CLASS TEST

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SOLID MECHANICS

CIVIL ENGINEERING

Date of Test : 04/03/2024

ANSWER KEY

1.	(d)	7.	(c)	13.	(c)	19.	(a)	25.	(b)
2.	(d)	8.	(a)	14.	(b)	20.	(b)	26.	(b)
3.	(d)	9.	(c)	15.	(c)	21.	(a)	27.	(a)
4.	(a)	10.	(d)	16.	(c)	22.	(b)	28.	(a)
5.	(a)	11.	(a)	17.	(a)	23.	(c)	29.	(d)
6.	(c)	12.	(a)	18.	(a)	24.	(a)	30.	(b)

DETAILED EXPLANATIONS

1. (d)

Radius of Mohr's circle, $R = \frac{\gamma_{max}}{2}$

2. (d)

- Vertical drop at B shows point load on the loaded beam having magnitude equal to (14 + 4) = 18 kN.
- 1° shear force diagram between B and D represents. Uniformly distributed load. Whose magnitude is

$$\frac{(-4)-(-16)}{8} = \frac{12}{8} = 1.5 \text{ kN/m}$$

or
$$\frac{9-(+3)}{4} = \frac{6}{4} = 1.5 \text{ kN/m}$$

3. (d)

As per maximum principal stress theory

$$\sigma_{p_1} = \frac{\sigma_y}{FOS}$$

$$150 = \frac{300}{FOS}$$

$$\text{FOS} = 2$$

4. (a)

$$E = 2G(1+\mu)$$

$$1.2 \times 10^5 = 2 \times 4.8 \times 10^4 (1+\mu)$$

$$\mu = 0.25$$

⇒ Now, Bulk modulus
$$K = \frac{E}{3(1-2\mu)}$$

$$= \frac{1.2 \times 10^5}{3(1-0.25 \times 2)}$$

$$= 8 \times 10^4 \text{ N/mm}^2 = 80 \text{ kN/mm}^2$$

5. (a)

The length of column is very large as compared to its cross-sectional dimensions.

6. (c)

As per Rankine theory:

$$\frac{1}{P_R} = \frac{1}{P_{CS}} + \frac{1}{P_E}$$

$$P_R = \frac{P_{CS} \times P_C}{P_{CS} + P_E} = \frac{P_{CS}}{1 + \frac{P_{CS}}{P_E}}$$

$\because$ $$P_{CS} = \sigma_{CS} A$$

$$P_E = \frac{\pi^2 EI}{Le^2}$$

$$P_R = \frac{P_{CS}}{1 + \dfrac{\sigma_{CS} A}{\dfrac{\pi^2 EI}{Le^2}}} = \frac{P_{CS}}{1 + \dfrac{\sigma_{CS} A Le^2}{\pi^2 EI}}$$

$\therefore$ $$k = \sqrt{\frac{I}{A}}$$

$\Rightarrow$ $$k^2 = \frac{I}{A}$$

Hence $$P_R = \frac{P_{CS}}{1 + \dfrac{\sigma_{CS}}{\pi^2 E}\left(\dfrac{Le}{k}\right)^2}$$

i.e., option (c) is correct.

7. (c)

$$d = 2 \text{ mm}$$
$$\sigma_{b(\max)} = 80 \text{ N/mm}^2$$
$$E = 100 \times 10^3 \text{ N/mm}^2$$

Distance between the neutral axis of wire and its extreme fibre

$$y = \frac{2}{2} = 1 \text{ mm}$$

So, minimum radius of the drum

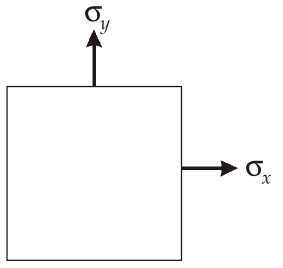
$$R = \frac{y}{\sigma_{b(\max)}} E \qquad \left(\because \frac{f}{y} = \frac{E}{R}\right)$$

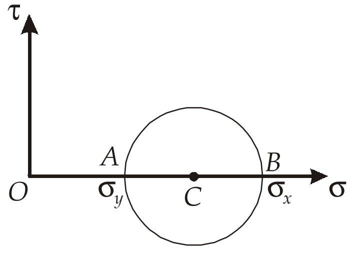
$$= \frac{1}{80} \times 100 \times 10^3$$

$$= 1.25 \times 10^3 \text{ mm} = 1.25 \text{ m}$$

8. (a)

Assuming ($\sigma_x > \sigma_y$)





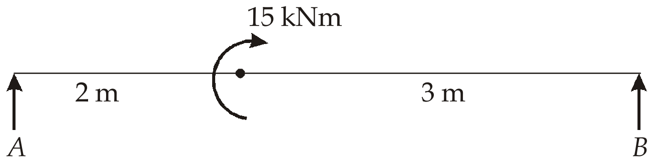
$$AB = \sigma_x - \sigma_y$$

$$AC = \frac{AB}{2} = \frac{\sigma_x - \sigma_y}{2}$$

$$\therefore \quad OC = OA + AC = \sigma_y + \frac{\sigma_x - \sigma_y}{2}$$

$$= \frac{\sigma_x + \sigma_y}{2}$$

9. (c)



$$R_A + R_B = 0$$

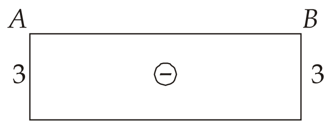
$$\Sigma M_A = 0$$

$$\Rightarrow \quad R_B \times 5 = 15$$

$$\Rightarrow \quad R_B = 3 \text{ kN}$$

$$R_A = -3 \text{ kN}$$

Now, SFD will be as shown in figure below,



10. (d)

As we know, $$\frac{dV}{dx} = w$$

$$\Rightarrow \quad w = \frac{d}{dx}\left(5x^2\right) = 10x$$

For midspan, $x = 1$ m

So, load intensity, $w = 10$ N/m

11. (a)

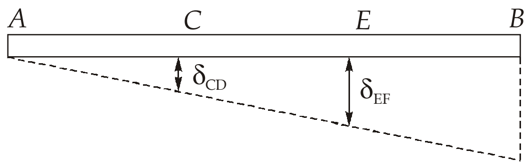
The maximum stress in the steel bar occurs at section *A-A*, the point of suspension because there the entire weight of steel and brass bar gives rise to normal stress whereas at any lower section only a portion of the weight and the steel would be effective in causing stress.

Weight of steel bar, $$W_s = 10 \times \frac{60}{10^4} \times \frac{76000}{10^3} = 4.56 \text{ kN}$$

Weight of brass bar, $$W_b = 6 \times \frac{50}{10^4} \times \frac{80000}{10^3} = 2.4 \text{ kN}$$

Stress across section *A-A*, $$\sigma = \frac{40 + 4.56 + 2.4}{60} = 0.78 \text{ kN/cm}^2$$

12. (a)



From similarity of ΔS,

$$\frac{\delta_{CD}}{1} = \frac{\delta_{EF}}{2}$$

$$\delta_{EF} = 2\delta_{CD}$$

$$\frac{F_{EF} \times L}{A_{EF} \times E_s} = \frac{2 \times F_{CD} \times L}{A_{CD} \times E_s}$$

$$\Rightarrow \quad F_{EF} = 2F_{CD} \quad \text{...(i)}$$

$$\Sigma M_A = 0$$

$$F_{CD} \times 1 + F_{EF} \times 2 = 40 \times 3$$

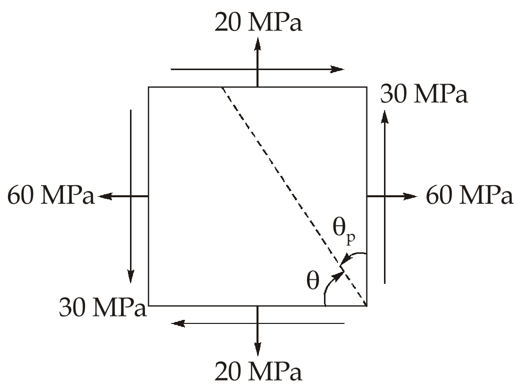
$$F_{CD} + 2 \times (2F_{CD}) = 120$$

$$5F_{CD} = 120$$

$$F_{CD} = 24 \text{ kN}$$

13. (c)

Plane having zero shear stress is called principal planes.



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2 \times 30}{60 - 20}$$

$$\tan 2\theta_p = 1.5$$

$$2\theta_p = \tan^{-1}(1.5)$$

$$\theta_p = 28.15°$$

Required angle from plane B, $\theta = 90° - 28.15° = 61.85°$ (Clockwise)

14. (b)

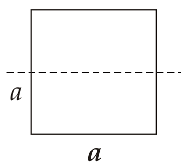
Normal stress, $$\sigma = \frac{32M}{\pi d^3} = \frac{32 \times (5 \times 200)}{\pi (50)^3} = 0.0815 \text{ N/mm}^2$$

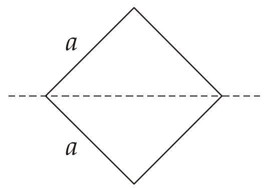
Shear stress, $$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16 \times (15 \times 100)}{\pi (50)^3} = 0.0611 \text{ N/mm}^2$$

$$\sigma_{p_1} = \frac{\sigma}{2} + \frac{1}{2}\sqrt{(\sigma)^2 + 4(\tau_{xy})^2}$$

$$= \frac{0.0815}{2} + \frac{1}{2}\sqrt{(0.0815)^2 + 4(0.0611)^2} = 0.114 \text{ N/mm}^2$$

15. (c)





$$\tau_{\max_1} = \frac{3}{2}\tau_{avg} \qquad \tau_{\max_2} = \frac{9}{8}\tau_{avg}$$

$$\tau_{\max_1} = \frac{3}{2}\frac{F}{a^2} \qquad \tau_{\max_2} = \frac{9}{8}\frac{F}{a^2}$$

$$\frac{\tau_{\max_1}}{B_1} = \frac{\frac{3}{2}\frac{F}{a^2}}{a} = \frac{3}{2}\frac{F}{a^3} \qquad \frac{\tau_{\max_2}}{B_2} = \frac{\frac{9}{8}\frac{F}{a^2}}{a\sqrt{2}} = \frac{9}{8\sqrt{2}}\frac{F}{a^3}$$

$\therefore$ $$\text{Ratio} = \frac{\frac{3}{2}}{\frac{9}{8\sqrt{2}}} = 1.89$$

16. (c)

When one end hinged and other end rigidly fixed against rotation and sway

$$P_{cr_1} = \frac{\pi^2 EI}{(L/\sqrt{2})^2}$$

$$120 = \frac{2\pi^2 EI}{L^2}$$

$$\frac{\pi^2 EI}{L^2} = 60 \qquad \text{...(i)}$$

When both ends are fixed against sway and rotation

$$P_{cr_2} = \frac{\pi^2 EI}{(L/2)^2}$$

$$P_{cr_2} = \frac{4\pi^2 EI}{L^2}$$

$$P_{cr_2} = 4 \times 60 = 240 \text{ kN}$$

17. (a)

Total deflection at free end, $\delta = \frac{wL^4}{8EI} = \frac{100(3)^4 \times 10^9}{8 \times 5 \times 10^{10}} = 20.25 \text{ mm}$

Reaction at roller support is produced due to the deflection resisted i.e., (20.25 – 3) mm = 17.25 mm

$$\Rightarrow \quad 17.25 = \frac{P \times (3)^3 \times 10^9}{3EI}$$

$$P = 95.83 \text{ N}$$

18. (a)

$$dL = \frac{PL}{Et(b-a)} \ln\left(\frac{b}{a}\right)$$

$$= \frac{40 \times 10^3 \times 3 \times 10^3}{2 \times 10^5 \times 15(80-20)} \ln\left(\frac{80}{20}\right)$$

$$= 0.924 \text{ mm}$$

19. (a)

20. (b)

21. (a)

Strain energy stored in hollow shaft, $U = \frac{\tau_{max}^2}{4G}\left[\frac{D^2 + d^2}{D^2}\right]V$

$$= \frac{80^2}{4 \times 100 \times 10^3}\left[\frac{80^2 + 60^2}{80^2}\right] \times 10^6$$

$$= \frac{10000 \times 10^6}{4 \times 10^5} = 2.5 \times 10^4 \text{ N-mm}$$

$$= 25 \text{ N-m}$$

22. (b)

We know deflection of spring,

$$\delta = \frac{64 WR^3 n}{Gd^4}$$

where, $W = 100$ N, $R = 25$ mm, $n = 12$, $G = 80$ GPa, $d = 5$ mm

So, $\delta = \frac{64 \times 100 \times (25)^3 \times 12}{80 \times 10^3 \times 5^4} = 24 \text{ mm}$

23. (c)

From bending equation, $$\frac{f}{y} = \frac{f_{max}}{y_{\max}}$$

$$\therefore \quad f = \frac{f_{max}}{y_{\max}} \times y$$

$$\therefore \quad \text{Force on shaded area} = \frac{f_{\max}}{y_{\max}} \times \Sigma A y$$

$$= \frac{f_{max}}{y_{\max}} (A\bar{y})$$

[where A is shaded area, $\bar{y}$ = distance of centroid of shaded area from N.A.]

$$= \frac{90}{12} \times \left[\frac{15}{2} \times 12\right] \times \frac{2}{3} \times 12$$

$$= 5400 \text{ kg}$$

24. (a)

$$\tau_{\max} = \frac{16}{\pi D^3}\sqrt{M^2 + T^2}$$

$$= \left[\frac{16}{\pi (100)^3}\sqrt{(8)^2 + (6)^2}\right] \times 10^6$$

$$= \frac{16}{\pi} \times \frac{10 \times 10^6}{10^6} = 50.93 \text{ MPa}$$

25. (b)

Principal strains, $$\varepsilon_{1/2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{800 + 200}{2} \pm \sqrt{\left(\frac{800 - 200}{2}\right)^2 + \left(\frac{-600}{2}\right)^2}\right] \times 10^{-6}$$

$$\varepsilon_1 = 924.264 \times 10^{-6}$$

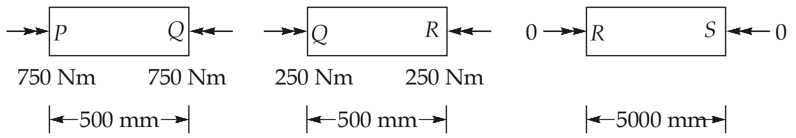
$$\varepsilon_2 = 75.74 \times 10^{-6}$$

Thus major principal stress is,

$$\sigma_1 = \frac{E}{1 - \mu^2}(\varepsilon_1 + \mu\varepsilon_2) = \frac{200 \times 10^3}{1 - 0.3^2}(924.264 + 0.3 \times 75.74) \times 10^{-6}$$

$$= 208.13 \text{ MPa} \simeq 208 \text{ MPa}$$

26. (b)



$$\text{Angle of twist } \phi = \frac{Tl}{GJ}$$

$$\phi_{PS} = \phi_{PQ} + \phi_{QR} + \phi_{RS}$$

$$= \frac{750 \times 10^3 \times 500}{80 \times 10^3 \times \frac{\pi}{32} \times 50^4} + \frac{250 \times 10^3 \times 500}{80 \times 10^3 \times \frac{\pi}{32} \times 50^4} + 0$$

$$= 10 \times 10^{-4} \times \frac{32}{\pi} \text{ rad}$$

$$= 0.58°$$

27. (a)

In pure bending case,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

So, $$R = \frac{EI}{M}$$

When same M is applied,

$$\frac{R_1}{R_2} = \frac{(EI)_1}{(EI)_2}$$

$$\Rightarrow \quad \frac{2}{R_2} = \frac{70 \times \frac{\pi}{4} \times 2.5^4}{120 \times \frac{\pi}{4} \times 2^4}$$

$$\Rightarrow \quad R_2 = 1.404 \text{ m}$$

28. (a)

As it is given that, $$\varepsilon = \frac{\sigma}{E} = \frac{y}{R} = 3.0 \times 10^{-5}$$

So, $$\frac{1}{R} = \frac{3.0 \times 10^{-5}}{30} \text{ mm}^{-1} = 10^{-6} \text{ mm}^{-1}$$

Also, in pure bending, $$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R} = \text{constant}$$

For σ_{max}, y_{max} has to be used

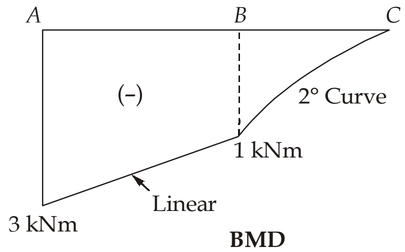
So, $\sigma_{max} = \frac{E}{R} y_{max} = \frac{200 \times 10^3}{R}$ MPa$\times y_{max}$

$\Rightarrow \sigma_{max} = 200 \times 10^3 \times 10^{-6} \times 50$ MPa

$\Rightarrow \sigma_{max} = 10$ MPa

29. (d)

There is no load on portion AB, so shear force remains constant throughout AB. Therefore, the shape of BMD between AB will be linearly varying with maximum value at A.



BMD

30. (b)

$$I_{NA} = \frac{200 \times 300^3}{12} - \frac{190 \times 260^3}{12} = 1.717 \times 10^8 \text{ mm}^4$$

Let point load at midspan $= W$ N

$\therefore$ Maximum shear force $= \frac{W}{2}$ N

$\therefore \tau_{max} = \frac{V a\bar{y}}{Ib} = 45 \text{ N/mm}^2$

$a\bar{y}$ = Moment of area above NA portion about NA

$= (200 \times 20) \times (130 + 10) + 10 \times 130 \times 65$

$= 644500 \text{ mm}^3$

$$\therefore \tau_{max} = \frac{W}{2} \times \frac{644500}{1.717 \times 10^8 \times 10} = 45$$

$\Rightarrow W = 239.77$ kN

$$M_{max} = \frac{WL}{4}$$

$$\because \sigma_{max} = \frac{M_{max}}{Z}$$

$$\Rightarrow 150 = \frac{WL}{4} \times \frac{y}{I}$$

$$\Rightarrow 150 = \frac{239.77 \times 10^3 \times L \times 10^3 \times 150}{4 \times 1.717 \times 10^8}$$

$\Rightarrow L = 2.864$ m

■■■■