20) = 240$ MPa

19. (d)

Hoop stress or circumferential stress in given by $\frac{pd}{2t} = \sigma_c$ longitudinal stress in given by $\frac{pd}{4t} = \sigma_l$

Volumetric strain $= \frac{dV}{V} = \frac{2d(d)}{d} + \frac{dl}{l}$

$$\in_V = 2\in_d + \in_l$$

Also, $\in_d = \in_c = \frac{\sigma_c}{E} - \frac{\nu\sigma_l}{E}$

and $\in_l = \frac{\sigma_l}{E} - \frac{\nu\sigma_c}{E}$

∴ $\in_V = \frac{1}{E}[2\sigma_c - 2\nu\sigma_l + \sigma_l - \nu\sigma_c]$

$$= \frac{1}{E}[(2 - \nu)\sigma_c + (1 - 2\nu)\sigma_l]$$

Here, $\sigma_c = \frac{pd}{2t} = \frac{3 \times 7.5}{2 \times 0.15} = 75$ MPa

$$\sigma_l = \frac{pd}{4t} = 37.5 \text{ MPa}$$

and $\nu = 0.3$

$$\in_v = \frac{1}{2\times10^5}[1.7\times75+0.4\times37.5] = 0.7125\times10^{-3}$$

∴ % change in volume = 0.07%

20. (b)

Modulus of resilience = Area under the $\sigma - \in$ curve till the yield point.

So, $$MOR = \frac{1}{2}\times2\times14 = 14$$

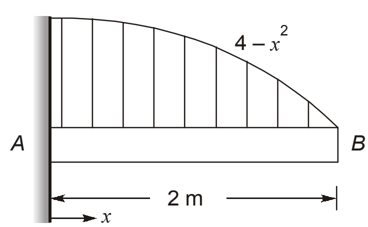
Modulus of toughness = Area under $\sigma - \in$ curve till the fracture point.

$$MOT = \frac{1}{2}\times2\times14+\frac{1}{2}\times4\times(20+14)+\frac{1}{2}\times2\times(20+12)$$

$$MOT = 14 + 68 + 32 = 114$$

So, $$\frac{MOR}{MOT} = \frac{14}{114} = 0.1228$$

21. (c)



According to the question,

$$\frac{dM}{dx} = 4 - x^2$$

So, $$\int_A^B dM = \int_0^2 (4-x^2)\,dx$$

So, $$M_B - M_A = 8-\frac{8}{3} = \frac{24-8}{3} = \frac{16}{3} = 5.33$$

22. (b)

Outer radius = R

Inner radius = $R - t$

So, Approximate polar moment of inertia

$$J = 2\pi R^3 t$$

Now, we know, $$\frac{T}{J} = \frac{\tau}{r}$$

For τ_{max}, $r = R$

So, $$T = \frac{\tau_{max} J}{R} = 2\pi R^2 t\,\tau_{max}$$

Now, weight of tube (W) = γLA, where A is the cross-section area,

So, $$A \approx 2\pi Rt$$

So, $$W = 2\pi RtL\gamma$$

$$\therefore \quad \frac{T}{W} = \frac{2\pi R^2 t\,\tau_{max}}{2\pi Rt\,L\gamma} = \frac{R\tau_{max}}{L\gamma}$$

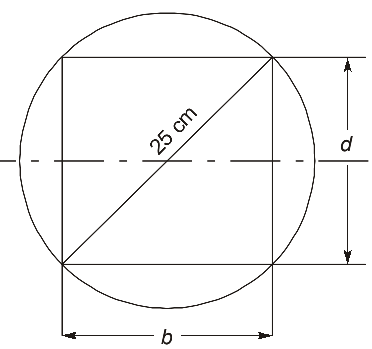
23. (d)

From the given figure,

$$\Rightarrow \quad b^2 + d^2 = (25^2)$$

$$\Rightarrow \quad d^2 = 625 - b^2 \qquad \text{...(1)}$$

Now for the strongest section, section modulus Z must be maximum.



$$\Rightarrow \quad Z = \frac{I}{Y} = \frac{bd^3}{12 \times d/2} = \frac{bd^2}{6}$$

$$\Rightarrow \quad = \frac{b}{6}(625 - b^2) = \frac{625}{6}b - \frac{b^3}{6}$$

For Z_{max}, $\frac{dZ}{db} = 0$

$$\Rightarrow \quad \frac{625}{6} - \frac{3b^2}{6} = 0$$

$$\Rightarrow \quad b^2 = \frac{625}{3}$$

$$\Rightarrow \quad b = 14.4 \text{ cm}$$

From equation (1)

$$\Rightarrow \quad d = 20.41 \text{ cm}$$

24. (b)

Outer radius = R

Inner radius = $R - t$

So, Approximate polar moment of inertia

$$J = 2\pi R^3 t$$

Now, we know, $$\frac{T}{J} = \frac{\tau}{r}$$

For τ_{max}, $r = R$

So, $$T = \frac{\tau_{max} J}{R} = 2\pi R^2 t\,\tau_{max}$$

Now, weight of tube (W) = γLA, where A is the cross-section area,

So, $$A \approx 2\pi Rt$$

So, $$W = 2\pi RtL\gamma$$

$$\therefore \quad \frac{T}{W} = \frac{2\pi R^2 t\,\tau_{max}}{2\pi Rt\,L\gamma} = \frac{R\tau_{max}}{L\gamma}$$

25. (a)

$$\frac{P_2}{P_1} = \frac{I_2}{I_1} = \left(\frac{d_2}{d_1}\right)^4$$

$$= (0.8)^4 = 0.4096$$

$$\frac{P_1 - P_2}{P_1} = 1 - 0.4096 = 0.5904 = 59.04\%$$

26. (b)

The strain energy per unit volume may be given as

$$u = \frac{1}{2} \times \frac{\sigma_y^2}{E} = \frac{1}{2} \times \frac{(250)^2}{2 \times 10^5}$$

$$= 0.156 \text{ Nmm/mm}^3$$

27. (a)

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

$$\frac{dV}{V} = \frac{r^3 - (r - \Delta r)^3}{r^3} = 1 - \left\{1 - \frac{\Delta r}{r}\right\}^3 = 1 - \left\{1 - \frac{\frac{0.55}{1000}}{2.5}\right\}^3$$

$$= 6.5985 \times 10^{-4}$$

$$K = \frac{250}{6.5985 \times 10^{-4}} = 378{,}871.2243 \text{ MPa}$$

$$= 378.871 \text{ GPa}$$

28. (b)

$$M = 80 \cdot x - 64(x - 1) \ \forall \ x \in (1, 4)$$

At centre

$$x = 4 \text{ m}$$

$$M = (80 \times 4) - 64(3) = 128 \text{ kNm}$$

29. (a)

Given,

$$\sigma_1 = 100 \text{ MPa}$$

$$\sigma_2 = 50 \text{ MPa},$$

$$\sigma_3 = 25 \text{ MPa}$$

$$S_{yt} = 220 \text{ MPa},$$

For maximum shear strain energy theory,

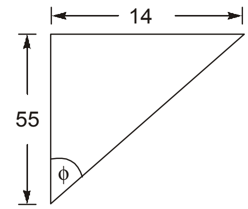
$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \le 2\left(\frac{S_{yt}}{N}\right)^2 \quad \text{[Where, } N = \text{factor of safety]}$$

$$(100 - 50)^2 + (50 - 25)^2 + (25 - 100)^2 = 2\left(\frac{220}{N}\right)^2$$

After solving,

$\because$ Factor of safety, $N = 3.326 \simeq \mathbf{3.33}$

30. (d)



$$\text{Shear strain, } \gamma = \tan\phi = \frac{14}{55} = 0.2545$$

$$\tau = \frac{P}{ab} = \frac{16000}{150 \times 225} = 0.474 \text{ MPa}$$

$$\text{Shear modulus, } G = \frac{\tau}{\gamma} = \frac{0.474}{0.2545} = 1.86 \text{ MPa}$$

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