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Detailed Solutions

ESE-2026 Mains Test Series

E & T Engineering Test No : 6

Section A : Electromagnetics + Basic Electrical Engineering

Q.1 (a) Solution:

Given data :

Directive gain, $G_d = 2500$

$f = 1300$ MHz

Transmitted Power, $P_{rad} = 150$ kW

Cross Section of object, $\sigma = 6\ m^2$

Distance, $r = 100$ km

(i) The power density incident at the object is given by

$$P_i = \frac{|E|^2}{2\eta_0} = \frac{P_{rad}G_d}{4\pi r^2}$$

$$|E| = \sqrt{\frac{2\eta_0 P_{rad}G_d}{4\pi r^2}}$$

$$= \frac{1}{r}\sqrt{\frac{2\times 120\pi\times P_{rad}G_d}{4\pi}}$$

$$= \frac{1}{r}\sqrt{60P_{rad}G_d}$$

$$= \frac{1}{100\times 10^3}\sqrt{60\times 150\times 10^3\times 2500}$$

$$|E| = 1.5\ \text{V/m}$$

(ii) Scattered electric field intensity at the radar,

$$|E_s| = \sqrt{\frac{|E|^2 \sigma}{4\pi r^2}} = \sqrt{\frac{1.5^2 \times 6}{4\pi \times 10^4 \times 10^6}} \qquad \left\{\because P_s = \frac{P_i \sigma}{4\pi r^2} = \frac{|E_s|^2}{2\eta_0}\right\}$$

$$= 1.036 \times 10^{-5} \text{ V/m}$$

$$|E_s| = 10.36 \text{ μV/m}$$

(iii) Power captured by object is P_c,

$$P_c = P_i \sigma = \frac{|E|^2 \sigma}{2\eta_0} = \frac{(1.5)^2}{240\pi} \times 6$$

$$P_c = 0.0179 \text{ Watt}$$

(iv) The scattered power density at radar,

$$P_s = \frac{|E_s|^2}{2\eta_0} = \frac{(10.36)^2 \times 10^{-12}}{240\pi} = 1.42 \times 10^{-13} \text{ W/m}^2$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{13 \times 10^8} = 0.23 \text{ m}$$

The power absorbed by the antenna from the scattered wave is given by

$$P_T = P_s \times A_e = P_s \times \frac{G_d \lambda^2}{4\pi}$$

$$P_T = 1.42 \times 10^{-13} \times \frac{2500 \times (0.23)^2}{4\pi}$$

$$P_T = 1.50 \times 10^{-12} \text{ W} = 1.50 \text{ pW}$$

Q.1 (b) Solution:

Since synchronous motors are not self-starting inherently, we should have some means for starting synchronous motors. It has to be rum upto synchronous (or near synchronous) speed by some means, before it can be synchronized to the supply.

The methods are given below:

1. **Induction Motor Start:** Damper windings, consisting of copper bronze bars embedded in slots in the pole faces and short-circuited at both ends are usually provided in synchronous motors which serve not only to damp out oscillations during hunting but are also suitably designed as a squirrel-cage winding for starting purposes. If the rotor reaches near synchronous speed, the field circuit is suddenly energised and the rotor and stator fields interlock each other.

2. **Auxiliary Motor Start:** Using an auxiliary motor, the synchronous motor is first run as an alternator and is synchronised with the three-phase bus-bars. After synchronisation, supply to auxiliary motor is cut, and the alternator is made to run as synchronous motor drawing power from three-phases AC mains.
3. **Using Resistor in the Field Circuit:** Instead of leaving the field circuit open during starting of synchronous motor, it can be short-circuited and can be used as a damper winding. When an additional resistance of value 100 times that of the field resistance, is inserted in series with the field, the starting torque will be large. The field resistance is reduced as the motor accelerates until it becomes three to four times that of field itself. This method can be used in machines where large starting torque are required.
4. **Super Synchronous Start:** The armature is not rigidly bolted to the bed-plates as in an ordinary motor. It is capable of rotating about the rotor shaft. But under normal conditions, it is held stationary by a brake band surrounding the frame of the armature core. The rotor is mechanically coupled to load. The brake band on stator is released and when line voltage is impressed on armature terminals, the stator will try to rotate. When the armature reaches near-synchronous speed, the rotor field is excited and the machine pulls into synchronism.

Q.1 (c) Solution:

For TE_{10} mode,

The cutoff frequency (f_c) in a rectangular waveguide for a TE_{mn} mode can be calculated as :

$$f_c = \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

where ε_r = Relative permittivity

m, n = Mode number

a, b = Waveguide dimensions

For TE_{10} mode,

$$f_{c(TE_{10})} = \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{1}{a}\right)^2} = \frac{c}{2a\sqrt{\varepsilon_r}}$$

$$f_c = \frac{3\times10^8}{2\times2.4\times10^{-2}\times\sqrt{2.11}}$$

$$f_{c(TE_{10})} = 4.3 \text{ GHz}$$

(i) Given that loss tangent = $\frac{\sigma}{\omega\varepsilon} = 3 \times 10^{-4}$

So, $\sigma = 3 \times 10^{-4} \times 2\pi f \times \varepsilon$

$$\sigma = 3\times10^{-4}\times2\pi\times5\times10^{9}\times2.11\times\frac{10^{-9}}{36\pi}$$

$$\sigma = 1.75 \times 10^{-4} \text{ S/m}$$

Intrinsic Impedance, $\eta' = \frac{120\pi}{\sqrt{\varepsilon_r}}$

$$= \frac{120\pi}{\sqrt{2.11}}$$

$$\eta' = 259.39\ \Omega$$

Now, as we know, α_d can be calculated as :

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{1.75\times10^{-4}\times259.39}{2\sqrt{1-\left(\frac{4.3}{5}\right)^2}} = \frac{226.96}{0.51}$$

$$\alpha_d = 445.01 \times 10^{-4} \text{ Np/m}$$

$$\alpha_d = 4.45 \times 10^{-2} \text{ Np/m}$$

(ii) The skin resistance (R_S) can be calculated as :

$$R_S = \frac{1}{\sigma\delta} = \sqrt{\frac{\mu f\pi}{\sigma_c}}$$

where $\sigma_c = 4.1 \times 10^7$ S/m

μ = Permeability = $4\pi \times 10^{-7}$

f = Frequency = 5 GHz

$$R_S = \sqrt{\frac{4\pi\times10^{-7}\times5\times10^{9}\times\pi}{4.1\times10^{7}}}$$

$$R_S = 0.0219$$

Now, α_c can be calculated as,

$$\alpha_c = \frac{2R_S}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[\frac{1}{2}+\frac{b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

$$= \frac{2\times 0.0219}{1.2\times 10^{-2}\times 259.39\sqrt{1-\left(\frac{4.3}{5}\right)^2}}\left[\frac{1}{2}+\frac{1}{2}\left(\frac{4.3}{5}\right)^2\right]$$

$$= \frac{0.014}{0.51}(0.8698)$$

$$\alpha_c = 0.0238 \text{ Np/m}$$

Q.1 (d) Solution:

Let $I_{\text{rated}} = I_{\text{base}} = 1.00$

$V_{\text{rated}} = V_{\text{base}} = 1.00$

Under short circuit, $I_{SC}Z_{e1} = V_{SC}$,

where Z_{e1} is the per-unit equivalent impedance of the 200 kVA transformer

where $I_{SC} = I_{\text{rated}}$

Thus, $1\ Z_{e1} = (0.03)\,(1)$

$Z_{e1} = 0.03$ p.u.

Short-circuit power factor,

$\cos\theta_{SC} = 0.25,$

$\therefore \quad \sin\theta_{SC} = 0.968$

In complex notation,

$$\bar{Z}_{e1} = Z_{e1}(\cos\theta_{sc} + j\sin\theta_{sc}) = 0.03(0.25 + j0.968)$$

$$= (0.0075 + j0.029) \text{ p.u.}$$

Similarly, $$\bar{Z}_{e2} = 0.04(0.3 + j0.953)$$

$$= 0.012 + j0.0381 \text{ p.u.}$$

Common base kVA may be 200 kVA, 500 kVA or any other suitable base kVA

Choosing 500 kVA base arbitrarity, we get

$$\bar{Z}_{e1} = \frac{500}{200}(0.0075 + j0.029) = 0.075\angle 75.52°$$

$$\bar{Z}_{e2} = \frac{500}{500}(0.012 + j0.0381) = 0.04\angle 72.54°$$

We have, $\bar{Z}_{e1} + \bar{Z}_{e2} = 0.03 + j0.11 = 0.114\angle 74.74°$

Total kVA, $S = \frac{P_L}{\cos\phi_L} = \frac{560}{0.8} = 700 \text{ kVA}$

$\therefore$ $\bar{S} = 700\angle\cos^{-1} 0.8 = 700\angle 36.9° \text{ kVA}$

We know that, in parallel connection of transformers, the load shared by each transformer is inversely proportional to impedance. Thus, power shared by transformer-1,

$$\bar{S}_1^* = \frac{\vec{Z}_{e2}}{\vec{Z}_{e1} + \vec{Z}_{e2}}\left(\vec{S}_L\right)^*$$

$$= (700\angle -36.9)° \frac{0.04\angle 72.54°}{0.114\angle 74.74°} = 245.6\angle -39.1° \text{ kVA}$$

$\therefore$ Load shared by transformer 1,

$$S_1 = (245.6)(\cos 39.1°) \text{ at power factor of } \cos 39.1° \text{ lag}$$

$$= 190.6 \text{ kW at } 0.776 \text{ p.f. lag}$$

Similarly, $\bar{S}_2^* = \frac{\vec{Z}_{e1}}{\vec{Z}_{e1} + \vec{Z}_{e2}} \times (\vec{S}_L)^* = (700\angle -36.90)° \frac{0.075\angle 75.52°}{0.114\angle 74.74°}$

$$= 460.5\angle -36.1° \text{ kVA}$$

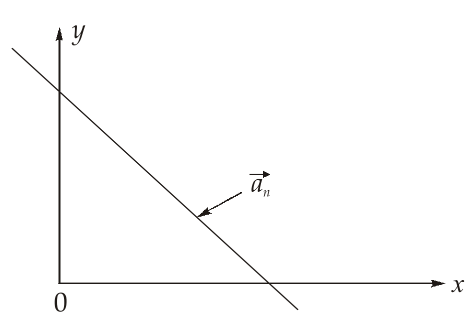
$\therefore$ Load shared by transformer 2,

$$S_2 = 460.5 \cos 36.1° \text{ at power factor of } \cos 36.1° \text{ lag}$$

$$= 372 \text{ kW at } 0.808 \text{ p.f. lag}$$

Q.1 (e) Solution:

Given that interface $x + y = 8$.



$$\vec{H}_1 = -3\hat{a}_x + 2\hat{a}_y - \hat{a}_2 \text{ A/m}$$

(i) The magnetic energy density in medium 1, W_{m_1} is given by

$$W_{m_1} = \frac{1}{2}\vec{B}_1 \cdot \vec{H}_1$$

$$= \frac{1}{2}\mu_0\mu_{r_1} \cdot \vec{H}_1 \cdot \vec{H}_1$$

$$= \frac{1}{2}\mu_o\left(\left|\vec{H}_1\right|\right)^2 \qquad (\mu_r = 1)$$

$$W_{m_1} = \frac{1}{2}\times 4\pi\times 10^{-7}\times 1\times\left(\sqrt{9+4+1}\right)^2$$

$$= 2\pi \times 10^{-7}(14)$$

$$W_{m_1} = 8.79\ \mu\text{J/m}^3$$

(ii) Assume $f(x, y) = x + y - 8 = 0$

$$\vec{\nabla} f = \vec{a}_x + \vec{a}_y$$

Unit vector normal to the interface,

$$\vec{a}_x = \frac{\vec{\nabla} f}{\left|\vec{\nabla} f\right|} = \frac{\vec{a}_x + \vec{a}_y}{\sqrt{2}}$$

Normal component of $\vec{H}_1$,

$$\vec{H}_{1n} = (\vec{H}_1 \cdot \vec{a}_n)\vec{a}_n$$

$$= \left[(-3\vec{a}_x + 2\vec{a}_y - \vec{a}_z)\cdot\left(\frac{\vec{a}_x + \vec{a}_y}{\sqrt{2}}\right)\right]\left(\frac{\vec{a}_x + \vec{a}_y}{\sqrt{2}}\right)$$

$$\vec{H}_{1n} = \left(\frac{-3+2}{\sqrt{2}}\right)\left(\frac{\vec{a}_x + \vec{a}_y}{\sqrt{2}}\right)$$

$$\vec{H}_{1n} = -\frac{1}{2}(\vec{a}_x + \vec{a}_y)$$

$$\vec{H}_{1n} = -0.5\vec{a}_x - 0.5\vec{a}_y\ \text{A/m}$$

Tangential component of $\vec{H}_1$,

$$\vec{H}_{1t} = \vec{H}_1 - \vec{H}_{1n}$$

$$= (-3\vec{a}_x + 2\vec{a}_y - \vec{a}_z) - (-0.5\vec{a}_x - 0.5\vec{a}_y)$$

$$\vec{H}_{1t} = -2.5\vec{a}_x + 2.5\vec{a}_y - \vec{a}_z$$

By applying boundary conditions,

$$\vec{H}_{1t} = \vec{H}_{2t}$$

$$\vec{B}_{2n} = \vec{B}_{1n}$$

$$\mu_2\vec{H}_{2n} = \mu_1\vec{H}_{1n}$$

$$\vec{H}_{2n} = \frac{\mu_1}{\mu_2}\vec{H}_{1n}$$

$$= \frac{1}{5}(-0.5\vec{a}_x - 0.5\vec{a}_y)$$

$$\vec{H}_{2n} = -0.1\vec{a}_x - 0.1\vec{a}_y$$

Now,
$$\vec{H}_2 = \vec{H}_{2t} + \vec{H}_{2n}$$

$$\vec{H}_2 = -2.5\vec{a}_x + 2.5\vec{a}_y - \vec{a}_z - 0.1\vec{a}_x - 0.1\vec{a}_y$$

$$\vec{H}_2 = -2.6\vec{a}_x + 2.4\vec{a}_y - \vec{a}_z$$

Thus,
$$\vec{M}_2 = \chi_{m_2}\vec{H}_2$$

$$= (\mu_{r_2} - 1)\vec{H}_2$$

$$= 4\vec{H}_2 \qquad (\mu_{r_2} = 5)$$

where, $\vec{M}_2$ = Magnetization

$\chi_{m_2} \Rightarrow$ Magnetic susceptibility

$$\vec{M}_2 = 4(-2.6\vec{a}_x + 2.4\vec{a}_y - \vec{a}_z)$$

$$\vec{M}_2 = -10.4\vec{a}_x + 9.6\vec{a}_y - 4\vec{a}_z \text{ A/m}$$

$$\vec{B}_2 = \mu_2\vec{H}_2 = 5\mu_0\vec{H}_2$$

$$= 5\times 4\pi\times 10^{-7}(-2.6\vec{a}_x + 2.4\vec{a}_y - \vec{a}_z)$$

$$\vec{B}_2 = -16.32\vec{a}_x + 15.07\vec{a}_y - 6.28\vec{a}_z \ \mu\text{Wb/m}^2$$

(iii) We have,
$$\vec{H}_1 \cdot \vec{a}_x = H_1 \cos\theta_1$$

$$\cos\theta_1 = \frac{\vec{H}_1 \cdot \vec{a}_x}{H_1}$$

Here, θ_1 is the angle that H_1 make with the normal to interface.

$$\cos\theta_1 = \frac{(-3\vec{a}_x + 2\vec{a}_y - \vec{a}_z)\cdot\left(\frac{\vec{a}_x + \vec{a}_y}{\sqrt{2}}\right)}{\sqrt{9+4+1}}$$

$$= \frac{-3+2}{\sqrt{2}\times\sqrt{14}} = \frac{-1}{1.414\times 3.74}$$

$$\cos\theta_1 = -0.189$$

$$\theta_1 = \cos^{-1}(-0.189) = 100.89°$$

Similarly, $$\cos\theta_2 = \frac{\vec{H}_2\cdot\vec{a}_x}{H_2}$$

Here, θ_2 is the angle, that H_2 make with the normal to interface.

$$\cos\theta_2 = \frac{(-2.6\vec{a}_x + 2.4\vec{a}_y - \vec{a}_z)\cdot\left(\frac{\vec{a}_x + \vec{a}_y}{\sqrt{2}}\right)}{\sqrt{(-2.6)^2 + (2.4)^2 + 1}}$$

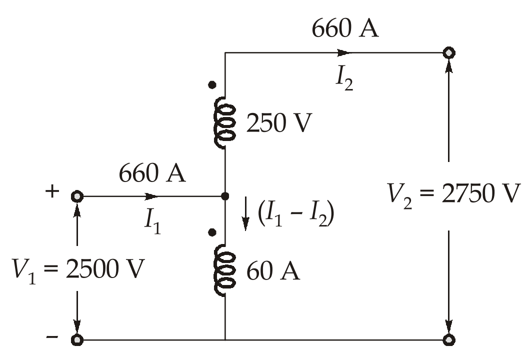
$$= \frac{-2.6+2.4}{3.67\times 1.414} = \frac{-0.2}{5.189}$$

$$\theta_2 = \cos^{-1}(-0.0385)$$

$$\theta_2 = 92.20°$$

Q.2 (a) Solution:

(i) To achieve auto transformer, the two winding transformer can be connected as shown in figure,



For two winding transformer primary rated current,

$$I_1 = \frac{150000}{2500} = 60 \text{ A}$$

Secondary rated current, $I_2 = \dfrac{150000}{250} = 600$ A

From figure, for auto transformer

$$\text{Primary voltage, } V_1 = 2500 \text{ V}$$

$$\text{Primary current, } I_1 = 600 + 60 = 660 \text{ A}$$

$$\text{Secondary voltage, } V_2 = 2750 \text{ V}$$

$$\text{Secondary current, } I_2 = 600 \text{ A}$$

(ii)

$$\text{kVA rating} = V_1 I_1 = V_2 I_2$$
$$= 2500 \times 660 = 2750 \times 600$$
$$= 1650 \text{ kVA}$$

(iii) Here transformation ratio, $k = \dfrac{2500}{2750} = 0.91$

Hence, percentage full load losses in auto transformer

$$= (1 - k) \text{ full load losses in two winding}$$
$$= (1 - 0.91) \times 2.5 = 0.225\%$$

Efficiency of auto transformer

$$= (100 - 0.225)\% = 99.775\%$$

(iv) Percentage impedance of auto-transformer

$$= (1 - k) \times Z_{PTW}$$
$$= (1 - 0.91) \times 4\% = 0.36\%$$

(v) Regulation of auto transformer

$$= (1 - k) \times \text{regulation of two winding transformer}$$
$$= (1 - 0.91) \times 3\% = 0.27\%$$

(vi) Short circuit current as auto transformer

$$= \frac{1}{(1-k)} \times \text{short circuit current of two windings transformer}$$

$$= \frac{1}{(1-0.91)} \times \frac{1}{\text{p.u. impedance of two winding transformer}}$$

$$= \frac{1}{0.09} \times \frac{1}{0.04} = 277.78 \text{ p.u.}$$

Primary current, $I_{sc,\,LV} = 660 \times 277.78 = 183.33$ kA

Secondary current $I_{sc,HV} = 600 \times 277.78 = 166.67$ kA

Q.2 (b) Solution:

(i) We know that,

Current, $$I_L = \frac{V_L}{Z_L}$$

Now, applying the equations of transmission lines

$$V_0^+ = \frac{1}{2}(V_L + Z_0 I_L)$$

$$V_0^+ = \frac{1}{2}\left(V_L + Z_0\frac{V_L}{Z_L}\right)$$

$$V_0^+ = \frac{V_L}{2Z_L}(Z_L + Z_0) \quad ...(1)$$

Similarly, $$V_0^- = \frac{V_L}{2Z_L}(Z_L - Z_0) \quad ...(2)$$

We have, $$I_S = \frac{V_0^+}{Z_0}e^{-\gamma z} - \frac{V_0^-}{Z_0}e^{\gamma z},$$

where z is the distance from the load and is negative towards the source

$$I_S = \frac{V_L}{2Z_L Z_0}(Z_L + Z_0)e^{-\gamma z} - \frac{V_L}{2Z_L Z_0}(Z_L - Z_0)e^{\gamma z}$$

$$I_S = \frac{V_L}{2Z_L Z_0}[(Z_L + Z_0)e^{-\gamma z} - (Z_L - Z_0)e^{\gamma z}]$$

$$I_S = \frac{\frac{V_L}{Z_0}(Z_L + Z_0)}{2Z_L}\left[e^{-\gamma z} - \frac{Z_L - Z_0}{Z_L + Z_0}e^{\gamma z}\right]$$

As we know, $$\frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma_L = \text{Reflection Coefficient}$$

$$1 + \Gamma_L = 1 + \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$1 + \Gamma_L = \frac{2Z_L}{Z_L + Z_0}$$

Hence, $$I_S = \frac{V_L / Z_0}{1 + \Gamma}[e^{-\gamma z} - \Gamma_L e^{\gamma z}]$$

At a distance $\lambda/8$ from the load,

$$z = -\frac{\lambda}{8}$$

$$-\gamma z = -j\left(\frac{2\pi}{\lambda}\right)\times\frac{-\lambda}{8} = j\frac{\pi}{4}$$

$$\left[\because \gamma = \alpha + j\beta;\ \alpha = 0;\ \gamma = j\beta;\ \beta = \frac{2\pi}{\lambda}\right]$$

Reflection coefficient Γ can be calculated as

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50e^{j25°} - 50}{50e^{j25°} + 50} = \frac{-4.68 + j21.13}{95.31 + j21.13}$$

$$\Gamma_L = 4.47 \times 10^{-5} + 0.221j \approx j0.221$$

Hence, I_S can be calculated as

$$I_S = \frac{20\angle 20°}{(1 + 0.221j)}\left(\frac{1}{50}\right)\left[e^{\frac{j\pi}{4}} - 0.221je^{\frac{-j\pi}{4}}\right]$$

$$I_S = \frac{18.79 + j6.84}{50 + j11.05}\times(0.707 + j0.707 - 0.156j - 0.156)$$

$$I_S = 0.39\angle 7.54° \times 0.78\angle 45°$$

$$I_S = 0.304\angle 52.54°\ \text{A}$$

(ii) Given that :

$$\vec{E} = 8\cos(wt - 4x - 3z)\hat{a}_y\ \text{V/m}$$

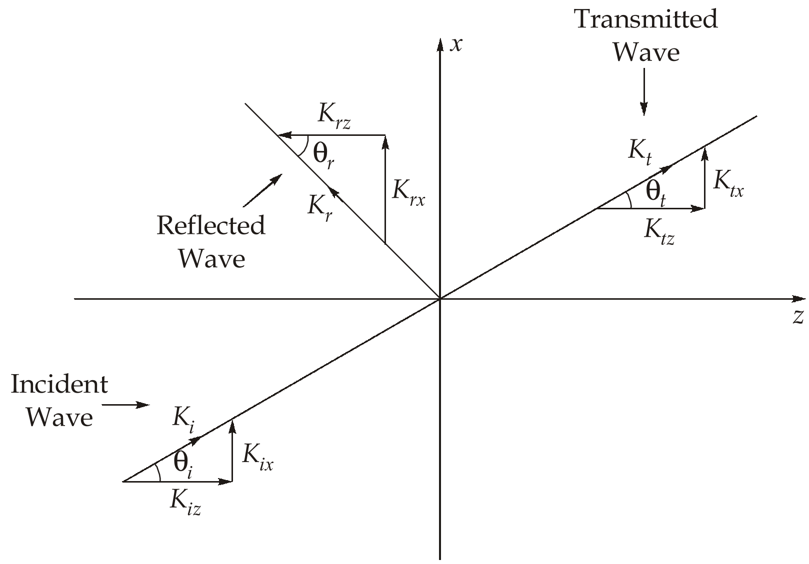
$$\vec{K}_i = 4\hat{a}_x + 3\hat{a}_z$$

$$\left|\vec{K}_i\right| = \sqrt{4^2 + 3^2} = 5$$

$$|K_i| = \frac{w}{c}$$

$$w = |K_i|c = 5 \times 3 \times 10^8$$

$$w = 15 \times 10^8\ \text{rad/sec}$$



$$\tan\theta_i = \frac{K_{ix}}{K_{iz}} = \frac{4}{3}$$

$$\theta_i = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$$

$$\theta_i = 53.13°$$

Reflected E-field, $$\vec{E}_r = E_{r0}\cos(wt - \vec{K}_r \cdot \vec{r})\hat{a}_y$$

$$\vec{K}_r = K_{rx}\cdot\hat{a}_x - K_{rz}\hat{a}_z$$

$$K_{rx} = K_r \sin\theta_r,\ K_{rz} = K_r\cos\theta_r$$

We have, $$\theta_i = \theta_r \text{ and } K_r = K_i = 5$$

$$\vec{K}_r = 4\hat{a}_x - 3\hat{a}_z$$

To find E_{r0}, we calculate θ_t from Snell's law

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i$$

$$\sin\theta_t = \frac{\sqrt{\varepsilon_{r1}}}{\sqrt{\varepsilon_{r2}}}\sin(53.13°)$$

$$\sin\theta_t = \frac{1}{\sqrt{2}}\sin(53.13°)$$

$$\sin\theta_t = 0.707 \times 0.799$$

$$\theta_t = \sin^{-1}(0.565) = 34.40°$$

The electric field vector is in the $\hat{a}_y$ direction, which is perpendicular to the plane of incidence (xz-plane). Therefore, this is Perpendicular Polarization. Hence,

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

where $\eta_1 = \eta_0 = 377\ \Omega$

$$\eta_2 = \frac{377}{\sqrt{\varepsilon_{r_2}}} = \frac{377}{\sqrt{2}} = 266.58\ \Omega$$

$$\Gamma_\perp = \frac{266.58\times\cos(53.13°) - 377\cos(34.40°)}{266.58\times\cos(53.13°) + 377\cos(34.4)}$$

$$= \frac{159.95 - 311.06}{159.95 + 311.06}$$

$$\Gamma_\perp = -0.32$$

$$E_{r0} = \Gamma_\perp E_{i0}$$

$$= -0.32 \times 8 = -2.56$$

Thus, $E_r = -2.56\cos(15\times10^8 t - 4x + 3z)\hat{a}_y$ V/m

Transmission coefficient,

$$T_1 = 1 + \Gamma_\perp = 1 - 0.32 = 0.68$$

We have, $\dfrac{E_{r0}}{H_{r0}} = \eta_1 \Rightarrow H_{r0} = \dfrac{-2.56}{377} = 6.8 \times 10^{-3}$ A/m

Reflected H field, $\vec{H}_r = H_{r0}\cos(15\times10^8 t - 4x + 3z)\hat{a}_{H_r}$ A/m

$$\hat{a}_{Hr} = \hat{a}_K \times \hat{a}_{E_r}$$

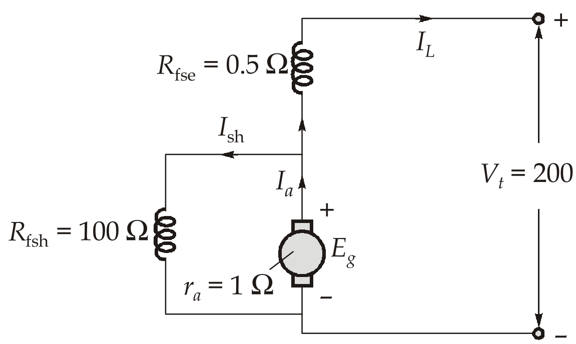
$$= \frac{4\hat{a}_x - 3\hat{a}_z}{5}\times\hat{a}_y = \left(\frac{4}{5}\hat{a}_z + \frac{3}{5}\hat{a}_x\right)$$

$$\vec{H}_r = -6.8\cos(15\times10^8 t - 4x + 3z)\left(\frac{4}{5}\hat{a}_z + \frac{3}{5}\hat{a}_x\right) \text{ mA/m}$$

Q.2 (c) Solution:

Calculation for short shunt:

The equivalent circuit for short shunt can be drawn as



$$\text{Load current} = \frac{\text{Load power}}{\text{Terminal voltage}} = \frac{4 \times 1000}{200} = 20 \text{ A}$$

(i) Voltage drop across series field resistance

$$V_{se} = I_{se} \times R_{fse} = 0.5 \times 20 = 10 \text{ V} \qquad [\because I_{se} = I_L]$$

Voltage drop across shunt field resistance

$$V_f = V_t + V_{se} = 200 + 10 = 210 \text{ V}$$

Shunt field current, $$I_{sh} = \frac{V_f}{R_{fsh}} = \frac{210}{100} = 2.1 \text{ A}$$

Series field current = Load current = 20 A

Armature current, $$I_a = I_{se} + I_{sh} = 20 + 2.1 = 22.1 \text{ A}$$

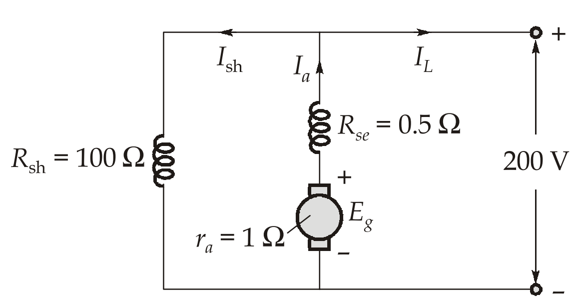
Total voltage drop across brushes = 1 × 2 V = 2 V

For generator, $$E_g = V_b + I_a r_a + I_{se} \times R_{fse} + V_t$$
$$= 2 + 22.1 \times 1 + 10 + 200 = 234.1 \text{ V}$$

Thus, generated emf = 234.1 V

Calculation for long shunt:

The equivalent circuit can be drawn as below



Shunt field current, $I_{sh} = \dfrac{200}{100} = 2\text{ A}$

Armature current (I_a) = Series field current $= I_{sh} + I_L = 2 + 20 = 22\text{ A}$

For generated emf, $E_g = V_t + V_b + I_a(R_{se} + r_a)$

$= 200 + 2 + 22\,(1 + 0.5) = 237\text{ V}$

Thus, generated emf = 235 V

(ii) To calculate flux per pole for short shunt connection:

Applying the equation, $E_g = \dfrac{\phi NZP}{60\,A}$

For lap winding, $A = P = 4$

$$\phi = \frac{E_g \times 60\,A}{Z \times P \times N} = \frac{234.1 \times 60 \times 4}{200 \times 4 \times 750} = 93.64\text{ mWb}$$

Thus, flux per pole = 93.64 mWb

Again, to calculate flux per pole for long shunt connection:

Applying the equation, $E_g = \dfrac{\phi NZP}{60\,A}$

For lap winding, $A = P = 4$

$$\Rightarrow \qquad \phi = \frac{E_g \times 60\,A}{Z \times P \times N}$$

$$= \frac{235 \times 60 \times 4}{750 \times 4 \times 200} = 94\text{ mWb}$$

Thus, flux per pole = 94 mWb

Q.3 (a) Solution:

(i) Consider $I_2 = k\, I_1 \angle\alpha$

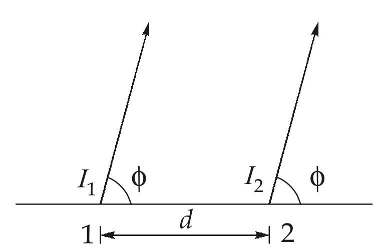
Given that

$I_2 = I_1 = I\angle 0°$

So, $\alpha = 0°$

$k = 1$

$d = 0.5\,\lambda$



For an n-element linear uniform array with $k = 1$, the normalized array factor is given by

$$\frac{E_T}{E_1} = \left|\frac{\sin\left(n\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right|$$

where $\psi = \beta d\cos\phi + \alpha = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\cos\phi + 0$

$$\psi = \pi\cos\phi$$

For $n = 2$,

$$\frac{E_T}{E_1} = \left|\frac{2\sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right| = \left|2\cos\left(\frac{\psi}{2}\right)\right|$$

$$= \left|2\cos\left(\frac{\pi}{2}\cos\phi\right)\right|$$

For maximum radiation,

$$\frac{\pi}{2}\cos\phi = 0$$

$$\cos\phi = 0$$

$$\phi = \pm\frac{\pi}{2}$$

(ii) For a two-port network:

$$b_1 = S_{11}a_1 + S_{12}a_2 \quad \text{...(i)}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \quad \text{...(ii)}$$

where $a_1, a_2 \rightarrow$ incident waves and $b_1, b_2 \rightarrow$ reflected waves

At port 2: $\Gamma_L = \frac{b_2}{a_2} \Rightarrow b_2 = \Gamma_L a_2$

Substituting in equation (ii),

$$\Gamma_L a_2 = S_{21}a_1 + S_{22}a_2$$

$$a_2(\Gamma_L - S_{22}) = S_{21}a_1$$

$$a_2 = \frac{S_{21}a_1}{\Gamma_L - S_{22}}$$

Substituting above in equation (ii),

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_1 = S_{11}a_1 + S_{12}\cdot\frac{S_{21}a_2}{\Gamma_L - S_{22}}$$

$$b_1 = a_1\left[S_{11} + \frac{S_{12}S_{21}}{\Gamma_L - S_{22}}\right]$$

The input reflection coefficient is thus given by

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

When the load is matched to the characteristics impedance ($Z_L = Z_0$), the load reflection coefficient becomes:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

The input reflection coefficient is thus given by:

$$\Gamma_{in} = S_{11} = 0.85\ \angle{-30°}$$

Q.3 (b) Solution:

We have,

Number of poles, $P = 4$

Number of conductors, $Z = 1200$

Rotational speed, $N = 500$ r.p.m.

Diameter of the pole shoe, $D = 0.35$ m

$$\frac{\text{Pole arc}}{\text{Pole pitch}} = 0.7$$

Length of the shoe, $l = 0.2$ m

Flux density, $B = 0.75$ T

Now, we know that

$$\text{Generated EMF, } E_a = \frac{NP\phi}{60} \times \frac{Z}{A} \quad \text{...(i)}$$

where Flux per pole, $\phi = B.a$ ...(ii)

As, Pole arc = 0.7 × Pole pitch

$$\text{Pole arc} = 0.7 \times \frac{\pi D}{P}$$

$$\text{Pole arc} = 0.7 \times \frac{\pi \times 0.35}{4}$$

Pole arc = 0.19 m

Area under pole, a = Pole arc × length (l)

$a = 0.19 \times 0.2$

$a = 0.038\ m^2$

Now from equation (ii), we get

$$\phi = 0.75 \times 0.038$$

$$\phi = 0.029 \text{ Wb} \quad \text{...(iii)}$$

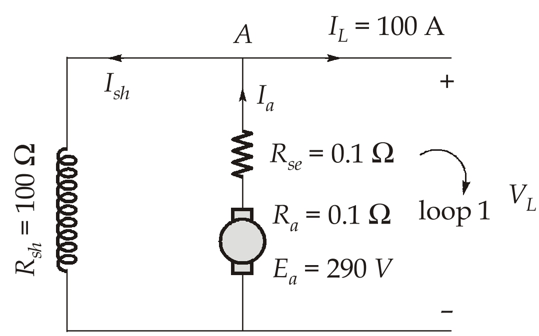
(i) If Armature Winding is Lap Connected, $A = P$

Using equation (i) and (iii), we get

$$E_a = \frac{500 \times 0.029 \times 1200}{60} \times \frac{A}{A}$$

$$E_a = 290 \text{ Volt}$$

The equivalent circuit of a cumulatively compounded dc generator with a long-shunt connection is shown below:



Given, $I_L = 100$A

On applying KCL at node A, we get

$$I_a = I_{sh} + I_L$$

$$I_a = \frac{V_L}{100} + 100$$

$$I_a = 0.01\ V_L + 100 \quad \text{...(iv)}$$

On applying KVL in loop 1, we get

$$-290 + 2 \text{ (brush drop)} + I_a [0.1 + 0.1] + V_L = 0$$

$$V_L = 288 - I_a [0.2]$$

Using equation (iv), we get

$$V_L = 288 - [0.01\ V_L + 100]\ 0.2$$

$$V_L = 288 - 0.002\ V_L - 20$$

$$V_L = 268 - 0.002\ V_L$$

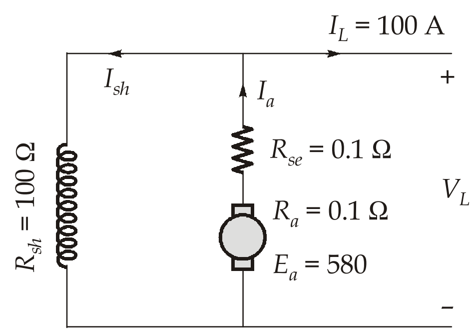
$$V_L = \frac{268}{1.002} = 267.465 \text{ volt}$$

(ii) If Armature Winding is Wave Connected, $A = 2$

Using equation (i) and (iii), we get

$$E_a = \frac{500 \times 0.029 \times 1200}{60} \times \frac{4}{2} = 580 \text{ Volt}$$

The equivalent circuit of a cumulatively compounded dc generator with a long-shunt connection is shown below:



On applying KCL at node A, we get

$$I_a = I_{sh} + I_L$$

$$I_a = \frac{V_L}{100} + 100$$

$$I_a = 0.01\ V_L + 100 \quad \text{...(v)}$$

On applying KVL in loop (i), we get

$$-580 + 2 \text{ (brush drop)} + I_a [0.1 + 0.1] + V_L = 0$$

$$V_L = 578 - I_a [0.2]$$

$$V_L = 578 - [0.01\ V_L + 100]0.2$$

$$V_L = 578 - 0.002\ V_L - 20$$

$$V_L + 0.002\ V_L = 558$$

$$V_L = 556.886 \text{ Volt}$$

Q.3 (c) Solution:

Given, η's phase $= 30°$

We have

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon\left(1 + \frac{\sigma}{j\omega\epsilon}\right)}}$$

$$\eta\text{'s phase} = 30° = \frac{\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}{2}$$

$\Rightarrow$ Loss tangent, $\dfrac{\sigma}{\omega\epsilon} = \sqrt{3}$

From the expression of electric field, we have

$$\beta = 1 \text{ rad/m}$$

Propagation constant, $\gamma = \alpha + j\beta$, where

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]}$$

We have, $$\frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1}}{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1}} = \frac{\sqrt{\sqrt{1+3}-1}}{\sqrt{\sqrt{1+3}+1}} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $\alpha = 0.577 \text{ m}^{-1}$

To obtain the expression for $\vec{H}$, we use

$$H = \frac{E}{\eta} = \frac{30}{300\angle 30^\circ} = 0.1\angle -30^\circ \text{ A/m}$$

The direction of H is obtained by,

$$\overrightarrow{a_E}\times\overrightarrow{a_H} = \hat{a}_z \qquad (\because \text{ The wave travels in } z\text{-direction})$$

$$\hat{a}_x\times\overrightarrow{a_H} = \hat{a}_z$$

$\therefore$ $\overrightarrow{a_H} = \hat{a}_y$

Hence, $\vec{H} = 0.1e^{-\alpha z}\sin(\omega t - z - 30^\circ)\,\hat{a}_y \text{ A/m}$

In phasor form, $\vec{H}$ can be expressed as

$$\vec{H} = 0.1e^{-\alpha z}\,\text{Re}\left\{e^{j(\omega t - z)}e^{j60^\circ}\right\}\hat{a}_y \text{ A/m}$$

The propagation constant, $\gamma = \alpha + j\beta = (0.577 + j1)\text{m}^{-1}$

The wave is linearly polarized as it has single E-field component and is polarised in x-direction.

Skin-depth, $$\delta = \frac{1}{\alpha} = \frac{1}{0.577} = 1.73 \text{ m}$$

To find the value of z where E is 1% of its initial value

$$0.01E_o = E_o e^{-\alpha z}$$

$$0.01 = e^{-\alpha z}$$

$$\alpha z = \ln(100)$$

$$z = \frac{1}{0.577}\ln 100 = 7.98 \text{ m}$$

Q.4 (a) Solution:

(i) For a transmission line, $$V(x) = \frac{V_L (Z_L + Z_0)}{2Z_L}\left[e^{\gamma x} + \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-\gamma x}\right]$$

$$I(x) = \frac{I_L (Z_L + Z_0)}{2Z_0}\left[e^{\gamma x} - \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-\gamma x}\right]$$

When $Z_L = Z_0$, the line is matched. Thus, we get

$$V(x) = V_L e^{\gamma x}$$

$$I(x) = I_L e^{\gamma x}$$

Thus, it represents a forward wave in the source - load direction.

The above equations when applied for an infinite line, $x = l = \infty$, gives

$$V(x) = \frac{V_L}{Z_L}\left[\frac{(Z_L + Z_0)e^{\gamma x}}{2} + \frac{(Z_L - Z_0)}{2}e^{-\gamma x}\right]$$

$$= \frac{V_L}{Z_L}e^{\gamma x}\left[\frac{(Z_L + Z_0)}{2} + \frac{(Z_L - Z_0)}{2}e^{-2\gamma x}\right]$$

$$= \frac{V_L (Z_L + Z_0)}{2Z_L}e^{\gamma x}$$

Similarly, $$I(x) = \frac{I_L (Z_L + Z_0)}{2Z_0}e^{\gamma x}$$

Both the equations contain only the forward wave but no reflected wave.

Similarly in case of a matched line,

$$Z(x) = \text{Impedance anywhere on the line} = Z_0 = Z_L$$

For an infinite line, $$\frac{V(x)}{I(x)} = Z(x) = \frac{V_L Z_0}{I_L Z_L} = Z_0$$

Thus, an infinite line is similar to a matched line.

(ii) Given mode is TM_2, and

$$a = 20 \text{ cm}$$

$$V_{p2} = 1.5c \text{ (for } TM_2 \text{ mode)}$$

The cut-off frequency for m^{th} mode is given as

$$f_{c_m} = \left(\frac{m}{a}\right) \times \frac{V}{2} \qquad ...(i)$$

where a : distance between parallel plates

and $$V = \frac{1}{\sqrt{\mu \in}}$$

Given that $V = c$ (i.e., air filled medium),

From equation (i),

$$f_{c_1} = \frac{V}{2a} = \frac{c}{2a} = \frac{3\times10^{10}}{2\times20}$$

$$f_{c_1} = \frac{3}{4} \text{ GHz}$$

Likewise $$f_{c_2} = 2\left(\frac{V}{2a}\right) = 2\times\frac{3}{4}\text{GHz}$$

$$f_{c_2} = 1.5 \text{ GHz}$$

* Guide phase velocity for m^{th} mode is given as

$$V_{pm} = \frac{V}{\sqrt{1-\left(\frac{f_{c_m}}{f}\right)^2}}$$

$$V_{p2} = \frac{V}{\sqrt{1-\left(\frac{f_{c_2}}{f}\right)^2}}$$

$$\sqrt{1-\left(\frac{f_{c_2}}{f}\right)^2} = \frac{V}{V_{p_2}}$$

$$\Rightarrow \qquad \sqrt{1-\left(\frac{f_{c_2}}{f}\right)^2} = \frac{c}{1.5c}$$

$$\left(\frac{f_{c_2}}{f}\right)^2 = 0.555$$

$$\frac{f_{c_2}}{f} = 0.7449$$

$$f = \frac{1.5\text{ GHz}}{0.7449} = 2.01\text{ GHz}$$

Hence, frequency (f) is 2.01 GHz

Guide wavelength for m^{th} mode is

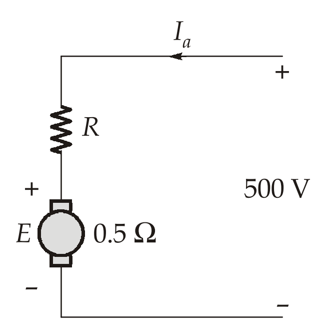
$$\lambda_{gm} = \frac{\lambda}{\sqrt{1-\left(\frac{f_{c_2}}{f}\right)^2}}$$

We have,
$$\lambda = \frac{V}{f} = \frac{c}{f} = \frac{3\times10^8}{2.01\times10^9} = 0.149\text{ m}$$

$$\lambda_{g2} = \frac{0.149}{0.66565} = 0.223\text{ m} \approx 22.38\text{ cm}$$

Q.4 (b) Solution:

Initially,
$$I_a = 60\text{ A}$$
$$N = N_1\text{ rpm}$$



Torque-speed Relation for fan load:

For a fan (or blower, pump), the load torque is proportional to the square of speed.

$$\tau \propto N^2$$

$$\tau_1 \propto N_1^2$$

$\because$ $\tau_1 \propto I_a^2$ as for series motor, $\tau \propto \phi I_a \propto I_a^2$. Thus,

$$I_{a_1}^2 \propto N_1^2 \quad \text{...(i)}$$

Now required speed, $N_2 = 0.75\, N_1$

$$\tau_2 \propto N_2^2$$

$$I_{a_2}^2 \propto (0.75 N_1)^2 \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\left(\frac{I_{a2}}{I_{a1}}\right)^2 = (0.75)^2 \Rightarrow I_{a_2} = 0.75 I_{a_1} = 0.75 \times 60 = 45A$$

Now,

We know that, $E = N\phi$

$$\frac{E_1}{E_2} = \frac{N_1\phi_1}{N_2\phi_2} \quad \text{...(iii)}$$

For series motor $\phi \propto I_a$, thus

$$\frac{I_{a_1}}{I_{a_2}} = \frac{\phi_1}{\phi_2} \Rightarrow \phi_2 = \frac{\phi_1 I_{a_2}}{I_{a_1}}$$

$$\phi_2 = \phi_1 \times \frac{45}{60}$$

$$\phi_2 = \frac{3\phi_1}{4} \quad \text{...(iv)}$$

Now, $E_1 = 500 - (60 \times 0.5)$

$$E_1 = 500 - 30 = 470 \text{ V}$$

$$E_2 = 500 - (45 \times R)$$

Using relation (iii), we can write

$$\frac{E_1}{E_2} = \frac{N_1\phi_1}{N_2\phi_2}$$

$$\frac{470}{500-45R} = \frac{N_1\phi_1}{0.75N_1\phi_2} \qquad \left[\frac{\phi_1}{\phi_2}=\frac{4}{3}\right]$$

$$\frac{470}{500-45R} = \frac{N_1}{0.75N_1}\times\frac{4}{3}$$

$$\frac{470}{500-45R} = \frac{4}{3\times 3}\times 4$$

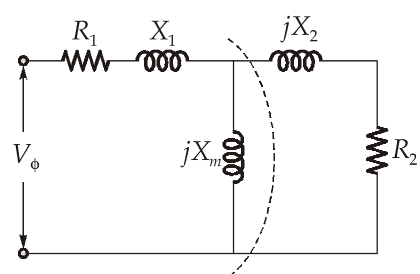
$$\frac{470}{500-45R} = \frac{16}{9}$$

$$470 \times 9 = 16 \times 500 - 16 \times 45R$$

$$R = 5.24\ \Omega$$

Q.4 (c) Solution:

The equivalent circuit of the motor referred to the stator side is as given below,



Taking Thevenin's equivalent across the magnetizing branch

$$V_{Th} = \frac{X_m}{\sqrt{R_1^2+(X_1+X_m)^2}}\times V_\phi$$

$$= \frac{266\times 26.3}{\sqrt{(0.641)^2+(1.106+26.3)^2}} = 255.2\ \text{V}$$

Thevenin Impedance, $Z_{th} = \dfrac{jX_m(R_1+jX_1)}{R_1+j(X_1+X_m)}\times\dfrac{R_1-j(X_1+X_m)}{R_1-j(X_1+X_m)}$

$$= \frac{R_1X_m^2}{R_1^2+(X_1+X_m)^2}+j\frac{X_mR_1^2+X_mX_1(X_1+X_m)}{R_1^2+(X_1+X_m)^2}$$

$$Z_{th} \simeq \frac{R_1X_m^2}{(X_1+X_m)^2}+j\frac{X_mX_1}{X_1+X_m}$$

The Thevenin resistance is,

$$R_{Th} \simeq R_1\left(\frac{X_m}{X_1+X_m}\right)^2 \simeq 0.641\times\left(\frac{26.3}{1.106+26.3}\right)^2 = 0.592\ \Omega$$

The Thevenin reactance is,

$$X_{Th} \simeq \frac{X_m X_1}{X_1+X_m} \simeq X_1 = 1.106\ \Omega$$

$$\text{Synchronous speed, } N_s = \frac{120f}{P} = \frac{120\times 60}{4} = 1800 \text{ rpm}$$

(i) The slip at which maximum torque occurs is given as :

$$s_{max} = \frac{R_2}{\sqrt{R_{Th}^2+(X_{Th}+X_2)^2}}$$

$$= \frac{0.332}{\sqrt{(0.592)^2+(1.106+0.464)^2}} = 0.198$$

The speed at maximum torque is given by

$$N_m = (1-s_{max})N_s = (1-0.198)\times 1800 = 1444 \text{ rpm}$$

The torque at this speed is,

$$T_{max} = \frac{3V_{Th}^2}{2\omega_s\left[R_{Th}+\sqrt{R_{Th}^2+(X_{Th}+X_2)^2}\right]},$$

where,

$$\omega_s = \frac{2\pi N_s}{60} = \frac{2\pi\times 1800}{60} = 188.5 \text{ rad/s}$$

$$T_{max} = \frac{3(255.2)^2}{2\times 188.5\left[0.592+\sqrt{(0.592)^2+(1.106+0.464)^2}\right]}$$

$$T_{max} \simeq 229 \text{ N-m}$$

(ii) The starting torque of this motor is found by setting $s = 1$, as below:

$$T_{st} = \frac{3V_{Th}^2\cdot R_2}{\omega_s[(R_{Th}+R_2)^2+(X_{Th}+X_2)^2]}$$

$$= \frac{3\times(255.2)^2\times 0.332}{188.5\times[(0.592+0.332)^2+(1.106+0.464)^2]}$$

$$T_{st} \simeq 104 \text{ N-m}$$

(iii) If the rotor resistance is doubled, then the slip at maximum torque doubles, too.

Therefore, $s_{max} = 0.396$

and the speed at maximum torque is

$$N_m = (1 - s_{max}).N_s = (1 - 0.396) \times 1800$$

$$N_m = 1087 \text{ rpm}$$

The maximum torque is still,

$$T_{max} = 229 \text{ N-m}$$

The starting torque now becomes

$$T_{st} = \frac{3V_{Th}^2 \cdot R_2'}{\omega_s[(R_{Th} + R_2')^2 + (X_{Th} + X_2)^2}$$

$$T_{st} = \frac{3 \times (255.2)^2 \times (0.664)}{(188.5) \times [(0.590 + 0.664)^2 + (1.106 + 0.464)^2]}$$

$$T_{st} \simeq 170 \text{ N-m}$$

Section B : Computer Organization and Architecture-1 + Materials Science-1 + Electronic Devices & Circuits-2 + Advanced Communications-2

Q.5 (a) Solution:

Given: 32-bit CPU

Therefore, word-size = 32 bit

Given, instruction size = 32 bits

Given: opcode size = 7 bit

Register file size = 32 registers

Number of bits required to represent register address

$$= \log_2 32 = 5 \text{ bit}$$

RAM size = 4 kB

Number of bits required to represent memory address

$$= \log_2 4 \text{ K} = \log_2 2^{12} = 12 \text{ bit}$$

Instruction format:

32-bit

Opcode	Register Address	Memory Address	Immediate field
7 bit	5 bit	12 bit	?

$$\text{Immediate field size} = 32 - (7 + 5 + 12)$$
$$= 8 \text{ bit}$$

So, for 8-bit immediate field size

$$\text{unsigned data range} = 0 \text{ to } 2^n - 1$$

i.e,
$$= 0 \text{ to } 2^8 - 1$$
$$= 0 \text{ to } 255$$

Hence, largest unsigned constant possible in the instruction = 255.

Q.5 (b) Solution:

(i) Given:

$$\text{Density of iron, } \rho = 7.87 \text{ g/cm}^3 = 7870 \text{ kg/m}^3$$
$$\text{Mass of one atom, } m = 9.27 \times 10^{-26} \text{ kg}$$

1. Volume of one iron atom

Ignoring the spaces between the spheres,

$$V = \frac{m}{\rho} = \frac{9.27 \times 10^{-26}}{7870}$$
$$V \approx 1.18 \times 10^{-29} \text{ m}^3$$

2. Distance between centers of adjacent atoms:

Given atoms are spherical and tightly packed:

$$\therefore \quad \text{Volume, } V = \frac{4}{3}\pi r^3$$

$$r = \left(\frac{3V}{4\pi}\right)^{1/3}$$

Substituting $V = 1.18 \times 10^{-29}$ m^3, we get

$$r \approx 1.41 \times 10^{-10} \text{ m}$$

Distance between centers of adjacent atoms:

$$d = 2r \approx 2.82 \times 10^{-10} \text{ m}$$

(ii) For FCC structure,
$$a = \frac{4r}{\sqrt{2}} = \frac{4 \times 1.44}{\sqrt{2}} = 4.073 \text{ Å}$$

The interplanar spacing for planes with Miller indices (*hkl*) in a cubic lattice is given by:

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

For (220) plane, $$d_{220} = \frac{4.073}{\sqrt{2^2+2^2+0^2}} = 1.44 \text{ Å}$$

For (200) plane, $$d_{200} = \frac{4.073}{\sqrt{2^2+0^2+0^2}} = 2.036 \text{ Å}$$

For (111) plane, $$d_{111} = \frac{4.073}{\sqrt{1^2+1^2+1^2}} = 2.35 \text{ Å}$$

Q.5 (c) Solution:

Given,

Short-circuit current, I_{sc} (300K) = 2 A

Open-circuit voltage, V_{oc} (300K) = 0.5 V

The open-circuit voltage in solar cell is given by

$$V_{oc} = V_T \ln\left(\frac{I_{sc}}{I_0}+1\right)$$

At 300 K, V_T (300K) ≈ 0.026 V. On solving, we get

$$0.5 = 0.026\ln\left(\frac{2}{I_0}+1\right)$$

$$I_0(300K) \approx 8.9 \times 10^{-9} \text{ A}$$

The saturation current doubles for every 10°C rise in temperature. Hence,

$$I_0(400K) = I_0(300K)\times 2^{\frac{T_2-T_1}{10}}$$

$$= I_0(300K)\times 2^{\frac{400-300}{10}} = 1024 I_s(300K)$$

At 400 K, thermal voltage incrases to

$$V_T(400K) = \frac{400}{300}\times 0.026 \approx 0.0347 \text{ V}$$

I_{SC} is weakly dependent on temperature, hence assuming it as constant, we get

$$V_{oc}(400K) \approx V_T(400K)\ln\left(\frac{I_{sc}}{I_0(400K)}\right)$$

$$= 0.0347\times\ln\left(\frac{2}{1024\times 8.9\times 10^{-9}}\right)$$

$$V_{oc}(400K) \approx 0.43 \text{ V}$$

Maximum output power, $P_{max} = V_{oc}I_{sc}$

Maximum power at 300 K,

$$P_{max(300K)} = 0.5 \times 2 = 1 \text{ W}$$

Maximum power at 400 K, $P_{max(400K)} = 0.43 \times 2 = 0.86$ W

Thus, $P_{max(400K)} = 0.86P_{max(300K)}$

Q.5 (d) Solution:

(i) Given: $n_1 = 1.5, \Delta = 3\%, \lambda = 0.82\ \mu\text{m}$,

To be calculated: Critical radius of curvature at which large bending losses occur.

The relative refractive index difference is given as:

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

Hence, $n_2^2 = n_1^2 - 2\Delta n_1^2 = 2.250 - 0.06 \times 2.250 = 2.115$

For the multimode fiber, critical radius of curvature

$$R_c = \frac{3n_1^2\lambda}{4\pi(n_1^2 - n_2^2)^{3/2}}$$

$$R_c = \frac{3 \times 2.250 \times 0.82 \times 10^{-6}}{4\pi \times (0.135)^{3/2}} = 8.88\ \mu\text{m}$$

(ii) Given: Core diameter single-mode fibre $(2a) = 8\ \mu$m, core refractive index $(n_1) = 1.5$, relative refractive index difference $(\Delta) = 0.3\%$ and wavelength $(\lambda) = 1.55\ \mu$m.

Again, $n_2^2 = n_1^2 - 2\Delta n_1^2 = 2.250 - (0.006 \times 2.250) = 2.237$

The cutoff wavelength for the single-mode fiber is given by equation as

$$\lambda_c = \frac{2\pi a n_1 (2\Delta)^{1/2}}{2.405}$$

$$= \frac{2\pi \times 4 \times 10^{-6} \times 1.500(0.06)^{1/2}}{2.405} = 3.84\ \mu\text{m}$$

The critical radius of curvature for the single-mode fiber gives

$$R_{cs} = \frac{20\lambda}{(n_1^2 - n_2^2)^{3/2}}\left[2.748 - 0.996\frac{\lambda}{\lambda_c}\right]^{-3}$$

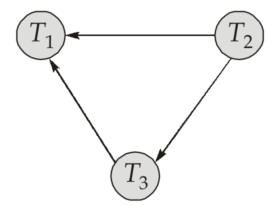
$$R_{cs} = \frac{20\times1.55\times10^{-6}}{(0.013)^{3/2}}\left(2.748-\frac{0.996\times1.55\times10^{-6}}{3.84\times10^{-6}}\right)^{-3}$$

$$= 1.62 \text{ mm}$$

Q.5 (e) Solution:

(i) Checking conflict serializability:

1. Draw precedence graph.
2. Check whether cycle exist in precedence graph.
    - if yes, then not conflict serializable
    - else conflict serializable



Since precedence graph does not contain cycle, the given schedule is conflict serializable.

Checking view serializability:

1. **Initial Read:** If transaction T_i read data item A from initial database in some serial schedule S, then in the given schedule also, T_i should read item A from initial database only.
2. **Write Read:** If transaction T_i reads data item A which is updated by T_j in some serial schedule S, then in the given schedule also T_i should read data item A which is modified by T_j only.
3. **Final write:** If transaction T_j, update database item A, finally in some serial schedule S, then in the given schedule also T_i should update data item finally. As

T_1	T_2	T_3
$r_1(X)$		
		$r_3(Y)$
		$r_3(X)$
	$r_2(Y)$	
	$r_2(Z)$	
		$w_3(Y)$
	$w_2(Z)$	
$r_1(Z)$		
$w_1(X)$		
$w_1(Z)$		

Every conflict serializable schedule is also view serializable. Since we have already proven conflict serializability, the schedule is automatically view serializable. However, to verify via the three conditions of View Equivalence against the serial schedule $T_2 \rightarrow T_3 \rightarrow T_1$.

Initial Read: X is first read by T_1 in S, Y is first read by T_3 in S and Z is first read by T_2 in S.

Write-Read: In S, $r_1(Z)$ reads the value of Z written by $w_2(Z)$. Therefore, T_2 must come before T_1, which is satisfied in $T_2 \rightarrow T_3 \rightarrow T_1$.

Final Write:

$X \rightarrow$ final write is $w_1(X) \rightarrow$ by T_1

$Y \rightarrow$ final write is $w_3(Y) \rightarrow$ by T_3

$Z \rightarrow$ final write is $w_1(Z) \rightarrow$ by T_1.

Therefore, T_1 must appear after T_2 and T_3, so it remains final writer of X and Z and T_3 must appear before T_1 (so it remains final writer of Y). This is satisfied in $T_2 \rightarrow T_3 \rightarrow T_1$.

Thus, the schedule is View Serializable with serial scheduling of $T_2 \rightarrow T_3 \rightarrow T_1$.

(ii) Given: $h_t = 49$ m, $h_r = 25$ m, $f = 100$ MHz, $P_t = 100$ Watts

To be calculated:

1. LOS distance
2. The received signal strength

1. The maximum line-of-sight distance between the two antennas having heights h_t (meters) and h_r (meters) above the earth is given by,

$$d = 4.12\left[\sqrt{h_t} + \sqrt{h_r}\right]\text{km}$$

$$\text{LOS distance} = 4.12(\sqrt{49} + \sqrt{25}) = 12 \times 4.12 = 49.44 \text{ km}$$

2.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{100 \times 10^6} = 3 \text{ m}$$

Electric field strength received (E_r) at a distance d from the transmitter is given by,

$$E_r = \left[88\sqrt{P} / [\lambda d^2]\right] h_t h_r \text{ V/m}$$

$$= \left[88\sqrt{100} / \{3 \times (49.44 \times 10^3)^2\}\right] 49 \times 25$$

$$= \left[880/[3\times(2444\times10^{6})]\right]\times1225 = 12\times10^{-8}\times1225$$

$$= 1.47 \times 10^{-4}\ \text{V/m}$$

Q.6 (a) Solution:

We have,

$$\text{Data} = -14.75 \times 2^{12}$$

Now, converting the number 14.75 into its binary equivalent, we get $-1110.11 * 2^{+12}$

S	BE	M
⇩	⇩	⇩
For –Ve sign = 1; and for +Ve sign = 0	BE = AE + Bias	Mantissa

- sign:- sign bit is 1 for negative number and 0 for positive number
- Bias Exponent (BE):- It is defined as sum of Actual Exponent (AE) + Bias

BE = AE + Bias

Normal Bias	Excess-3 Bias
⇩	⇩
Bias = $(2^{n-1} - 1)$	Bias = 2^{n-1}

(n is the number of bits used for exponent)

If nothing is mentioned, then by default go for normal bias.

$$\text{Bias} = (2^{n-1} - 1) = (2^{7-1} - 1) = 63$$

In binary, $\text{Bias} = (0111111)_2$

- Mantissa (M):– bits after the decimal point is called as mantissa or it is defined as the significant digits of a floating point number.

(i) Without Normalisation:

⇒ The binary equivalent of $(-14.75) * 2^{12} = -0.111011 * 2^{+16}$

- Sign $(S) = 1$
- Bias Exponent (BE) = Actual Exponent (AE) + Bias

$$= (0010000)_2 + (2^{n-1} - 1)_2 \qquad (\text{Here, } n = 7)$$

$$= (0010000)_2 + (0111111)_2 = (1001111)_2$$

- Mantissa : $0.\underbrace{111011}_{\Downarrow \; M}$

$$M = 111011$$

S	BE	M
1	1001111	1110 1100 0000

1 1 0 0 | 1 1 1 1 | 1 1 1 0 | 1 1 0 0 | 0 0 0 0

C *F* *E* *C* 0

Hence, without normalisation (–14.75) $*$ 2^{12} is represented as $(\text{CFEC0})_H$ in the above mentioned format.

(ii) with normalisation:

The binary equivalent of (–14.75) $*$ 2^{12} = –1110.11 $*$ 2^{12}.

After normalisation, we can represent it as –1.11011 $*$ 2^{15}.

- Sign Bit, $S = 1$
- Bias Exponent (BE) = Actual Exponent (AE) + Bias

  = 0001111 + 0111111 = $(1001110)_2$
- Mantissa (M) = $1.\underbrace{11011}_{\Downarrow \; M}$

$$M = 11011$$

Sign	BE	M
1	1001110	11011 0000 000

1 1 0 0 | 1 1 1 0 | 1 1 0 1 | 1 0 0 0 | 0 0 0 0

C *E* *D* 8 0

Hence, with normalisation (–14.75) $*$ 2^{12} is represent as $(\text{CED80})_H$, in the above mentioned format.

Denormalization (Reversing to original form):

Now, write the $(\text{CED80})_H$, in binary form as 1100 1110 11011000 0000.

On Comparing the data with given format, we have

Sign	BE	M
1	1001110	11011 0000 000
1 bit	7 bits	12 bits

- Sign bit is 1 $\Rightarrow$ number is negative
- BE = $(1001110)_2$

We know that $BE = AE + Bias$

$AE = BE - Bias$

$= (1001110 - 0111111)_2 = (0001111)_2$

The binary equivalent of AE is 15.

- Mantissa, M = 11011

Combining all the information, we have

Number as $(-1.11011)_2 * 2^{15} = (-1.11011)_2 * 2^{12}$

Now, converting the binary number in decimal form:

$$(1110.11)_2 = 2^0(0) + 2^1(1) + 2^2(1) + 2^3(1) + 2^{-1}(1) + 2^{-2}(1)$$

$$= 0 + 2 + 4 + 8 + \frac{1}{2} + \frac{1}{4}$$

$$= 14.75$$

Hence, we get data as $(-14.75) * 2^{12}$ from normalised form $(CED80)_H$.

Q.6 (b) Solution:

(i) Substrate: p-type Si with

$$N_A = 5 \times 10^{16} \text{ cm}^{-3}$$

Oxide thickness, $t_{ox} = 10 \text{ nm} = 1 \times 10^{-6} \text{ cm}$

Gate: n^+ poly silicon,

Constants: $\varepsilon_{ox} = 3.45 \times 10^{-13}$ F/cm, $\varepsilon_s = 1.04 \times 10^{-12}$ F/cm,

$$q = 1.6 \times 10^{-19} \text{ C}$$

Oxide capacitance, $C_{ox} = \dfrac{\varepsilon_{ox}}{t_{ox}} = \dfrac{3.45\times10^{-13}}{10^{-6}} = 3.45\times10^{-7}\text{ F/cm}^2$

Fermi potential, $\phi_F = V_T \ln\left(\dfrac{N_A}{n_i}\right) = 0.026\ln\left(\dfrac{5\times10^{16}}{10^{10}}\right)$

$$= 0.026 \ln(5 \times 10^6) = 0.026 \times 15.43$$

$\therefore$ $\phi_F \approx 0.40$ V

Flat-band voltage (V_{fb})

For n^+ poly gate: $V_{fb} = \phi_m - \phi_s$

For n^+ poly: $\phi_m \approx \chi = 4.05$ V

For p-type substrate: $\phi_s = \chi + \dfrac{E_g}{2} + \phi_F = 4.05 + 0.56 + 0.40 = 5.01$ V

$\therefore$ $V_{fb} \approx -0.96$ V

Threshold voltage (V_t),

$$V_t = V_{fb} + 2\phi_F + \frac{\sqrt{2q\varepsilon_s N_A(2\phi_F)}}{C_{ox}}$$

$$V_t = -0.96 + 0.8 + \frac{\sqrt{2(1.6\times10^{-19})(1.04\times10^{-12})(5\times10^{16})(0.8)}}{3.45\times10^{-7}}$$

$$V_t = 0.174 \text{ V}$$

(ii) At $V_g = V_{fb} - 1$ V,

Accumulation charge,

$$Q_A = -C_{ox}(V_g - V_{fb})$$

$$= -3.45 \times 10^{-7}(-1)$$

$$\therefore \quad Q_A \approx +3.45 \times 10^{-7} \text{ C/cm}^2$$

This is a positive charge density representing the holes accumulated at the surface.

(iii) Depletion and inversion charge at $V_g = 2$ V

Depletion charge at strong inversion:

$$Q_d = -\sqrt{2q\varepsilon_s N_A(2\phi_F)}$$

$$= -\sqrt{2(1.6\times10^{-19})(1.04\times10^{-12})\times5\times10^{16}\times0.8}$$

$$Q_d \approx -1.15 \times 10^{-7} \text{ C/cm}^2$$

Inversion charge: Any gate voltage applied beyond the threshold voltage creates the inversion layer charge given by,

$$Q_{\text{inv}} = -C_{ox}(V_g - V_t)$$

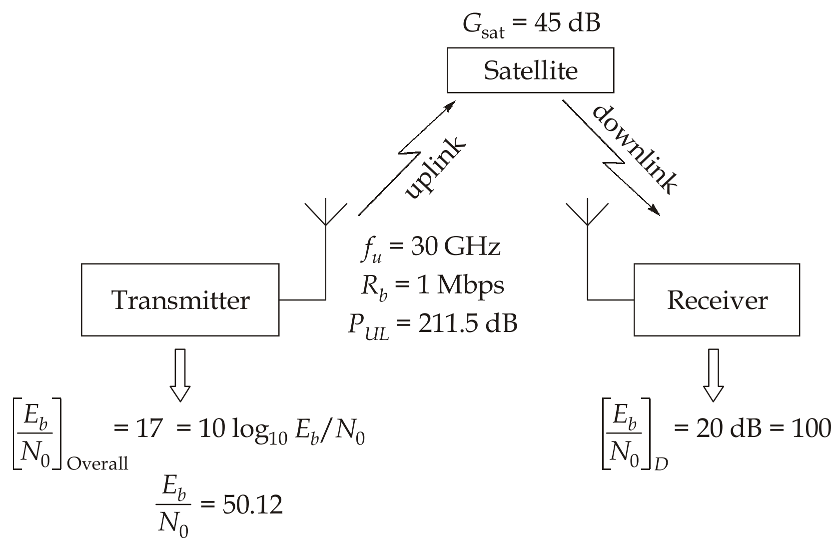
$$= -3.45 \times 10^{-7}(2 - 0.176)$$

$$= -3.45 \times 10^{-7}(1.824)$$

$$Q_{\text{inv}} \approx -6.29 \times 10^{-7} \text{ C/cm}^2$$

Q.6 (c) Solution:

On the basis of given information, we can draw the whole link as



- Antenna efficiency = 60%
- Satellite receive noise density, N_0 = –169 dBm/Hz

$$-169 = 10 \log_{10} N_0$$

$$N_0 = 1.25892 \times 10^{-17} \text{ mW}$$

We know that,

$$\frac{1}{\left[\frac{E_b}{N_0}\right]_{overall}} = \frac{1}{\left[\frac{E_b}{N_0}\right]_U} + \frac{1}{\left[\frac{E_b}{N_0}\right]_D}$$

$$\frac{1}{50.12} = \frac{1}{\left[\frac{E_0}{N_0}\right]_U} + \frac{1}{100}$$

$$\left[\frac{E_b}{N_0}\right]_U = 100.48$$

We know that,

$$\left[\frac{C}{N_0}\right]_U = \frac{E_b}{N_0} \times R_b$$

$$\left[\frac{C}{N_0}\right]_U = 100.48 \times 10^6$$

In decibels, $\left[\frac{C}{N_0}\right]_U = 10 \log_{10}(100.48 \times 10^6) = 80.02 \text{ dB}$

Now, $T_{SAT} = \frac{N_0}{K} = \frac{1.25892 \times 10^{-20} \text{ W/Hz}}{1.38 \times 10^{-23} \text{ J/K}}$

$$T_{SAT} = 912.26°\text{K}$$

In decibels, $T_{SAT} = 10 \log_{10}(912.26°) = 29.6 \text{ dB}$

$$\left[\frac{G}{T}\right]_{SAT} = [G]_{SAT} - [T]_{SAT}$$

$$= 45 - 29.6 = 15.4 \text{ dB}$$

and, $\left[\frac{C}{N_0}\right]_U = [\text{EIRP}]_U - [\text{Path loss}]_U + \left[\frac{G}{T}\right]_{SAT} + 228.6$

$$[\text{EIRP}]_U = 80.02 + 211.5 - 15.4 - 228.6 = 47.52 \text{ dB}$$

And, we know that,

$$[\text{EIRP}]_U = [P]_U + [G_{ES}]$$

$$47.52 = -6.989 + [G_{ES}],$$

where $[P]_U = 10\log_{10}(200 \times 10^{-3}) = -6.989 \text{ dB}$

$$G_{ES} = 54.509 \text{ dB}$$

$$G_{ES} = 0.28 \times 10^6$$

Gain of antenna is given by, $G_{ES} = \frac{4\pi A}{\lambda^2} \times \eta_A$

$$0.28 \times 10^6 = \frac{4\pi \times (\pi D^2 / 4)}{(0.01)^2} \times 0.6, \text{ where } \lambda = \frac{c}{f} = \frac{3 \times 10^8}{30 \times 10^9} = 0.01 \text{ m}$$

$$3.71 = \frac{\pi D^2}{4}$$

$$D^2 = 4.723$$

$$D = 2.17 \text{ m}$$

Q.7 (a) Solution:

(i) Ceramic materials used in engineering are broadly classified into Traditional ceramics and Engineering (Advanced) Ceramics based on composition, processing and applications.

1. Traditional Ceramics

Definition

Traditional ceramics are made from naturally occurring raw materials, mainly clay, silica, and feldspar.

Characteristics

- Low cost and easily available
- Processed by conventional methods (molding, firing)
- Moderate mechanical strength
- Brittle and porous (in some cases)
- Limited high-temperature and advanced performance

Examples

- Bricks
- Tiles
- Pottery
- Porcelain
- Cement

Applications

- Building and construction (bricks, roofing tiles)
- Household items (utensils, sanitary ware)
- Electrical insulators (porcelain)

2. Engineering (Advanced) Ceramics

Definition

Engineering ceramics are made from high-purity synthetic materials with controlled composition and microstructure.

Characteristics

- High strength and hardness
- Excellent wear and corrosion resistance
- High-temperature stability

- Good electrical and thermal properties
- More expensive due to advanced processing

Examples

- Alumina (Al_2O_3)
- Silicon carbide (SiC)
- Silicon nitride (Si_3N_4)
- Zirconia (ZrO_2)

Applications

- Aerospace components
- Cutting tools
- Biomedical implants (bone replacements, dental ceramics)
- Electronic devices (semiconductors, capacitors)
- Engine parts (turbines, bearings)

(ii) Methods for Preparing Ceramic Raw Materials

Ceramic raw materials are prepared mainly by two methods:

1. Dry Processing Method

- Raw materials are crushed, ground, and mixed in dry form
- Powders are blended uniformly without adding water
- Suitable for simple shapes and large-scale production

Features:

- Low cost
- Faster process
- Less control over uniformity compared to wet method

2. Wet Processing Method

- Raw materials are mixed with water to form a slurry (slip)
- The slurry is thoroughly blended for better homogeneity
- Used in processes like slip casting

Features:

- Better mixing and uniform composition
- Suitable for complex shapes
- Requires drying step $\rightarrow$ increases time and cost

Types of Ingredients Added

During preparation, different additives are mixed to improve processing and properties:

1. **Plasticizers**
   - Improve moldability and workability
   - Example: Clay, organic materials
2. **Binders**
   - Provide strength to green (unfired) body
   - Example: Starch, resins
3. **Lubricants**
   - Reduce friction during shaping
   - Example: Waxes
4. **Deflocculants**
   - Improve fluidity in slurry (wet method)
   - Example: Sodium silicate
5. **Fluxes**
   - Lower melting temperature during firing
   - Example: Feldspar
6. **Fillers/Reinforcements**
   - Improve strength and stability
   - Example: Silica

Q.7 (b) Solution:

(i) We have,

$$\text{Acceptance angle, } \theta_A = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right) = 8°$$

$$\therefore \quad n_0 = 1$$

$$\sin 8° = \sqrt{n_1^2 - n_2^2}$$

$$0.319 = \sqrt{n_1^2 - n_2^2}$$

$$0.019 = (n_1 + n_2)(n_1 - n_2)$$

where, n_1 = refractive index of core

n_2 = refractive index of cladding

Given, $n_1 + n_2 = 3.046$

$$\therefore \quad n_1 - n_2 = \frac{0.019}{3.046}$$

$$n_1 - n_2 = 6.239 \times 10^{-3}$$

On solving, we get,

$$n_1 = 1.526 \text{ and } n_2 = 1.52$$

Relative refractive index,

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} = \frac{(\sin\theta_A)^2}{2(1.526)^2} = \frac{(\sin 8°)^2}{2(1.526)^2} = 0.00416$$

$$\Delta = 0.416\%$$

(ii) Solid acceptance angle, $\Omega = \pi(\text{NA})^2$ steradian

$$\Omega = \pi(\sin\theta_a)^2$$

$$\Omega = \pi(\sin 8°)^2$$

$$\Omega = 0.061 \text{ steradian}$$

RI of cladding, $n_2 = 1.52$

We know that, Normalised frequency,

$$V = \frac{2\pi}{\lambda} a(n_1^2 - n_2^2)^{1/2} = \frac{2\pi a}{\lambda}(NA)$$

$$V = \frac{2\pi}{1.2\times10^{-6}} \times \frac{40}{2} \times 10^{-6} (\sin 8°) = 14.57$$

Number of modes, $N = \left(\frac{\alpha}{\alpha+2}\right)\frac{V^2}{2}$

For parabolic refractive index profile, $\alpha = 2$

$$N = \frac{V^2}{4} = \frac{(14.57)^2}{4} = 53.07$$

Thus, the number of guided modes supported by the fibre is approximately 53.

Q.7 (c) Solution:

(i) We have, $n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4 \text{ cm}^{-3}$

At $x = 0$, $n_B(0) = n_{B0}\exp\left(\frac{V_{BE}}{V_t}\right)$

Thus, $$V_{BE} = V_t \ln\left(\frac{n_B(0)}{n_{B0}}\right), \text{ where } n_B(0) = 0.1N_B$$

$$V_{BE} = (0.0259)\ln\left(\frac{10^{15}}{2.25\times10^4}\right)$$

$$V_{BE} = 0.635 \text{ V}$$

(ii) At $x' = 0$,

$$p_E(0) = p_{E0}\exp\left(\frac{V_{BE}}{V_t}\right)$$

where, $$p_{E0} = \frac{n_i^2}{N_E} = \frac{(1.5\times10^{10})^2}{10^{17}} = 2.25\times10^3 \text{ cm}^{-3}$$

Thus, $$p_E(0) = 2.25\times10^3\exp\left(\frac{0.635}{0.0259}\right) = 10^{14}\text{cm}^{-3}$$

(iii) In the B-C space charge region,

$$x_{p1} = \left[\frac{2\varepsilon_{Si}(V_{b1}+V_A)}{q}\left(\frac{N_C}{N_B}\right)\cdot\frac{1}{N_C+N_B}\right]^{1/2}$$

where, $$V_{b1} = 0.0259\ln\left[\frac{10^{16}\times10^{15}}{(1.5\times10^{10})^2}\right] = 0.635 \text{ V}$$

and V_A = 3 V is the applied reverse bias voltage at B-C junction

thus, $$x_{p1} = \left[\frac{2(11.7)(8.854\times10^{-14})(0.635+3)}{1.6\times10^{-19}}\times\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{15}+10^{16}}\right)\right]^{1/2}$$

We get, x_{p1} = 0.207 μm = depletion width inside base at collector base junction.

For, the base emitter junction, the depletion width in base side can be found similarly as

$$V_{b2} = (0.0259)\ln\left[\frac{(10^{17})(10^{16})}{(1.5\times10^{10})^2}\right] = 0.754 \text{ V}$$

We have, $x_{p2} = \sqrt{\frac{2\epsilon_{si}(V_{b2}-V_F)}{q}\left(\frac{N_E}{N_B}\right)\cdot\frac{1}{N_E+N_B}}$, where V_F is the applied forward bias voltage at B-E junction.

then, $$x_{p2} = \left[\frac{2(11.7)(8.854\times10^{-14})(0.754-0.635)}{1.6\times10^{-19}}\times\left(\frac{10^{17}}{10^{16}}\right)\left(\frac{1}{10^{17}+10^{16}}\right)\right]^{1/2}$$

$$x_{p2} = 0.118\ \mu\text{m}$$

∴ Neutral base width,

$$x_B = x_{B0} - x_{p1} - x_{p2}\ ; \quad \text{where, } x_{B0} = \text{metallurgical length}$$

$$= 1.1 - 0.207 - 0.118$$

$$x_B = 0.775\ \mu\text{m}$$

Q.8 (a) Solution:

(i) Instruction design

Opcode	Register operand 1	Register operand 2	Memory operand	Immediate field

Each instruction has a unique opcode. Thus, Opcode size determine the possible number of instructions in the CPU.

Given,

Number of instructions or Instruction size = 120

So, Number of bits required to represent the opcode = $\log_2 120$ = 7 bit

Since the CPU support 64 registers, number of bits required to represent the register operand 1 = $\log_2 64$ = 6 bit

Number of bits required to represent the register operand 2 = $\log_2 64$ = 6 bit

For a 2 kB memory, number of bits required to represent memory operand = $\log_2 2$ K = 11 bit

Given that Immediate field size = 7 bit

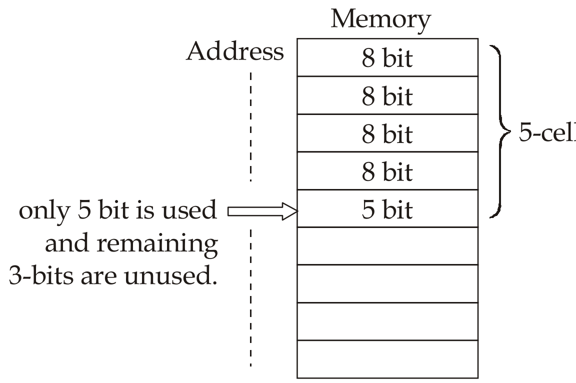
Instruction design:

Opcode	Register operand 1	Register operand 2	Memory operand	Immediate field
7 bit	6 bit	6 bit	11 bit	7 bit

Hence instruction size = 7 + 6 + 6 + 11 + 7 = 37 bit

i.e., 37 bit instruction is stored in the memory

Since the memory is byte addressable, cell size = 8 bit



$$\text{Instruction length} = 8 \times 4 + 5 = 37 \text{ bit}$$

Hence one instruction require 5 memory cells.

$$\text{Given program size} = 200 \text{ instructions}$$

Thus,

$$\text{Storage space required} = 200 \text{ (5 cells)} \qquad \text{[1 instruction require 5 cells]}$$
$$= 1000 \text{ cells}$$
$$= 1000 \text{ Bytes} \qquad \text{[1 cell = 8 bit = 1 byte]}$$

So, 1000 Byte storage space is required to store the program.

(ii) Critical frequency f_c is the highest frequency that returns from an ionospheric layer when launched at a vertical incidence. When the frequency exceeds f_c, the return will depend upon the angle of incidence at a particular ionospheric layer. Thus, for a specified angle of incidence, the highest frequency that can be reflected back to Earth from the ionospheric layer is known as the Maximum Usable Frequency (MUF) i.e. the maximum possible value of frequency for which reflection takes place for a given distance of propagation is termed as maximum usable frequency (MUF) for that distance and for the given ionospheric layer. Hence, beyond MUF, the wave will not return. At the point of reversal of path to return to the ground, the sky wave must undergo total internal reflection i.e. angle of refraction is 90° at f_{MUF}. Thus, if $\angle\phi_i$ is the incident angle and angle $\angle\phi_r$ is the refraction angle, the refractive index n at $f = f_{\text{MUF}}$ can be written as

$$n = \frac{\sin\phi_i}{\sin\phi_r} = \frac{\sin\phi_i}{\sin 90^\circ} = \sin\phi_i = \sqrt{1 - \frac{81 N_{\max}}{f_{MUF}^2}}$$

$$n = \sqrt{1 - \frac{81 N_{\max}}{f_{MUF}^2}}$$

We have,

$$f_c^2 = 81\, N_{\max}$$

Thus, $\sin^2\phi_i = 1-\frac{f_c^2}{f_{MUF}^2}$ or $\frac{f_c^2}{f_{MUF}^2} = 1-\sin^2\phi_i = \cos^2\phi_i$

$$f_{MUF}^2 = \frac{f_c^2}{\cos^2\phi_i} = f_c^2\sec^2\phi_i$$

Thus, we get $f_{\text{MUF}} = f_c \sec\phi_i$

Q.8 (b) Solution:

(i) Given,

$$d = 2.82\ \text{Å} = 2.82 \times 10^{-10}\ \text{m},\ n = 1,\ \theta = 10°$$

1. According to Bragg's law

$$2d\sin\theta = n\lambda, \text{ where } n \text{ is the order of reflection}$$

$$\therefore \quad \lambda = \frac{2d\sin\theta}{n} = \frac{2\times 2.82\times 10^{-10}\times\sin 10°}{1}$$

$$= 0.979 \times 10^{-10} = 0.979\ \text{Å}$$

2. For $n = 2$,

$$2d\sin\theta = n\lambda$$

$$\therefore \quad \sin\theta = \frac{n}{2}\frac{\lambda}{d} = \frac{2\times 0.979\times 10^{-10}}{2\times 2.82\times 10^{-10}} = 0.347$$

$$\therefore \quad \theta = \sin^{-1}(0.347) = 20.31°$$

3. For the highest order, reflection maximum value of $\sin\theta = 1$

$$\therefore \quad n_{\text{max}} = \frac{2d(\sin\theta)_{\text{max}}}{\lambda} = \frac{2\times 2.82\times 10^{-10}\times 1}{0.979\times 10^{-10}} = 5.76$$

Thus, highest order that can be seen is $n_{\text{max}} = 5$.

(ii) Given $W(x) = W_0 + x$ ...(i)

Channel from $x = 0$ (source) to $x = L$ (drain)

1. drift current, $I_D = \mu_n C_{ox} W(x)[(V_{GS}-V_T)-V(x)]\frac{dV}{dx}$

Since current is constant, by rearranging:

$$\frac{I_D}{\mu_n C_{ox}} = (W_0+x)[(V_{GS}-V_T)-V(x)]\frac{dV}{dx}$$

$$\frac{I_D}{\mu_n C_{ox}}dx = (W_0+x)[(V_{GS}-V_T)-V]dV$$

$$\frac{I_D}{W_0 + x}dx = \mu_n C_{ox}(V_{GS} - V_T - V(x))dV$$

Integrating both sides from the source ($x = 0$, $V = 0$) to the drain ($x = L$, $V = V_{DS}$), we get

$$\int_0^L \frac{I_D dx}{W_0 + x} = \mu_n C_{ox} \int_0^{V_{DS}} (V_{GS} - V_T - V(x))dV$$

$$I_D \ln\left(\frac{W_0 + L}{W_0}\right) = \mu_n C_{ox}(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}$$

$$I_D = \frac{\mu_n C_{ox}}{\ln\left(1 + \frac{L}{W_0}\right)}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

2. Saturation Current:

At saturation, $V_{DS} = V_{GS} - V_T$. Thus,

$$I_{Dsat} = \frac{\mu_n C_{ox}}{2\ln\left(1 + \frac{L}{W_0}\right)}(V_{GS} - V_T)^2$$

Q.8 (c) Solution:

(i) Given: edge of cube a = 0.281 nm

1. Distance between opposite corner ions (body diagonal)

In a cube, the body diagonal is:

$$d = \sqrt{a^2 + a^2 + a^2} = \sqrt{3}a$$

$$d = \sqrt{3}a$$

Substitute a = 0.281 nm:

$$d = \sqrt{3} \times 0.281 \approx 1.732 \times 0.281$$

$$d \approx 0.487 \text{ nm}$$

2. Angle θ is given by the angle between the body diagonal, and face diagonal.

Let:

Body diagonal vector = (a, a, a)

Face diagonal vector = (a, a, 0)

Using dot product:

$$\cos\theta = \frac{(a,a,a)\cdot(a,a,0)}{|a,a,a||a,a,0|}$$

$$= \frac{a^2+a^2+0}{\sqrt{3}a\cdot\sqrt{2}a} = \frac{2a^2}{a^2\sqrt{6}} = \frac{2}{\sqrt{6}} = \sqrt{\frac{2}{3}}$$

$$\theta = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

$$\theta \approx 35.26°$$

(ii) Given data,

Density ρ = 7.19 g/cm^3 = 7190 kg/m^3

Atomic weight A = 52 g/mol = 0.052 kg/mol

Number of vacancies, N_v = 2.574 × 10^{22} atoms/m^3

Temperature T = 670°C = 943 K

Boltzmann constant, k = 1.38 × 10^{-23} J/K

Avogadro's number N_A = 6.022 × 10^{23}

The equilibrium number of vacancies in a crystalline material at a specific temperature is determined by the Arrhenius relation:

$$N_v = Ne^{-Q_v/(kT)},$$

where Q_v is the activation energy and N is the total number of atomic sites.

Number of atomic sites N,

$$N = \frac{\rho N_A}{A} = \frac{7190\times6.022\times10^{23}}{0.052}$$

$$N \approx 8.33\times10^{28}\ \text{atoms/m}^3$$

By using vacancy equation,

$$\frac{N_v}{N} = e^{-Q_v/(kT)}$$

$$\ln\left(\frac{N_v}{N}\right) = -\frac{Q_v}{kT}$$

$$Q_v = -kT\ln\left(\frac{N_v}{N}\right)$$

We have, $$\frac{N_v}{N} = \frac{2.574\times10^{22}}{8.33\times10^{28}} \approx 3.09\times10^{-7}$$

Thus, $$Q_v = (1.38 \times 10^{-23})(943)(14.99)$$

$$Q_v \approx 1.95 \times 10^{-19}\ \text{J}$$

$$Q_v = \frac{1.95\times10^{-19}}{1.6\times10^{-19}} \approx 1.22\ \text{eV}$$

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