DETAILED SOLUTIONS

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ESE 2026 : Prelims Exam
CLASSROOM TEST SERIES

GS & ENGINEERING APTITUDE

Test 1

Section A : Reasoning & Aptitude [All Topics]

Section B : Engineering Mathematics [All Topics]

Answer Key

1.	(a)	11.	(d)	21.	(a)	31.	(c)	41.	(b)
2.	(d)	12.	(a)	22.	(b)	32.	(b)	42.	(d)
3.	(c)	13.	(b)	23.	(c)	33.	(b)	43.	(a)
4.	(c)	14.	(a)	24.	(b)	34.	(c)	44.	(b)
5.	(b)	15.	(b)	25.	(b)	35.	(a)	45.	(d)
6.	(b)	16.	(a)	26.	(c)	36.	(a)	46.	(d)
7.	(a)	17.	(c)	27.	(d)	37.	(a)	47.	(b)
8.	(b)	18.	(c)	28.	(d)	38.	(d)	48.	(c)
9.	(a)	19.	(c)	29.	(c)	39.	(c)	49.	(b)
10.	(a)	20.	(b)	30.	(c)	40.	(c)	50.	(c)

Section A : Reasoning & Aptitude

1. (a)

The last digit of 2016 is 6 and the cyclicity of 6 is one hence, last digit of $(2016)^{2016}$ will be 6. Similarly, the last digit of 2017 is 7 and the cyclicity of 7 is four. Dividing by four gives the remainder 1, so, the last digit of $(2017)^{2017}$ will be 7.

$\therefore$ Last digit of $(2016)^{2016} + (2017)^{2017} =$ Last digit of $(6 + 7) = 3$

2. (d)

Let the original earnings of A and B be ₹$4x$ and ₹$7x$.

$$\text{New earnings of } A = 150\% \text{ of ₹}4x = \text{₹}6x$$

$$\text{New earnings of } B = 75\% \text{ of ₹}7x = \text{₹}\frac{21x}{4}$$

Now, according to question,

$$6x : \frac{21x}{4} = 8 : 7$$

i.e.

$$\frac{24x}{21x} = \frac{8}{7}$$

$$\frac{8}{7} = \frac{8}{7}$$

Hence, data is inadequate to determine x.

3. (c)

Let initial volume of cube is x.

$$\text{Final volume} = x \times 1.05 \times 1.05 \times 1.05$$

$$= 1.1576x$$

$$\text{So, percentage change} = \frac{1.1576x - x}{x} \times 100 = 15.76\%$$

4. (c)

Using the above result, if there is a profit of x% and loss of y%, then the resultant profit or loss is given by the equation.

$$\text{Profit/Loss} = \left(x - y - \frac{xy}{100}\right)\%$$

$$= \left(20 - 20 - \frac{20 \times 20}{100}\right) = -4\%$$

Alternatively,

The selling price of both cycles is same i.e. $(SP)_1 = (SP)_2 =$ ₹1000. However, the cost price difference. We have,

$$20\% = \frac{(SP)_1 - (CP)_1}{(CP)_1} \times 100$$

$$\Rightarrow \quad (CP)_1 = \frac{5}{6}(SP)_1$$

$$-20\% = \frac{(SP)_2 - (CP)_2}{(CP)_2} \times 100$$

$$\Rightarrow \quad (CP)_1 = \frac{5}{4}(SP)_2$$

$$\%\frac{\text{Profit}}{\text{Loss}} = \frac{(SP)_1 + (SP)_2 - (CP)_1 + (CP)_2}{(CP)_1 + (CP)_2} \times 100$$

$$\%\frac{\text{Profit}}{\text{Loss}} = \frac{2000 - \frac{2500}{3} - 1250}{\frac{2500}{3} + 1250} \times 100 = \frac{-250}{6250} \times 100 = -4\%(\text{Loss})$$

5. (b)

Let the annual interest rate is r% and principal amount is P, then

Total amount after 2 years,

$$9680 = P\left(1 + \frac{r}{100}\right)^2 \quad \text{...(i)}$$

Total amount after 3 years,

$$10648 = P\left(1 + \frac{r}{100}\right)^3 \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\left(1 + \frac{r}{100}\right) = \frac{10648}{9680}$$

$$r = \left(\frac{10648}{9680} - 1\right) \times 100 = 10\%$$

6. (b)

$$\frac{a+b}{2} = \text{AM}$$

$$\sqrt{ab} = \text{GM}$$

Here it is given that GM is 20% less than AM, i.e.

$$\text{GM} = \frac{4}{5}AM$$

$$\sqrt{ab} = \frac{4}{5} \times \frac{a+b}{2}$$

From the given options, the only possibility is a = 1, b = 4 and a = 4, b = 1.

So, the required ratio is 4 : 1.

7. (a)

If D occupies the first place, number of possible arrangements,

$$n(A) = 7!$$

$$\text{Total number of arrangements} = n(s) = 8!$$

$$\therefore \quad \text{Required probability, } P(A) = \frac{n(A)}{n(s)} = \frac{7!}{8!} = \frac{1}{8}$$

8. (b)

If we multiply the given expression by 3, we get

$$\left(3x + \frac{1}{3x}\right) = 3$$

Squaring both sides,

$$9x^2 + \frac{1}{9x^2} + 2 = 9$$

$$9x^2 + \frac{1}{9x^2} = 7$$

$$\text{Required expression,} \quad 27x^3 + \frac{1}{27x^3} = (3x)^3 + \frac{1}{(3x)^3}$$

$$= \left(3x + \frac{1}{3x}\right)\left((3x)^2 + \frac{1}{(3x)^2} - 1\right) = (3)(7-1)$$

$$= 3 \times 6 = 18$$

9. (a)

There are three vowels "A, I and U" in the given word.

3	3	2	2	1	1
odd	even	odd	even	odd	even

First odd place can be filled with 3 vowels.

Second odd place can be filled with 2 vowels.

Third odd place can be filled with 1 vowels.

Similarly, even places will be filled by consonants.

$$\text{Number of ways} = 3 \times 3 \times 2 \times 2 \times 1 \times 1$$

$$= 36 \text{ ways}$$

10. (a)

$$\text{Let upstream speed} = x \text{ km/hr}$$

$$\text{and downstream speed} = y \text{ km/hr}$$

Then
$$\frac{40}{x} + \frac{55}{y} = 13 \quad \text{...(i)}$$

and
$$\frac{30}{x} + \frac{44}{y} = 10 \quad \text{...(ii)}$$

Multiplying (ii) by 4 and (i) by 3 and subtracting, we get

$$\frac{11}{y} = 1$$

$$y = 11$$

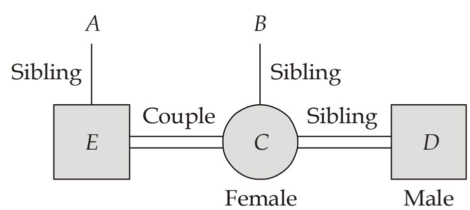
Substituting $y = 11$ in equation (i), we get

$$x = 5$$

$$\text{Speed of man in still water} = \frac{1}{2}(y+x) = \frac{1}{2}(11+5) = 8 \text{ kmph}$$

$$\text{Speed of current} = \frac{1}{2}(y-x) = \frac{1}{2}(11-5) = 3 \text{ kmph}$$

11. (d)



D is brother in law of *E*.

12. (a)

It is given that B and G sits in the same vehicle. Further, since two persons belonging to the same profession do not travel in the same vehicle, hence the doctors D and C travel in different cars. Since C does not travel with the pair of sisters, A and F, hence A and F travel with D. Since A is not an engineer and cannot be a doctor, thus A is Teacher and F is Student. The profession and vehicle for each person is obtained as below:

Vehicle	Person	Profession
Maruti	B	Engineer
	G	Teacher
Santro	A	Teacher
	F	Engineer
	D	Doctor
Opel	C	Doctor
	E	Teacher

13. (b)

The required region is the one common to the square, triangle and circle i.e. 3.

14. (a)

Grandmother is one female, mother is another, wives of fours sons are 4 females and two daughters of all four sons are 8 females. Hence, the total number of females are

$$1 + 1 + 4 + 8 = 14 \text{ females}$$

15. (b)

$$\text{Sushil} + \text{Mukesh} = 200 \quad \ldots(1)$$
$$\text{Asim} + \text{Rajesh} = \text{Sushil} \quad \ldots(2)$$
$$\text{Mukesh} = 4\ \text{Rajesh} \quad \ldots(3)$$
$$\text{Rajesh} = \text{Asim} - 20$$
$$\Rightarrow \quad \text{Asim} = \text{Rajesh} + 20 \quad \ldots(4)$$

From equation (1) and (3),

$$\text{Sushil} + 4\ \text{Rajesh} = 200 \quad \ldots(5)$$

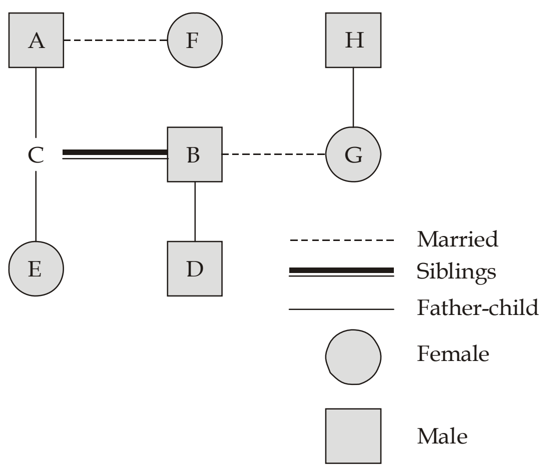
From equation (2) and (4),

$$\text{Rajesh} + \text{Rajesh} + 20 = \text{Sushil}$$
$$\text{Sushil} = 2\ \text{Rajesh} + 20 \quad \ldots(6)$$

From equation (5) and (6),

$$6\ \text{Rajesh} + 20 = 200$$
$$6\ \text{Rajesh} = 180$$
$$\text{Rajesh} = 30$$
$$\text{Asim} = 30 + 20 = 50$$

16. (a)



The gender of C is not confirmed and it is given that B and C are siblings. Thus, B is the son of A (Father) and F (Mother).

17. (c)

$$2^{280} = (2^5)^{56} = 32^{56}$$
$$3^{168} = (3^3)^{56} = 27^{56}$$
$$4^{140} = (2^5)^{56} = 32^{56}$$
$$5^{112} = (5^2)^{56} = 25^{56}$$

Hence, 3^{168} is the smallest among the given numbers.

18. (c)

At 3 ‘o’ clock, the hands of the watch are 15 min spaces apart. To be in opposite direction, they must be 30 min spaces apart.

Thus, minute hand will have to gain 45 min spaces

The minute hand moves a distance of 60 minute spaces in an hour whereas the hour hand moves a distance of 5 minute spaces. Thus, the minute hand gains (60 – 5) = 55 minutes in an hour.

45 min spaces are gained in $\frac{60}{55} \times 45$ or $49\frac{1}{11}$ minutes

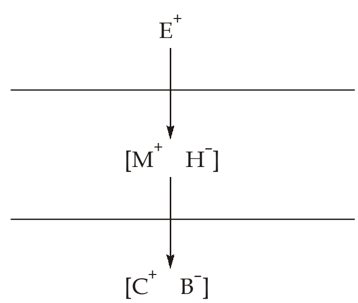
$$\text{Required time} = 49\frac{1}{11} \text{ minutes past 3}$$

19. (c)

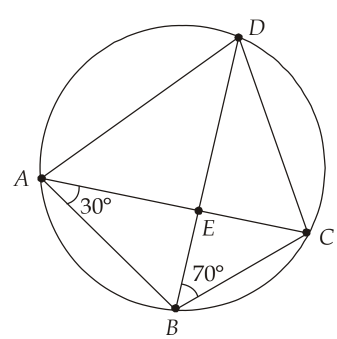
Difference between CI and SI for two years $(\text{CI-SI})_{2y} = P\left(\frac{R}{100}\right)^2$ (which is interest on first year interest)

$$(\text{CI-SI})_{2y} = 400\left(\frac{12}{100}\right)^2 = ₹\,5.76$$

20. (b)



21. (a)



$$\angle CAD = \angle DBC = 70° \quad \text{(Angles in the same segment are equal)}$$

$$\therefore \quad \angle DAB = \angle CAD + \angle BAC = 70° + 30° = 100°$$

But, $\angle DAB + \angle BCD = 180°$ (Opposite angles of cyclic quadrilateral)

So, $\angle BCD = 180° - 100° = 80°$

Now, we have $AB = BC$

Therefore, $\angle BCA = 30°$ (Angles opposite to equal sides of an isosceles triangle are equal)

Again, $\angle DAB + \angle BCD = 180°$ (Opposite angles of cyclic quadrilateral are supplementary)

$$100° + \angle BCA + \angle ECD = 180°$$
$$100° + 30° + \angle ECD = 180°$$
$$\angle ECD = 50°$$

22. (b)

The area of the triangle formed by joining the middle points of the sides of a triangle is equal to one-fourth area of the given triangle.

$$\text{Let area of } 15^{th}\ \Delta = A$$
$$\text{Then area of } 18^{th}\ \Delta = \frac{A}{64}$$
$$\text{So, required ratio} = \frac{64}{1} = 64$$

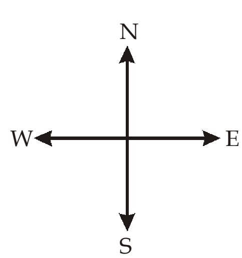
23. (c)

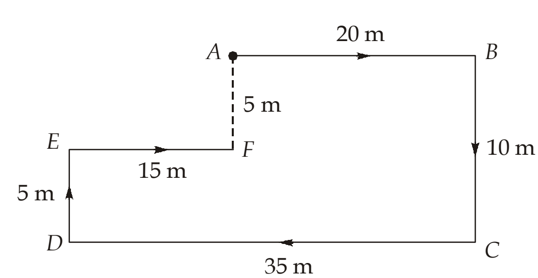
As per the given information, we obtain,

Courses	January	February	March	April	May
A			Q		
B		P			
C					
D					
E	R				

Hence, either course C or D is taught by S in April or May.

24. (b)





Clearly, $DC = AB + EF$

$\therefore$ F is in line with A.

Also, $AF = BC - DE = 5$ m

So, Neelesh is 5 m away from his initial position.

25. (b)

Ratio of number of men, women and children = $\frac{18}{6}:\frac{10}{5}:\frac{12}{3}$ = $3x : 2x : 4x$

$$\therefore \quad (3x + 2x + 4x) = 18$$

$$x = 2$$

Therefore, number of women = 4

$$\text{Share of all women} = \frac{10}{40}\times 4000 = ₹1000$$

$$\therefore \quad \text{Share of each women} = \frac{1000}{4} = ₹250$$

Section B : Engineering Mathematics

26. (c)

Let X denotes the number of sixes in 6 throws of a single die. It follows the binomial distribution with $p = \frac{1}{6}$ and $q = 1 - p = \frac{5}{6}\cdot$ Thus, the required probability is

$$P = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= {}^6C_0\left(\frac{5}{6}\right)^6 + {}^6C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5 + {}^6C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^6 + 6\times\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5 + \frac{6\times 5}{2\times 1}\left(\frac{1}{36}\right)\left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^4\left[\frac{25}{36}+\frac{5}{6}+\frac{5}{12}\right]$$

$$P = \frac{35}{18}\left(\frac{5}{6}\right)^4$$

27. (d)

We can write,

$$y = \sqrt{a^x + y}$$

$$y^2 = a^x + y$$

Differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = a^x \ln a + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{a^x \ln a}{(2y-1)}$$

28. (d)

$$|A + B| \neq |A| + |B|$$

29. (c)

X	$P(X)$	$XP(X)$	$X^2P(X)$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
	$\sum P(X)=1$	$\mu=\sum XP(X)=\frac{3}{2}$	$\sum X^2P(X)=3$

$$\sigma^2 = \sum X^2P(X)-\mu^2 = 3-\frac{9}{4}=\frac{3}{4}$$

30. (c)

$$\frac{d^2x}{dt^2} = -4x$$

$$\frac{d^2x}{dt^2}+4x = 0$$

$$(D^2+4) = 0$$

Auxiliary equation is $m^2 + 4 = 0$, which gives

$$m = \pm 2i$$

The solution is of the form, $x = C_1 \cos 2t + C_2 \sin 2t$ ... (i)

Given, $x(0) = 2$ i.e. $x \to 2$ when $t \to 0$

$$2 = C_1$$

We have, $\frac{dx}{dt} = -2C_1 \sin 2t + 2C_2 \cos 2t$

$x'(0) = 6$ i.e. $x' \to 6$ when $t \to 0$

$$6 = 2C_2$$

$$C_2 = 3$$

$\therefore$ $x = 2\cos 2t + 3\sin 2t$

31. (c)

$$\lim_{x\to\infty}\frac{x^n}{e^x} = \lim_{x\to\infty}\frac{x^n}{x+\frac{x^2}{2!}+\frac{x^3}{3!}+----\infty}$$

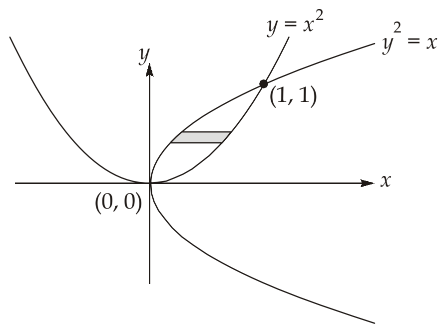
$$= \lim_{x\to\infty}\frac{x^n}{x^n\left(\frac{1}{x^{n-1}}+\frac{1}{x^{n-2}}+......+\frac{1}{x}+1+x+x^2+....\infty\right)}=\frac{1}{\infty}=0$$

32. (b)

$$I = \int_C \left(2xy - x^2\right)dx - \left(x^2 + y^2\right)dy$$

By Green's theorem,

$$I = \int_C F_1 dx + F_2 dy = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dxdy$$



Here,
$$F_1 = 2xy - x^2$$
$$F_2 = -(x^2 + y^2)$$

$\therefore$
$$I = \iint_R (-2x - 2x)dxdy = \iint_R (-4x)dxdy$$

$\Rightarrow$
$$I = -4\int_0^1 \int_{x=\sqrt{y}}^{x=y^2} x \, dxdy$$

$$= -4\int_0^1 \left[\frac{x^2}{2}\right]_{\sqrt{y}}^{y^2} dy = \int_0^1 \frac{4}{2}\left[y - y^4\right]dy = 2\left[\frac{y^2}{2} - \frac{y^5}{5}\right]_0^1$$

$$= 2\left[\frac{1}{2} - \frac{1}{5}\right] = 2 \times 0.3 = 0.60$$

33. (b)

Given: $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

$\because$ Given matrix is upper triangular matrix, hence its eigen values are given by diagonal elements i.e. $\lambda = 1, 3, -2$

The eigen values of $f(A) = 3A^3 + 5A^2 - 6A + 2I$ will be $f(\lambda_1)$, $f(\lambda_2)$ and $f(\lambda_3)$

(i) First eigen value = $3(1)^3 + 5(1)^2 - 6(1) + 2(1) = 4$

(ii) Second eigen value = $3(3)^3 + 5(3)^2 - 6(3) + 2(1) = 110$

(iii) Third eigen value = $3(-2)^3 + 5(-2)^2 - 6(-2) + 2(1) = 10$

$\therefore$ Sum of the eigen values = 4 + 110 + 10 = 124

34. (c)

$$A = \begin{bmatrix} 2 & -3 & 4 \\ 3 & -2 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 0$$

$$\therefore \quad \rho(A) < 3$$

$$\text{Minor of the element } 2 = \begin{vmatrix} -2 & 5 \\ 1 & 1 \end{vmatrix}$$

$$= -7 \neq 0$$

Thus, there is a minor of order 2, which is not zero. Hence, $\rho(A) = 2$.

35. (a)

Let $$A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$$

Since the given matrix is a upper triangular matrix, hence the diagonal elements provides the eigen values. Thus, the eigen values of A are 1, 2

The eigen vectors for $\lambda = 1$ are given by

$$[A - I]x = 0$$

$$\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad y = 0$$

$$\therefore \quad x_1 = C_1\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The eigen vectors for $\lambda = 2$ are given by

$$[A - 2I]x = 0$$

$$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x + 2y = 0$$

$$\Rightarrow \quad x = 2y$$

$$\therefore \quad x_2 = C_2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\therefore$ The eigen vector pair is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

36. (a)

$$f(x) = 3 + x \text{ when } x \geq 0$$
$$= 3 - x \text{ when } x < 0$$
$$\text{L.H.L.} = \underset{x\to 0^-}{lt} f(x) = \underset{x\to 0}{lt}(3+x) = 3$$
$$\text{R.H.L.} = \underset{x\to 0^+}{lt} f(x) = \underset{x\to 0}{lt}(3-x) = 3$$
$$f(0) = 3$$

Since L.H.L = R.H.L = $f(0)$, hence

$$f(x) \text{ is continuous at } x = 0$$
$$f'(0^+) = \underset{h\to 0^+}{lt}\frac{f(0+h)-f(0)}{h} = \underset{h\to 0}{lt}\frac{(3-h)-3}{h} = -1$$
$$f'(0^-) = \underset{h\to 0^-}{lt}\frac{f(0+h)-f(0)}{h} = \underset{h\to 0}{lt}\frac{(3+h)-3}{h} = 1$$

$\therefore$ $f'(0)$ does not exist.

Hence, $f(x)$ is not differentiable at $x = 0$.

37. (a)

Let
$$I = \int_0^{\pi/2} \log(\tan x)\,dx$$
$$= \int_0^{\pi/2} \log\tan\left(\frac{\pi}{2}-x\right)dx = \int_0^{\pi/2} \log\frac{1}{\tan x}$$
$$= -\int_0^{\pi/2} \log\tan x\,dx$$
$$= -I$$
$\therefore$
$$I = 0$$

38. (d)

The directional derivative of
$$f(x, y, z) = x^2 - y^2 + 2z^2 \text{ at } P(1, 2, 3)$$
along z-axis i.e., in the direction of $\vec{a} = \vec{k}$ is
$$(\text{grad } f)_P \frac{\vec{k}}{|\vec{k}|} = \left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}\right)_{P(1,2,3)} \cdot \frac{\vec{k}}{|\vec{k}|}$$
$$= (2\hat{i} - 4\hat{j} + 12\hat{k})\frac{\vec{k}}{|\vec{k}|} = 12$$

39. (c)

If $\overline{V}$ is solenoidal, then

$$\text{div}\,\overline{V} = 0$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0$$

$$1 + 1 + a = 0$$

$$a = -2$$

40. (c)

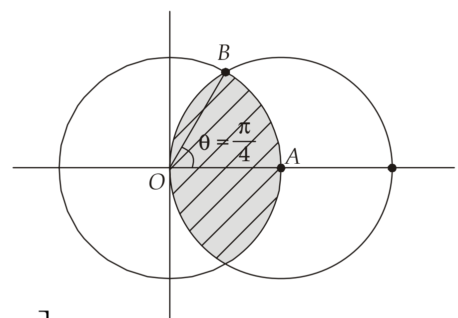
Let event A = First card drawn is queen; B = Second card drawn is queen

$$\text{Required probability} = P(A \cap B) = P(A)P\left(\frac{B}{A}\right) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$$

41. (b)

Given: $r = \sqrt{2}a$ and $r = 2a\cos\theta$

Point of intersection,



$$\sqrt{2}a = 2a\cos\theta$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

$$\text{Area, } A = 2\left[\frac{1}{2}\int_0^{\pi/4} r^2 d\theta + \frac{1}{2}\int_{\pi/4}^{\pi/2} r^2 d\theta\right]$$

$$A = \int_0^{\pi/4} \left(\sqrt{2}a\right)^2 d\theta + \int_{\pi/4}^{\pi/2} 4a^2\cos^2\theta\, d\theta$$

$$A = 2a^2\,[\theta]_0^{\pi/4} + \frac{4a^2}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_{\pi/4}^{\pi/2}$$

$$A = 2a^2 \times \frac{\pi}{4} + 2a^2\left(\frac{\pi}{2} - \frac{\pi}{4} - \frac{1}{2}\right)$$

$$A = a^2(\pi - 1)$$

42. (d)

The given differential equation $Mdx + Ndy = 0$ is an exact differential equation since

$$\frac{\partial M}{\partial y} = 12xy^2 = \frac{\partial N}{\partial x}$$

Hence, the solution is given by

$$\int M dx + \int (\text{Terms of N not containing } x) dy = C$$

$$\int (x^4 + 4xy^3) dx + \int y^4 dy = C$$

$$\frac{1}{5}(x^5 + 10x^2y^3 + y^5) = C$$

43. (a)

Consider, $F_1 = 2xy^3 + y, \quad F_2 = 3x^2y^2 + 2x$

$$\frac{\partial F_1}{\partial y} = 6xy^2 + 1, \quad \frac{\partial F_2}{\partial x} = 6xy^2 + 2$$

By Green's theorem,

$$\int_c F_1 dx + F_2 dy = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dxdy$$

$$= \iint (6xy^2 + 2 - 6xy^2 - 1) dxdy$$

$$= \iint dxdy$$

$= \text{Area of the circle } x^2 + y^2 = 1$

$= \pi$

44. (b)

$$t^2 \frac{dy}{dt} = ty + y^2$$

$$\frac{dy}{dt} = \frac{y}{t} + \left(\frac{y}{t}\right)^2 \quad \text{...(i)}$$

Assuming, $u = \frac{y}{t}$

$$\frac{du}{dt} = \frac{-y}{t^2} + \frac{1}{t}\frac{dy}{dt}$$

$$\frac{dy}{dt} = t\frac{du}{dt} + u \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{y}{t} + \left(\frac{y}{t}\right)^2 = t\frac{du}{dt} + u$$

$$t\frac{du}{dt} = u^2$$

$$\int \frac{du}{u^2} = \int \frac{dt}{t}$$

$$\frac{1}{u} = -\ln|t| + c$$

$$y = \frac{t}{[c - \ln|t|]}$$

45. (d)

Let λ_1 and λ_2 be the eigen values of matrix A. Thus,

$$\lambda_1 + \lambda_2 = 1$$
$$\lambda_1 \cdot \lambda_2 = -6$$

On solving, we get

$$\lambda_1 = 3$$
$$\lambda_2 = -2$$

Eigen values of A^4 are λ_1^4 and λ_2^4, i.e. 81 and 16.

Eigen values of A^3 are λ_1^3 and λ_2^3, i.e. 27 and –8.

Thus, $$\text{Trace of } (A^4 - A^3) = \text{Trace of } (A^4) - \text{Trace of } (A^3)$$
$$= (81 + 16) - (27 - 8)$$
$$= 78$$

46. (d)

The above system of equations can be written as

$$AX = B$$

where, $$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 3 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 5 \\ 6 \\ 4 \\ -1 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc:c} 1 & 2 & 2 & 5 \\ 2 & 1 & 3 & 6 \\ 3 & -1 & 2 & 4 \\ 1 & 1 & 1 & -1 \end{array}\right]$$

Applying, $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$ and $R_4 \to R_4 - R_1$

$$[A|B] = \left[\begin{array}{ccc:c} 1 & 2 & 2 & 5 \\ 0 & -3 & -1 & -4 \\ 0 & -7 & -4 & -11 \\ 0 & -1 & -1 & -6 \end{array}\right]$$

Applying, $R_3 \to 3R_3 - 7R_2$ and $R_4 \to 3R_4 - R_2$

$$[A\,|\,B] = \left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 0 & -3 & -1 & -4 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -2 & -14 \end{array}\right]$$

Applying, $R_4 \to 5R_4 - 2R_3$

$$[A\,|\,B] = \left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 0 & -3 & -1 & -4 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & 0 & -60 \end{array}\right]$$

$\Rightarrow$ $\det(A\,|\,B) \neq 0$

$\therefore$ $\text{Rank}(A) = 3 \neq \text{Rank}(A\,|\,B) = 4$

$\therefore$ The system is inconsistent and has no solution.

47. (b)

Using Taylor's series expansion,

$$\frac{\sin z}{z^8} = \frac{1}{z^8}\left\{z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + ...\right\}$$

$$= \frac{1}{z^7} - \frac{1}{3!z^5} + \frac{1}{5!z^3} - \frac{1}{7!z} + ...$$

Residue at $z = 0$ is the coefficient of $(1/z)$ in the expansion of $f(z)$. Thus,

$$\underset{z=0}{\text{Res.}} f(z) = -\frac{1}{7!}$$

48. (c)

Since $\sum P(x) = 1$, therefore

$$P(0) + P(1) + P(2) + P(3) = 1$$

$$k + 2k + 3k + 4k = 1$$

$$k = 0.1$$

We have, $P(x < 2) = P(0) + P(1) = k + 2k = 0.3$

$$P(x \le 2) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k = 0.6$$

$$P(x < 2) + P(x \le 2) = 0.9$$

49. (b)

Let, $f(x) = x^2 - 15$

For the Newton's Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 2x$$

First iteration: $f(x_0) = f(3.5) = 3.5^2 - 15 = -2.75$

$$f'(x_0) = f'(3.5) = 7$$

$$x_1 = 3.5 + \frac{2.75}{7} = 3.8929$$

Second iteration:

$$f(x_1) = f(3.8929) = 0.1543$$

$$f'(x_1) = f'(3.8929) = 7.7857$$

$$x_2 = 3.8929 - \frac{0.1543}{7.7857} = 3.873$$

50. (c)

Given: $$\frac{dy}{dx} = \left(\frac{y}{x}\right) + \tan\left(\frac{y}{x}\right)$$

It is homogeneous differential equation,

Put $$\frac{y}{x} = v$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$$

$$v + x\frac{dv}{dx} = v + \tan(v)$$

$$x\frac{dv}{dx} = \tan v$$

$$\int \cot v \, dv = \int \frac{dx}{x}$$

$$\log(\sin v) = \log cx$$

$\therefore$ $$\sin\left(\frac{y}{x}\right) = cx \text{ is the required solution}$$