

# CLASS TEST

S.No. : 01 ND\_EE\_NW\_220919

Signal and Systems



## MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Noida | Bhopal | Hyderabad | Jaipur | Lucknow | Indore | Pune | Bhubaneswar | Kolkata | Patna

Web: [www.madeeasy.in](http://www.madeeasy.in) | E-mail: [info@madeeasy.in](mailto:info@madeeasy.in) | Ph: 011-45124612

# CLASS TEST 2019-2020

## ELECTRICAL ENGINEERING

Date of Test : 22/09/2019

### ANSWER KEY > Signal and Systems

1. (c)	7. (c)	13. (c)	19. (c)	25. (d)
2. (d)	8. (a)	14. (d)	20. (c)	26. (a)
3. (c)	9. (d)	15. (d)	21. (b)	27. (c)
4. (b)	10. (b)	16. (c)	22. (b)	28. (a)
5. (a)	11. (b)	17. (d)	23. (a)	29. (b)
6. (d)	12. (d)	18. (a)	24. (a)	30. (b)

**DETAILED EXPLANATIONS**

1. (c)

From the given figure of  $x(t)$  and  $y(t)$ , we get conclude that,

$$x(0) = y(5)$$

at  $t = 5$  sec,  $y(5) = x(-5a + 20) = x(0)$

$$\therefore -5a + 20 = 0$$

$$\therefore a = 4$$

2. (d)

Given, signal  $x(t)$  has energy 'E'

for  $ax(t) \xrightarrow{E} a^2 E$

$$\therefore ax(bt + c) \xrightarrow{E} \frac{a^2 E}{b}$$

From the given signal  $a = 2, b = 5$  and hence

$\therefore$  The energy of signal  $2x(5t - 6)$  is

$$E = \frac{(2)^2 \times 10}{5} = 8 \text{ J}$$

3. (c)

We know that, unit impulse let  $x(t)$ ,

$$x(t) = \delta(t)$$

for  $\delta(t) \xrightarrow{LT} 1$

for  $\frac{d}{dt} x(t) \xrightarrow{LT} sX(s)$

$$\frac{d}{dt} \delta(t) \xrightarrow{LT} s$$

$$\frac{d^2}{dt^2} \delta(t) \xrightarrow{LT} s^2$$

4. (b)

Given,  $x(t) = \frac{\sin(10\pi t)}{\pi t}$

Taking Fourier transform

$$X(j\omega) = \begin{cases} 1 & ; \quad |\omega| \leq 10\pi \\ 0 & ; \quad |\omega| > 10\pi \end{cases}$$

or

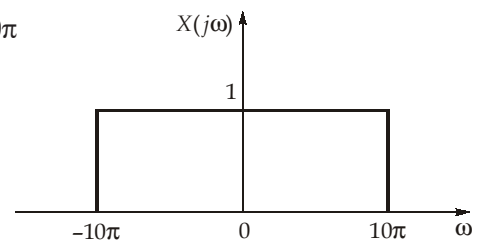
$\therefore$  The maximum frequency ' $\omega_m$ ' present in  $x(t)$  is  $\omega_m = 10\pi$

Hence we require,

$$\frac{2\pi}{T_s} > 2\omega_m$$

$$\frac{2\pi}{T_s} > 20\pi$$

$$\therefore T_s < \frac{1}{10}$$



6. (d)

Given,  $x(t) = \delta(t) * 2\delta(t - 1) * 3\delta(t - 2)$

From the convolution property of impulse,

$$x(t) * \delta(t - t_0) = x(t - t_0)$$

$$\therefore \begin{aligned} x(t) &= \delta(t) * 6\delta(t - 3) \\ x(t) &= 6\delta(t - 3) \end{aligned}$$

7. (c)

Given,  $H(z) = \frac{z}{z-0.2} = \frac{1}{1-0.2z^{-1}}$  ROC:  $|z| > 0.2$

Since the ROC:  $|z| > 0.2$ , which includes unit circle.

$\therefore$  The impulse response will be stable.

9. (d)

$$\int_{-\infty}^{\infty} \delta(at) dt = \frac{1}{a}$$

Since  $\delta(at) = \frac{1}{|a|} \delta(t)$

10. (b)

For sequence  $X_1[n] \xrightarrow{Z} X_1[z]$  ; ROC = R

For sequence  $X_2[n] = X_1[-n] \xrightarrow{Z} X_1[1/z]$  ; ROC = 1/R

$\therefore$  ROC's are reciprocal of each other.

11. (b)

By the differentiation property of Fourier transform,

$$\begin{aligned} \frac{dx(t)}{dt} &\xrightarrow{\text{F.T}} j\omega X(\omega) \\ \frac{d^2x(t)}{dt^2} &\xrightarrow{\text{F.T}} -\omega^2 X(\omega) \end{aligned}$$

By the time shifting property,

$$-\frac{d^2x(t+2)}{dt^2} \xrightarrow{\text{F.T}} \omega^2 e^{j2\omega} X(\omega)$$

12. (d)

Let us consider two signals,

$$x_1(t) = 1, \quad \forall t$$

$$x_2(t) = -1, \quad \forall t$$

Clearly  $x_1(t) \neq x_2(t)$  but  $(x_1(t))^2 = (x_2(t))^2$

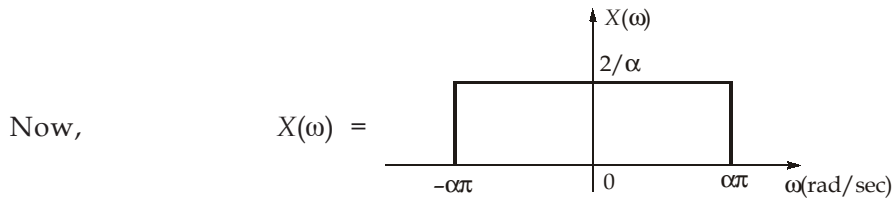
Therefore different inputs gives the same output hence the system is non invertible.

And also it is non linear system.

13. (c)

We know that,

Energy, 
$$E = \int_{-\infty}^{\infty} x^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$



$$E = \frac{1}{2\pi} \int_{-\alpha\pi}^{\alpha\pi} \left| \frac{2}{\alpha} \right|^2 d\omega$$

$$0.5 = \frac{1}{2\pi} \times \frac{4}{\alpha^2} [2\alpha\pi]$$

$$0.5 = \frac{4}{\alpha}$$

$\therefore \alpha = 8$

14. (d)

Given, 
$$x[n] = \cos \frac{\pi}{4} n + \sin \left( \frac{\pi}{3} n + \frac{1}{2} \right)$$

Let  $x[n] = x_1[n] + x_2[n]$

Let  $N_1$  be the period of  $x_1[n]$ .

$$\frac{\pi/4}{2\pi} = \frac{m}{N_1}$$

$\Rightarrow N_1 = 8$

Let  $N_2$  be the period of  $x_2[n]$ .

$$\frac{\pi/3}{2\pi} = \frac{m}{N_2}$$

$\Rightarrow N_2 = 6$

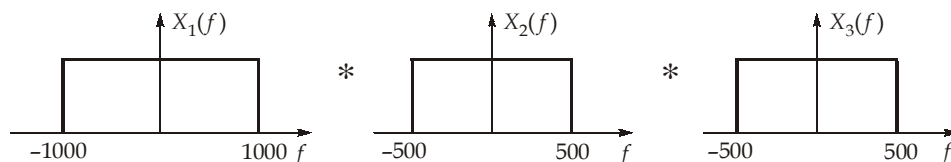
Overall time period,  $N = \text{LCM}(N_1, N_2) = \text{LCM}(6, 8)$

$\therefore N = 24$

15. (d)

We know that,

For 
$$X(f) = X_1(f) * X_2(f) * X_3(f)$$



Sampling frequency,  $f_s = 2(1000 + 500 + 500)$

$f_s = 4000$  samples/sec

16. (c)

Consider a right sided signal  $x(t)$ So, that  $x(t) = 0$  for  $t < t_0$  and  $X(s)$  converges for  $\text{Re}\{s\} = \sigma_0$ , then

$$|X(s)| \leq \int_{-\infty}^{\infty} |x(t)e^{-st}| dt = \int_{-\infty}^{\infty} |x(t)| e^{-\sigma_0 t} dt = \int_{t_1}^{\infty} |x(t)| e^{-\sigma_0 t} dt < \infty$$

Let  $\text{Re}\{s\} = \sigma_1 > \sigma_0$ . Then

$$\begin{aligned} \int_{t_1}^{\infty} |x(t)| e^{-\sigma_1 t} dt &= \int_{t_1}^{\infty} |x(t)| e^{-\sigma_0 t} e^{-(\sigma_1 - \sigma_0)t} dt \\ &< e^{-(\sigma_1 - \sigma_0)t_1} \int_{t_1}^{\infty} |x(t)| e^{-\sigma_0 t} dt < \infty \end{aligned}$$

Thus,  $X(s)$  converges for  $\text{Re}\{s\} = \sigma_1$  and the ROC of  $X(s)$  is of the form  $\text{Re}\{s\} > \sigma_0$ . Since the ROC of  $X(s)$  cannot include any poles of  $X(s)$ , we conclude that it is of the form  $\text{Re}\{s\} > \sigma_{\max}$ .

17. (d)

Given signal, Let  $x(t) = \frac{1}{\pi(1+t^2)}$

We know that,

$$e^{-a|t|} \xleftrightarrow{\text{FT}} \frac{2a}{a^2 + \omega^2}$$

Put  $a = 1$ 

$$e^{-|t|} \xleftrightarrow{\text{FT}} \frac{2}{1 + \omega^2}$$

By using duality property,

$$\begin{aligned} \frac{2}{1+t^2} &\xleftrightarrow{\text{FT}} 2\pi e^{-|\omega|} \\ \frac{2}{1+t^2} &\xleftrightarrow{\text{FT}} 2\pi e^{-|\omega|} \\ \frac{1}{\pi(1+t^2)} &\xleftrightarrow{\text{FT}} e^{-|\omega|} \end{aligned}$$

18. (a)

Given,  $y(t) = \frac{1}{2}x\left(\frac{t}{2}\right) * \delta(t-4) = \frac{1}{2}x\left(\frac{t-4}{2}\right) = \frac{1}{2}x\left(\frac{1}{2}t-2\right)$

at  $t = 2$ ;  $y(2) = \frac{1}{2}x\left(\frac{1}{2}(2)-2\right) = \frac{1}{2}x(-1)$

Slope of given signal  $x(t)$  is 2.

$$x(-1) = -2$$

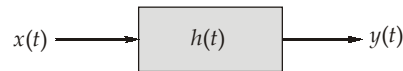
$$y(2) = \frac{1}{2}(-2) = -1$$

19. (c)

Given, the Causal LTI system,

$$H(j\omega) = \frac{1}{3 + j\omega}$$

and output,  $y(t) = e^{-3t} u(t) - e^{-4t} u(t)$



We know that,  $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

$$\therefore X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4+j\omega}$$

By inverse Fourier transform of  $X(j\omega)$ , we have,

$$x(t) = e^{-4t} u(t)$$

20. (c)

Given, sinusoidal pulse

$$z(t) = \begin{cases} e^{j10t} & ; |t| < \pi \\ 0 & ; |t| > \pi \end{cases}$$

We may express  $z(t)$  as the product of a complex sinusoid  $e^{j10t}$  and a rectangular pulse  $x(t)$ .

Let,  $x(t) = \begin{cases} 1 & ; |t| < \pi \\ 0 & ; |t| > \pi \end{cases}$

Fourier transform of  $x(t)$  is  $X(j\omega)$

$$\begin{aligned} \therefore X(j\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt = \int_{-\pi}^{\pi} 1 \cdot e^{-j\omega t} dt = \left[ \frac{e^{-j\omega t}}{-j\omega} \right]_{-\pi}^{\pi} \\ &= -\frac{1}{j\omega} [e^{-j\omega\pi} - e^{+j\omega\pi}] = \frac{e^{j\omega\pi} - e^{-j\omega\pi}}{j\omega} = \frac{2}{\omega} \left[ \frac{e^{j\omega\pi} - e^{-j\omega\pi}}{2j} \right] \end{aligned}$$

$$\therefore X(j\omega) = \frac{2}{\omega} \sin(\omega\pi)$$

By using frequency shift property of Fourier transform, we get,

$$z(t) = e^{j10t} \cdot x(t) \xrightarrow{\text{FT}} X(j(\omega - 10))$$

$$\therefore z(t) \xrightarrow{\text{FT}} \frac{2}{\omega - 10} \sin((\omega - 10)\pi)$$

21. (b)

Given,  $X(s) = \log(s + 2) - \log(s + 3)$

Differentiating both the sides with respect to  $s$

$$\frac{d}{ds} X(s) = \frac{1}{s+2} - \frac{1}{s+3} \quad \dots(i)$$

From the properties of Laplace transform, we know that,

$$tx(t) \longleftrightarrow -\frac{d}{ds} X(s)$$

Thus equation (i) can be written as,

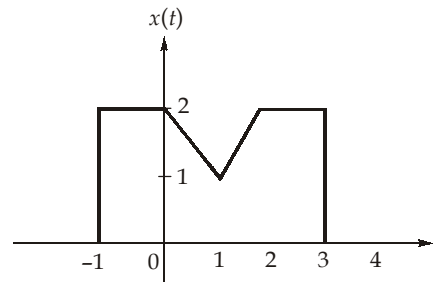
$$-tx(t) = [e^{-2t} - e^{-3t}]u(t)$$

or,

$$x(t) = \left[ \frac{e^{-3t} - e^{-2t}}{t} \right] u(t)$$

22. (b)

Given,



By the definition of Fourier transform,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \cdot e^{j\omega t} d\omega$$

at  $t = 0$ ,

$$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) d\omega$$

$$\therefore \int_{-\infty}^{\infty} X(j\omega) d\omega = 2\pi x(0) = 2\pi(2) = 4\pi \approx 12.57$$

23. (a)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} \frac{2^{-n}}{n!} z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{(2z)^{-n}}{n!} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2z}\right)^n}{n!}$$

$$X(z) = 1 + \frac{1}{2z} + \frac{\left(\frac{1}{2z}\right)^2}{2!} + \frac{\left(\frac{1}{2z}\right)^3}{3!} + \dots$$

$$X(z) = e^{1/2z}$$

$$X(1) = e^{1/2} = \sqrt{e} = 1.648 \approx 1.65$$

24. (a)

By Parseval's theorem,

$$\text{Energy of a signal } x[n] \text{ is, } E = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

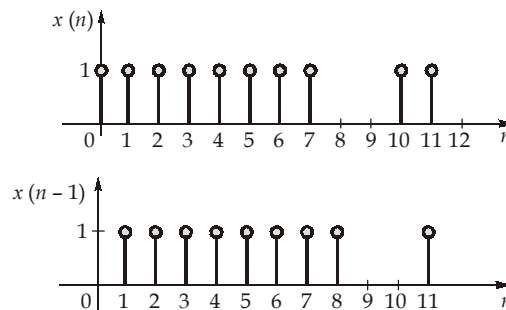
where  $X(e^{j\omega})$  is discrete time Fourier transform of  $x[n]$ ,

$$\text{So, } \frac{\sin(2n)}{\pi n} \xleftrightarrow{\text{DFT}} X(e^{j\omega}) = \begin{cases} 1; & |\omega| \leq 2 \\ 0; & \text{otherwise} \end{cases}$$

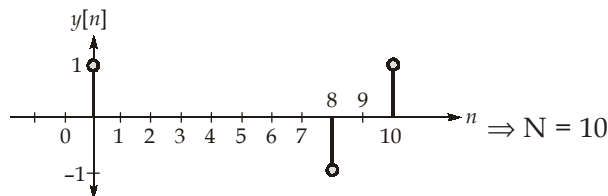
$$\therefore \text{Energy, } E = \frac{1}{2\pi} \int_{-2}^2 1 \cdot d\omega = \frac{1}{2\pi} [4]$$

$$\therefore E = \frac{2}{\pi} = 0.64 \text{ J}$$

25. (d)



Subtracting the two signal, we get



26. (a)

$$\text{Given, } x[n] = [1, 2, 3, 4]$$

$$h[n] = \delta[n - 1]$$

$$y[n] = x[n] * h[n] \quad [* \text{ denotes convolution}]$$

$$= x[n] * \delta[n - 1] = x[n - 1]$$

$$\therefore y[0] = x[-1] = 1 \quad (\because n = 0)$$

27. (c)

We know that the Laplace transform of

$$\cos(at)u(t) = \frac{s}{s^2 + a^2}$$

$$\therefore \cos(\pi t)u(t) = \frac{s}{s^2 + \pi^2}$$

now, the given function  $x(t)$  can be written as,



$$\begin{aligned}
 &= \cos \pi t [u(t) - u(t-1)] \\
 &= \cos(\pi t)u(t) - \cos \pi t u(t-1) \\
 &= \cos \pi t u(t) - \cos \pi(t-1+1)u(t-1) \\
 &= \cos \pi t u(t) - \cos[\pi(t-1) + \pi]u(t-1) \\
 x(t) &= \cos(\pi t)u(t) + \cos[\pi(t-1)]u(t-1)
 \end{aligned}$$

By taking Laplace transform,

$$X(s) = \frac{s}{s^2 + \pi^2} + \frac{s e^{-s}}{s^2 + \pi^2} \quad [\because x(t-t_0) = X(s) \cdot e^{-st_0}, \text{ by shifting property}]$$

$$X(s) = \frac{s[1 + e^{-s}]}{s^2 + \pi^2}$$

28. (a)

We know that,

$$x[n] \xleftrightarrow{\text{DTFT}} X(\omega)$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{n=-2}^3 x[n] e^{-j\omega n}$$

$$X(\omega) = e^{j2\omega} + 2e^{j\omega} + 3 + 2e^{-j\omega} + e^{-j2\omega}$$

$$X(\omega) = 3 + 4\cos\omega + 2\cos 2\omega \quad \dots(i)$$

$$H(k) = \sum_{n=0}^5 h[n] e^{-j\frac{2\pi}{6}nk}$$

$$H(k) = \sum_{n=0}^5 h[n] e^{-j\frac{\pi}{3}nk} = 3 + 2e^{-j\frac{\pi}{3}k} + e^{-j\frac{\pi}{3}2k} + 0 + e^{-j\frac{\pi}{3}4k} + 2e^{-j\frac{\pi}{3}5k}$$

$$= 3 + 2e^{-j\frac{\pi}{3}k} + 2e^{+j\frac{\pi}{3}k} + e^{-j\frac{2\pi}{3}k} + e^{j\frac{2\pi}{3}k}$$

$$= 3 + 4\cos \frac{\pi k}{3} + 2\cos \frac{2\pi k}{3} \quad \dots(ii)$$

By comparing equation (i) and equation (ii), we get,

$$\omega = \frac{\pi k}{3}$$

29. (b)

Given signals,  $x(t) = \sin \omega_0 t$

$h(t) = \text{sgnt}$

from the multiplication property of Fourier transform,

$$x(t)h(t) = \frac{1}{2\pi} [X(\omega) * H(\omega)]$$

Fourier transform of  $x(t)$  is  $X(\omega)$ ,

$$X(\omega) = \frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$$

Fourier transform of  $h(t)$  is  $H(\omega)$ ,

$$H(\omega) = \frac{2}{j\omega}$$

$$\begin{aligned} \therefore x(t) h(t) &\xrightarrow{\text{FT}} \frac{1}{2\pi} \left[ \frac{\pi}{j} (\delta(\omega - \omega_0) - \delta(\omega + \omega_0)) * \frac{2}{j\omega} \right] \\ &\xrightarrow{\text{FT}} \frac{1}{2\pi} \left[ \left[ \frac{\pi}{j} \times \frac{2}{j(\omega - \omega_0)} \right] - \left[ \frac{\pi}{j} \times \frac{2}{j(\omega + \omega_0)} \right] \right] \\ &\quad (\because X(\omega) * \delta(\omega - \omega_0) = X(\omega - \omega_0)) \\ &\xrightarrow{\text{FT}} \left[ \frac{-1}{\omega - \omega_0} + \frac{1}{\omega + \omega_0} \right] = \frac{-\omega - \omega_0 + \omega - \omega_0}{\omega^2 - \omega_0^2} \\ \therefore x(t)h(t) &\xrightarrow{\text{FT}} \frac{-2\omega_0}{\omega^2 - \omega_0^2} \end{aligned}$$

30. (b)

Given, 
$$X(z) = \frac{1}{1 - 2.5z^{-1} + z^{-2}} = \frac{1}{(z^{-1} - 2)\left(z^{-1} - \frac{1}{2}\right)}$$

$$\frac{1}{(z^{-1} - 2)\left(z^{-1} - \frac{1}{2}\right)} = \frac{A}{z^{-1} - 2} + \frac{B}{z^{-1} - \frac{1}{2}}$$

$$\therefore A = \frac{1}{2 - \frac{1}{2}} = \frac{1}{3/2} = \frac{2}{3}$$

$$B = \frac{1}{\frac{1}{2} - 2} = -\frac{2}{3}$$

$$\begin{aligned} \therefore X(z) &= \frac{\frac{2}{3}}{z^{-1} - 2} + \frac{-\frac{2}{3}}{z^{-1} - \frac{1}{2}} \\ &= \frac{-\frac{1}{3}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{4}{3}}{1 - 2z^{-1}} \end{aligned}$$

Given  $X(z)$  is a causal system, the ROC is right of the right most pole.

$$\therefore |z| > 2$$

hence, 
$$x[n] = -\frac{1}{3} \left(\frac{1}{2}\right)^n u[n] + \frac{4}{3} (2)^n u[n]$$

$$\therefore x(0) = -\frac{1}{3} + \frac{4}{3} = \frac{3}{3} = 1$$

