Detailed Solutions

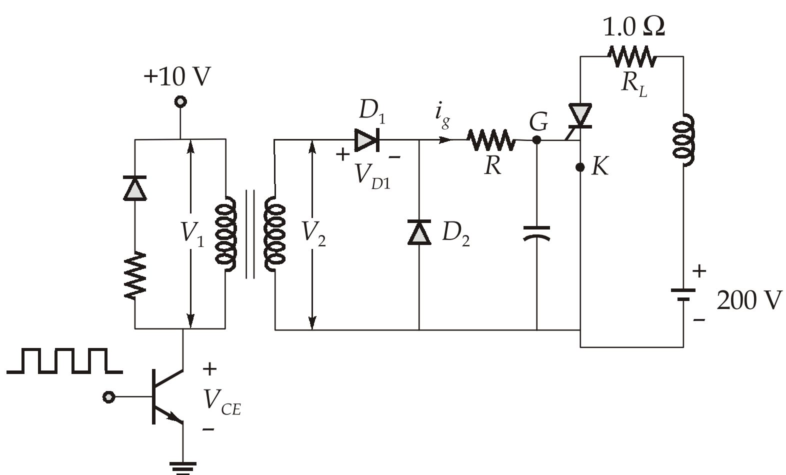
ESE-2026 Mains Test Series

Electrical Engineering Test No : 6

Section A : Power Electronics & Drives + Engineering Mathematics

Q.1 (a) Solution:

(i)



When the pulses are applied to the base of the transistor. Transistor operates in ON state. So, the forward voltage drop in transistor $V_{CE} = 1$ V.

$$V_1 = 10 - V_{CE} = 10 - 1 = 9 \text{ V}$$

$$V_2 = V_1\left(\frac{1}{1}\right) = V_1 = 9 \text{ V} \qquad \text{[turn ratio 1 : 1]}$$

D_1 is forward biased and voltage drop in diode $V_{D1} = 1$ V.

D_2 is reversed biased and acts as open circuit.

Capacitor behaves as open circuit for DC voltage. Forward voltage drop of gate cathode junction.

$$V_{gk} = 1 \text{ V}$$

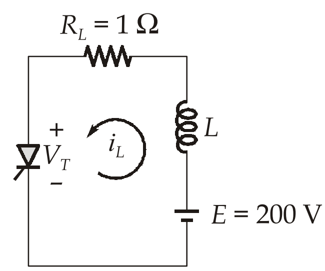
Voltage drop across resistor R,

$$V_R = V_2 - V_{D1} - V_{gk} = 9 - 1 - 1 = 7 \text{ V}$$

To ensure turn –ON of SCR,

$$R = \frac{V_R}{I_{g(\max)}} = \frac{7}{150 \text{ mA}} \approx 47\ \Omega$$

(ii)



Forward voltage drop of SCR during ON-state

$$V_T = 1 \text{ V}$$

$$E - \frac{Ldi_a}{dt} - Ri_a - V_T = 0$$

$$\Rightarrow \quad 200 - 0.15\frac{di_a}{dt} - i_a - 1 = 0$$

$$\Rightarrow \quad i_a = 199(1 - e^{-t/0.15})$$

Gate pulse width required = time taken by i_a to rise upto $I_L = T$

$$I_L = i_a$$

$$\Rightarrow \quad 250 \times 10^{-3} = 199(1 - e^{-T/0.15})$$

$$T = 188.56\ \mu\text{s}$$

Width of the pulse, $T = 188.56\ \mu\text{s}$

Magnitude of voltage, $V = 10$ V

Voltage second rating of PT

$$VT = T = 10 \times 188.56\ \mu\text{s} = 1885.6 \text{ V-s} \approx 2000\ \mu\text{s}$$

Q.1 (b) Solution:

$$y^2 + z^2 = 2ax \qquad \text{...(i)}$$

$$y^2 = ax \qquad \text{...(ii)}$$

$$x = a \qquad \text{...(iii)}$$

Differentiating equation (i), we get

$$2z\frac{\partial z}{\partial x} = 2a,$$

$$\frac{\partial z}{\partial x} = \frac{a}{z}$$

$$2y + 2z\frac{\partial z}{\partial y} = 0,$$

$$\frac{\partial z}{\partial y} = \frac{-y}{z}$$

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1 = \frac{a^2}{z^2} + \frac{y^2}{z^2} + 1 = \frac{a^2 + y^2}{z^2} + 1$$

$[y^2 + z^2 = 2ax \,;\, z^2 = 2ax - y^2]$

$$= \frac{a^2 + y^2}{2ax - y^2} + 1 = \frac{a^2 + y^2 + 2ax - y^2}{2ax - y^2}$$

$$= \frac{a^2 + 2ax}{2ax - y^2}$$

$$S = \int_0^a \int_{-\sqrt{ax}}^{\sqrt{ax}} \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\; dx\, dy$$

$$= \int_0^a \int_{-\sqrt{ax}}^{\sqrt{ax}} \sqrt{\frac{a^2 + 2ax}{2ax - y^2}}\; dx\, dy \qquad [y^2 = ax \,;\, y = \pm\sqrt{ax}\,]$$

$$= \sqrt{a}\int_0^a \int_{-\sqrt{ax}}^{\sqrt{ax}} \sqrt{\frac{a + 2x}{2ax - y^2}}\; dx\, dy$$

$$= \sqrt{a}\int_0^a \sqrt{a + 2x}\, dx \int_{-\sqrt{ax}}^{\sqrt{ax}} \frac{1}{\sqrt{2ax + y^2}}\, dy$$

$$= \sqrt{a}\int_0^a \sqrt{a + 2x}\, dx \left[\sin^{-1}\frac{y}{\sqrt{2ax}}\right]_{-\sqrt{ax}}^{\sqrt{ax}}$$

$$= \sqrt{a}\int_0^a \sqrt{a + 2x}\, dx \left[\sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right]$$

$$= \sqrt{a}\int_0^a \sqrt{a + 2x}\, dx \left[\frac{\pi}{4} - \left(\frac{\pi}{4}\right)\right]$$

$$= \sqrt{a}\frac{\pi}{2}\int_0^a \sqrt{a+2x}\,dx$$

$$= \frac{\pi}{2}\times\frac{\sqrt{a}}{2}\times\frac{2}{3}\left[(a+2x)^{3/2}\right]_0^a$$

$$= \frac{\pi\sqrt{a}}{6}\left[(3a)^{3/2}-a^{3/2}\right]$$

$$= \frac{\pi a^2}{6}\left[3\sqrt{3}-1\right]$$

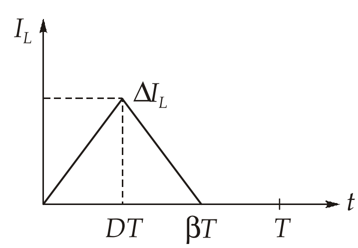
Q.1 (c) Solution:

The critical value of inductance for buck converter

$$\Rightarrow \quad L_{cr} = \frac{(1-D)R}{2f} = \frac{(1-0.4)\times 20}{2\times 10\text{k}} = 600\ \mu\text{H}$$

Since, $L_{\text{given}} < L_{cr}$

Hence, the converter is operating in discontinous mode



$$\Rightarrow \quad (I_{sw})_{\text{avg}} = \frac{1}{2}\times\Delta I_L\times\frac{DT}{T} = \frac{D\Delta I_L}{2}$$

$$\Rightarrow \quad V_0 = \frac{DV_{DC}}{\beta},$$

$$\Delta I_L = \frac{D(1-D/\beta)V_{DC}}{fL}$$

$\Rightarrow$ By power balancing

$$V_{DC}\times(I_{sw})_{\text{avg}} = \frac{(V_0)^2}{R}$$

$$= 24\times\left[0.4\times\frac{0.4\left(1-\frac{0.4}{\beta}\right)\times 24}{2\times 10\text{k}\times 200\mu}\right] = \frac{\left(\frac{0.4\times 24}{\beta}\right)^2}{20}$$

$$\Rightarrow \qquad 23.04\left(1-\frac{0.4}{\beta}\right) = \frac{4.608}{\beta^2}$$

$$\Rightarrow \qquad \beta^2 - 0.4\beta - 0.2 = 0$$

$$\Rightarrow \qquad \beta = 0.69, -0.289$$

Since $\beta > D$ so $\qquad \beta = 0.69$

So, $\qquad V_0 = \dfrac{DV_{DC}}{\beta} = 13.91$ Volts

Q.1 (d) Solution:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{-\pi}^{0} f(x)dx + \frac{1}{\pi}\int_{0}^{\pi} f(x)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} (-x-\pi)dx + \frac{1}{\pi}\int_{0}^{\pi}(x+\pi)dx$$

$$= \frac{1}{\pi}\left(\frac{-x^2}{2} - \pi x\right)_{\pi}^{0} + \frac{1}{\pi}\left(\frac{x^2}{2} + \pi x\right)$$

$$= \frac{1}{\pi}\left(\frac{\pi^2}{2} - \pi^2\right) + \frac{1}{\pi}\left(\frac{\pi^2}{2} + \pi^2\right)$$

$$= \pi\left(\frac{1}{2} - 1\right) + \pi\left(\frac{1}{2} + 1\right) = \pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} (-x-\pi)\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}(x-\pi)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[(-x-\pi)\frac{\sin nx}{n} - (-1)\left\{\frac{-\cos nx}{n^2}\right\}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[(x+\pi)\frac{\sin nx}{n} - (1)\left\{-\frac{\cos nx}{n^2}\right\}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{1}{n^2} + \frac{(-1)^n}{n^2}\right] + \frac{1}{\pi}\left[-\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right]$$

$$= \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

$$a_n = \frac{-4}{n^2\pi}, \quad \text{If } n \text{ is odd}$$

and $a_n = 0$, If n is evens

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} (-x-\pi)\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}(x+\pi)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[(-x-\pi)\left(-\frac{\cos nx}{n}\right) - (-1)\left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{1}{\pi}\left[(x+\pi)\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\right] + \frac{1}{\pi}\left[\frac{-2\pi}{n}(-1)^n + \frac{\pi}{n}\right]$$

$$= \frac{1}{n}\left[(1) - 2(-1)^n + (1)\right]$$

$$= \frac{2}{n}\left[1 - (-1)^n\right]$$

$$= \frac{4}{n}, \quad \text{if } n \text{ is odd}$$

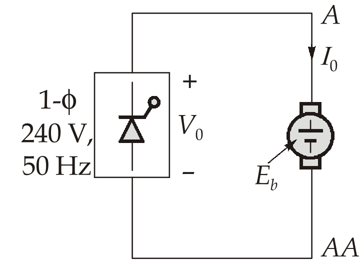
$$= 0, \quad \text{if } n \text{ is even}$$

Fourier series is

$$f(x) = \frac{a_0}{2} + a_1\cos x + a_2\cos 2x + \ldots + b_1\sin x + b_2\sin 2x + \ldots$$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \ldots\right) + 4\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \ldots\right)$$

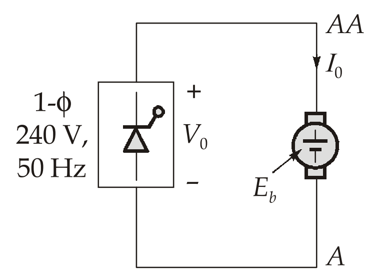
Q.1 (e) Solution:



$$\text{AC} \xrightarrow{P} \text{DC}$$
Motoring

Rating of DC machine
210 V, 10 A, 1200 rpm,

$$R_a = 1\ \Omega$$
$$I_F \to \text{constant}$$
$$\therefore \qquad \phi \to \text{constant}$$
$$\therefore \qquad E_b = K_m\omega = K_m \cdot \frac{2\pi \cdot N}{60}$$
$$V_0 = E_b + I_0R_a$$
$$210 = K_m \cdot \frac{2\pi N}{60} + I_0R_a$$
$$210 = K_m \cdot \frac{2\pi \cdot 1200}{60} + (10 \times 1)$$
$$K_m = 1.59\ \text{(V-s)/rad}$$



$$\text{AC} \xleftarrow{P} \text{DC}$$
Inv

Armature terminals reversed
$$\therefore \qquad E_b \to \text{Negative}$$
For regenerative braking,
$$V_0 = -E_b + I_0R_a$$
$$\frac{2V_m}{\pi}\cos\alpha = -E_b + I_0R_a$$
$$\frac{2.240\sqrt{2}}{\pi}\cos\alpha = -K_m \cdot \frac{2\pi N}{60} + I_0R_a$$

$$\frac{2\cdot 240\sqrt{2}}{\pi}\cdot\cos\alpha = -1.59\left(\frac{2\pi\cdot 600}{60}\right)+10$$
$$\alpha = 114.58°$$

Q.2 (a) (i) Solution:

The output voltage V_0 can be represented by Fourier series as under:

$$V_0 = \sum_{n=1,3,5,...}^{\infty} a_n\cos n\omega t + b_n\sin n\omega t$$

where,
$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, d(\omega t)$$
$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, d(\omega t)$$

$$a_n = \frac{1}{\pi}\left[\int_0^{\alpha}100\cos\omega t\, d(\omega t) + \int_{\alpha}^{180°-\alpha}(-100)\cos\omega t\, d(\omega t) + \int_{180°-\alpha}^{180°}100\cos\omega t\, d(\omega t)\right.$$
$$\left. + \int_{180°}^{180°+\alpha}-100\cos\omega t\, d(\omega t) + \int_{180°+\alpha}^{360°-\alpha}100\cos\omega t\, d(\omega t) + \int_{360°-\alpha}^{360°}-100\cos\omega t\, d(\omega t)\right]$$

$$a_n = \frac{100}{\pi}[\sin\alpha - \sin(180° - \alpha) + \sin\alpha + \sin(180°) - \sin(180° - \alpha) - \sin(180° + \alpha)$$
$$+ \sin 180° + \sin(360° - \alpha) - \sin(180° + \alpha) - \sin 360° + \sin(360° - \alpha)]$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi}\left[\int_0^{\alpha}100\sin\omega t\, d(\omega t) + \int_{\alpha}^{180°-\alpha}-100\sin\omega t d(\omega t) + \int_{180°-\alpha}^{180°}100\sin\omega t\, d(\omega t)\right.$$
$$\left. + \int_{180°}^{180°+\alpha}-100\sin\omega t\, d(\omega t) + \int_{180°+\alpha}^{360°-\alpha}100\sin\omega t\, d(\omega t) + \int_{360°-\alpha}^{360°}-100\sin\omega t\, d(\omega t)\right]$$

$$b_n = \frac{100}{\pi}[-\cos\alpha + 1 + \cos(180° - \alpha) - \cos\alpha - \cos 180° + \cos(180° - \alpha)$$
$$+ \cos(180° + \alpha) - \cos 180° - \cos(360° - \alpha) + \cos(180° + \alpha)$$
$$+ \cos 360° - \cos(360° - \alpha)]$$

$$b_n = \frac{100}{\pi}[4 - 8\cos\alpha] = 50\sqrt{2}$$

$$\cos\alpha = 0.22231$$
$$\alpha = 77.15°$$

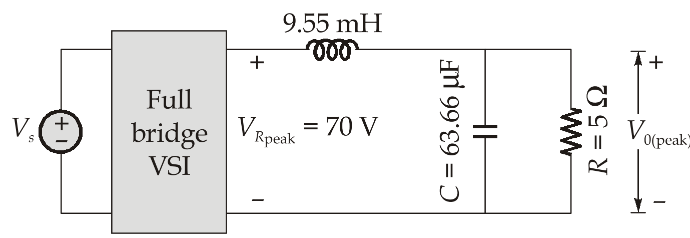
Q.2 (a) (ii) Solution:

The rms value of the fundamental component of output voltage,

$$V_{R1} = \frac{V_s}{\sqrt{2}} \times m_i = \frac{100}{\sqrt{2}} \times 0.7 = 49.49 \text{ V}$$

Amplitude of the value is peak value (i.e.),

$$V_{R_{\text{peak}}} = \sqrt{2} \times 49.49 = 70 \text{ V}$$



$V_{R_{\text{peak}}}$ is the amplitude of fundamental voltage at the VSI. $V_{0(\text{peak})}$ is the amplitude of fundamental voltage at the resistor,

$$Z = \frac{R(-jX_C)}{R - jX_C} = \frac{5(-j50)}{5 - j50} = 4.97\angle -5.71°\ \Omega$$

where, $$X_C = \frac{1}{\omega C}$$

$$= \frac{1}{2\pi \times 50 \times 63.66 \times 10^{-6}} = 50\ \Omega$$

$$X_L = \omega L = 2\pi \times 50 \times 9.55 \times 10^{-3} = 3\ \Omega$$

$$V_{0(\text{peak})} = V_{R_{\text{peak}}} \times \frac{4.97\angle -5.71°}{(4.97\angle -5.71°) + j3} = 62.75\angle{-32.5°} \text{ V}$$

∴ The amplitude of the fundamental component in the output voltage,

$$V_0 = 62.75 \text{ V}$$

Power absorbed by load, $$P = \frac{V_{01,\text{rms}}^2}{R} = \frac{(62.75)^2}{2 \times 5} = 397.37 \text{ Watts}$$

Q.2 (b) (i) Solution:

We have,

$$\frac{dx}{dt} + y = \sin t$$

$$Dx + y = \sin t \quad \text{...(i)}$$

$$\frac{dy}{dt} + x = \cos t$$

$$Dy + x = \cos t \quad \text{...(ii)}$$

Multplying (ii) by D, we get

$$D^2y + Dx = D\cos t$$

$$D^2y + Dx = -\sin t \quad \text{...(iii)}$$

Subtracting (i) from (iii), we have

$$D^2y - y = -2\sin t$$

$$(D^2 - 1)y = -2\sin t$$

A.E. is $\quad m^2 - 1 = 0$

$$m^2 = 1$$

$$m = \pm 1$$

$$\text{C.F.} = C_1e^t + C_2e^{-t}$$

$$\text{P.I.} = \frac{1}{D^2 - 1}(-2\sin t)$$

$$\text{P.I.} = \frac{1}{-1-1}(-2\sin t) = \sin t$$

$$\text{Complete solution} = \text{C.F.} + \text{P.I.}$$

$$y = C_1e^t + C_2e^{-t} + \sin t \quad \text{...(iv)}$$

Putting $y = 0$ and $t = 0$ in (iv), we get

$$0 = C_1 + C_2$$

$$C_2 = -C_1$$

On putting, $\quad C_2 = -C_1$ in equation (iv), we get

$$y = C_1e^t - C_1e^{-t}\sin t$$

On putting the value of y in (ii), we get

$$D(C_1e^t - C_1e^{-t} + \sin t) + x = \cos t$$

$$C_1e^t + C_1e^{-t} + \cos t + x = \cos t$$

$$x = -C_1e^t - C_1e^{-t} \quad \text{...(v)}$$

On putting $x = 2$, $t = 0$ in equation (v), we get

$$2 = -C_1 - C_1$$

$$C_1 = -1$$

Putting the value of C_1 in (v) and (iv), we have

$$x = e^t + e^{-t}$$

$$y = -e^t + e^{-t} + \sin t$$

Which is the required solution.

Q.2 (b) (ii) Solution:

Let, matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is orthogonal matrix

$$\therefore \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a^2 + b^2 = 1,$$

$$c^2 + d^2 = 1,$$

$$ac + bd = 0$$

$$\frac{a}{d} = -\frac{b}{c} = K \text{ (let)}$$

$$c^2 + d^2 = \frac{a^2+b^2}{k} = \frac{1}{k^2}$$

$$K^2 = 1$$

or $$K = \pm 1$$

$$\frac{a}{d} = \frac{-b}{c} = \pm 1$$

Also, we must have $a, b, c, d, \in [-1, 1]$ for equations (i) and (ii) to get defined,
Hence, without loss of generality, we can assume $a = \cos\theta$ and $b = \sin\theta$

So, for $\frac{a}{d} = \frac{-b}{c} = 1$

We have, $$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

For $\frac{a}{d} = \frac{-b}{c} = -1,$

We have, $$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$$

Q.2 (c) (i) Solution:

We have, $$f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$$

To find $y(0.2)$:

Here, $x_0 = 0, \quad y_0 = 1, \quad h = 0.2$

$$k_1 = h\, f(x_0, y_0) = 0.2 f(0, 1)$$
$$= 0.2000$$
$$k_2 = hf\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_1\right) = 0.2f(0.1, 1.1)$$
$$= 0.19672$$
$$k_3 = hf\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2\right) = 0.2f(0.1, 1.09836)$$
$$= 0.1967$$
$$k_4 = hf(x_0 + h, y_0 + k_3) = 0.2f(0.2, 1.1967)$$
$$= 0.1891$$
$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$= \frac{1}{6}[0.2 + 2(0.19672) + 2(0.1967) + 0.1891]$$
$$= 0.19599$$

Hence, $$y(0.2) = y_0 + k = 1.196$$

To find $y(0.4)$:

Here, $x_1 = 0.2, \quad y_1 = 1.196, \quad h = 0.2$

$$k_1 = h\, f(x_1, y_1)$$
$$= 0.1891$$
$$k_2 = hf\left(x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}k_1\right) = 0.2f(0.3, 1.2906)$$
$$= 0.1795$$
$$k_3 = hf\left(x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}k_2\right) = 0.2f(0.3, 1.2858)$$
$$= 0.1793$$
$$k_4 = hf(x_1 + h, y_1 + k_3) = 0.2f(0.4, 1.3753) = 0.1688$$

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= \frac{1}{6}[0.1891 + 2(0.1795) + 2(0.1793) + 0.1688]$$

$$= 0.1792$$

Hence, $y(0.4) = y_1 + k = 1.196 + 0.1792 = 1.3752$

Q.2 (c) (ii) Solution:

We constructed the following difference table from the given data:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	$y_0 = 1$				
		–2			
1	$y_1 = -1$		4		
		2		–8	
2	$y_2 = 1$		–4		16
		–2		8	
3	$y_3 = -1$		4		16
		2		$\Delta^3 y_2'$	
4	$y_4 = 1$		$\Delta^2 y_3$		16
		Δy_4		$\Delta^3 y_3'$	
5	y_5		$\Delta^2 y_4$		16
		Δy_5		$\Delta^3 y_4'$	
6	y_6		$\Delta^2 y_5$		
		Δy_6			
7	y_7				

Since the values of y belong to a polynomial of degree 4, the fourth differences must be constant. But $\Delta^4 y = 16$.

$\therefore$ The other fourth order differences must also be 16,

Thus, $\Delta^4 y_1 = 16 = \Delta^3 y_2 - \Delta^3 y_1$

i.e, $\Delta^3 y_2 = \Delta^3 y_1 + \Delta^4 y_1 = 8 + 16 = 24$

$$\Delta^2 y_3 = \Delta^2 y_2 + \Delta^3 y_2 = 4 + 24 = 28$$

$$\Delta y_4 = \Delta y_3 + \Delta^2 y_3 = 2 + 28 = 30$$

and $y_5 = y_4 + \Delta y_4 = 1 + 30 = 31$

Similarly starting with $\Delta^4 y_2 = 16,$

We get, $\Delta^3 y_3 = 40, \quad \Delta^2 y_4 = 68, \quad \Delta y_5 = 98, \quad y_6 = 129$

Starting with $\Delta^4 y_3 = 16,$

We obtain, $\Delta^3 y_4 = 56, \quad \Delta^2 y_5 = 124, \quad \Delta y_6 = 222, \quad y_7 = 351$

Q.3 (a) (i) Solution:

Let, $$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} \quad \text{(i.e } A\text{)},$$

So that,

(i) $$u_{11} = 3, \quad u_{12} = 2,$$
$$u_{13} = 7$$

(ii) $$l_{21}u_{11} = 2,$$
$$\therefore \quad l_{21} = \frac{2}{3}$$
$$l_{31}u_{11} = 3$$
$$\therefore \quad l_{31} = 1$$

(iii) $$l_{21}u_{12} + u_{22} = 3,$$
$$\therefore \quad l_{22} = \frac{5}{3}$$
$$l_{21}u_{13} + u_{23} = 1$$
$$\therefore \quad u_{23} = -\frac{11}{3}$$

(iv) $$l_{31}u_{12} + l_{32}u_{22} = 4,$$
$$\therefore \quad l_{32} = \frac{6}{5}$$

(v) $$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 1$$
$$\therefore \quad u_{33} = -\frac{8}{5}$$

Thus $$A = \begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ 1 & \frac{6}{5} & 0 \end{bmatrix}\begin{bmatrix} 3 & 2 & 7 \\ 0 & \frac{5}{3} & -\frac{11}{3} \\ 0 & 0 & -\frac{8}{5} \end{bmatrix}$$

Writing $UX = V$,

The given system becomes,

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ 1 & \frac{6}{5} & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

Solving this system, we have $v_1 = 4$,

$$\frac{2}{3}v_1 + v_2 = 5 \quad \text{or} \quad v_2 = \frac{7}{3}$$

$$v_1 + \frac{6}{5}v_2 + v_3 = 7 \quad \text{or} \quad v_3 = \frac{1}{5}$$

Hence the original system becomes

$$\begin{bmatrix} 3 & 2 & 7 \\ 0 & \frac{5}{3} & -\frac{11}{3} \\ 0 & 0 & -\frac{8}{5} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 7/3 \\ 1/5 \end{bmatrix}$$

i.e,
$$3x + 2y + 7z = 4;$$

$$\frac{5}{3}y - \frac{11}{3}z = \frac{7}{3}; \quad -\frac{8}{5}z = \frac{1}{5}$$

By back-substitution, we have $z = \frac{-1}{8}$, $y = \frac{9}{8}$ and $x = \frac{7}{8}$

Q.3 (a) (ii) Solution:

We have,

$$x + y + z = 9 \quad \text{...(i)}$$

$$2x - 3y + 4z = 13 \quad \text{...(ii)}$$

$$3x + 4y + 5z = 40 \quad \text{...(iii)}$$

Step-I : Operate (ii) - 2(i) and (iii) - 3(i) to eliminate x from (ii) and (iii),

$$x + y + z = 9 \quad \text{...(iv)}$$

$$-5y + 2z = -5 \quad \text{...(v)}$$

$$y + 2z = 13 \quad \text{...(vi)}$$

Step-II : Operate (iv) + $\frac{1}{5}$ (v) and (vi) + $\frac{1}{5}$ (v) to eliminate y from (iv) and (vi),

$$x + \frac{7}{5}z = 8 \quad \text{...(vii)}$$

$$-5y + 2z = -5 \quad \text{...(viii)}$$

$$\frac{12}{5}z = 12 \quad \text{...(ix)}$$

Step-III : Operate (vii) – $\frac{7}{12}$ (ix) and (viii) – $\frac{5}{6}$ (ix) to eliminate z from (vii) and (viii),

$$x = 1$$

$$-5y = -15$$

$$\frac{12}{5}z = 12$$

Hence the solution is $x = 1$, $y = 3$, $z = 5$

Otherwise : Rewriting the equations as

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ 40 \end{bmatrix}$$

Operate $R_2 - 2R_1$, $R_3 - 3R_1$,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ 13 \end{bmatrix}$$

Operate $R_3 + \frac{1}{5}R_2$,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & 0 & 12/5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ 12 \end{bmatrix}$$

Operate $-R_2$, $5R_3$,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 5 & -2 \\ 0 & 0 & 12 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 60 \end{bmatrix}$$

Operate $R_2 + \frac{1}{6}R_3, \frac{1}{12}R_3$,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \\ 5 \end{bmatrix}$$

Operate $\frac{1}{5}R_2$,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 5 \end{bmatrix}$$

Operate $R_1 - R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

Hence, $x = 1, y = 3, z = 5$

Q.3 (b) (i) Solution:

Let $f(x) = x^4 - x - 10$

So that, $f(1) = -10 = -\text{ve}$

$f(2) = 16 - 2 - 10 = 4 = +\text{ve}$

$\therefore$ A root of $f(x) = 0$ lies between 1 and 2. Let us take $x_0 = 2$.

Also, $f'(x) = 4x^3 - 1$

Newton-Raphson's formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Putting $n = 0$, the first approximation is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)}$$

$$= 2 - \frac{4}{4 \times 2^3 - 1} = 2 - \frac{4}{31} = 1.871$$

Putting $n = 1$, the second approximation is given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.871 - \frac{f(1.871)}{f'(1.871)}$$

$$= 1.871 - \frac{(1.871)^4 - (1.871) - 10}{4(1.871)^3 - 1}$$

$$= 1.871 - \frac{0.3835}{25.199} = 1.856$$

Putting $n = 2$, the third approximation is

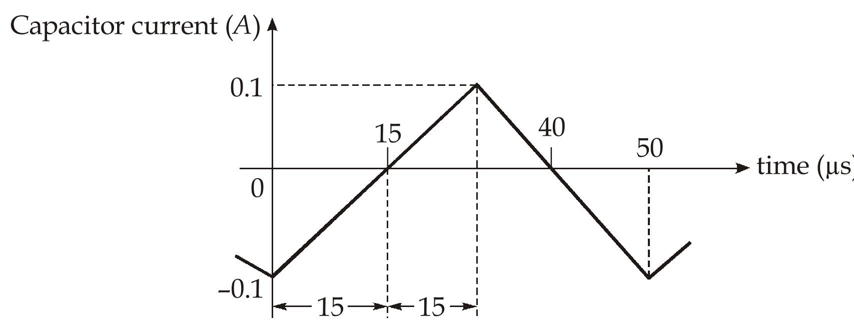
$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.856 - \frac{(1.856)^4 - (1.856) - 10}{4(1.856)^3 - 1}$$

$$= 1.856 - \frac{0.010}{24.574} = 1.856$$

Here, $x_2 = x_3$

Hence, the desired root is 1.856 correct to three decimal places.

Q.3 (b) (ii) Solution:



DC-DC Buck Converter :

Inductor ripple current,

$$\Delta I_L = \Delta I_C = (0.1) - (-0.1) = 0.2 \text{ A}$$

Chopping frequency,

$$f = \frac{1}{T} = \frac{1}{50.10^{-6}}$$

$\Rightarrow$ $$f = 20 \text{ kHz}$$

Duty Ratio, $$\alpha = \frac{T_{ON}}{T} = \frac{30}{50} = 0.6$$

We know that,

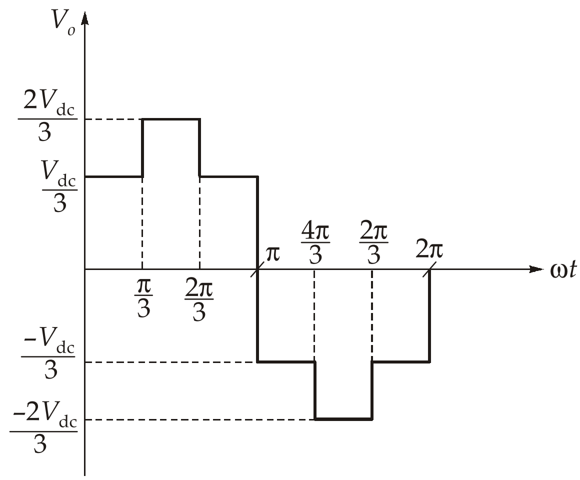
$$\Delta I_L = \frac{\alpha(1-\alpha)V_s}{fL}$$

$$\Rightarrow \qquad L = \frac{\alpha(1-\alpha)V_s}{\Delta I_L \times f} = \frac{0.6(1-0.6)\times 30}{0.2\times 20.10^3}$$

$$L = 1.8 \text{ mH}$$

Q.3 (c) Solution:

(i) The output phase voltage waveform of 6-step, 3-ϕ, inverter is given as



Since given output voltage is having quarter wave symmetry, i.e., it is symmetric about 90°, therefore, fourier series expression of output phase voltage can be given as

$$b_n = \frac{8}{T_o}\left[\int_0^{\pi/2} V_o(t)\sin(n\omega t)d(\omega t)\right]$$

$$= \frac{4}{\pi}\left[\int_0^{\pi/3} \frac{V_{dc}}{3}\sin(n\omega t)d(\omega t) + \int_{\pi/3}^{\pi/2} \frac{2V_{dc}}{3}\sin(n\omega t)d(\omega t)\right]$$

$$= \frac{4V_{dc}}{3\pi}\left[-\frac{\cos n\omega t}{n}\Big|_0^{\pi/3} + 2\left[-\frac{\cos n\omega t}{n}\right]_{\pi/3}^{\pi/2}\right]$$

$$= \frac{4V_{dc}}{3\pi}\left[-\frac{\cos n\frac{\pi}{3}}{n} + \frac{1}{n} - \frac{2}{n}\cos\frac{n\pi}{2} + \frac{2\cos n\frac{\pi}{3}}{n}\right]$$

For odd harmonics, $\qquad n = 1, 3, 5, 7, ...$

$$\cos\frac{n\pi}{2} = 0$$

Therefore, $$b_n = \frac{4V_{dc}}{3n\pi}\left[1+\cos\frac{n\pi}{3}\right]$$

Therefore, output voltage expression,

$$V_o(t) = \sum_{n=6k\pm1}^{\infty} b_n \sin n\omega t$$

$$= \sum_{n=6k\pm1}^{\infty} \frac{4V_{\text{dc}}}{3n\pi}\left(1+\cos\frac{n\pi}{3}\right)\sin n\omega t$$

Fundamental voltage, $$V_{o1} = \frac{4V_{\text{dc}}}{3\pi}\left(1+\cos\frac{\pi}{3}\right)\sin\omega t$$

$$V_{o1} = \frac{2V_{\text{dc}}}{\pi}\sin\omega t$$

$$V_{o1(\text{rms})} = \frac{\sqrt{2}}{\pi}V_{\text{dc}} \text{ (rms per phase)}$$

(ii) For 60 Hz output,

$$V_o = 400\times\frac{60}{50} = 480 \text{ V (line)}$$

Therefore, Inverter output voltage = Motor input voltage

$$\frac{\sqrt{2}}{\pi}V_{\text{dc}} = \frac{480}{\sqrt{3}}$$

$$V_{\text{dc}} = 615.62 \text{ Volt}$$

For 50 Hz output,

$$\frac{\sqrt{2}}{\pi}V_{\text{dc}} = \frac{400}{\sqrt{3}}$$

$$V_{\text{dc}} = 513 \text{ Volt}$$

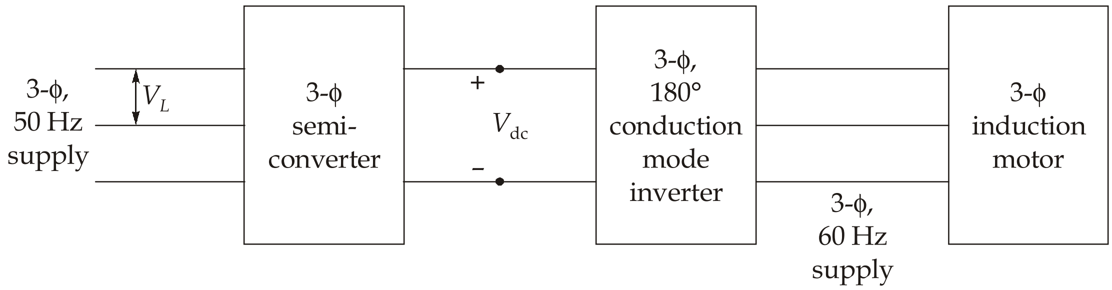
For 40 Hz output,

$$V = 400\times\frac{40}{50} = 320 \text{ V}$$

Therefore, $$\frac{\sqrt{2}}{\pi}V_{\text{dc}} = \frac{320}{\sqrt{3}}$$

$$V_{\text{dc}} = \frac{320\pi}{\sqrt{6}} = 410.41 \text{ Volt}$$

(iii) Now, inverter is fed through 3-ϕ, semi-converter



Given : $V_{dc} = 615.62$ V

Corresponds to 60 Hz and constant $\left(\dfrac{V}{f}\right)$ ratio.

$$V_L = 500 \text{ V [L – L]}$$

The average output voltage of 3-ϕ semi-converter is given as

$$V_{dc} = \frac{3V_{ml}}{2\pi}(1+\cos\alpha)$$

$$615.62 = \frac{3\sqrt{2}\times 500}{2\pi}(1+\cos\alpha)$$

$$1 + \cos\alpha = \frac{513\times 2\pi}{1500\sqrt{2}} = 1.8234$$

$$\cos\alpha = 0.8234$$

Firing angle, $\alpha = \cos^{-1}(0.8234) = 34.57°$

Q.4 (a) (i) Solution:

1. The voltage waveform is symmetrical about π as well as $\dfrac{\pi}{2}$.

As this voltage waveform has quarter-wave symmetry,

$$a_n = 0$$

$$b_n = \frac{4V_s}{\pi}\left[\int_0^{\alpha_1} \sin n\omega t\, d(\omega t) - \int_{\alpha_1}^{\alpha_2} \sin n\omega t\, d\omega t + \int_{\alpha_2}^{\pi/2} \sin n\,\omega t\, d(\omega t)\right]$$

$$b_n = \frac{4V_s}{\pi}\left[-\left(\frac{\cos n\omega t}{n}\right)_0^{\alpha_1} + \left(\frac{\cos n\,\omega t}{n}\right)_{\alpha_1}^{\alpha_2} - \left(\frac{\cos n\,\omega t}{n}\right)_{\alpha_2}^{\pi/2}\right]$$

$$= \frac{4V_s}{n\pi}\left[-\cos n\alpha_1 + 1 + \cos n\alpha_2 - \cos n\alpha_1 + \cos n\alpha_2 - \cos\frac{n\pi}{2}\right]$$

$$b_n = \frac{4V_s}{n\pi}\left[1 - 2\cos n\alpha_1 + 2\cos n\alpha_2\right],\ n = \text{odd}$$

$$b_3 = b_5 = 0$$

$$b_7 = \frac{4\times V_S}{7\pi}\left[1 - 2\cos(7\times 23.62°) + 2\cos(7\times 33.3°)\right] = 0.3154\ V_s$$

$$b_9 = \frac{4V_s}{9\pi}\left[1 - 2\cos(9\times 23.62°) + 2\cos(9\times 33.3°)\right] = 0.5201\ V_s$$

$$b_{11} = \frac{4V_s}{11\pi}\left[1 - 2\cos(11\times 23.62°) + 2\cos(11\times 33.3°)\right] = 0.3867\ V_s$$

2.

$$b_1 = \frac{4V_s}{\pi}\left[1 - 2\cos(23.62°) + 2\cos(33.3°)\right] = 1.0684\ V_s$$

Amplitude of fundamental component of unmodulated output voltage wave is,

$$b_{1n} = \frac{4V_s}{\pi} = 1.2732\ V_s$$

$$\frac{b_1}{b_{1n}} = \frac{1.0684}{1.2732} = 0.8391$$

$$\%\ \text{deration} = (1 - 0.8391) \times 100 = 16.09\%$$

Main disadvantages:

- Amplitude of fundamental wave form is less than that of unmodulated waveform. Thus inverter is derated.
- No. of commutations per cycle is very large as compared to unmodulated waveform. So, it leads to more switching losses in the thyristors.

Q.4 (a) (ii) Solution:

The average inductor current, $I_L = \dfrac{I_{L\max} + I_{L\min}}{2} = \dfrac{10+6}{2} = 8$ Amp

For boost converter, the output current

$$I_0 = I_L(1 - D) = 8(1 - D)$$

The ripple voltage in output,

$$\Delta V_C = \frac{DI_0}{fC} \quad ...(i)$$

From figure duty cycle, $$D = \frac{T_{ON}}{T}$$

$$D = \frac{10 \times 10^{-6}}{T} \text{ and } f = \frac{1}{T}$$

So equation (i) becomes, $$\Delta V_C = \frac{D I_L (1-D)}{fC} = \frac{10\times10^{-6}\times 8\left(1-\dfrac{10\times10^{-6}}{T}\right)}{150\times10^{-6}} = 0.2$$

$$\left(1-\frac{10\times10^{-6}}{T}\right) = 0.375$$

$$T = 1.6 \times 10^{-5} \text{ sec}$$

So, $$\text{frequency}, f = \frac{1}{1.6\times10^{-5}} = 62.5 \text{ kHz}$$

Q.4 (b) (i) Solution:

The fundamental peak voltage of output voltage by Fourier analysis is given as

$$\hat{V}_{01} = \frac{4V_{dc}}{\pi}\sin d$$

We know modulation index, $$M_a = \frac{\hat{V}_{01}}{V_{dc}} = \frac{4V_{dc}\sin d}{\pi V_{dc}} = \frac{4}{\pi}\sin d$$

Given, $$M_a = 0.8 = \frac{4}{\pi}\sin d$$

$$\sin d = \frac{\pi}{5}$$

$$d = \sin^{-1}\left(\frac{\pi}{5}\right) = 38.93°$$

So rms value of output voltage,

$$V_{rms} = \sqrt{\frac{2d}{180°}}V_{dc} = \sqrt{\frac{2\times 38.93}{180}}\times 300 = 197.297 \text{ volt}$$

The power lost in resistor, $$P_0 = \frac{V_{rms}^2}{R} = \frac{197.297^2}{10} = 3.893 \text{ kW}$$

For input current, $$P_{in} = P_{out}$$

$$V_{dc}I_{dc} = P_{out}$$

$$I_{dc} = \frac{3.893\times 1000}{300} = 12.98 \text{ Amp}$$

Q.4 (b) (ii) Solution:

1. Let

$$V_{AN} = 240\sqrt{2}\sin(\omega t - 30°)\text{ V}$$

$$V_{BN} = 240\sqrt{2}\sin(\omega t - 150°)\text{ V}$$

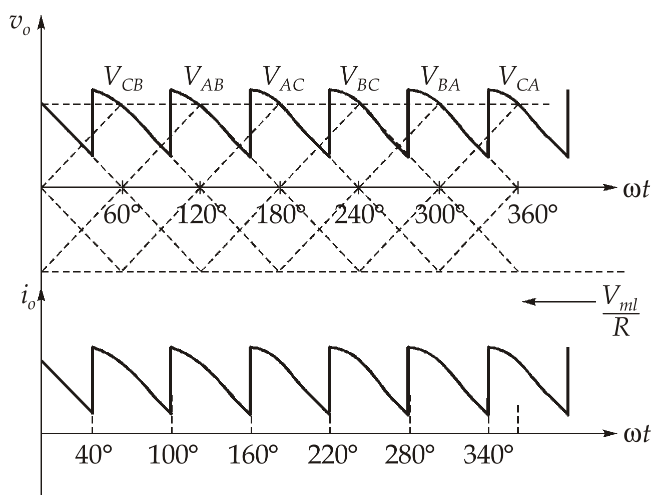
$$V_{CN} = 240\sqrt{2}\sin(\omega t - 270°)\text{ V}$$

Therefore,

$$V_{AB} = 240\sqrt{6}\sin(\omega t)\text{ Volt}$$

$$V_{BC} = 240\sqrt{6}\sin(\omega t - 120°)\text{ Volt}$$

$$V_{CA} = 240\sqrt{6}\sin(\omega t + 120°)\text{ Volt}$$



2. Average output voltage is given as

$$V_o = \frac{3}{\pi}\int_{60+\alpha}^{120+\alpha} V_{ml}\sin\omega t\cdot d\omega t$$

$$V_o = \frac{3}{\pi}V_{ml}[\cos(60°+\alpha) - \cos(120°+\alpha)] \qquad (\text{for } \alpha \le 60°)$$

Now,

$$\alpha = 40°,\ V_{ml} = 240\sqrt{6}$$

Therefore,

$$V_o = \frac{3}{\pi}\times 240\sqrt{6}[\cos(100°) - \cos(160°)]$$

$$V_o = 430\text{ Volt}$$

and average output current,

$$I_o = \frac{V_o}{R_L} = \frac{430}{240} = 1.79\text{ A}$$

Q.4 (c) (i) Solution:

$$f(x) = e^x - 3x = 0$$
$$f(0) = 1$$
$$f(1) = e^1 - 3 = -0.2817$$

The middle point of the interval (0, 1) is 0.5

$$f(0.5) = e^{0.5} - 3(0.5) = 1.649 - 1.5 = 0.149$$
$$f'(x) = e^x - 3, f'(0.5) = e^{0.5} - 3$$
$$= 1.649 - 3 = -1.351$$

By Newton-Raphson method,

$$a_1 = a - \frac{f(a)}{f'(a)}$$
$$a_1 = 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.5 - \frac{0.149}{-1.351}$$
$$= 0.5 + 0.11 = 0.61$$
$$f(0.61) = e^{0.61} - 3(0.61) = 1.84 - 1.83 = 0.01$$
$$f'(0.6) = e^{0.61} - 3 = 1.84 - 3 = -1.16$$
$$a_2 = 0.61 - \frac{f(0.61)}{f'(0.61)} = 0.61 - \frac{0.01}{-1.16}$$
$$= 0.61 + 0.0086 = 0.6186$$
$$f(0.6186) = e^{0.6186} - 3(0.6186)$$
$$= 1.8563 - 1.8558 = 0.0005$$

Q.4 (c) (ii) Solution:

In the tennis match of best of 5 sets, A can win the match, if score of A against the score of B is (3, 0), (3, 1), (3, 2)

The probability of A's doing the score of (3, 0) is

$$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

The probability of A's winning by the score of (3, 1) is

P(A loses I, wins II, III, IV sets) + P(A wins I , loses II and winds III, IV sets) + P(A wins sets I, II and loses III and wins set IV)

$$= \frac{1}{2}\times\frac{1}{4}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{4} = \frac{3}{32}$$

The probability of A's winding by the score of (3, 2) is

P(A looses I and II sets) + P(A looses I and III sets) + P(A looses I and IV sets) + P(A looses II and III sets) + P(A looses II and IV sets) + P(A looses III and IV sets)

$$= \frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)$$

$$+\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = \frac{12}{128}$$

Therefore, the probability that A wins the match is

$$\frac{1}{8} + \frac{3}{32} + \frac{12}{128} = \frac{16+12+12}{128} = \frac{40}{128} = \frac{5}{16}$$

Section B : Basic Electronics Engineering-1 + Analog Electronics-1 + Electrical Materials-1 + Electrical Machines-2

Q.5 (a) Solution:

Given,
$$\rho_p = 2\ \Omega\text{-cm}$$
$$\rho_n = 1\ \Omega\text{-cm}$$

Assume for Ge ,
$$\mu_n = 3800\ \text{cm}^2/\text{V.s}$$
$$\mu_p = 1800\ \text{cm}^2/\text{V.s}$$
$$\sigma_p = \rho\ \mu_p\ q \approx N_A\ \mu_P\ q$$
$$N_A = \frac{\sigma_p}{\mu_p q} = \frac{1}{\rho_p \mu_p q} = \frac{1}{2\times 1800\times 1.6\times 10^{-19}} = 1.74 \times 10^{15}\ \text{atoms/cm}^3$$

Similarly,
$$N_D = \frac{1}{\rho_n \mu_n q} = \frac{1}{2\times 3800\times 1.6\times 10^{-19}} = 1.64 \times 10^{15}\ \text{atoms/cm}^3$$

Built-in potential,
$$(V_0) = \text{kT}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$
$$n_i \text{ for Ge} = 2.25 \times 10^{13}/\text{cm}^3$$

$\Rightarrow$
$$V_0 = 0.026\ln\left(\frac{1.74\times 10^{15}\times 1.64\times 10^{15}}{6.25\times 10^{26}}\right) = 0.22\ \text{V}$$

For Si :
$$\mu_n = 1300\ \text{cm}^2/\text{V.s}$$
$$\mu_n = 500\ \text{cm}^2/\text{V.s}$$
$$N_A = \frac{1}{1\times 500\times 1.6\times 10^{-19}} = 6.25 \times 10^{-15}\ \text{atoms/cm}^3$$

$$N_D = \frac{1}{1\times1300\times1.6\times10^{-19}} = 4.8 \times 10^{15} \text{ atoms/cm}^3$$

$$V_0 = 0.026\ln\left(\frac{6.25\times10^{15}\times4.8\times10^{15}}{2.25\times10^{20}}\right) = 0.667 \text{ V}$$

Q.5 (b) Solution:

Given : 28 slots, 2 pole, 16 turns/coil

Length, $l = 20$ cm

Radius, $r = \frac{l}{2} = 10$ cm

As the winding is double layered, therefore,

$$\text{Number of coils} = \text{Number of slots}$$

$$\text{Total number of turns} = 28 \times 16 = 448 \text{ turns}$$

Total number of conductors, $Z = 2 \times 448 = 896$

Effective area per pole :

$$A = \frac{\pi DL}{P} \times \frac{\text{Pole arc}}{\text{Pole pitch}} = \frac{\pi\times0.2\times0.2}{2}\times0.75$$

$$A = 0.047 \text{ m}^2$$

$$\text{Average flux/pole} = B_{avg} \times A$$

$$\phi_{avg} = 1.08 \times 0.047 = 0.0509 \text{ Wb/pole}$$

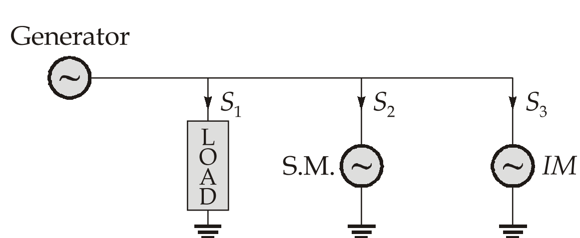
Now, induced emf in machine is given as

$$E = \frac{ZNP\phi}{60\ A} \qquad [\because \text{ for lap winding, } A = P]$$

$$= \frac{896\times1750\times0.0509}{60}$$

$$E = 1330.247 \text{ Volts}$$

Q.5 (c) Solution:



Lighting and heating load, $S_1 = P_1 + jQ_1 = (100 + j0)$

$= 100$ kVA

Induction motor load, $P_{IM} = 1000 \times 746 = 746$ kW

$$S_{IM} = \frac{746\angle\phi}{0.7} = 1065.71\angle\cos^{-1}(0.7)$$

$$= 1065.71\angle 45.6° \text{ kVA}$$

Input to induction motor, $S_3 = \frac{S_{IM}}{\eta} = \frac{1065.71\angle 45.6°}{0.85} = 1253.78\angle 45.6°$ kVA

Synchronous motor load, $P_2 = \frac{300\times 746}{0.92} = 243.26$ kW

$$\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$$

$$|S|\angle\cos^{-1} 0.95 = 100 + 243.26 + jQ_2 + 877.22 + j895.79$$

$$|S|\angle 18.2° = 1220.48 + j(Q_2 + 895.79)$$

$$|S| \times 0.95 = 1220.48$$

$$|S| = 1284.71$$

$$|S| \sin\phi = Q_2 + 895.79$$

$$1284.71 \sin(18.2) = Q_2 + 895.79$$

$$Q_2 = -494.52 \text{ kVAR}$$

This much kVAR is delivered by synchronous motor

$$S_2 = 243.26 + j494.52$$

$$= 551.81\angle 63.81° \text{ kVA}$$

and power factor $= \cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right)$

$$= \cos\left(\tan^{-1}\left(\frac{494.52}{243.26}\right)\right) = 0.442 \text{ leading}$$

Q.5 (d) Solution:

For active and reactive power, we need generated emf,

$$E' = \sqrt{(V_t\cos\phi + I_aR_a)^2 + (V_t\sin\phi + I_aX_q)^2}$$

$$= \sqrt{(440\times 0.85)^2 + (231.784 + \frac{1000}{\sqrt{3}}\times 0.075)^2}$$

$$E' = 464.271 \text{ V}$$

$$\tan\psi = \frac{V_t \sin\phi + I_a X_q}{V_t \cos\phi + I_a R_a} = 0.7355$$

$$\psi = 36.335°$$

As we know,

$$\psi = \delta + \phi$$

$$\delta = 36.335° - 31.78 = 4.555$$

Now, $$I_d = I_a \sin\psi = \frac{1000}{\sqrt{3}} \times \sin(36.335°)$$

$$I_d = 342.083 \text{ A}$$

Now, generated emf

$$|E_f| = E' + I_d(X_d - X_q)$$

$$= 464.271 + 342.083(0.025) = 472.823 \text{ V}$$

$$\vec{E}_f = 472.823 \text{ V}$$

Now, active developed power in generator,

$$P_{\text{dev}} = \left[\frac{E_f V_t}{X_d}\sin\delta + \frac{V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta\right] \times 3$$

$$= \left[\frac{472.823 \times 440}{0.1}\sin(4.555°) + \frac{440^2}{2}\left(\frac{1}{0.075} - \frac{1}{0.1}\right) \times \sin(2 \times 4.555°)\right] \times 3$$

$$P_{\text{dev}} = 648.92 \text{ kW}$$

$$Q_{\text{dev}} = \left[\frac{VE_f}{X_d}\cos\delta - \frac{V^2}{2X_d X_q}[(X_d + X_q) - (X_d - X_q)\cos 2\delta]\right] \times 3$$

$$Q_{\text{dev}} = \left[\frac{472.823 \times 440}{0.1}\cos 4.555 - \frac{440^2}{2 \times 0.1 \times 0.075}[0.175 - 0.025\cos(2 \times 4.555)]\right] \times 3$$

$$Q_{\text{dev}} = 401.34 \text{ kVAr}$$

Q.5 (e) Solution:

(i) In body centered cubic structure, the atoms touch each other along the diagonal of the cube as shown in figure. Therefore, the length of the diagonal of cube is $4r$.

$$AC = \sqrt{AB^2 + BC^2}$$

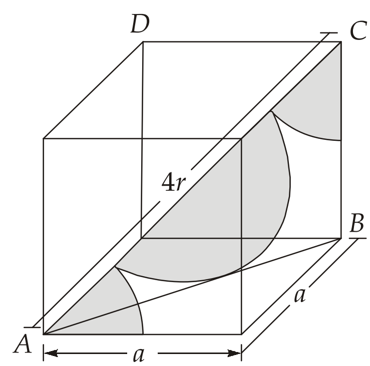
$$AB = \sqrt{x^2 + x^2} = x\sqrt{2}$$

$$BC = x \qquad \text{(Let } a = x\text{)}$$

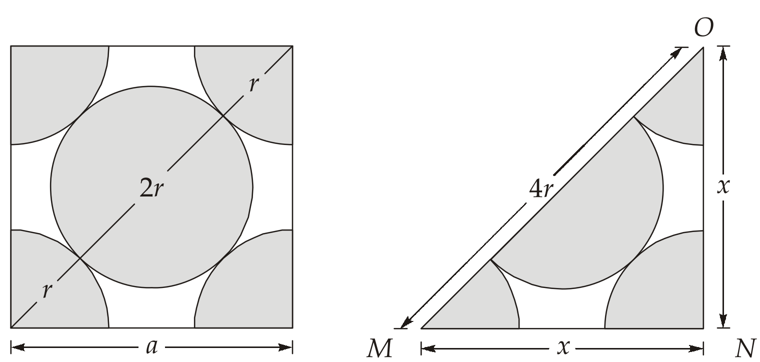
$$AC = 4r = \sqrt{(x\sqrt{2})^2 + x^2} = x\sqrt{3}$$

$$r = \frac{x\sqrt{3}}{4}$$

$$x = \frac{4r}{\sqrt{3}}$$



In face centered cubic structure, the atoms touch each other along the diagonal of any face of the cube as shown in figure. The length of the diagonal of the face = 4r as shown in figure below,



(a) (b)

$$MO^2 = MN^2 + ON^2$$

$$(4r)^2 = x^2 + x^2 = 2x^2$$

or, $$x = \frac{4r}{\sqrt{2}}$$

or, $$r = \frac{x\sqrt{2}}{4}$$

or, $$r^2 = \frac{2x^2}{16} = \frac{x^2}{8}$$

$$r = \frac{x}{2\sqrt{2}}$$

So, $$x = 2\sqrt{2}r$$

(ii) Imperfections result due to deviation from an orderly periodic array. The study of defect is especially useful when studying the properties of materials which are structure sensitive.

This effects the properties like fracture strength plasticity, electrical resistivity and thermal conduction.

Point Defects:

A point defect is a very localized defect in the regularity of a lattice and usually does not spread over more than one or two lattice spacing. The structural point defect can be classified into three types, i.e., vacancies, interstitial, and impurities.

- Vacancies defect: In vacancies defect, there may be missing atom in a crystal. A vacancy may result because of imperfect packing during original crystallization or may arise from thermal vibrations of atoms at elevated temperature.
- Interstitial defect: In interstitial defect, an atom occupies a definite position in the lattice such that it is normally not occupied in a perfect crystal. The position occupied by a normal atom or a foreign atom lies between the atoms of the ideal crystal.
- Impurity defect: In impurity defect, foreign atoms are introduced into a crystal lattice either as an interstitial atom or as a substitution atom. Therefore defects may be classified as interstitial impurity defect and substitution impurity defect.

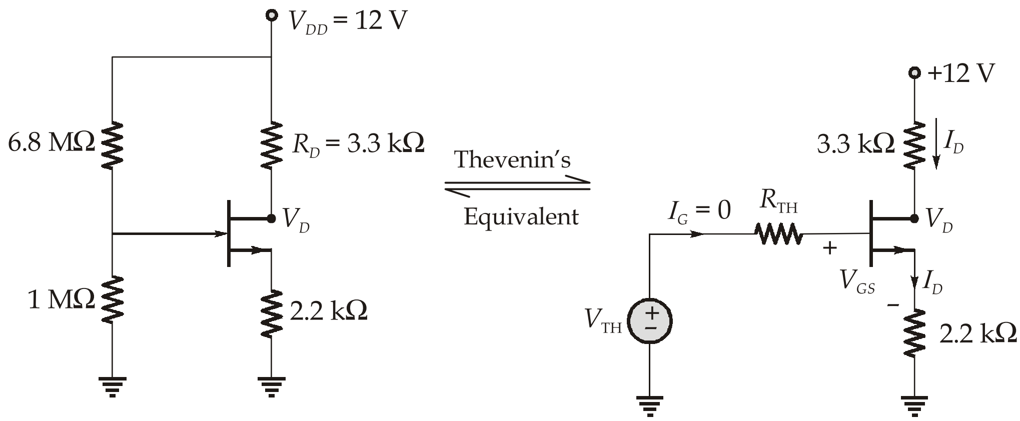
Line Defects:

Line defects are basically one-dimensional defects or dislocations. Due to dislocations, lattice distortions are centered around a line. Dislocations are produced due to solidification of the crystalline solids by vacancy condensation or by atomic mismatch in the solid solutions. Dislocations are mainly of two types i.e., the edge dislocation and the screw dislocation.

- Edge dislocation: In edge dislocation an extra half plane of atoms is inserted in a crystal either above or below the slip plane. Edge dislocation may be either positive or negative.
- Screw dislocation: The screw dislocation can be created in a perfect crystal by applying upward or downward shear stresses to the regions of perfect crystal, which have been separated by a cutting plane. Screw dislocation may be either positive or negative.

Q.6 (a) (i) Solution:

Given JFET circuit, $V_D = 7\text{ V}$ (Given)



where, $R_{TH} = 1\text{ M}\Omega || 6.8\text{ M}\Omega = 0.8718\text{ M}\Omega$

$$V_{TH} = 12 \times \left(\frac{1\text{ M}\Omega}{1\text{ M}\Omega + 6.8\text{ M}\Omega}\right) = 1.538\text{ V}$$

Now, $$I_D = \left(\frac{12 - V_D}{3.3\text{ k}\Omega}\right) = 1.515\text{ mA}$$

KVL in input loop:

$$-V_{TH} + V_{GS} + (2.2\text{ k})\, I_D = 0$$

$\Rightarrow$ $$V_{GS} = V_{TH} - 2.2 \times 1.515$$
$$= -1.795\text{ V}$$

Check: $V_s = (2.2\text{k})\, I_D = 3.33\text{ V}$

$\therefore$ $V_{DS} = V_D - V_s = 3.67\text{ V}$

Q.6 (a) (ii) Solution:

Given, Critical temperature $= T_C = 7.2°\text{K}$

Critical Magnetic field for the metal at 0°K $= H_{CO} = 7.8 \times 10^5$ Amp/m

$T = 5°\text{K}$

We know that, $$H_{CT} = H_{CO}\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

Where, H_{CT} = Critical magnetic field for the metal to be usable as superconductor at 5° K

So, $$H_{CT} = 7.8 \times 10^5 \left[1 - \left(\frac{5}{7.2}\right)^2\right] = 7.8 \times 10^5 \left[1 - \frac{25}{51.84}\right]$$

$$H_{CT} = 7.8\times10^5\left[\frac{51.84-25}{51.84}\right]$$

$$= 7.8\times10^5\times\frac{26.84}{51.84} = 4.038 \times 10^5 \text{ Amp/m}$$

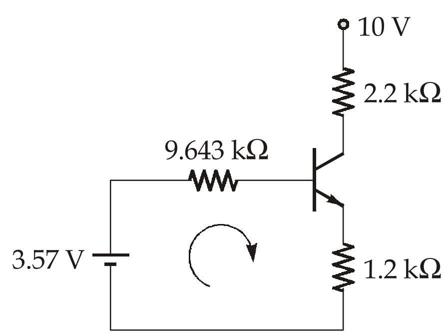
Q.6 (b) Solution:

For given amplifier network:

$$V_{th} = \frac{R_2}{R_1+R_2}\times V_{CC} = \frac{15}{15+27}\times10 = 3.57 \text{ V}$$

$$R_{th} = (R_1 || R_2) = \frac{15\times27}{15+27} = 9.643 \text{ k}\Omega$$

So, we redraw transistor configuration,



By KVL in above shown loop,

$$-3.57 + 9.643\,kI_B + 0.7 + 1.2\,kI_E = 0$$

$$-3.57 + 9.643\,kI_B + 0.7 + 1.2\,k(1+\beta)\,I_B = 0$$

$$-2.87 + 9.643\,kI_B + 1.2\,kI_B + 1.2\,k\beta I_B = 0$$

$$130.843\,kI_B = 2.87$$

$$I_B = 21.934\ \mu\text{A}$$

$$I_E = (1+\beta)\,I_B$$

$$= 101 \times 21.934\ \mu\text{A} = 2.215 \text{ mA}$$

$$I_E = I_B + I_C$$

So, $I_C = I_E - I_B = 2.193 \text{ mA}$

So for hybrid-π model,

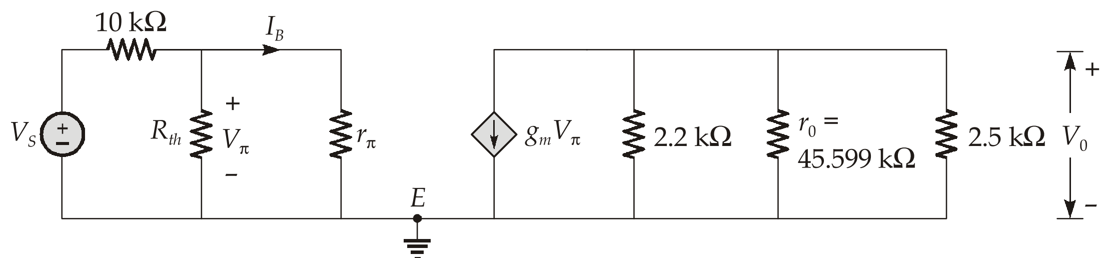
$$r_0 = \frac{V_A}{I_C} = \frac{100}{2.193 \text{ mA}} = 45.599 \text{ k}\Omega$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{2.215 \text{ mA}} = 11.286\ \Omega$$

$$r_\pi = (1+\beta)\, r_e = 101 \times 11.286\ \Omega$$

$$= 1.139\ \text{k}\Omega$$

Drawing hybrid-π model,



$$g_m = \frac{I_C}{V_T} = \frac{2.193}{25} = 0.0877 \text{ A/V}$$

$\therefore$
$$V_0 = -g_m V_\pi (2.2\,k \,||\, 45.599\,k \,||\, 2.5\,k)$$

$$= -g_m V_\pi\,(2.2\,k \,||\, 2.37\,k)$$

$$= -g_m V_\pi (1.1409\,k)$$

$$= -(0.0877) \times 1.1409 \times 10^2\ V_\pi$$

$$= -100.08\ V_\pi$$

$$V_0 = -(100.08) \times \left(\frac{R_i}{R_i + R_s}\right) V_s$$

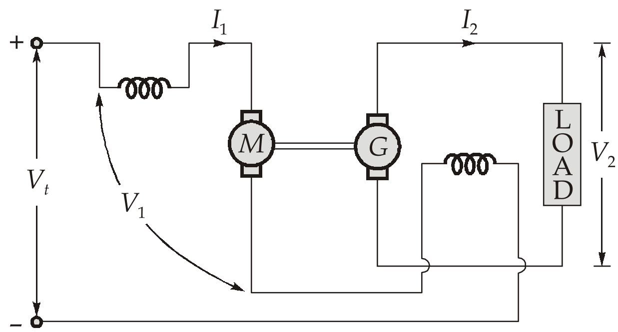
$$R_i = R_{th} \,||\, r_\pi$$

$$= 9.643\,k \,||\, 1.139\,k = 1.018\ \text{k}\Omega$$

$$V_0 = -100.08 \times \left(\frac{1.018\,k}{1.018\,k + 10\,k}\right) \cdot V_s$$

$$\frac{V_0}{V_s} = -9.246$$

Q.6 (c) (i) Solution:



$$\text{Power input to whole set} = V_t I_1$$
$$= (400 + 25 + 25) \times 50$$
$$= 450 \times 50 = 22.5 \text{ kW}$$

$$\text{Total losses in the whole set, } W = V_t I_1 - V_2 I_2$$
$$= 22.5 - 350 \times 40 \times 10^{-3}$$
$$W = 8.5 \text{ kW}$$

$$\text{No load rotational losses} = W - \text{ohmic losses}$$
$$W_0 = 8500 - 50^2 \times 0.5 - 25 \times 50 - 25 \times 50 - 40^2 \times 0.5$$
$$W_0 = 3950 \text{ W}$$

$$\text{Motor efficiency, } \eta_m = 1 - \frac{\frac{W_0}{2} + I_1^2 r_{am} + V_f I_L}{V_1 I_1}$$

$$\eta_m = \left[1 - \frac{\left(\frac{3950}{2} + 50^2 \times 0.5 + 25 \times 50\right)}{425 \times 50}\right] \times 100 = 78.94\%$$

Generator efficiency,

$$P_{\text{losses}} = 40^2 \times 0.5 + 25 \times 50 + 3950 \times 0.5$$
$$= 4025 \text{ W}$$
$$P_{\text{in}} = 350 \times 40 + 4025 = 18025 \text{ W}$$
$$\eta_G = \left(1 - \frac{4025}{18025}\right) \times 100 = 77.67\%$$

Q.6 (c) (ii) Solution:

$$\text{Rated current of the motor} = \frac{20.100}{088 \times 220} = 103.3\,\text{A}$$

$$\text{Rated torque of the motor} = \frac{20.100 \times 60}{2\pi \times 1200} = 159.15\,\text{A}$$

1. E of the motor at rated current $= 220 - 103.3 \times 0.1$

$= 209.67$ V

Voltage across armature circuit as soon as braking starts

$= 220 + 209.67 = 429.67$ V

Braking current at the beginning of braking

$= 2.103 \times 3$ A

Hence, resistance in the armature circuit

$$= \frac{429.67}{206.6} = 2.08\ \Omega$$

Therefore, extra resistance to be added to the motor armature

$= 2.08 - 0.1$

$= 1.98\ \Omega$

2. Being a shunt motor, the flux per pole will be constant. Hence, torque $\propto$ current. Therefore, initial braking torque will be twice the full load torque,

i.e, $= 2.159 \times 15 = 318.30$ N-m

3.
$$E \text{ at 400 rpm} = \frac{209.67}{1200} \times 400 = 69.89\ \text{V}$$

Hence, braking current at 400 rpm

$$= \frac{220 + 69.89}{2.08} = 139.37\ \text{A}$$

$$\text{Braking torque} = \frac{159.15}{103.3} \times 139.37 = 214.72\ \text{N-m}$$

Q.7 (a) Solution:

(i) According to given condition for synchronous motor.

Reluctance power will be maximum at $\delta = 45°$

$$\frac{540 \times 10^3}{3} = \frac{V_t^2}{2 \times 25.4 \times 15.4}(25.4 - 15.4)$$

$$V_t = 3752.567 \text{ V/phase}$$

Power, $$P = T \times \omega_s$$

$$= 3.5\times10^3 \times \frac{2\pi\times1000}{60}$$

$$= 366.519 \text{ kW}$$

Now, $$P = \sqrt{3}VI\cos\phi$$

$$366.519 \times 10^3 = \sqrt{3}\times3752.567\times I_a\times0.8$$

$$I_a = \frac{366.519\times10^3}{3\times3752.567\times0.8} = 40.696 \text{ A}$$

Also, $$\tan\Psi = \frac{V_t\sin\phi - I_aX_q}{V_t\cos\phi - I_aR_a}$$

$$= \frac{-3752.567\times0.6 - 40.696\times15.4}{3752.567\times0.8}$$

$$= \frac{-2251.540 - 626.718}{3002.053} = -0.9587$$

$$\Psi = -43.79°$$

$$\delta = \phi - \Psi$$

$$= 43.79 - 36.86 = 6.93° \quad [\because \text{ p.f. is leading}]$$

$$E = V\cos\delta - I_dX_d - I_qR_a$$

As $R_a = 0$, $$I_d = I_a\sin\Psi$$

$$= 40.696\sin(-43.79°) = -28.162 \text{ A}$$

$$E = 3752.567 \times \cos(6.93°) - (-28.162)(25.4)$$

$$= 4440.41 \text{ V (per phase) or 7691 V (line)}$$

(ii) Power-angle Characteristic:

Fig. (a) shows the one-line diagram of a salient pole synchronous machine connected to infinite bus-bars of voltage V_b through a line of series reactance $X_{ext.}$ (per phase).

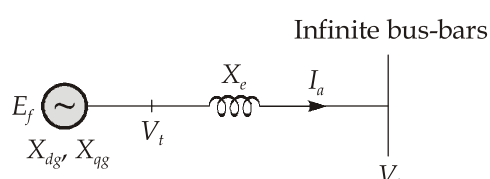


Fig. (a)

The total d- and q-axis reactance are then,

$$X_d = X_{dg} + X_e$$

$$X_q = X_{qg} + X_e$$

The resistance of the machine armature and line are assumed negligible.

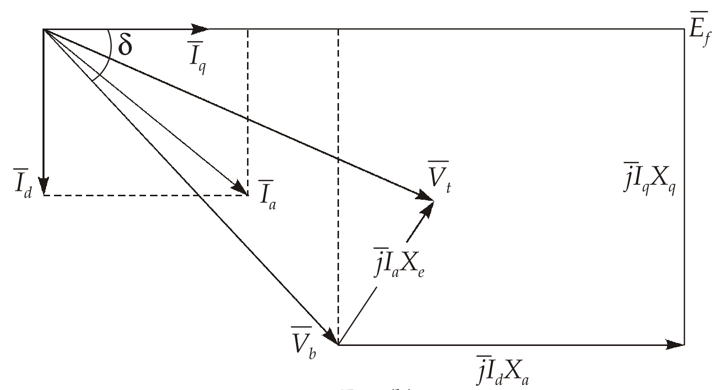


Fig. (b)

Fig. (b) gives the phasor diagram when the machine is generating. It is easy to see from this figure that the real power delivered to the bus-bars is,

$$P_e = I_d V_b \sin\delta + I_q V_b \cos\delta \quad \text{...(i)}$$

Now, $$I_d = \frac{E_f - V_b \cos\delta}{X_d} \quad \text{...(ii)}$$

and $$I_q = \frac{V_b \sin\delta}{X_q} \quad \text{...(iii)}$$

Substituting in equation (i),

$$P_e = \left[\frac{E_f V_b}{X_d}\sin\delta + V_b^2 \underbrace{\frac{X_d - X_q}{2X_d X_q}}_{\text{Reluctance power}} \sin 2\delta\right] \quad \text{...(iv)}$$

Equation (iv) gives the expression for the electrical power output of a salient pole generator. The same expression will give the electrical power input of a motoring machine wherein E_f lags V_b by angle δ.

The second term in equation (iv) compared to equation of a cylindrical motor arises on account of saliency and is known as the reluctance power (torque). The reluctance power varies as sin2δ with a maximum value at δ = 45°.

The power angle plots of both the terms of equation (iii) along with the form of the resultant power angle curve are shown in Fig. (c).

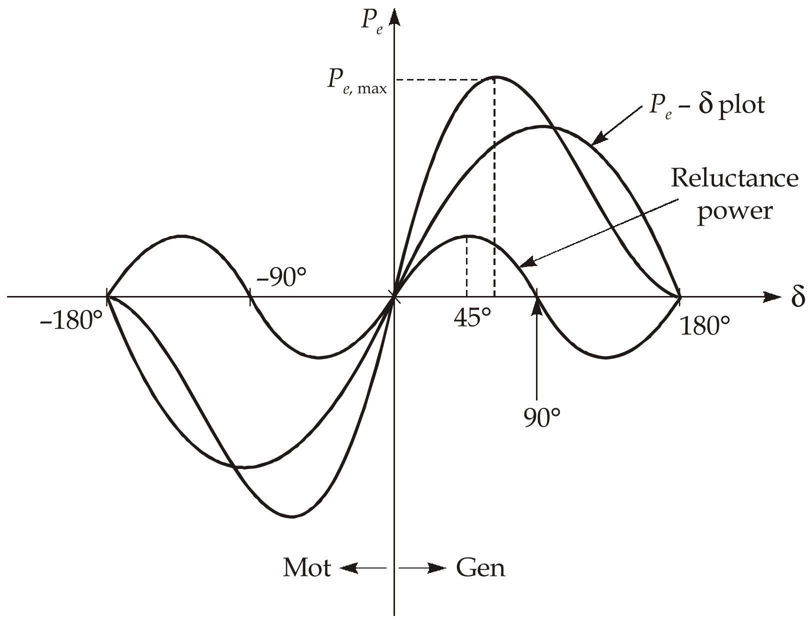
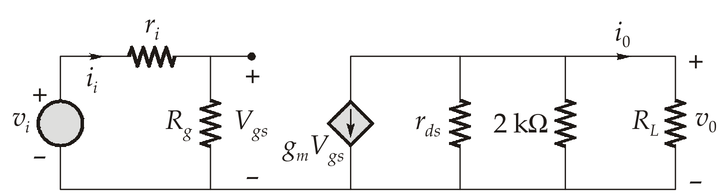


Fig. (c)

(Power-angle characteristic of salient pole synchronous machine)

Q.7 (b) Solution:

AC equivalent circuit is drawn in figure below,



In the above circuit,
$$R_g = \frac{800k \times 200k}{800k + 200k} = 160 \text{ k}\Omega$$

$$Z_{in} = \frac{v_i}{i_i} = r_i + R_g$$

$$Z_{in} = 165 \text{ k}\Omega$$

From above circuit,

$$v_0 = -g_m V_{gs}\left(r_{ds} \parallel 2k \parallel 2k\right)$$

$$V_{gs} = \frac{v_i \times R_g}{R_g + r_i} = \frac{v_i \times 160k}{160k + 5k} = \frac{v_i \times 160}{165}$$

So we have,

$$v_0 = \frac{-v_i \times 160}{165} \times g_m \times (r_{ds} \parallel 2k \parallel 2k)$$

$$\frac{v_0}{v_i} = \frac{-160}{165} \times 2 \times 10^{-3} \times (30k \parallel 2k \parallel 2k)$$

$$A_V = \frac{v_0}{v_i} = -1.8768$$

$$i_0 = \frac{v_0}{R_L}$$

and
$$i_1 = \frac{v_i}{R_g + r_i}$$

$$A_I = \frac{i_0}{i_1} = \frac{v_0 / R_L}{v_i / (R_g + r_i)}$$

$$A_I = A_V \times \frac{R_g + r_i}{R_L}$$

$$A_I = -1.8768 \times \frac{160k + 5k}{2k}$$

$$A_I = -1.8768 \times \frac{165}{2}$$

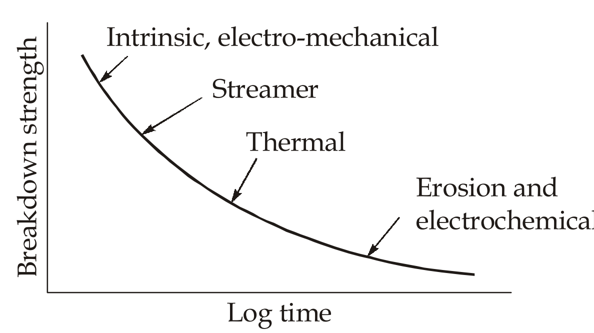
$$A_I = -154.836$$

Q.7 (c) Solution:

Dielectric strength is defined as the maximum voltage required to produce a dielectric breakdown through the material and is expressed as volts per unit thickness. The higher the dielectric strength of a material the better its quality as an insulator.

In the case of solids, mechanism of breakdown is a complex phenomenon , and varies depending on the time of application of voltage as shown in the figure. The various breakdown mechanisms can be classified as follows:

(a) Intrinsic or ionic breakdown
(b) Electromechanical breakdown
(c) Thermal breakdown
(d) Electrochemical breakdown
(e) Treeing and tracking
(f) Internal discharges



(a) Intrinsic Breakdown:

- When voltage is applied only for short durations of the order of 10^{-8} sec, the dielectric strength of a solid dielectric increases very rapidly to an upper limit called the intrinsic electric strength.
- Maximum strength usually obtainable ranges from 5-10 MV/cm.
- Intrinsic breakdown depends upon the presence of free electrons which are capable of migration through the lattice of the dielectric. Usually, small numbers of conduction electrons are present, along with some structural imperfections and small amounts of impurities. The impurity atoms or molecules act as traps for the conduction electrons up to certain ranges of electric fields and temperatures. When these ranges are exceeded, additional electrons in addition to trapped electrons are released and participate in the conduction process.
- Based on this principle, there are two types of intrinsic breakdown mechanisms:

(i) Electronic breakdown:

- Intrinsic breakdown occurs in time of the order of 10^{-8} s and therefore, is assumed to be electronic in nature.
- Initial density of conduction (free) electrons is assumed to be large and electron-electron collisions occurs.
- When electric field is applied, electrons gain energy and cross the forbidden gap from the valence band to the conduction band. When this process is repeated, more and more electrons become available in conduction band, eventually leading to breakdown.

(ii) Avalanche or streamer breakdown:

- This is similar to breakdown in gases due to cumulative ionization.
- Conduction electrons gain sufficient energy above a certain critical electric field and cause liberation of electrons from the lattice atoms by collision.
- The electron within dielectric moving from cathode to anode will gain energy from the field and loses it during collisions. When the energy gained by an electron exceeds the lattice ionization potential, an additional electron will be liberated due to collision of the first electron. This process repeats itself resulting in the formation of an electron avalanche, and breakdown will occur when the avalanche exceeds a certain critical size.
- In practice, breakdown does not occur by the formation of a single avalanche, but occurs as a result of many avalanches formed and extending step by step through the entire thickness of the material.

(b) Breakdown due to Treeing and Tracking

- When a solid dielectric subjected to electrical stresses for a long time fails, two kinds of visible marking are observed.

a. Presence of conducting path across the surface of the insulation.

b. Mechanism whereby leakage current passes through the conducting path, finally leading to the formation of spark. Insulation deterioration occurs as a result of these sparks.

- Spreading of spark channels during tracking, in the form of the branches of a tree is called treeing.
- Consider a system of a solid dielectric having a conducting film and two electrodes on its surface. In practice, the conducting film very often is formed due to moisture. On application of voltage, the film starts conducting, resulting in generation of heat, and the surface starts becoming dry. The conducting film becomes separate due to drying, and so sparks are drawn damaging the dielectric surface. With organic insulating material, the dielectric carbonizes at the region of sparking, and carbonized regions act as permanent conducting channels resulting in increased stress over the rest of the region. This is a cumulative process, and insulation failure occurs when carbonized tracks bridge the distance between the electrodes. This phenomenon is called tracking.
- Treeing occurs due to the erosion of material at the tips of the spark and results in the roughening of the surface and hence, becomes a source of dirt and contamination. This causes increased conductivity resulting either in the formation of conducting path bridging the electrodes or in a mechanical failure of the dielectric.
- Usually tracking occurs even at very low voltages, whereas treeing requires high voltage.

(c) Breakdown due to Internal Discharges:

- Solid insulating materials contain voids or cavities within the medium or at the boundaries between the dielectric and the electrodes. These voids are generally filled with a medium of lower dielectric strength, and the dielectric constant of the medium in the voids is lower than that of the insulation.
- The electric field strength in the voids is higher than that across the dielectric. Therefore, even under normal working voltages the field in the voids may exceed their breakdown value, and breakdown may occur.

(d) Electromechanical Breakdown:

- When solid dielectrics are subjected to high electric fields, failure occurs due to electrostatic compressive forces, which can exceed the mechanical compressive strength.
- If the thickness of specimen is d_0 and is compressed to a thickness d under an applied voltage V, the highest apparent electric stress before breakdown is:

$$E_{max} = \frac{V}{d_0} = 0.5\sqrt{\left(\frac{Y}{\varepsilon_0 \varepsilon_r}\right)}$$

where, Y - Young's modulus

- Mechanical instability occurs when $d/d_0 = 0.6$
- The above equation is only approximate as Y depends on the mechanical stress. Also when the material is subjected to high stresses, the theory of elasticity does not hold good and plastic deformation has to be considered.

(e) Thermal Breakdown:

- When an electric field is applied to a dielectric, conduction current flows through the material. Current heats up the specimen and the temperature rises. Equilibrium is reached when the heat used to raise the temperature of the dielectric, plus the heat radiated out, equals the heat generated, causing breakdown.
- Thermal breakdown sets-up an upper limit for increasing the breakdown voltage when the thickness of insulation is increased.
- Heat generated is proportional to the frequency and hence thermal breakdown is more serious at high frequency.
- Thermal breakdown stresses (MV/cm) are lower under a.c. condition than under d.c.

(f) Chemical and Electrochemical Deterioration and Breakdown:

- In presence of air and other gases, dielectric materials undergo chemical changes when subjected to continuous electrical stresses. Some of the reactions that occur are:

  Oxidation: In the presence of air or oxygen, materials such as rubber and polyethylene undergo oxidation giving rise to surface cracks.

  Hydrolysis: When moisture or water vapour is present on the surface of the solid dielectric, hydrolysis occurs and the materials lose their electrical and mechanical properties. Electrical properties of materials such as paper, cotton tape and other cellulose materials deteriorate very rapidly due to hydrolysis.

  Chemical Action: Progressive chemical degradation can occur due to a variety of processes such as chemical instability at high temperature, oxidation, cracking in the presence of air and hydrolysis due to moisture and heat.
- Chemical and electrochemical deterioration increases very rapidly with temperature.

Q.8 (a) Solution:

(i) The rated phase voltage of this generator is $\frac{14.4\,\text{kV}}{\sqrt{3}} = 8313$ V. The base impedance of this generator is

$$Z_{base} = \frac{3V_{\phi,base}^2}{S_{base}} = \frac{3(8313\,\text{V})^2}{10000000\,\text{VA}} = 2.07\,\Omega$$

Therefore, $R_A \approx 0\ \Omega$ (negligible)

$$X_S = (1.0)(2.07\ \Omega) = 2.07\ \Omega$$

(ii) The rated armature current is

$$I_A = I_L = \frac{S}{\sqrt{3}V_T} = \frac{100\ \text{MVA}}{\sqrt{3}(14.4\,\text{kV})} = 4009\ \text{A}$$

The power factor is 0.8 lagging,

So, $I_A = 4009\angle{-36.87°}$ A

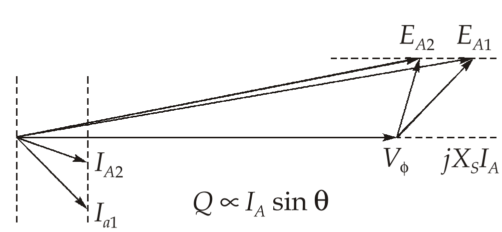
Therefore, the internal generated voltage is

$$E_A = V_\phi + R_A I_A + jX_S I_A$$
$$= 8313\angle 0° + j(2.07\ \Omega)(4009\angle{-36.87°}\ \text{A})$$
$$E_A = 14858\angle 26.54°\ \text{V}$$

(iii) From the above calculations,

$$I_A = 4009\angle{-36.87°}\ \text{A}$$

(iv) If E_A is decreased by 5%, the armature current will change as shown below, note that the infinite bus will keep V_ϕ and ω_m constant. Also, since the prime mover hasn't changed, the power supplied by the generator will be constant



$$P = \frac{3V_\phi E_A}{X_s}\sin\delta = \text{constant},$$

So, $E_{A1}\sin\delta_1 = E_{A2}\sin\delta_2$

with a 5% decrease, $E_{A2} = 14115$ V

and
$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right) = \sin^{-1}\left(\frac{14858\text{ V}}{14115\text{ V}}\sin 26.54°\right)$$
$$= 28.0°$$

Therefore, the new armature current is
$$I_A = \frac{E_{A2} - V_\phi}{jX_s} = \frac{14115\angle 28.0° - 8313\angle 0°}{j2.07}$$
$$= 3777\angle{-32.1°}\text{ A}$$

(v) Repeating part (iv):

With a 10% decrease, $E_{A2} = 13372$ V

and
$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right)$$
$$= \sin^{-1}\left(\frac{14858\text{ V}}{13372\text{ V}}\sin 26.54°\right) = 29.8°$$

Therefore, the new armature current is
$$I_A = \frac{E_{A2} - V_\phi}{jX_s} = \frac{13372\angle 29.8° - 8313\angle 0°}{j2.07}$$
$$= 3582\angle{-26.3°}\text{ A}$$

With a 15% decrease, $E_{A2} = 12629$ V

and
$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right)$$
$$= \sin^{-1}\left(\frac{14858\text{ V}}{12629\text{ V}}\sin 26.54°\right) = 31.7°$$

Therefore, the new armature current is
$$I_A = \frac{E_{A2} - V_\phi}{jX_s} = \frac{12629\angle 31.7° - 8313\angle 0°}{j2.07}$$
$$= 3414\angle{-20.1°}\text{ A}$$

With a 20% decrease, $E_{A2} = 11886$ V

and
$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right)$$

$$= \sin^{-1}\left(\frac{14858\text{ V}}{11886\text{ V}}\sin 26.54°\right) = 34.0°$$

Therefore, the new armature current is

$$I_A = \frac{E_{A2} - V_\phi}{jX_s} = \frac{11886\angle 34.0° - 8313\angle 0°}{j2.07}$$

$$= 3296\angle{-13.1°}\text{ A}$$

With a 25% decrease, $E_{A2} = 11144\text{ V}$

and $\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right)$

$$= \sin^{-1}\left(\frac{14858\text{ V}}{11144\text{ V}}\sin 26.54°\right) = 36.6°$$

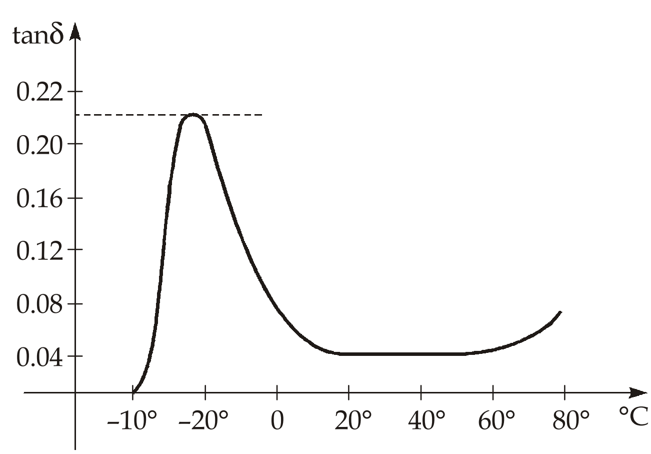
Therefore, the new armature current is

$$I_A = \frac{E_{A2} - V_\phi}{jX_s} = \frac{11144\angle 36.6° - 8313\angle 0°}{j2.07}$$

$$= 3224\angle{-5.4°}\text{ A}$$

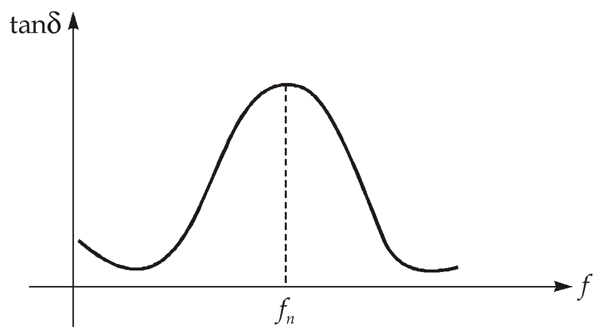
Q.8 (b) (i) Solution:

The dielectric loss of polar dielectric consists of two components. Those due to leakage current and those resulting from dipole polarization. The dependence of tanδ on temperature is shown below:



At low temperature, the loss due to dipole polarization is greater than that due to the leakage current. At temperature much below 0°C due to less thermal motion orientation of dipoles is limited. With increase in temperature the dipoles acquire greater mobility hence it increase in temperature causes the loss tangent to drop off due to enhanced thermal agitation. After falling to a minimum, tanδ begins to increase, this time due to an increase in the leakage current.

The loss tangents of polar dielectrics of polar dielectrics depends on frequency. The variation of loss tangent tanδ with frequency at a constant temperature is shown below.



At low frequency the dipoles make less number of rotations per second resulting in a small amount of power loss. As the frequency is increased beyond a certain limit the dipolar polarization ceases because the molecules will not be able to keep up with increased rate of field reversal. At same mid frequency depending on the temperature the loss tangent will be maximum as shown in above figure.

At zero frequency the loss is due to leakage current only and hence tanδ is minimum. At infinite frequency the losses due to both polarization and leakage current become zero.

Q.8 (b) (ii) Solution:

Given,

Magnetic moment per atom $= 7.1\ \mu_B$

Magnetic moment per gram $= \text{N}\ 7.1\mu B$

Where, N = number of atoms of Gd in one gram

157.3 gm of Gd contains 6.023×10^{23} atoms

1 gm of Gd contains,

$$N = \frac{6.023 \times 10^{23} \times 1}{157.3}$$

$$= 3.836 \times 10^{21}$$

Magnetic moment per gm, $N = 3.836 \times 10^{21} \times 7.1\ \mu_B$

$$= 3.836 \times 10^{21} \times 7.1 \times 9.27 \times 10^{-24}$$

$$= 0.253\ \text{Am}^2$$

Q.8 (b) (iii) Solution:

The paramagnetic susceptibility is given by

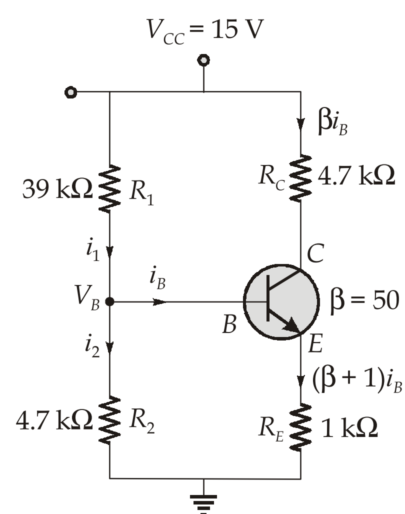
$$\chi = \frac{N\mu_0\mu_r\mu_B^2}{3kT}$$

For a pasramagnet, $\mu_r = 1$

$$\chi = \frac{1.2\times10^{28}\times4\pi\times10^{-7}\times9.28^2\times10^{-48}}{3\times1.38\times10^{-23}\times300}$$

$$= 1.05 \times 10^{-5}$$

Q.8 (c) Solution:



Let us assume that the BJT is working in active mode.

$$i_1 = i_B + i_2$$

$$i_1 = i_B + \frac{V_B}{R_2}$$

$$V_B = V_{BE} + (1+\beta)i_B R_E$$

$$i_1 = i_B + \frac{1}{R_2}[0.6 + (\beta+1)i_B R_E]$$

$\Rightarrow$ $$i_1 = i_B + \frac{1}{4.7\,K}[0.6 + 51\,K\,i_B]$$

$\Rightarrow$ $$i_1 = \left(i_B + \frac{51}{4.7}i_B\right) + \frac{0.6}{4.7\,K}$$

$\Rightarrow$ $$i_1 = 11.85\,i_B + 0.13\text{ mA} \qquad \text{...(i)}$$

$$V_{CC} = R_1 i_1 + V_B$$

$$15 = 39\text{ K}\,[11.85\, i_B + 0.13\text{ mA}] + 0.6 + 51\text{ K}\, i_B$$

$$15 = 462.15\text{ K}\, i_B + 5.07 + 0.6 + 51\text{ K}\, i_B$$

$$15 = 513.15\text{ K}\, i_B + 5.67$$

$$i_B = 0.0182\text{ mA}$$

So, $$i_C = \beta\, i_B$$

$\Rightarrow$ $$i_C = 50 \times 0.0182 \times 10^{-3}$$

$$i_C = 0.91\text{ mA}$$

$$i_E = (\beta + 1)\, i_B$$

$\Rightarrow$ $$i_E = 0.93\text{ mA}$$

$$V_{CE} = V_{CC} - i_C R_C - i_E R_E$$

$\Rightarrow$ $$V_{CE} = 15 - (0.91)\,(4.7) - 0.93\,(1) = 9.793\text{ V}$$

So, our assumption of active mode is correct.

Operating point, (V_{CE}, i_C) is (9.793 V, 0.91 mA).

$$\text{Stability factor} = \frac{1+\beta}{1-\beta\dfrac{di_B}{di_C}}$$

Now, we have to find a relationship between i_B and i_C which is always valid.

$$V_B = 0.6 + (i_B + i_C)\, R_E$$

$$i_2 = \frac{V_B}{R_2} = \frac{0.6}{R_2} + (i_B + i_C)\frac{R_E}{R_2}$$

$$i_1 = i_2 + i_B = \frac{0.6}{R_2} + i_B\left(\frac{R_E}{R_2} + 1\right) + \frac{i_C R_E}{R_2}$$

$$V_{CC} = V_B + i_1 R_1$$

$$V_{CC} = 0.6 + (i_B + i_C) R_E + 0.6\frac{R_1}{R_2} + i_B\left(\frac{R_E}{R_2} + 1\right) R_1 + i_C\frac{R_E R_1}{R_2}$$

Differentiate it with respect to i_C,

$\Rightarrow$ $$0 = 0 + \left(\frac{di_B}{di_C} + 1\right) R_E + \left(\frac{R_E}{R_2} + 1\right) R_1 \frac{di_B}{di_C} + \frac{R_E R_1}{R_2}$$

$\Rightarrow$ $$0 = \frac{di_B}{di_C}\left[R_E + \frac{R_E R_1}{R_2} + R_1\right] + R_E + \frac{R_E R_1}{R_2}$$

$\Rightarrow$ $$\frac{di_B}{di_C} = -\frac{\left[R_E + \frac{R_E R_1}{R_2}\right]}{\left[R_E + R_1 + \frac{R_E R_1}{R_2}\right]} = -\frac{\left[1 + \frac{39}{4.7}\right]}{\left[1 + 39 + \frac{39}{4.7}\right]} = -0.1925$$

So, $$S = \frac{1+\beta}{1+0.1925\beta} = \frac{51}{1+9.625} = 4.8$$

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