Electronics Engineering

Network Theory

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Network Theory

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CHAPTER 1

Basic and Electric Circuits

1.1 Introduction

In Electronics Engineering, we are often interested in communicating or transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an *electric circuit*, and each component of the circuit is known as an *element*.

An **electric circuit** is an interconnection of electrical elements.

In circuit analysis we need to calculate the voltage across some component or the current through other component or the power absorbed and delivered by different elements. In this chapter we will study about basic electrical variables such as charge, current, voltage, power and energy which will be used throughout the book. We initially focus on the *resistor*, a simple passive component, and a range of idealized active sources of voltage and current. As we move forward, new components will be added to the inventory to allow more complex (and useful) circuits to be considered.

A quick word of advice before we begin: Pay close attention to the role of "+" and "–" signs when labelling voltages, and the significance of the arrow in defining current; they often make the difference between wrong and right answers.

1.2 Charge

The concept of electric charge is the underlying principle for explaining all electrical phenomena. Also the most basic quantity in an electric circuit is the *electric charge*.

Charge is an electrical property of the atomic particles of which matter consists, measured in Coulombs (C).

The smallest amount of charge that exists is the charge carried by an electron, equal to -1.6×10^{-19} Coulomb. While, a proton carries a charge of $+1.6 \times 10^{-19}$ Coulomb.

The following points should be noted about electric charge:

1. The Coulomb is a large unit for charges. In 1 C of charge, there are $1 / (1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons.
2. According to experimental observations, the only charges that occur in nature are integral multiple of the electronic charge $e = -1.602 \times 10^{-19}$ C.
3. The law of conservation of charge states that charge can neither be created nor be destroyed,can be only transferred. Thus the algebraic sum of the electric charges in a system does not change.

1.3 Current

Electric current is the time rate of change of charge, measured in amperes (A). Mathematically, the relationship between current i, charge q, and time t is

$$i(t) = \frac{dq(t)}{dt} \quad ...(1.1)$$

The net movement of 1 Coulomb (1C) of charge through a cross section of a conductor in 1 second (1s) produces an electric current of 1 amperes (1A).

The charge transferred between time t_0 and t is obtained by integrating both sides of Equation (1.1). We gets

$$q(t) = \int_{t_0}^{t} i(t)\,dt \quad ...(1.2)$$

1.3.1 Reference Direction for Current

The direction of current flow is conventionally taken as the direction of positive charge movement. Figure 1.1 shows the convention that we use to describe a current. The current i_1 is the rate of flow of electric charge from left to right, while the current i_2 is the flow of charge from right to left. Both have same value but opposite direction.

$$i_1 = -i_2$$

A current can be completely described by a value (which can be positive or negative) and a direction (indicated by an arrow).

***Figure*-1.1 :** *Current in a circuit element*

For **example**, a current of 5 A may be represented positively or negatively as shown in Figure 1.2. In other words, a negative current of –5 A flowing in one direction as shown in Figure 1.2 (b) is the same as a current of +5 A flowing in the opposite direction as shown in Figure 1.2 (a).

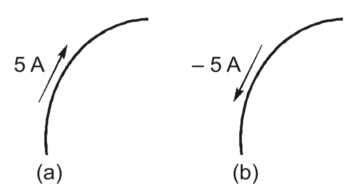


***Figure*-1.2 :** *Conventional current flow (a) Positive current flow (b) Negative current flow*

1.3.2 Types of Current

Different types of current are illustrated in Figure 1.3

- A current that is constant in time is termed as direct current, or simply dc, and is shown by Figure 1.3 (a).
- A current that vary sinusoidally with time [Figure 1.3 (b)]; is often referred as alternating current, or ac.

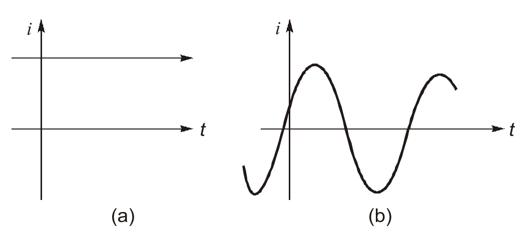


***Figure*-1.3 :** *(a) Direct current (dc) (b) Sinusoidal current (ac)*

1.4 Voltage

To move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery. This emf is also known as *voltage* or *potential difference*. The voltage v_{ab} between two points *a* and *b* in an electric circuit is the energy (or work) needed to move a unit charge from *a* to *b*; mathematically,

$$v_{ab}(t) = \frac{dw}{dq} \qquad ...(1.3)$$

where *w* is energy in joules (J), *q* is charge in Coulombs (C) and voltage v_{ab} is measured in volts (V). It is evident that

1 volt = 1 Joule/Coulomb = 1 Newton-meter/Coulomb

Thus, **voltage** (or **potential difference**) is the energy required to move a unit charge through an element, measured in volts (V).

Remember

Voltage does not exist at a point by itself; it is always determined with respect to some other point. For this reason, voltage is also called potential difference. We often use the terms interchangeably.

1.4.1 Reference Polarity for Voltage

Figure 1.4 shows the voltage across an element (represented by a rectangular block) connected to points *a* and *b*. The plus (+) and minus (–) signs are used to define reference direction or voltage polarity. The v_{ab} can be interpreted in two ways; (1) point a is at a potential of v_{ab} volts higher than point *b*, or (2) the potential at point *a* with respect to point *b* is v_{ab}. It follows logically that general

$$v_{ab} = -v_{ba}$$
$$v_{ab} = v_a - v_b$$
$$v_{ba} = v_b - v_a$$

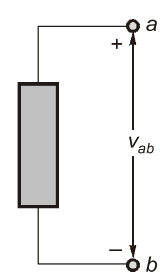


***Figure-1.4 :** Polarity of Voltage v_{ab}*

For **example**, in Figure 1.5, we have two representation of the same voltage. In Figure 1.5 (a), point *a* is +9 V above point *b*; in Figure 1.5 (b), point *b* is –9 V above point *a*. We may say that in Figure 1.5 (a), there is a 9 V *voltage drop* from *a* to *b* or equivalently a 9 V *voltage rise* from *b* to *a*. In other words, a voltage drop from *a* to *b* is equivalent to a voltage rise from *b* to *a*.

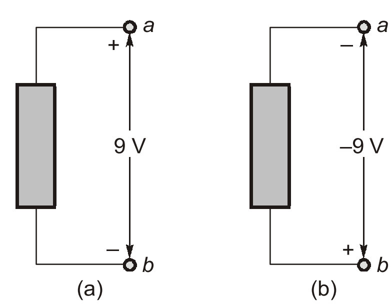


***Figure-1.5 :** Two equivalent representations of the same voltage v_{ab}:*
(a) Point a is 9 V above point b,
(b) Point b is –9 V above point a

NOTE: Keep in mind that electric current is always through an element and that electric voltage is always across the element or between two points.

1.4.2 Types of Voltage

Types of voltage are:

- A voltage that is constant in time is termed as dc voltage.
- A voltage that vary sinusoidally with time is referred as ac voltage.

1.5 Power

Power is the time rate of expending or absorbing energy, measured in watts (W). Thus, in terms of energy, power is defined as

$$p(t) = \frac{dw}{dt} \quad ...(1.4)$$

$$p(t) = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = v(t)i(t)$$

$$p(t) = v(t)\, i(t) \quad ...(1.5)$$

We see that power is simply the product of the voltage across an element and the current through the element. This is a relation which we shall have frequent use in this book.

1.5.1 Passive Sign Convention for Power Calculation

Current direction and voltage polarity are important in determining the sign of power. According to passive sign convention, the current enters the circuit element at the + terminal of the voltage and exit at the – terminal as shown in Figure 1.6(a). In this case power absorbed by the circuit element is

$$p = vi$$

This power is also called power dissipated by the element or power received by the element.

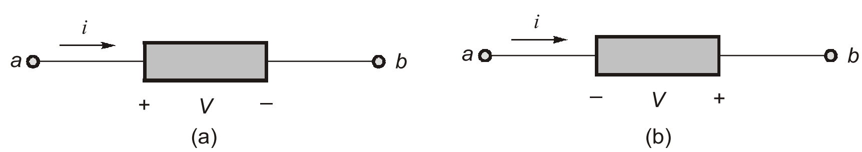


Figure-1.6 : *Passive Sign convention for power (a) Absorbing power (b) Supplying power*

If the current enters the circuit element at the – terminal of the voltage and exit at the + terminal as shown in Figure 1.6 (b), the power absorbed will be

$$p = -vi$$

If the absorbed power p is negative, then the circuit element actually generates power or , equivalently, delivers power to the rest of the circuit. The power absorbed by an element and the power supplied by the same element are related by

Power absorbed = –Power supplied

For example, the element in both circuits of Figure 1.7 has an absorbing power of +12 W because a positive current enters the positive terminal in both cases. In Figure 1.8 however, the element is absorbing power of –12 W is equivalent to a supplying power of +12 W.

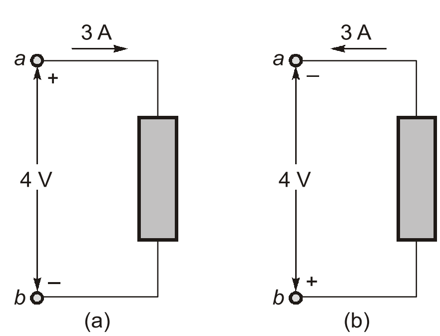


***Figure*-1.7 :** *Two cases of an element with an absorbing power of* 12 W

(a) $p = 4 \times 3 = 12$ W *and (b)* $p = 4 \times 3 = 12$ W

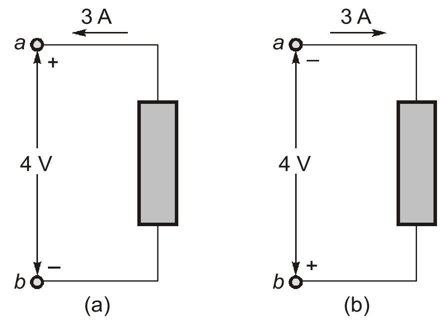


***Figure*-1.8 :** *Two cases of an element with a supplying power of* 12 W
(a) $p = -4 \times 3 = -12$ W *and (b)* $p = -4 \times 3 = -12$ W

1.5.2 Law of Conservation of Energy

Law of conservation of energy must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\Sigma p = 0 \qquad \text{...(1.6)}$$

Thus, sum of the absorbed power is always equal to sum of delivered power in a circuit. Mathematically we can write

$$\Sigma p_{\text{absorbed}} = \Sigma p_{\text{supplied}}$$

1.6 Energy

Energy is the capability to perform work. The energy over a time interval is found by integrating the power. The energy absorbed or supplied by an element from time t_0 to t is

$$w(t) = \int_{t_0}^{t} p(\tau)\,d\tau \qquad \text{...(1.7)}$$

Which is expressed in watt-seconds or Joules (J). The electric power utility companies measure energy in watt-hours (Wh), where

$$1 \text{ Wh} = 3{,}600 \text{ J}$$

NOTE: The electric bill that we pay to electric utility companies is paid for electric energy consumed over a certain period of time.

1.7 Circuit Elements

An element is the basic building block of a circuit. By definition, a simple circuit element is the mathematical model of a two-terminal electrical device, and it can be completely characterized by its voltage-current relationship; it cannot be subdivided into other two-terminal devices.

For **example**,

- If the voltage across the element is linearly proportional to the current through it, then element is called as a **resistor**.
- If the terminal voltage is proportional to *derivative of current* with respect to time, then element is called as a **inductor**.
- If the terminal voltage is proportional to *integral of current* with respect to time, then element is called as a **capacitor**.
- If the terminal voltage is completely independent of current, or the current is completely independent of voltage, then element is called as an **independent source**.

- The element for which either the voltage or current depend upon a current or voltage elsewhere in the circuit; such elements are called as **dependent source**.

1.7.1 Classification of Network Elements

Active and Passive Elements

- If we have a network element that is absorbing power i.e., energy delivered to the element $\left(\int_{-\infty}^{t} v(t)i(t)\,dt\right)$ is positive then the element is **passive element**. Example of passive elements are resistor, inductor, diodes and capacitor.
- If we have a network element that is delivering power i.e., energy delivered to the element $\left(\int_{-\infty}^{t} v(t)i(t)\,dt\right)$ is negative then the element is **active element**. Op-amps, generators and independent sources are the example of active elements.
- The active element can provide power to the circuit, or provide power gain in the circuit.

Remember

- The transistor provide power gain so they are active element, but transformer have same power at input and output they are not active element.
- The active element should be able to provide power/power gain to the circuit for infinite duration of time, that is why the charged capacitor or inductor are not active elements.

Bilateral and Unilateral Elements

- For a **Bilateral element**, the voltage-current relationship is the same for current flowing in either direction. Resistors, inductors and capacitors are the examples of bilateral elements.
- For a **Unilateral element**, the voltage-current relationship is different for two directions of current flow. Diode is an Unilateral elements.

Lumped and Distributed Elements

- **Lumped elements** are considered as the separate elements which are very small in size. For example resistor, inductors, capacitors.
- **Distributed elements** are not electrically separable. These are distributed over the entire length of the circuit. For example, transmission lines.

NOTE: The size of Lumped element is small with respect to signal wavelength. At steady state we can consider distributed element as Lumped element.

Linear and Non-linear Elements

Linearity is the property of an element describing a linear relationship between excitation and response. The property is a combination of both the homogeneity (scaling) property and the additivity property.

- The **homogeneity property** requires that if the input (also called the *excitation*) is multiplied by a constant, then the output (also called the *response*) gets multiplied by the same constant.
  For **example**, if for excitation (voltage or current) $E(t)$ we get response (voltage or current) $R(t)$. Then for excitation $cE(t)$ we will get response $cR(t)$.
- The **additivity property** requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

For **example**, if for excitation (voltage or current) $E_1(t)$ we get response (voltage or current) $R_1(t)$ and for excitation (voltage or current) $E_2(t)$ we get response (voltage or current) $R_2(t)$ then for excitation $E_1(t) + E_2(t)$ we get response (voltage or current) $R_1(t) + R_2(t)$.

The element that follows **additivity** and **homogeneity property** for relationship between excitation and response is called a **linear element**.

The element that does not follow **additivity** and **homogeneity property** for relationship between excitation and response is called a **non-linear element**.

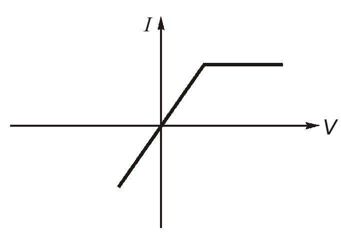
1.7.2 I-V Characteristic Curves for Different Elements

Following are given some I-V characteristic curves for different elements, looking at these characteristics we can find the type of element.

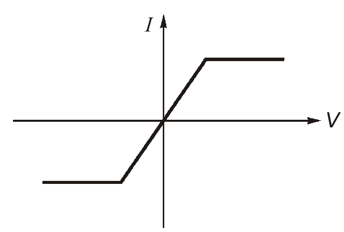
Remember

- If the characteristic curve is similar in opposite quadrants then the element is bidirectional otherwise it is unidirectional.
- If ratio of voltage to current at any point on characteristic curve is negative then the element is active otherwise it is passive.
- Every linear element must exhibit bidirectional property.

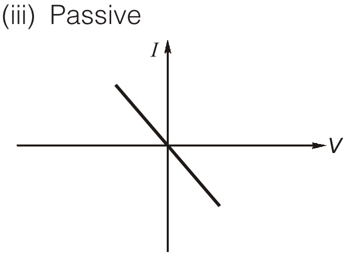
Characteristics

1.


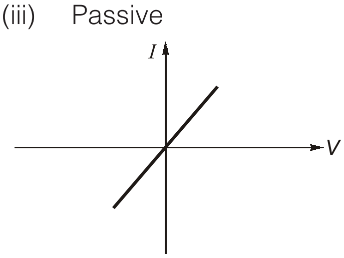
(i) Non-linear
(ii) Unidirectional
(iii) Passive

2.


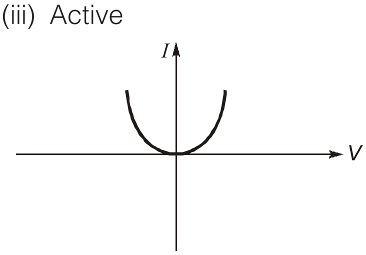
(i) Non-linear
(ii) Bidirectional
(iii) Passive

3.


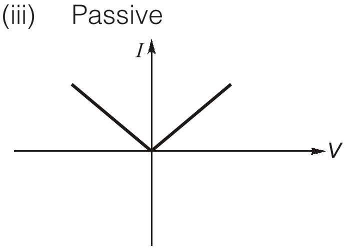
(i) Linear
(ii) Bidirectional
(iii) Active

4.


(i) Linear
(ii) Bidirectional
(iii) Passive

5.


(i) Non-linear
(ii) Unidirectional
(iii) Active

6.


(i) Non-linear
(ii) Unidirectional
(iii) Active

1.8 Sources

In this section we introduce a basic circuit element called a source. The other elements such as resistors, inductors and capacitors are discussed in next chapters.

Sources are classified as voltage sources and current sources. Further it may be classified as dependent and independent sources.

1.8.1 Independent Voltage Sources

An independent voltage source is characterized by a terminal voltage which is completely independent of the current through it. The circuit symbol is shown in Figure 1.9; the subscript *'s'* merely identifies the voltage as a "source" voltage, and is common but not required.

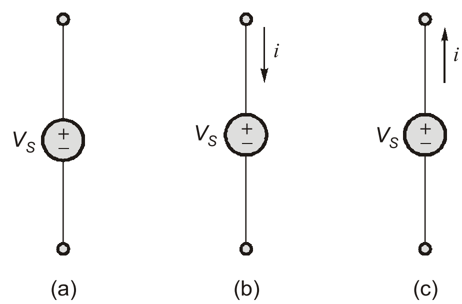
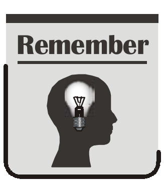


***Figure*-1.9 :** *Circuit symbol of the independent voltage source*



- The presence of the plus sign at the upper end of the symbol of the independent voltage source in Figure 1.9 does not necessarily mean that the upper terminal is numerically positive with respect to the lower terminal. Instead, it means that the upper terminal is V_s volts positive with respect to the lower.
- If at some instant V_s happens to be negative, then the upper terminal is actually negative with respect to the lower at that instant.

An independent voltage source with a constant terminal voltage is often termed an independent dc voltage source and can be represented by either of the symbols shown in Figure 1.10 (a) and (b). The symbol for an independent ac voltage source is shown in Figure 1.10 (c).

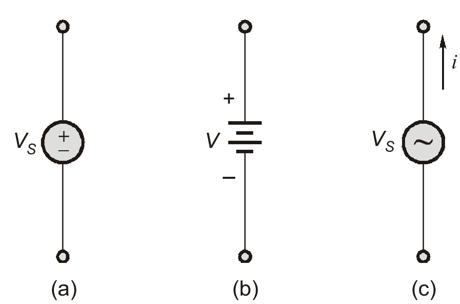


***Figure*-1.10 :** *(a) dc voltage source symbol; (b) battery symbol and (c) ac voltage source symbol*

1.8.2 Independent Current Source

In the independent current source the current through the element is completely independent of the voltage across it.

An independent current source with a constant terminal current is often termed as independent dc current source and can be represented by symbol shown in Figure 1.11 (a). The symbol for an independent ac current source is shown in Figure 1.11 (b).

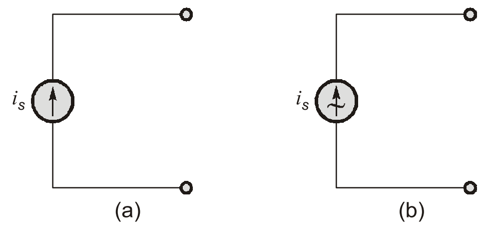


Figure-1.11 : *(a) dc current source symbol (b) ac current source symbol*

Remember

It is a common mistake to view an independent current source as having zero voltage across its terminals while providing a fixed current. In fact, we do not know a priori what the voltage across a current source will be—it depends entirely on the circuit to which it is connected.

NOTE: Generally, independent sources supply power to the circuit. However, they can be connected into a circuit in such a way that they absorb power from the circuit.

1.8.3 Dependent Source

Dependent sources are the sources whose output depend on some other voltage or current in a circuit. Both voltage and current types of sources may be dependent, and either may be controlled by a voltage or a current. Thus, there are four types of dependent sources:

1. A voltage-controlled voltage source (VCVS); $v = Av_x$
2. A current-controlled voltage source (CCVS); $v = Ai_x$
3. A voltage-controlled current source (VCCS); $i = Av_x$
4. A current-controlled current source (CCCS); $i = Ai_x$

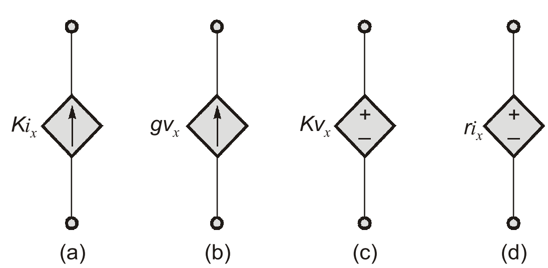


***Figure*-1.12 :** *The four different types of dependent sources*
(a) A current-controlled current source (b) A voltage-controlled current source
(c) A voltage-controlled voltage source (d) A current-controlled voltage source

NOTE

- A different symbol, in the shape of a diamond, is used to represent dependent sources.
- Dependent sources are very useful in describing certain types of electronic circuits.
- A dependent source may absorb or supply power.

Example - 1.1 How much charge is represented by 4600 electron?

Solution:

Each electron has -1.602×10^{-19} C.

Hence 4600 electrons will have, -1.602×10^{-19} C/electron $\times$ 4600 electrons $= -7.369 \times 10^{-16}$ C

Example - 1.2 The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ *s*.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t\sin 4\pi t)\text{ mC/s} = (5\sin 4\pi t + 20\pi t\cos 4\pi t)\text{ mA}$$

at $t = 0.5$, $\quad i = 5\sin 2\pi + 10\pi\cos 2\pi = 0 + 10\pi = 31.42$ mA

Example - 1.3 Determine the total charge entering a terminal between $t = 1s$ and $t = 2s$ if the current passing the terminal is $i = (3t^2 - t)\ A$.

Solution:

$$Q = \int_{t=1}^{2} i\,dt = \int_{1}^{2}(3t^2 - t)\,dt = \left(t^3 - \frac{t^2}{2}\right)\Bigg|_1^2 = (8-2) - \left(1 - \frac{1}{2}\right) = 5.5\text{ C}$$

Example - 1.4 An energy source forces a constant current of 2*A* for 10*s* to flow through a lightbulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:

The total charge is $\quad \Delta q = i\Delta t = 2 \times 10 = 20$ C

The voltage drop is $\quad v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115$ V

Example - 1.5 Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is $i = 5\cos 60\pi t\ A$ and the voltage is: (a) $v = 3i$, (b) $v = 3\,di/dt$.

Solution:

(a) The voltage is $v = 3i = 15\cos 60\pi t$; hence, the power is

$$p = vi = 75\cos^2 60\pi t\text{ W}$$

At $t = 3$ ms, $\quad p = 75\cos^2(60\pi \times 3 \times 10^{-3})$

$$= 75\cos^2 0.18\pi = 53.48\text{ W}$$

(b) We find the voltage and the power as

$$v = 3\frac{di}{dt} = 3(-60\pi)\,5\sin 60\pi t = -900\pi\sin 60\pi t\text{ V}$$

$$p = vi = -4500\pi\sin 60\pi t\cos 60\pi t\text{ W}$$

At $t = 3$ ms, $\quad p = -4500\pi\sin 0.18\pi\cos 0.18\pi$ W

$$= -6.396\text{ kW}$$