Electrical Engineering

Electric Circuits

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Electric Circuits

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CHAPTER 1

Basic and Electric Circuits

1.1 Introduction

In Electronics Engineering, we are often interested in communicating or transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an *electric circuit*, and each component of the circuit is known as an *element*.

An **electric circuit** is an interconnection of electrical elements.

In circuit analysis we need to calculate the voltage across some component or the current through other component or the power absorbed and delivered by different elements. In this chapter we will study about basic electrical variables such as charge, current, voltage, power and energy which will be used throughout the book. We initially focus on the *resistor*, a simple passive component, and a range of idealized active sources of voltage and current. As we move forward, new components will be added to the inventory to allow more complex (and useful) circuits to be considered.

A quick word of advice before we begin: Pay close attention to the role of "+" and "–" signs when labelling voltages, and the significance of the arrow in defining current; they often make the difference between wrong and right answers.

1.2 Charge

The concept of electric charge is the underlying principle for explaining all electrical phenomena. Also the most basic quantity in an electric circuit is the *electric charge*.

Charge is an electrical property of the atomic particles of which matter consists, measured in Coulombs (C).

The smallest amount of charge that exists is the charge carried by an electron, equal to -1.6×10^{-19} Coulomb. While, a proton carries a charge of $+1.6 \times 10^{-19}$ Coulomb.

The following points should be noted about electric charge:

1. The Coulomb is a large unit for charges. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons.
2. According to experimental observations, the only charges that occur in nature are integral multiple of the electronic charge $e = -1.602 \times 10^{-19}$ C.
3. The law of conservation of charge states that charge can neither be created nor be destroyed,can be only transferred. Thus the algebraic sum of the electric charges in a system does not change.

1.3 Current

Electric current is the time rate of change of charge, measured in amperes (A). Mathematically, the relationship between current i, charge q, and time t is

$$i(t) = \frac{dq(t)}{dt} \quad ...(1.1)$$

The net movement of 1 Coulomb (1C) of charge through a cross section of a conductor in 1 second (1s) produces an electric current of 1 amperes (1A).

The charge transferred between time t_0 and t is obtained by integrating both sides of Equation (1.1). We gets

$$q(t) = \int_{t_0}^{t} i(t)\,dt \quad ...(1.2)$$

1.3.1 Reference Direction for Current

The direction of current flow is conventionally taken as the direction of positive charge movement. Figure 1.1 shows the convention that we use to describe a current. The current i_1 is the rate of flow of electric charge from left to right, while the current i_2 is the flow of charge from right to left. Both have same value but opposite direction.

$$i_1 = -i_2$$

A current can be completely described by a value (which can be positive or negative) and a direction (indicated by an arrow).

***Figure*-1.1 :** *Current in a circuit element*

For **example**, a current of 5 A may be represented positively or negatively as shown in Figure 1.2. In other words, a negative current of –5 A flowing in one direction as shown in Figure 1.2 (b) is the same as a current of +5 A flowing in the opposite direction as shown in Figure 1.2 (a).

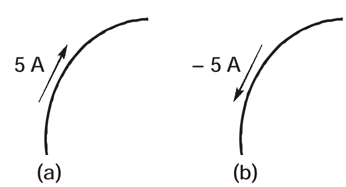


***Figure*-1.2 :** *Conventional current flow (a) Positive current flow (b) Negative current flow*

1.3.2 Types of Current

Different types of current are illustrated in Figure 1.3

- A current that is constant in time is termed as direct current, or simply dc, and is shown by Figure 1.3 (a).
- A current that vary sinusoidally with time [Figure 1.3 (b)]; is often referred as alternating current, or ac.

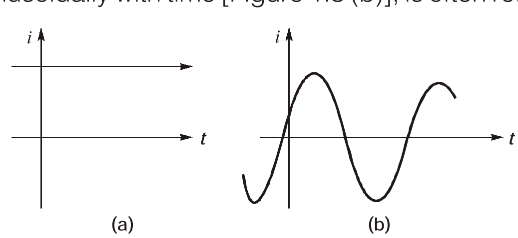


***Figure*-1.3 :** *(a) Direct current (dc) (b) Sinusoidal current (ac)*

1.4 Voltage

To move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery. This emf is also known as *voltage* or *potential difference*. The voltage v_{ab} between two points *a* and *b* in an electric circuit is the energy (or work) needed to move a unit charge from *a* to *b*; mathematically,

$$v_{ab}(t) = \frac{dw}{dq} \quad ...(1.3)$$

where w is energy in joules (J), q is charge in Coulombs (C) and voltage v_{ab} is measured in volts (V). It is evident that

1 volt = 1 Joule/Coulomb = 1 Newton-meter/Coulomb

Thus, **voltage** (or **potential difference**) is the energy required to move a unit charge through an element, measured in volts (V).

Remember

Voltage does not exist at a point by itself; it is always determined with respect to some other point. For this reason, voltage is also called potential difference. We often use the terms interchangeably.

1.4.1 Reference Polarity for Voltage

Figure 1.4 shows the voltage across an element (represented by a rectangular block) connected to points *a* and *b*. The plus (+) and minus (–) signs are used to define reference direction or voltage polarity. The v_{ab} can be interpreted in two ways; (1) point a is at a potential of v_{ab} volts higher than point *b*, or (2) the potential at point *a* with respect to point *b* is v_{ab}. It follows logically that general

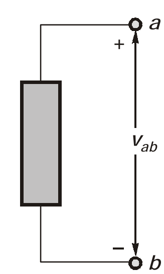


***Figure*-1.4 :** *Polarity of Voltage* v_{ab}

$$v_{ab} = -v_{ba}$$
$$v_{ab} = v_a - v_b$$
$$v_{ba} = v_b - v_a$$

For **example**, in Figure 1.5, we have two representation of the same voltage. In Figure 1.5 (a), point *a* is +9 V above point *b*; in Figure 1.5 (b), point *b* is –9 V above point *a*. We may say that in Figure 1.5 (a), there is a 9 V *voltage drop* from *a* to *b* or equivalently a 9 V *voltage rise* from *b* to *a*. In other words, a voltage drop from *a* to *b* is equivalent to a voltage rise from *b* to *a*.

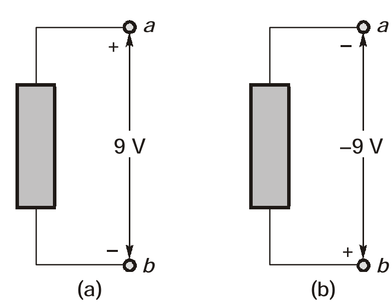


***Figure*-1.5 :** *Two equivalent representations of the same voltage* v_{ab}*:*
(a) Point a is 9 V above point b,
(b) Point b is –9 V above point a

NOTE: Keep in mind that electric current is always through an element and that electric voltage is always across the element or between two points.

1.4.2 Types of Voltage

Types of voltage are:

- A voltage that is constant in time is termed as dc voltage.
- A voltage that vary sinusoidally with time is referred as ac voltage.

1.5 Power

Power is the time rate of expending or absorbing energy, measured in watts (W). Thus, in terms of energy, power is defined as

$$p(t) = \frac{dw}{dt} \quad ...(1.4)$$

$$p(t) = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = v(t)i(t)$$

$$p(t) = v(t)\, i(t) \quad ...(1.5)$$

We see that power is simply the product of the voltage across an element and the current through the element. This is a relation which we shall have frequent use in this book.

1.5.1 Passive Sign Convention for Power Calculation

Current direction and voltage polarity are important in determining the sign of power. According to passive sign convention, the current enters the circuit element at the + terminal of the voltage and exit at the – terminal as shown in Figure 1.6(a). In this case power absorbed by the circuit element is

$$p = vi$$

This power is also called power dissipated by the element or power received by the element.

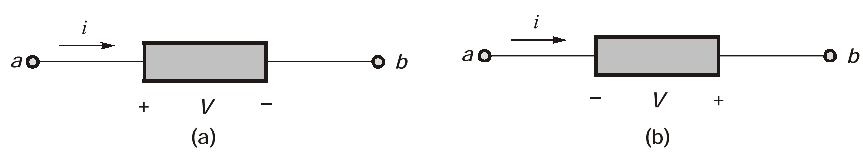


Figure-1.6 : *Passive Sign convention for power (a) Absorbing power (b) Supplying power*

If the current enters the circuit element at the – terminal of the voltage and exit at the + terminal as shown in Figure 1.6 (b), the power absorbed will be

$$p = -vi$$

If the absorbed power p is negative, then the circuit element actually generates power or , equivalently, delivers power to the rest of the circuit. The power absorbed by an element and the power supplied by the same element are related by

Power absorbed = –Power supplied

For **example**, the element in both circuits of Figure 1.7 has an absorbing power of +12 W because a positive current enters the positive terminal in both cases. In Figure 1.8 however, the element is absorbing power of –12 W is equivalent to a supplying power of +12 W.

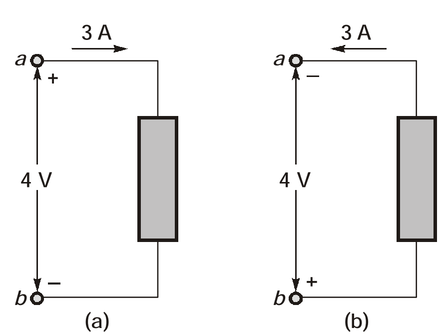


***Figure*-1.7 :** *Two cases of an element with an absorbing power of* 12 W

(a) $p = 4 \times 3 = 12$ W *and (b)* $p = 4 \times 3 = 12$ W

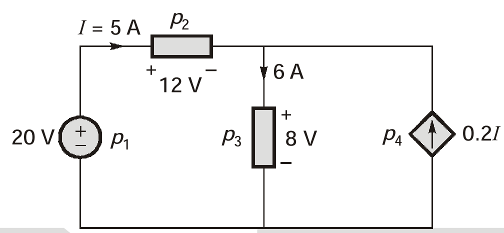
Example - 1.6 How much energy does a 100 W electric bulb consume in two hours?

Solution:

$$w = pt = 100\text{ (W)} \times 2\text{ (h)} \times 60\text{ (min/h)} \times 60\text{ (s/min)}$$
$$= 720000\text{ J} = 720\text{ kJ}$$

This is the same as $w = pt = 100\text{ W} \times 2\text{h} = 200\text{ Wh}$

Example - 1.7 Calculate the power supplied or absorbed by each element in figure.



Solution:

We apply the sign convention for power shown in figure. For p_1, the 5 A current is out of the positive terminal (or into the negative terminal); hence,

$$p_1 = 20(-5) = -100\text{ W} \quad \text{Supplied power}$$

For p_2 and p_3 the current flows into the positive terminal of the element in each case.

$$p_2 = 12(5) = 60\text{ W} \quad \text{Absorbed power}$$
$$p_3 = 8(6) = 48\text{ W} \quad \text{Absorbed power}$$

For p_4, we should note that the voltage is 8 V (positive at the top), the same as the voltage for p_3, since both the passive element and the dependent source are connected to the same terminals. (Remember that voltage is always measured across an element in a circuit). Since the current flows out of the positive terminal,

$$p_4 = 8(-0.2I) = 8(-0.2 \times 5) = -8\text{ W} \quad \text{Supplied power}$$

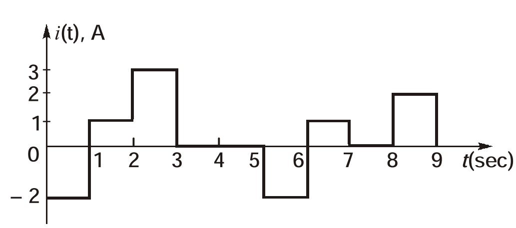
We should observe that the 20 V independent voltage source and $0.2I$ dependent current source are supplying power to the rest of the network, while the passive elements are absorbing power. Also

$$p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0$$

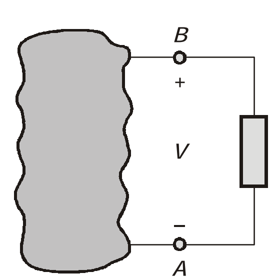
The total power supplied equals the total power absorbed.

Student's Assignments 1

Q.1 The current in an ideal conductor is plotted in the figure below. How much charge is transferred in the interval $0 < t < 6$ sec (in C)?

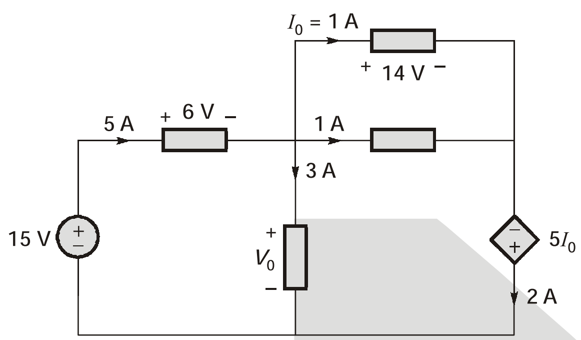


Q.2 In the figure shown below, when 12 coulombs of charge passes through the element from A to B, the energy absorbed by the element is 60 J. What is the voltage V across the element (in volts)?



Q.3 An automobile battery is charged with a constant current of 4 A. The terminal voltage of the battery is $v(t) = 10 + 2t$ V, where t is in hours. How much amount of energy (in kJ) is delivered to the battery during 3 hours?

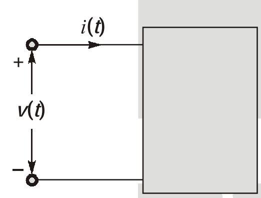
Q.4 In the circuit shown in figure, voltage V_0 is ______ Volts.



Q.5 For the circuit element shown in figure voltage and current are given as

$v(t) = 200e^{-50t} \sin 150t$ V and

$i(t) = 10e^{-50t} \sin 150t$ A



What is the power absorbed by the element at $t = 20$ ms (in watts)?

Q.6 If $q = (10 - 10e^{-2t})$ mC, find the current at $t = 0.5$s.

Q.7 In an electron beam in a TV picture tube carries 10^{13} electrons/second and is passing through plates maintained at a potential difference of 30 kV, calculate the power in the beam.

Answers:

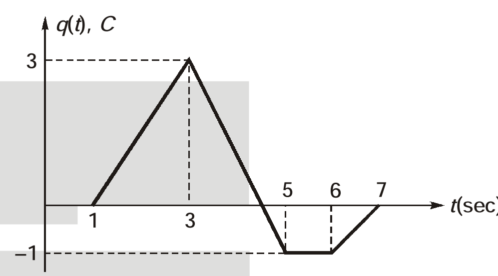
1. (0)	**2.** (5)	**3.** (561.6 kJ)
4. (9)	**5.** (5.4)	
6. (7.36 mA)	**7.** 48 mW	

Student's Assignments 2

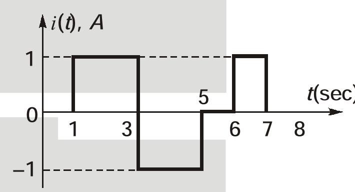
Q.1 Which of the following amount of electrons is equivalent to –3.941 C of charge?

(a) 1.628×10^{20} (b) 1.24×10^{18}
(c) 6.482×10^{17} (d) 2.46×10^{19}

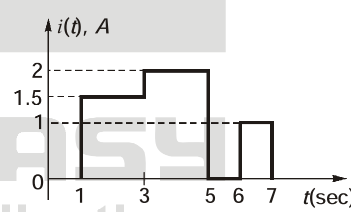
Q.2 The charge flowing in a circuit element is plotted in the following figure. The plot for current $i(t)$ will be



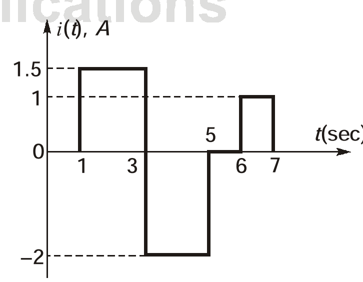
(a)



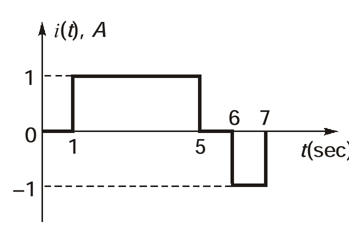
(b)



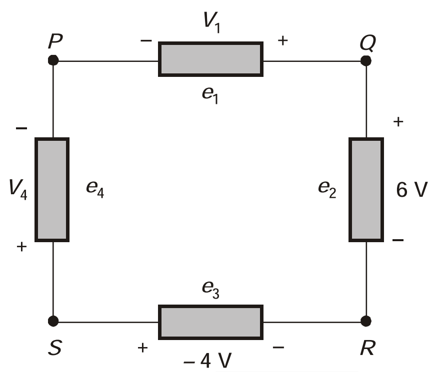
(c)



(d)



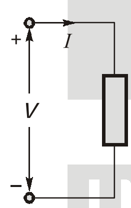
Q.3 A circuit consisting of four circuit elements is shown in the figure below. The voltage drops and polarity are indicated in the figure.



It is given that $V_{PQ} = 8$ V, then what are the values of V_1 and V_{QR}?

(a) 8 V, 6 V (b) –8 V, 6 V
(c) –8 V, –6 V (d) 8 V, –6 V

Q.4 Match **List-I (V, I)** with **List-II (Power absorbed or supplied)** for the element shown in figure and choose the correct answer using the codes given below the list.

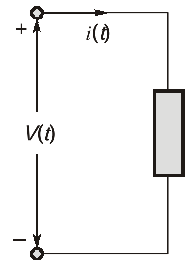


List-I	List-II
A. $V = 5$ V, $I = 3$ A	**1.** 15 W, supplied
B. $V = 5$ V, $I = -3$ A	**2.** 16 W, absorbed
C. $V = -8$ V, $I = 2$ A	**3.** 15 W, absorbed
D. $V = -8$ V, $I = -2$ A	**4.** 16 W, supplied

Codes:

	A	B	C	D
(a)	1	3	2	4
(b)	1	3	4	2
(c)	3	1	4	2
(d)	3	1	2	4

Q.5 The current through and voltage across the element shown in Figure are given as $v(t) = 4 \cos 3t$ V, $i(t) = 5 \sin 3t$ A.

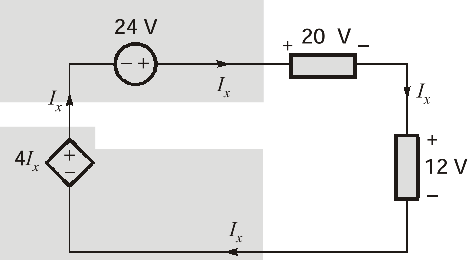


Consider the following statements:
1. The maximum value of power absorbed is 10 W.
2. At $t = 0.5$ sec, element absorbs power.
3. At $t = 1$ sec, element delivers power.

Which of the above statements are true?

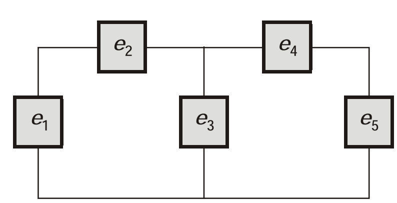
(a) 1 and 3 (b) 2 and 3
(c) 1 and 2 (d) 1, 2 and 3

Q.6 In the circuit shown below, if dependent source supplies 16 W of power, then the 24 V source



(a) Supplies 48 W of power
(b) Absorbs 48 W of power
(c) Supplies 96 W of power
(d) Absorbs 64 W of power

Q.7 Let P_i denotes the power absorbed by element e_i in the following circuit. If $P_1 = -210$ W, $P_2 = 45$ W, $P_4 = 50$ W and $P_5 = 20$ W then element e_3

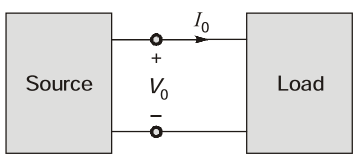


(a) receives 325 W (b) delivers 95 W
(c) receives 95 W (d) delivers 325 W

Common Data Questions (8 to 9):

In the electric circuit shown in Figure, voltage V_0 and I_0 are related as follows

$$V_0 = \begin{cases} 25 - I_0^2, & 0 \le I_0 \le 5 \\ 0, & I_0 \ge 5 \end{cases}$$



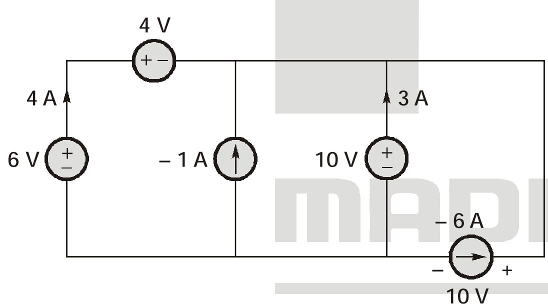
Q.8 The power absorbed by the load when I_0 = 3 A and I_0 = 4 A are respectively

(a) 16 W, 9 W
(b) 64 W, 27 W
(c) 48 W, 36 W
(d) 66 W, 84 W

Q.9 The value of current I_0, such that power absorbed by load is maximum, will be

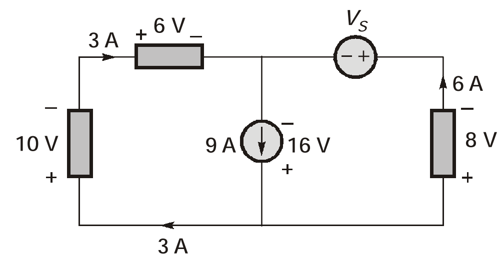
(a) 2.88 A
(b) 5 A
(c) 1 A
(d) 1.69 A

Q.10 In the circuit shown below, which of the following sources are being charged?



(a) 6 V, 4 V and 10 V source
(b) –1 A and –6 A source
(c) 6 V and –1 V source
(d) 4 V and 10 V source

Q.11 The voltage source V_S in the figure



(a) supplies 48 W
(b) absorbs 216 W
(c) absorbs 48 W
(d) supplies 216 W

Answer Key:

1. (d)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (a)
6. (a)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (b)
11. (c)				

CHAPTER 4

Circuit Theorems

4.1 Introduction

The techniques of nodal and mesh analysis described in Chapter 3 are reliable and extremely powerful methods. However, both require that we develop a complete set of equations to describe a particular circuit as a general rule, even if only one current, voltage, or power quantity is of interest. In this chapter, we investigate several different techniques for isolating specific parts of a circuit in order to simplify the analysis. We shall learn some of the circuit theorems which are used to reduce a complex circuit into a simple equivalent circuit. This includes Thevenin theorem and Norton theorem. These theorems are applicable to linear circuit, so we first discuss the concept of circuit linearity.

4.2 Linearity and Superposition

All of the circuits which we plan to analyze can be classified as linear circuits. Having done this, we can then consider the most important consequence of linearity, the principle of superposition. This principle is very basic and will appear repeatedly in our study of linear circuit analysis.

4.2.1 Linear Elements and Linear Circuits

Linearity is the property of an element describing a linear relationship between cause and effect. The property is a combination of both the homogeneity (scaling) property and the additivity property.

Homogeneity Property

The homogeneity property requires that if the input (excitation) is multiplied by a constant, then the output (response) gets multiplied by the same constant. For a resistor, for example, Ohm's law relates the input I to the output V,

$$V = IR$$

If the current is increased by a constant k, then the voltage increases correspondingly by k, that is,

$$kIR = kV$$

Additivity Property

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately. Using the voltage current relationship of a resistor, if

$$V_1 = I_1 R \quad \text{(Voltage due to current } I_1\text{)}$$

and $$V_2 = I_2 R \quad \text{(Voltage due to current } I_2\text{)}$$

then, applying current $(I_1 + I_2)$ gives

$$V = (I_1 + I_2)R = I_1 R + I_2 R$$
$$= V_1 + V_2$$

NOTE

These two properties defining a linear system can be combined into a single statement as:

For any linear resistive circuit, any output voltage or current, denoted by the variable y, is related linearly to the independent sources (inputs), i.e.,

$$y = a_1 x_1 + a_2 x_2 + ... + a_n x_n$$

where $x_1, x_2 .... x_n$ are the voltage and current values of the independent sources in the circuit and a_1 through a_m are properly dimensioned constants.

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

We define a linear dependent source as a dependent current or voltage source whose output current or voltage is proportional only to the first power of a specified current or voltage variable in the circuit (or to the sum of such quantities).

We now define a linear circuit as a circuit composed entirely of independent sources, linear dependent sources, and linear elements. From this definition, it is possible to show that "the response is proportional to the source," or that multiplication of all independent source voltages and currents by a constant K increases all the current and voltage responses by the same factor K (including the dependent source voltage or current outputs).

To illustrate the linearity principle, consider the linear circuit shown in Figure 4.1. The linear circuit has no independent sources inside it. It is excited by a voltage source v_s, which serves as the input. The circuit is terminated by a load R. We may take the current 'i' through R as the output. Suppose v_s = 10 V gives i = 2 A. According to the linearity principle, v_s = 1 V will give i = 0.2 A. By the same token, i = 1 mA must be due to v_s = 5 mV.

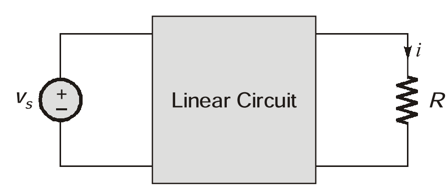


Figure-4.1 : *A linear circuit with input v_s and output i*

NOTE

For example, when current i_1 flows through resistor R, the power is $p_1 = Ri_1^2$, and when current i_2 flows through R, the power is $p_2 = Ri_2^2$. If current $i_1 + i_2$ flows through R, the power absorbed is $p_3 = R(i_1 + i_2)^2 \neq p_1 + p_2$. Thus, the power relation is non-linear.

4.2.2 The Superposition Principle

If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis. Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the superposition.

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages (or currents through) that element due to each independent source acting alone. The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition principle, we must keep two things in mind: