CLASS TEST

S.No.: 01SK_ECEE_A_17012024

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SIGNAL & SYSTEM

EC-EE

Date of Test : 17/01/2024

ANSWER KEY

1.	(b)	7.	(b)	13.	(c)	19.	(a)	25.	(c)
2.	(d)	8.	(c)	14.	(d)	20.	(c)	26.	(c)
3.	(d)	9.	(c)	15.	(a)	21.	(c)	27.	(a)
4.	(c)	10.	(b)	16.	(a)	22.	(c)	28.	(a)
5.	(c)	11.	(a)	17.	(c)	23.	(c)	29.	(b)
6.	(b)	12.	(a)	18.	(b)	24.	(c)	30.	(b)

DETAILED EXPLANATIONS

1. (b)

Energy over one period, $$E_{period} = \int_0^{T_0} \left|e^{j\omega_0 t}\right|^2 dt$$

$$= \int_0^{T_0} 1 \cdot dt = T_0$$

Average power over one period,

$$P_{period} = \frac{1}{T_0} \times E_{period}$$

$$= \frac{1}{T_0} \times T_0 = 1$$

2. (d)

Let $X(\omega)$ and $Y(\omega)$ be the Fourier transform of $x(t)$ and $y(t)$ respectively. Then,

$$X(\omega) = \frac{1}{2 + j\omega}$$

$$Y(\omega) = \frac{1}{1 + j\omega}$$

Frequency response, $$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2 + j\omega}{1 + j\omega}$$

$$= \frac{1 + 1 + j\omega}{1 + j\omega}$$

$$H(\omega) = 1 + \frac{1}{1 + j\omega}$$

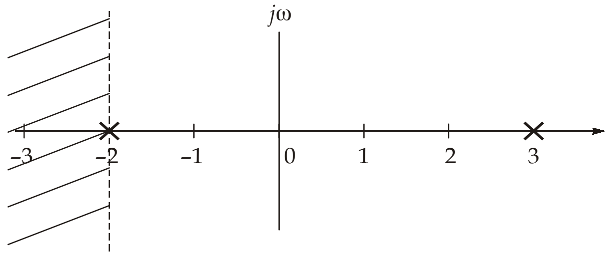
Taking inverse Fourier transform of $H(\omega)$ yields the impulse response $h(t)$.

$$h(t) = \delta(t) + e^{-t}u(t)$$

3. (d)

$$H_1(s) = \frac{1}{s^2 - s - 6} = \frac{1}{(s-3)(s+2)}, \text{Re}\{s\} < -2$$

Since given system $H_1(s)$ is rational and its ROC is to the left of the left most pole, therefore, corresponding LTI system is anticausal.



Also, ROC doesn't consists of $j\omega$ axis, therefore, system is unstable.

4. (c)

Let $h_1(n) = \left(\frac{1}{2}\right)^n u(n)$, it has Fourier transform

$$H_1(\Omega) = \frac{1}{1-\frac{1}{2}e^{-j\Omega}}$$

Using modulation theorem,

$$H(\Omega) = \frac{1}{2}\left[\frac{1}{1-\frac{1}{2}e^{-j\left(\Omega-\frac{\pi}{2}\right)}} + \frac{1}{1-\frac{1}{2}e^{-j\left(\Omega+\frac{\pi}{2}\right)}}\right]$$

5. (c)

6. (b)

7. (b)

$$y[n] = \sum_{k=-\infty}^{\infty} u(k+3)\,u[n-k-3]$$

$$u(k+3) = 1, \quad \text{for } k+3 \geq 0 \text{ or } k \geq -3$$
$$u[n-k-3] = 1, \quad \text{for } n-k-3 \geq 0 \text{ or } k \leq n-3$$

So, $$y[n] = \sum_{k=-3}^{n-3} 1 = n+1$$

$\therefore$ $$y[n] = (n+1)\,u(n)$$

Given, $$y[n] = (n+k)\,u[n+k-1]$$

By comparing $k = 1$.

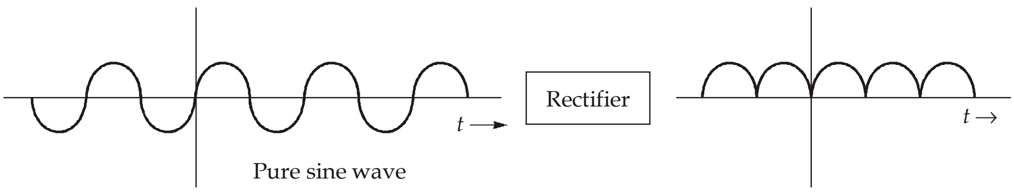
8. (c)

The impulse response of a high pass filter is simply obtained from the impulse response of the low pass filter by changing the signs of the odd numbered samples in $h_{IP}(n)$.

Thus, $$h_{HP}(n) = (-1)^n\, h_{IP}(n)$$
$$= (e^{j\pi})^n\, h_{IP}(n)$$

Thus the frequency response of the high pass filter is obtained as $H_{IP}(\omega - \pi)$.

9. (c)



The rectified signal is even signal.

$\therefore$ $$a_n \neq 0,\ b_n = 0$$

10. (b)

$\because$ $x[n]$ is real and odd, the Fourier transform $X(e^{j\omega})$ will be purely Imaginary and odd function. Thus, $\text{Re}\{X(e^{j\omega})\} = 0$ and the discrete time sequence corresponding to $\text{Re}\{X(e^{j\omega})\} = 0$.

11. (a)

With $N = 4$ we obtain the transfer function

$$H(z) = \frac{1}{4}\left(z^{-1} + z^{-2} + z^{-3} + z^{-4}\right)$$

After writing,
$$H(z) = \frac{1}{4}\left[\frac{z^3 + z^2 + z + 1}{z^4}\right]$$

Clearly there are 4 poles at $z = 0$, and there are three zeros from the solution

i.e.
$$z^3 + z^2 + z + 1 = \frac{1 - z^4}{1 - z} = 0$$

$\therefore$ Zeros must be such that $z^4 = 1$, with exclusion of $z = 1$.

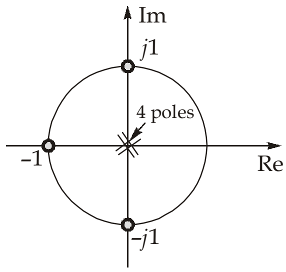
This is to say $\quad z^4 = e^{jk2\pi} \quad$ for $k = 1, 2, 3$

$$z = e^{jk\frac{\pi}{2}} \quad \text{for } k = 1, 2, 3$$

For $k = 1$,
$$z = e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = j$$

$k = 2$,
$$z = e^{j\pi} = \cos\pi + j\sin\pi = -1$$

$k = 3$,
$$z = e^{j\frac{3\pi}{2}} = \cos\left(\frac{3\pi}{2}\right) + j\sin\left(\frac{3\pi}{2}\right) = -j$$



12. (a)

Given:
$$x(t)\cos^2 t = x(t)\left[\frac{1}{2} + \frac{1}{2}\cos 2t\right] = \frac{1}{2}x(t) + \frac{1}{2}\cos 2t\, x(t)$$

Now,
$$g(t) = x(t)\cos^2 t * \frac{\sin t}{\pi t} = x(t)\left[\frac{1}{2} + \frac{\cos 2t}{2}\right] * \frac{\sin t}{\pi t}$$

$$G(j\omega) = \left[\frac{1}{2}X(j\omega) + \frac{1}{4}X(\omega - 2) + \frac{1}{4}X(\omega + 2)\right] \times rect\left(\frac{\omega}{2}\right)$$

Thus, the given solution will be

$$G(j\omega) = \frac{1}{2}X(j\omega)$$

$$g(t) = \frac{1}{2}x(t)$$

Thus, to get the desired result,

$$h(t) = \frac{1}{2}\delta(t)$$

13. (c)

Given that,

Let, $$y_1(t) = 2\pi X(-\omega)\big|_{\omega=t}$$

We have, $$y_1(t) = 2\pi \int_{u=-\infty}^{\infty} x(u)e^{jut}du$$

Similarly, let $y_2(t)$ be the output due to passing $x(t)$ through 'F' twice.

$$y_2(t) = 2\pi \int_{v=-\infty}^{\infty} 2\pi \int_{u=-\infty}^{\infty} x(u)e^{juv}du\, e^{jtv}dv$$

$$= (2\pi)^2 \int_{u=-\infty}^{\infty} x(u) \int_{v=-\infty}^{\infty} e^{j(t+u)v}dv\, du$$

$$= (2\pi)^2 \int_{u=-\infty}^{\infty} x(u)(2\pi)\delta(t+u)du$$

$$= (2\pi)^3 X(-t)$$

Finally, let $y_3(t)$ be the output due to passing $x(t)$ through F three times

$$y_3(t) = 2\pi \int_{u=-\infty}^{\infty} (2\pi)^3 x(-u)e^{jtu}du$$

$$= (2\pi)^4 \int_{-\infty}^{\infty} e^{-jtu}x(u)du = (2\pi)^4 X(t)$$

14. (d)

Given, $$x_1(n) = x(2n)$$

From the definition of z-transform,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$\therefore$ $$X_1(z) = \sum_{n=-\infty}^{\infty} x(2n)z^{-n}$$

$$K = 2n \Rightarrow n = \frac{K}{2}$$

at $$n = -\infty \Rightarrow K = -\infty$$

$$n = +\infty \Rightarrow K = +\infty$$

$$= \sum_{K=-\infty}^{\infty} x(K)z^{-\frac{K}{2}}$$

$$= \sum_{K=-\infty}^{\infty} \left[\frac{x(K)+(-1)^K x(K)}{2}\right] z^{-\frac{K}{2}}; \; K \text{ even}$$

$$= \frac{1}{2}\sum_{K=-\infty}^{\infty} x(K) z^{-\frac{K}{2}} + \frac{1}{2}\sum_{K=-\infty}^{\infty} x(K)\left(-z^{\frac{1}{2}}\right)^{-K}$$

From the definition of z-transform

$$X_1(z) = \frac{1}{2}\left[X(\sqrt{z}) + X(-\sqrt{z})\right]$$

15. (a)

Given, impulse response of the LTI system

$$h(t) = e^{-|t|}$$

by using the definition of Laplace transform

$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$$

$$\therefore \quad H(s) = \int_{-\infty}^{0} e^{t} e^{-st} dt + \int_{0}^{\infty} e^{-t} e^{-st} dt$$

$$= \left.\frac{e^{(1-s)t}}{1-s}\right|_{-\infty}^{0} + \left.\frac{e^{-(1+s)t}}{-(1+s)}\right|_{0}^{\infty}$$

$$= \frac{1}{1-s} + \frac{1}{1+s} = \frac{1+s+1-s}{1-s^2}$$

$$H(s) = \frac{2}{1-s^2} = \frac{Y(s)}{X(s)}$$

$$\therefore \quad 2X(s) = (1-s^2)Y(s) \text{ which can also write,}$$

$$y(t) - \ddot{y}(t) = 2x(t)$$

16. (a)

$$F_2(s) = F_1(s) \cdot e^{-s\tau}$$

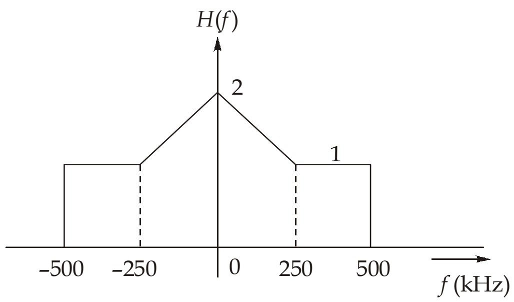
$$\therefore \quad G(s) = e^{-s\tau} \cdot \frac{F_1(s) \cdot F_1^*(s)}{|F_1(s)|^2} = e^{-s\tau} \cdot \frac{|F_1(s)|^2}{|F_1(s)|^2}$$

$$= e^{-s\tau}$$

$$\therefore \quad g(t) = \delta(t-\tau)$$

17. (c)

$\Rightarrow$



$$f_s = 4 f_{max}$$
$$= 4 \times 450 \text{ kHz}$$
$$= 1800 \text{ kHz}$$
$$f_c \text{ of LPF} = 500 \text{ kHz}$$

The spectrum of the sampled signal is given as

$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

$$X_s(f) = f_s[.....X(f - 1800k) + X(f) + X(f + 1800k) + ...]$$

After passing through LPF, $X_s(f) = f_s\, X(f)$

Frequency components passed through LPF are

$$f_{m1} = 200 \text{ kHz}$$
$$f_{m2} = 450 \text{ kHz}$$

Normalizing gain of filter by f_s,

Gain provided by filter to $f_{m2} = 1$

$$\text{Gain provided to } f_{m1} = 2 - \frac{f_{m1}}{250 \text{ kHz}}$$
$$= 2 - \frac{200 \text{ K}}{250 \text{ K}}$$
$$= 2 - \frac{4}{5}$$
$$= 1.2$$

$\therefore$ $y(t) = 2.4 \cos(400\pi \times 10^3 t) + 8 \cos(900\pi \times 10^3 t)$

18. **(b)**

Message signal has frequency components, $f_{m1} = 50$ Hz, $f_{m2} = 100$ Hz.

The pulse train is having fundamental frequency

$$f_s = \frac{1}{2 \times 10^{-3}} = 500 \text{ Hz}$$

Pulse train has half-wave symmetry.

Hence, even harmonics are 0.

$\therefore$ Sampling frequencies are 500 Hz, 1500 Hz.

Therefore, spectral components after sampling

$$f_s \pm f_{m1}, \quad f_s \pm f_{m2}$$
$$3f_s \pm f_{m1}, \quad 3f_s \pm f_{m2}$$
$$500 + f_{m1} = 500 + 50 = 550 \text{ Hz}$$
$$500 + f_{m2} = 500 + 100 = 600 \text{ Hz}$$
$$3f_s - f_{m1} = 1500 - 50 = 1450 \text{ Hz}$$
$$3f_s - f_{m2} = 1500 - 100 = 1400 \text{ Hz}$$

Therefore only 2 frequency components 550 Hz and 600 Hz exits in the range 500 Hz to 1000 Hz.

19. **(a)**

Given, Pole, $P = \frac{-1}{2} + \frac{1}{2}j$

Complex poles are always present in conjugate pairs.

$\therefore$ $$\bar{P} = \frac{-1}{2} - \frac{1}{2}j$$

For an all pass filter,

$$\text{Zero} = \frac{1}{(\text{pole*})}$$

$$Z = \frac{1}{\left(\frac{-1}{2} + \frac{1}{2}j\right)^*}$$

$$Z = \frac{1}{\frac{-1}{2} - \frac{1}{2}j} = \frac{2\ (-1+j)}{(-1-j)(-1+j)}$$

$$Z = \frac{2(-1+j)}{2}$$

$$Z = -1 + j$$

$$\bar{Z} = -1 - j$$

20. **(c)**

For a causal periodic signal, the Laplace transform is given as

$$X(s) = \frac{X_1(s)}{1-e^{-sT}}\ ;\ \text{for } \left|e^{-sT}\right| < 1 \qquad \text{where } T \text{ is time period}$$

For given case $T = 13$

Now, $$x_1(t) = 4u(t) - 5u(t-3) + u(t-8)$$

$\therefore$ $$X_1(s) = \frac{4}{s} - 5e^{-3s} \cdot \frac{1}{s} + \frac{e^{-8s}}{s}$$

$\therefore$ $$X(s) = \frac{(4 - 5e^{-3s} + e^{-8s})}{s(1-e^{-13s})}$$

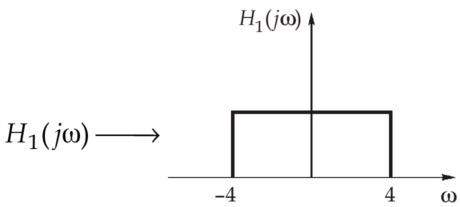
which can be rewritten as,

$$X(s) = \frac{e^{-4s}(4e^{4s} - 5e^{s} + e^{-4s})}{s(1-e^{-13s})}$$

21. **(c)**

Let, $$h_1(t) = \frac{\sin 4t}{\pi t}$$

It's Fourier transform $H_1(j\omega)$ will be



So, $$H(j\omega) \xleftrightarrow{\text{FT}} e^{-j\omega}H_1(j\omega) \qquad (\because \text{ Using time shift property})$$

and $$h(t) = h_1(t-1)$$

$\therefore$ $$H(j\omega) = \begin{cases} e^{-j\omega} & ; \quad |\omega| < 4 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Now, $$X(j\omega) = \frac{\pi}{j}\left[\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k \{\delta(\omega-3k)-\delta(\omega+3k)\}\right]$$

∴ $$Y(j\omega) = X(j\omega)H(j\omega) = \frac{\pi}{j}\left[\frac{1}{2}\{\delta(\omega-3)-\delta(\omega+3)\}e^{-j\omega}\right]$$

∴ $$y(t) = \frac{1}{2}\sin(3(t-1))$$

22. **(c)**

$$y(t) = x(t) * h(t)$$
$$= e^{-t}u(t) * \sum_{k=-\infty}^{\infty}\delta(t-3k)$$

∴ $$y(t) = \ldots e^{-(t+3)}u(t+3) + e^{-t}u(t) + e^{-(t-3)}u(t-3) + \ldots$$

In the range $0 \le t < 3$ we may write $y(t)$ as

$$y(t) = e^{-t} + e^{-(t+3)} + e^{-(t+6)} + \ldots = e^{-t}(1+e^{-3}+e^{-6}+\ldots)$$
$$= \frac{1}{1-e^{-3}}\cdot e^{-t}$$

∴ $$A = \frac{1}{1-e^{-3}} = 1.052$$

23. **(c)**

$$x[k] = \sum_{n=0}^{N-1} x[n]\exp\left[-j\frac{2\pi}{N}\cdot nk\right]$$
$$g[n] = x[n-2]_{\text{mod}\,N} + x[-n]_{\text{mod}\,N}$$
$$G[k] = \exp\left[-j\frac{2\pi}{N}(2)k\right]X[k] + X[-k]_{\text{mod}\,N}$$
$$G[1] = \exp\left[-j\frac{2\pi}{4}\cdot 2\right]X[1] + X[-1]$$

and since $$X[-1] = X[3]$$

So, $$G[1] = e^{-j\pi}X[1] + X[3] = -X[1] + X[3] = -7 + 9 = 2$$

24. **(c)**

The output of the equalizer $x'[n]$

Now, if there is no linear distortion, then

$$x'[n] = x[n-n_0]$$

∴ $$H_{eq}(z) = \frac{X'(z)}{Y(z)}$$

$$H_{eq}(z) = z^{-n_0}\frac{X(z)}{Y(z)} = \frac{z^{-n_0}}{H(z)}$$

Thus, $$H_{eq}(e^{j\Omega}) = \frac{e^{-jn_0\Omega}}{H(e^{j\Omega})}$$

25. (c)

$$h(t) = \delta(t) * h_1(t) * [1 - h_3(t) - h_4(t)] * h_2(t)$$
$$= \delta(t) * u(t) * [1 - 2\delta(t - \tau)] * \delta(t)$$
$$h(t) = u(t) - 2u(t - \tau)$$

26. (c)

$$h(t) = \cos\pi t\,(u(t + 3) - u(t - 3))$$
$$h(t) = \cos\pi t\,\{x(t)\}$$

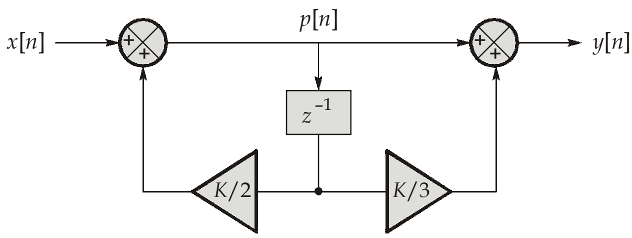
where, $$X(j\omega) = \frac{2\sin(3\omega)}{\omega}$$

Using modulation property of Fourier transform

$$H(j\omega) = \frac{1}{2}[X(j(\omega-\lambda)) + X(j(\omega+\lambda))]$$

So, $$H(j\omega) = \frac{2\omega\sin 3\omega}{\pi^2 - \omega^2}$$

27. (a)



$$p[n] = x[n] + \frac{K}{2}p[n-1]$$

and $$y[n] = p[n] + \frac{K}{3}p[n-1]$$

Taking Z-transform, we get,

$$P(z) = X(z) + \frac{K}{2}z^{-1}P(z)$$
$$Y(z) = P(z) + \frac{K}{3}z^{-1}P(z)$$
$$H(z) = \frac{Y(z)}{X(z)}$$

and $$X(z) = \left(1 - \frac{K}{2}z^{-1}\right)P(z)$$
$$Y(z) = \left(1 + \frac{K}{3}z^{-1}\right)P(z)$$

$\therefore$ $$H(z) = \left[\frac{1 + \frac{K}{3}z^{-1}}{1 - \frac{K}{2}z^{-1}}\right]; \quad |z| > \left|\frac{K}{2}\right|$$

For system to be stable $\Rightarrow$ $$1 > \frac{|K|}{2}$$
$$|K| < 2$$

28. **(a)**

Here, $\omega_o = 2\pi$

and $a_k = 1$ for $\lambda = 0, 1, 2, 3, \ldots$

Also, $$H(j\omega) = \frac{4}{4+\omega^2} = \frac{1}{2-j\omega} + \frac{1}{2+j\omega}$$

Now, $$b_k = a_k H(jk\omega_o) = \frac{1}{2+j2\pi k} + \frac{1}{2-j2\pi k}$$

29. **(b)**

By taking Z-transform on both the side of difference equation

$$Y(z) - \frac{1}{2}z^{-1}[Y(z) + y[-1]z] = X(z)$$

$$Y(z) - \frac{1}{2}z^{-1}Y(z) - \frac{1}{2}\times 3 = X(z)$$

For, $x(n) = \delta(n)$; $Y(z) = H(z)$

So, $$H(z)\left[1 - \frac{1}{2}z^{-1}\right] = \frac{5}{2}$$

$$H(z) = \frac{5/2}{1-\frac{1}{2}z^{-1}} = \frac{5}{2}\left(\frac{z}{z-1/2}\right)$$

$\therefore$ $$h[n] = \frac{5}{2}\left(\frac{1}{2}\right)^n \; ; \; n \geq 0$$

30. **(b)**

$$y[n] = \alpha y[n-1] + \beta x[n]$$

Put, $x[n] = \delta[n]$ to obtain impulse response

$$h[n] = \alpha h[n-1] + \beta\delta[n]$$

$n = 0$, $$h[0] = \alpha h[-1] + \beta\delta[0]$$
$$= \beta$$ For causal system $h[-1] = 0$

$n = 1$, $$h[1] = \alpha h[0] + \beta\delta[1]$$
$$= \alpha\beta$$
$$h[2] = \alpha h(1) + \beta\delta[2] = \alpha^2\beta$$

In general $h[n] = \alpha^n \beta$

Given that $$\sum_{n=0}^{\infty} h[n] = 1$$

$$\sum_{n=0}^{\infty} \alpha^n\beta = 1$$

$$\frac{\beta}{1-\alpha} = 1$$

$$\alpha + \beta = 1$$

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