MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

Civil Engineering Test No : 6

Section A : Structural Analysis + CPM PERT

1. (a) Solution:

Resource Levelling in Construction Project Management

Resource levelling is a technique used in project management to adjust the project schedule so that the demand for resources does not exceed their available supply. In this method, activities are rescheduled within their available float to reduce peak demand and achieve a more uniform distribution of resources over time.

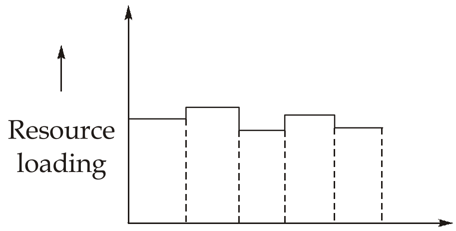
The main objective of resource levelling is to balance resource utilization and avoid situations where resources are either over-allocated or underutilized. It is particularly useful when resources are limited and cannot be increased beyond a certain level. The project duration may increase if resource constraints are strict. Resources are assumed to be limited. The critical path may change after levelling.

Resource levelling attempts to smooth out large fluctuations in resource demand by shifting non-critical activities within their available slack.

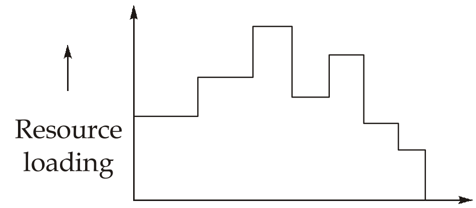
Resource Loading

Resource loading refers to the assignment of resources to various project activities as per the project schedule. It represents the amount of resources required at different time periods and is often displayed graphically.

A resource loading graph shows variation of resource demand over time and helps in planning, monitoring, and budgeting of resources. It does not necessarily attempt to modify the schedule to reduce fluctuations.



Resource loading after levelling



Resource loading graph

Difference between Resource Levelling and Resource Loading

1. Resource loading is the process of allocating resources to activities, whereas resource levelling is the process of adjusting the schedule to minimize fluctuations in resource demand.
2. Resource loading may result in peaks and valleys in resource usage; resource levelling aims to reduce these variations.
3. Resource loading does not change activity timings, but resource levelling may shift activities within their float.
4. Resource levelling may increase project duration, while resource loading does not affect project duration directly.
5. Resource levelling improves resource utilization efficiency under limited availability conditions

1. (b) Solution:

Given data

Total weight of floor, $W = 222411.08$ N

$g = 9.81 \text{ m/s}^2$

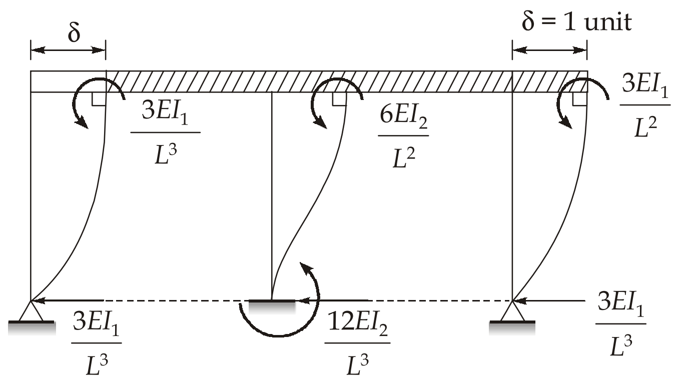
Mass of floor, $m = \frac{W}{g} = \frac{222411.08}{9.81} = 22671.874$ kg

$L = 3.658$ m

$E = 2.1 \times 10^{11} \text{ N/m}^2$

$I_1 = 2235 \text{ cm}^4 = 2.235 \times 10^{-5} \text{ m}^4$

$I_2 = 5131.6 \text{ cm}^4 = 5.1316 \times 10^{-5} \text{ m}^4$



For one outer ISMB 200 column (fixed at top and pinned at base), lateral stiffness is

$$k_{outer} = \frac{3EI_1}{L^3}$$

$$k_{outer} = \frac{3\times2.1\times10^{11}\times2.235\times10^{-5}}{(3.658)^3}$$

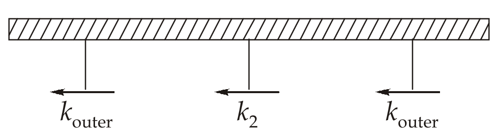
$$\Rightarrow \quad k_{outer} = 287664.966 \text{ N/m}$$

For the central ISMB 250 column (fixed at both ends), lateral stiffness is

$$k_2 = \frac{12EI_2}{L^3}$$

$$\Rightarrow \quad k_2 = \frac{12\times2.1\times10^{11}\times5.1316\times10^{-5}}{(3.658)^3}$$

$$\Rightarrow \quad k_2 = 2641935.641 \text{ N/m}$$



Total stiffness of the system is

$$k = k_2 + 2\,K_{outer}$$

$$\Rightarrow \quad k = 3217265.573 \text{ N/m}$$

Natural frequency is given by

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3217265.573}{22671.874}}$$

$$\omega_n = 11.912 \text{ rad/s}$$

Natural frequency in Hertz is

$$f_n = \frac{w_n}{2\pi} = \frac{11.912}{2 \times 3.14159}$$

$\Rightarrow$ $$f_n = 1.896 \text{ Hz}$$

The natural frequency of horizontal vibration of the platform is 1.896 Hz.

1. (c) Solution:

Given data

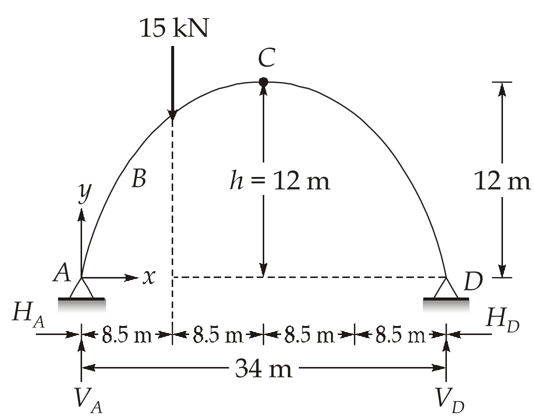
Span of arch, $L = 34$ m

Rise of arch, $h = 12$ m

Point load, $W = 15$ kN

Position of load from left support A, $a = 8.5$ m

Position of load from right support D, $b = 25.5$ m



Vertical Reactions:

Taking moment about 'C' (considering left side)

$$V_A \times 17 - H_A \times 12 - 15 \times 8.5 = 0$$

$\Rightarrow$ $$V_A = \frac{12H_A + 127.5}{17} \quad \text{...(i)}$$

Taking moment about 'C' (considering right side)

$$V_D \times 17 - H_D \times 12 = 0$$

$$V_D = \frac{12H_D}{17} \quad \text{...(ii)}$$

$\Sigma F_x = 0 \quad \Rightarrow \quad H_A = H_D = H$

$\Sigma F_y = 0 \quad \Rightarrow \quad V_A + V_D = 15 \quad \text{...(iii)}$

From equation (i), (ii) and (iii)

$$\frac{12H + 127.5}{17} + \frac{12H}{17} = 15$$

$$\Rightarrow \quad H = 5.3125 \text{ kN}$$

$$\therefore \quad V_A = \frac{12 \times 5.3125 + 127.5}{17} = 11.25 \text{ kN}$$

and $$V_D = \frac{12 \times 5.3125}{17} = 3.75 \text{ kN}$$

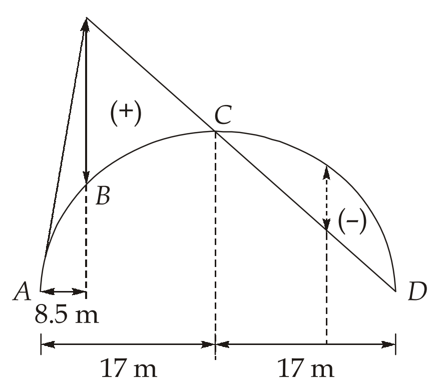
Parabolic Arch Equation:

The vertical distance y from the horizontal line joining the supports to any point at a distance x from support A:

$$y = \frac{4hx(L-x)}{L^2} = \frac{4 \times 12 \times x(34-x)}{34^2} = \frac{48}{1156}(34x - x^2)$$

Maximum Positive Bending Moment:

$BM = BM_{\text{beam}} - H \times y$: on this basis of this BMD may be drawn as follows



Max + ve BM Occurs directly under the point load at $x = 8.5$ m. Calculate y at this location:

$$y = \frac{48}{1156}(34 \times 8.5 - 8.5^2) = \frac{48}{1156}(289 \times 72.25) = 9 \text{ m}$$

Bending moment under the load:

$$M_B = V_A \times x - H \times y$$

$$\Rightarrow \quad M_B = 11.25 \times 8.5 - 5.3125 \times 9$$

$$\Rightarrow \quad M_B = 95.625 - 47.8125$$

$$\Rightarrow \quad M_{\text{max positive}} = 47.813 \text{ kNm}$$

1. (d) Solution:

Given data

Uniformly distributed load, $w = 5$ kN/m

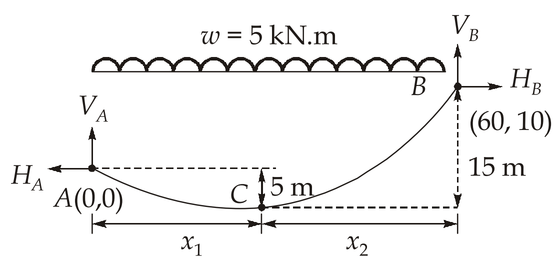
Horizontal span, $L = 60$ m

Vertical distance from support A to the lowest point, $h_1 = 5$ m

Vertical distance from support B to the lowest point, $h_2 = 15$ m (since $h_2 = h_1 + 10$)

Vertical difference between supports: 10 m

Determining the Position of the Lowest Point:



Let x_1 be the horizontal distance from support A to the lowest point C, and x_2 be the horizontal distance from support B to C Then:

$$l_1 + l_2 = 60 \text{ m}$$

For a cable under UDL, the horizontal tension H is constant. The relationship between sag and horizontal distance is:

$$h = \frac{wx^2}{2H}$$

From this, the ratio of distances is:

$$\frac{l_1}{l_2} = \sqrt{\frac{h_1}{h_2}} = \sqrt{\frac{5}{15}}$$

$$\Rightarrow \quad l_2 = \sqrt{3}l_1$$

Substitute into the span equation:

$$l_1 + \sqrt{3}l_1 = 60$$

$$\Rightarrow \quad l_1 = 21.96 \text{ m}$$

$$\Rightarrow \quad l_2 = 60 - 21.96 = 38.04 \text{ m}$$

Horizontal Reaction (H):

Using the sag equation at support A:

$$H = \frac{wl_1^2}{2h_1}$$

$$\Rightarrow \quad H = \frac{5\times(21.96)^2}{2\times 5} = 241.12 \text{ kN}$$

Vertical Reactions (V_A and V_B):

The vertical reaction at each support equals the weight of the load acting on the cable segment from the lowest point to that support:

$$V_A = w\,x_1 = 5 \times 21.96 = 109.8 \text{ kN}$$

$$V_B = w\,x_2 = 5 \times 38.04 = 190.2 \text{ kN}$$

Maximum Tension (T_{max}):

The tension is maximum at the highest support (Support B): ($V_B > V_A$)

$$T_{max} = \sqrt{H^2 + V_B^2} = \sqrt{(241.12)^2 + (190.2)^2}$$

$$\Rightarrow \quad T_{max} = 307.12 \text{ kN}$$

The tension is minimum at the lowest point (c)

$$T_{min} = H = 241.12 \text{ kN}$$

Alternate solution for Horizontal and vertical reaction

Here, $l = 60$ m, $h_1 = 5$ m, $h_2 = 15$ m

$$l_1 + l_2 = 60 \text{ m}$$

$$\because \quad l_1 = \frac{l\sqrt{h_1}}{\left(\sqrt{h_1}+\sqrt{h_2}\right)} \quad \text{and} \quad l_2 = \frac{l\sqrt{h_2}}{\left(\sqrt{h_1}+\sqrt{h_2}\right)}$$

$$l_1 = \frac{60\sqrt{5}}{\left(\sqrt{5}+\sqrt{15}\right)} \quad \text{and} \quad l_2 = \frac{60\sqrt{15}}{\left(\sqrt{5}+\sqrt{15}\right)}$$

$$l_1 = 21.96 \text{ m} \quad \text{and} \quad l_2 = 38.04 \text{ m}$$

Horizontal thrust,

$$H = \frac{wl^2}{2\left(\sqrt{h_1}+\sqrt{h_2}\right)^2}$$

$$\Rightarrow \quad H = \frac{5\times 60^2}{2\left(\sqrt{5}+\sqrt{15}\right)^2} = 241.15 \text{ kN}$$

Taking moment about c (considering left side)

$$V_A \times 21.96 - 5\times\frac{21.96^2}{2} - 241.15 \times 5 = 0$$

$$V_A = 109.8 \text{ kN}$$

Taking moment about c (considering right side)

$$V_B \times 38.04 - 5\times\frac{38.04^2}{2} - 241.15 \times 15 = 0$$

$$V_B = 190.2 \text{ kN}$$

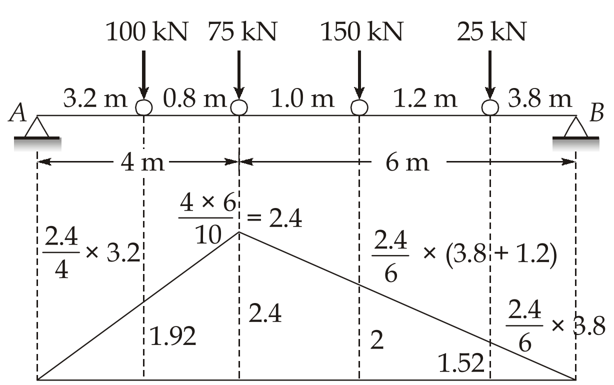
1. (e) Solution:

Let's the loads to cross the given section one after another and find the average loads on *AD* and *BD*. The calculation is shown in the following table:

Load crossing the section *D*	Average load on *AD*	Average load on *BD*	Remarks
25 kN	$\frac{325}{4}$	$\frac{25}{6}$	Average load on AD is greater than the average load on BD.
150 kN	$\frac{175}{4}$	$\frac{175}{6}$	Average load on AD is greater than the average load on BD.
75 kN	$\frac{100}{4}$	$\frac{250}{6}$	Average load on BD is greater than the average load on AD

Hence, for the maximum B.M. at *D*. We will place the 75 kN load exactly at *D* and the other loads relative to this load. (see figure)

I.L.D. for the bending moment at the section *D*.

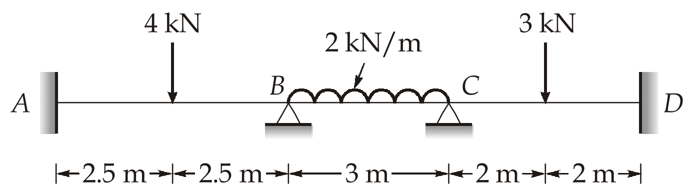


(ILD for *BM* at *D*)

$$\text{Maximum B.M. at } D = 100 \times 1.92 + 75 \times 2.4 + 150 \times 2.0 + 25 \times 1.52 \text{ kNm}$$

$$= 710 \text{ kNm}$$

2. (a) Solution:



Kinematic indeterminacy, $D_k = 2\ (\theta_B, \theta_C)$

Fixed end moment,

$$M_{FAB} = -\frac{Pl}{8} = -\frac{4\times5}{8} = -2.5 \text{ kN-m}$$

$$M_{FBA} = +\frac{Pl}{8} = +\frac{4\times5}{8} = +2.5 \text{ kN-m}$$

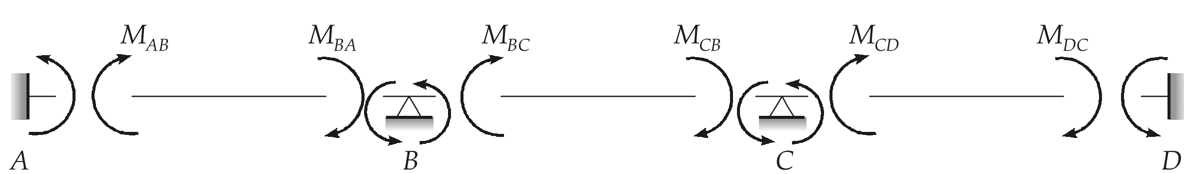
$$M_{FBC} = -\frac{wl^2}{12} = -\frac{2\times3^3}{12} = -1.5 \text{ kN-m}$$

$$M_{FCB} = +\frac{wl^2}{12} = +\frac{2\times3^3}{12} = +1.5 \text{ kN-m}$$

$$M_{FCD} = -\frac{Pl}{8} = -\frac{3\times4}{8} = -1.5 \text{ kN-m}$$

$$M_{FDC} = +\frac{Pl}{8} = +\frac{3\times4}{8} = +1.5 \text{ kN-m}$$

$$\theta_A = \frac{1}{500}\ (\circlearrowright),\quad \Delta_B = 15 \text{ mm } (\downarrow),\quad \Delta_C = 10 \text{ mm } (\downarrow)$$



$$\Sigma M_B = 0$$

$\Rightarrow$ $$-M_{BA} - M_{BC} = 0$$

$\Rightarrow$ $$-M_{BA} - M_{BC} = 0 \quad \text{...(i)}$$

$$\Sigma M_C = 0$$

$\Rightarrow$ $$-M_{CB} - M_{CD} = 0$$

$\Rightarrow$ $$-M_{CB} - M_{CD} = 0 \quad \text{...(ii)}$$

Now, from slope deflection equation

For span *AB*:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$M_{AB} = -2.5 + \frac{2EI}{5}\left[2\left(\frac{+1}{500}\right) + \theta_B - \frac{3\times(+15\times10^{-3})}{5}\right]$$

$$M_{BA} = +2.5 + \frac{2EI}{5}\left[2\theta_B + \left(\frac{+1}{500}\right) - \frac{3(+15\times10^{-3})}{5}\right]$$

For span *BC*:

$$M_{BC} = -1.5 + \frac{2EI}{3}\left[2\theta_B + \theta_C - \frac{3(-5\times10^{-3})}{3}\right]$$

$$M_{CB} = +1.5 + \frac{2EI}{3}\left[2\theta_C + \theta_B - \frac{3(-5\times10^{-3})}{3}\right]$$

For span *CD*:

$$M_{CD} = -1.5 + \frac{2EI}{4}\left[2\theta_C + \cancelto{0}{\theta_D} - \frac{3(-10\times10^{-3})}{4}\right]$$

$$M_{DC} = +1.5 + \frac{2EI}{4}\left[\cancelto{0}{2\theta_D} + \theta_C - \frac{3(-10\times10^{-3})}{4}\right]$$

From equation (i)

$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow \quad +2.5 + \frac{2EI}{5}\left(2\theta_B + \frac{1}{500} - 0.009\right) - 1.5 + \frac{2EI}{3}(2\theta_B + \theta_C + 0.005) = 0$$

$$\Rightarrow \quad 2.133\,(EI\,\theta_B) + 0.667\,(EI\,\theta_C) = 2.5 - \frac{2\times764}{5\times500} + \frac{2\times764\times0.009}{5} + 1.5 - \frac{2\times764\times0.005}{3}$$

$$\Rightarrow \quad 2.133\,(EI\,\theta_B) + 0.667\,(EI\,\theta_C) = -1.408 \qquad \text{...(iii)}$$

From equation (ii)

$$M_{CB} + M_{CD} = 0$$

$$\Rightarrow \quad +1.5 + \frac{2EI}{3}(2\theta_C + \theta_B + 0.005) - 1.5 + \frac{2EI}{4}(2\theta_C + 0.0075) = 0$$

$$\Rightarrow \quad 0.667\,(EI\,\theta_B) + 2.333\,(EI\,\theta_C) = -\frac{2\times764\times0.005}{3} - \frac{2\times764\times0.0075}{4}$$

$$\Rightarrow \quad 0.667\,(EI\,\theta_B) + 2.333\,(EI\,\theta_C) = -5.412 \qquad \text{...(iv)}$$

On solving equation (iii) and (iv)

$$\theta_B = \frac{+0.0717}{EI}, \theta_C = \frac{-2.340}{EI}$$

Now, Final end moments.

$$M_{AB} = -2.5 + \frac{2}{5}\left[\frac{2\times764}{500} + (+0.0717) - 0.009\times764\right] = -4 \text{ kN-m}$$

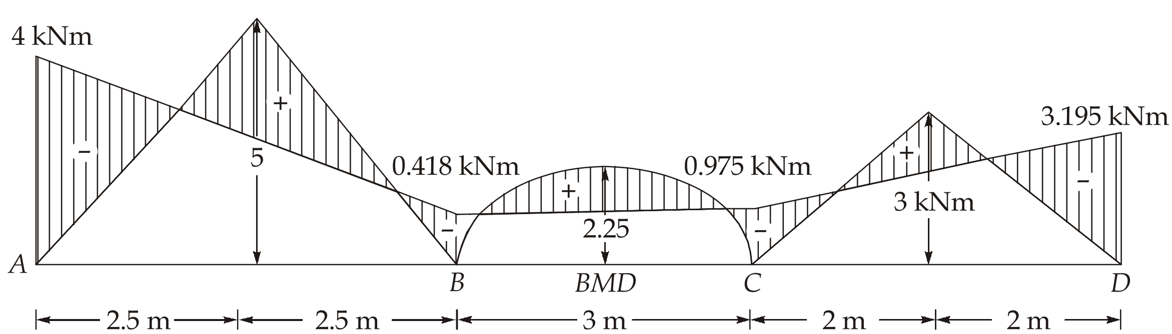
$$M_{BA} = +2.5 + \frac{2}{5}\left[2(+0.0717) + \frac{764}{500} - 0.009\times764\right] = 0.418 \text{ kN-m}$$

$$M_{BC} = -1.5 + \frac{2}{3}\left[2(+0.0717) + (-2.34) + 0.005\times764\right] = -0.418 \text{ kN-m}$$

$$M_{CB} = +1.5 + \frac{2}{3}\left[2(-2.34) + (+0.0717) + 0.005\times764\right] = 0.975 \text{ kN-m}$$

$$M_{CD} = -1.5 + \frac{1}{2}[2(-2.34) + 0.0075\times764] = -0.975 \text{ kN-m}$$

$$M_{DC} = +1.5 + \frac{1}{2}[(-2.34) + 0.0075\times764] = 3.195 \text{ kN-m}$$



2. (b) Solution:

1. **Notice Inviting Tender (NIT):** Notice Inviting Tender is an official announcement issued by the client inviting bids from eligible contractors. It contains brief details of the project such as name of work, estimated cost, completion period, eligibility criteria, and key dates. It ensures transparency and fair competition in procurement. NIT also specifies where tender documents can be obtained and submission procedures. It is the first formal step in the tendering process.
2. **Earnest Money Deposit (EMD):** EMD is a refundable deposit submitted along with the tender to demonstrate the bidder's seriousness. It protects the employer against withdrawal or modification of bids during the validity period. If the bidder withdraws

or fails to sign the contract, the EMD may be forfeited. Unsuccessful bidders receive a refund after finalization. It discourages non-serious or speculative bids.

3. **Bid Security:** Bid Security is a guarantee provided by bidders to secure their commitment to the tender conditions. It may be in the form of a bank guarantee, demand draft, or online transfer. It serves the same purpose as EMD in many cases. It is forfeited if the bidder fails to honor the bid terms. It enhances reliability in the bidding process.
4. **Performance Guarantee:** Performance Guarantee is submitted by the successful bidder after award of work. It ensures satisfactory execution of the contract as per specifications and conditions. It is usually a percentage of the contract value. It may be in the form of a bank guarantee. It is released after successful completion of the project or after the defect liability period.
5. **Letter of Acceptance (LOA):** Letter of Acceptance is issued by the employer to the successful bidder. It formally communicates acceptance of the bid and contract price. LOA signifies formation of a binding agreement between both parties. It mentions important details such as contract amount and time for completion. The contractor must acknowledge and comply with its terms.
6. **Mobilization Advance:** Mobilization Advance is a payment made to the contractor at the beginning of the project. It helps in mobilizing resources like labor, machinery, and materials. It is usually interest-bearing and recovered in installments from running bills. It improves cash flow for the contractor. Proper safeguards are taken before releasing this advance.
7. **Variation Order:** A variation order is issued when there is a change in scope, quantity, or specification of work. It may increase or decrease the contract value. Such changes are formally recorded and approved. Payment adjustments are made accordingly. It ensures flexibility during project execution.
8. **Defect Liability Period (DLP):** Defect Liability Period is the time after completion during which the contractor is responsible for rectifying defects. It ensures quality workmanship and material performance. Any defects arising during this period must be corrected at the contractor's cost. It typically ranges from 6 to 24 months. Final payment is often linked to completion of DLP.
9. **Liquidated Damages (LD):** Liquidated damages are predetermined penalties imposed for delay in completion. They are calculated as a percentage of contract value per day or week of delay. *LD* compensates the employer for losses due to delay. It is limited to a maximum specified amount. It encourages timely completion of the project.

10. **Escalation Clause:** An escalation clause provides for adjustment of contract price due to changes in material, labor, or fuel costs. It protects contractors from abnormal price rise during long-term projects. The formula for escalation is defined in the contract. It ensures fairness to both parties. It is commonly applied in large infrastructure works.

2. (c) (i) Solution:

A rule devised by D. R. Fulkerson reduces this sequential numbering to the following steps:

(i) An "initial" event is one which has arrows coming out of it and none entering it. In any network, there will be one such event. Number it "1".

(ii) Delete all arrows emerging from event 1. This will create at least one more "initial event".

(iii) Number these new initial events as "2, 3, …".

(iv) Delete all emerging arrows from these numbered events which will create new initial events.

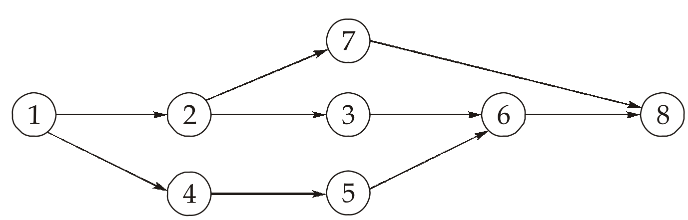
(v) Follow step (iii).

(vi) Continue until the last event which has no arrows emerging from it is obtained.

(vii) A restriction is a prerequisite relationship which establishes the sequence of activities. When one activity must be completed before a second activity can begin, the first is considered to be restraint or restriction on the second.

2. (c) (ii) Solution:

(a) Network diagram for the given project



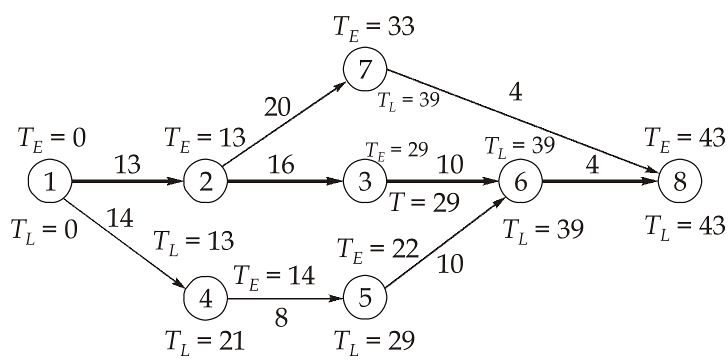
We know, the expected time of the project

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

$$\text{Std. deviation } \sigma = \frac{t_p - t_o}{6}$$

Activity	Optimistic Time (t_o)(weeks)	Most Likely Time (t_m) (Weeks)	Pessimistic Time (t_p) (Weeks)	Expected Time (t_e) (Weeks)	Standard duration (σ) (Weeks)
1 – 2	10	12	20	13	10/6
1 – 4	5	15	19	14	14/6
2 – 3	10	15	26	16	16/6
2 – 7	15	20	25	20	10/6
3 – 6	5	10	15	10	10/6
4 – 5	4	8	12	8	8/6
5 – 6	5	10	15	10	10/6
6 – 8	2	4	6	4	4/6
7 – 8	2	4	6	4	4/6

Calculation of project completion time is done in network diagram below.



Hence slack is zero for all events on path

$$1 - 2 - 3 - 6 - 8$$

∴ Critical path = $1 - 2 - 3 - 6 - 8$

Critical duration μ = 43 weeks = 301 days

(b) Standard deviation of project,

$$\sigma = \sqrt{\sigma_i^2} \qquad \text{where } \sigma_i = \text{std. deviation of critical activity}$$

$$\Rightarrow \quad \sigma = \sqrt{\left(\frac{10}{6}\right)^2 + \left(\frac{16}{6}\right)^2 + \left(\frac{10}{6}\right)^2 + \left(\frac{4}{6}\right)^2}$$

$$\Rightarrow \quad \sigma = 3.621 \text{ weeks}$$

For probability of 95%

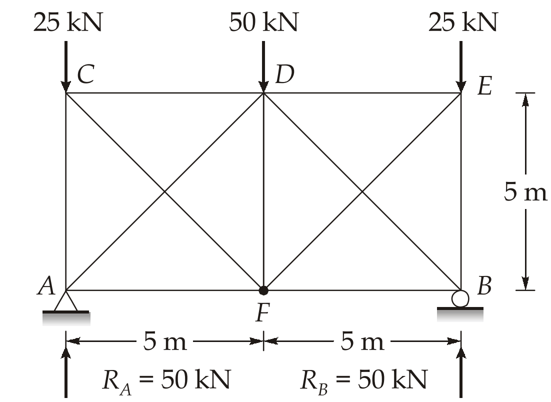
$$Z = 1.5 + \frac{(2.0 - 1.5)}{(97.92 - 93.92)} \times (95 - 93.92) = 1.635$$

So, $$Z = \frac{x - \mu}{\sigma}$$

$$\Rightarrow \qquad 1.635 = \frac{x - 43}{3.621}$$

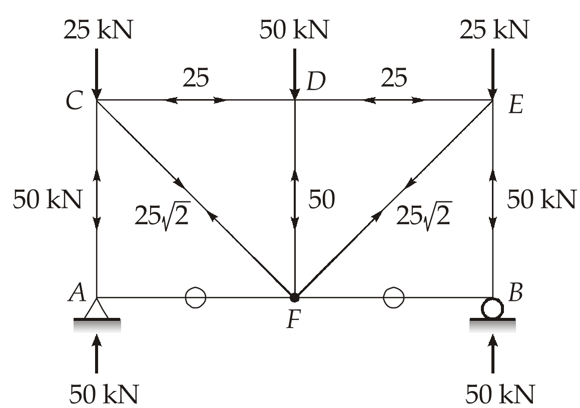
$$t_p = 48.920 \text{ weeks}$$

3. (a) Solution:

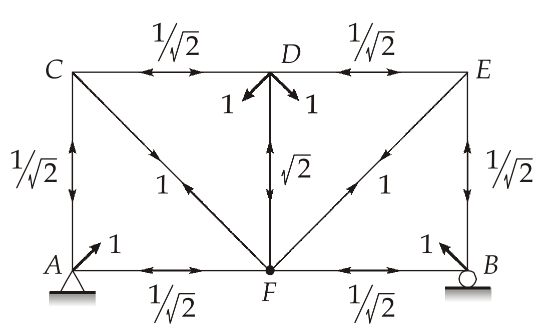


Given truss is internally indeterminate with 2 degree of indeterminacy

Let AD and BD be the redundant forces calculating the forces in members (P) due to applied load after releasing redundant force.



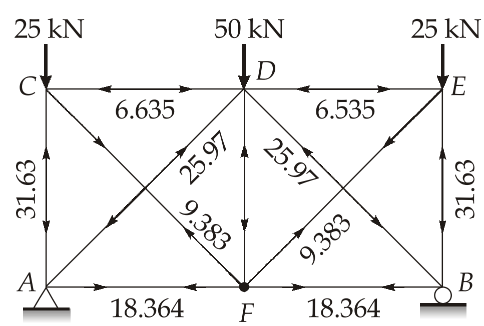
Due to symmetry of truss, consider two members to be treated as redundant and after applying unit load for these redundant member force (K) are calculated.



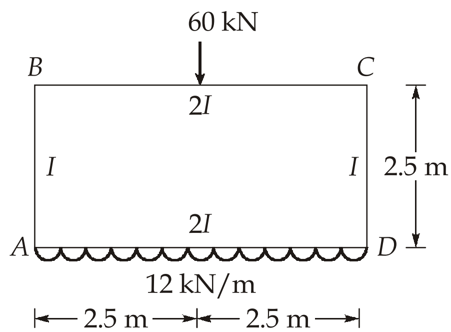
Member	P (Member force due to external load)	K (Member force due to unit load)	l (m)	PK l	$K^2 l$	P + KX (kN)
AC	–50	$-\frac{1}{\sqrt{2}}$	5	$125\sqrt{2}$	2.5	–31.63
CD	–25	$-\frac{1}{\sqrt{2}}$	5	$62.5\sqrt{2}$	2.5	–6.635
DE	–25	$-\frac{1}{\sqrt{2}}$	5	$62.5\sqrt{2}$	2.5	–6.635
BE	–50	$-\frac{1}{\sqrt{2}}$	5	$125\sqrt{2}$	2.5	–31.63
BF	0	$-\frac{1}{\sqrt{2}}$	5	0	2.5	18.364
AF	0	$-\frac{1}{\sqrt{2}}$	5	0	2.5	18.364
CF	$25\sqrt{2}$	+1	$5\sqrt{2}$	250	$5\sqrt{2}$	9.383
AD	0	+1	$5\sqrt{2}$	0	$5\sqrt{2}$	–25.97
BD	0	+1	$5\sqrt{2}$	0	$5\sqrt{2}$	–25.97
EF	$25\sqrt{2}$	+1	$5\sqrt{2}$	250	$5\sqrt{2}$	9.383
DF	–50	$-\sqrt{2}$	5	$250\sqrt{2}$	10	–13.27

$$X = \frac{\frac{-\Sigma PKl}{AE}}{\Sigma\frac{K^2 l}{AE}} = \frac{-\Sigma PKl}{\Sigma K^2 l} = -\frac{(883.883+500)}{(20\sqrt{2}+25)} = -25.972 \text{ kN}$$

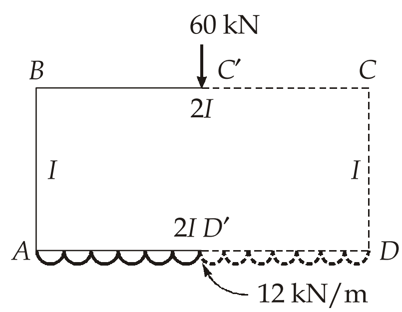
Hence force in all other members = $P + KX$ (As given in table)



3. (b) Solution:



The structure is symmetrical, hence symmetrical beam approach will be used. We will consider only half of the structure *i.e.*,



Joint	Member	Stiffness	T.S.	D.F.
B	BC'	$\frac{4E(2I)}{5\times2}$	$\frac{12EI}{5}$	0.33
	BA	$\frac{4EI}{2.5}$		0.67
A	AB	$\frac{4E(2I)}{5\times2}$	$\frac{12EI}{5}$	0.67
	AD'	$\frac{4EI}{2.5}$		0.33

Fixed end moments:

$$M_{FBC} = -\frac{PL}{8} = \frac{60\times5}{8} = -37.25 \text{ kN-m}$$

$$M_{FBA} = 0$$

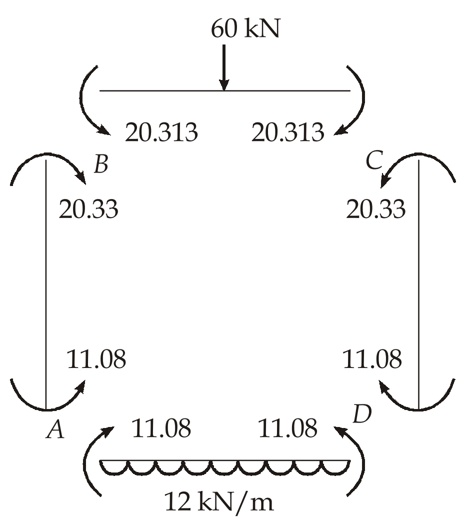
$$M_{FAB} = 0$$

$$M_{FAD} = \frac{wL^2}{12} = \frac{12\times5^2}{12} = 25 \text{ kN-m}$$

Moment distribution table

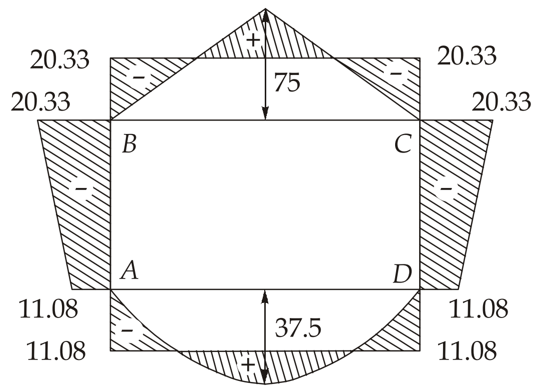
Joint	A		B	
Member	**AD′**	**AB**	**BA**	**BC′**
D.F.	0.33	0.67	0.67	0.33
FEM	+25	0	0	–37.25
Balancing	–8.25	–16.75	+24.95	+12.30
C.O.M		12.475	–8.375	
Balancing	–4.12	–8.36	+5.61	+2.76
C.O.M.		+2.8	–4.18	
Balancing	–0.924	–1.876	+2.8	+1.38
C.O.M.		+1.4	–0.938	
Balancing	–0.462	–0.938	+0.628	+0.31
C.O.M.		+0.314	–0.469	
Balancing	–0.10	–0.21	+0.31	+0.15
C.O.M.		+0.15	–0.10	
Balancing	–0.049	–0.10	+0.067	+0.033
C.O.M.		+0.0335	–0.05	
Balancing	–0.011	–0.022	+0.0335	+0.0165
Final moment	+11.08	–11.08	+20.33	–20.33

The free body diagram:



BMD will be (considering sagging as positive and hogging as negative)

$$M = \frac{PL}{4} = \frac{60 \times 5}{4} = 75 \text{ kN-m}, \qquad \left[M = \frac{wL^2}{8} = \frac{12 \times 25}{8} = 37.5 \text{ kN-m}\right]$$



BMD (All in kN-m)

3. (c) Solution:

Given: Initial cost, C_i = Rs.2200

Salvage value, C_s = Rs.200

Useful life, n = 4 years

Interest rate, i = 10% = 0.10

Total depreciable amount, $C - S$ = 2200 – 200 = Rs.2000

(i) Straight Line Method

Annual depreciation $D = \dfrac{C_i - C_s}{n} = \dfrac{2000}{4}$ = Rs. 500

Year	Annual Depreciation (Rs.)	Book Value (Rs.)
0	–	2200
1	500	1700
2	500	1200
3	500	700
4	500	200

(ii) Declining Balance Method

Depreciation factor, $r = 1 - \left(\dfrac{C_s}{C_i}\right)^{1/n}$

$\Rightarrow$ $r = 1 - \left(\dfrac{200}{2200}\right)^{1/4} \approx 0.451$

Depreciation in a given year = book value at the end of previous year × r

Year	Annual Depreciation (Rs.)	Book Value (Rs.)
0	–	2200
1	2200 × 0.451 = 992.2	1207.8
2	1207.8 × 0.451 = 544.7	663.1
3	663.1 × 0.451 = 299.1	364.0
4	364.0 × 0.451 = 164.0	200

(iii) Sum of Years Digits Method

$$S_y = 1 + 2 + 3 + 4 = 10$$

Year	Annual Depreciation (Rs.)	Book Value (Rs.)
0	–	2200
1	$2000 \times \frac{4}{10} = 800$	1400
2	$2000 \times \frac{3}{10} = 600$	800
3	$2000 \times \frac{2}{10} = 400$	400
4	$2000 \times \frac{1}{10} = 200$	200

(iv) Sinking Fund Method

$$D = (C_i - C_S)\left[\frac{i}{(1+i)^n - 1}\right]$$

$$\Rightarrow \quad D = 2000\left[\frac{0.10}{(1.10)^4 - 1}\right] = \text{Rs. } 430.942$$

Depreciation in a given year

$$D_m = D \times (1 + i)^{m-1}$$

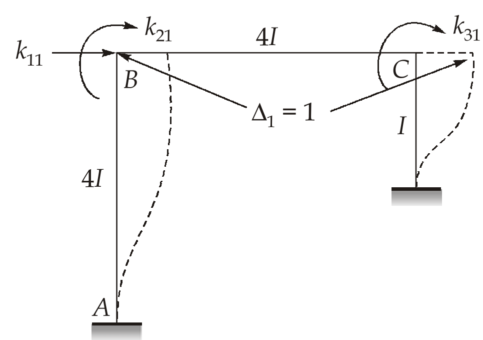
Year	Annual Depreciation (Rs.)	Book Value (Rs.)
0	–	2200
1	430.942	1769.058
2	474.036	1295.022
3	522.440	772.582
4	573.584	200

4. (a) Solution:

The stiffness matrix can be developed by giving a unit displacement successively at coordinates 1, 2 and 3 without any displacement at other coordinates and determining the forces required at all the coordinates.

To generate the first column of the stifness matrix, give a unit displacement at coordinate. 1 as shown in figure.

For 1st column of stiffness matrix



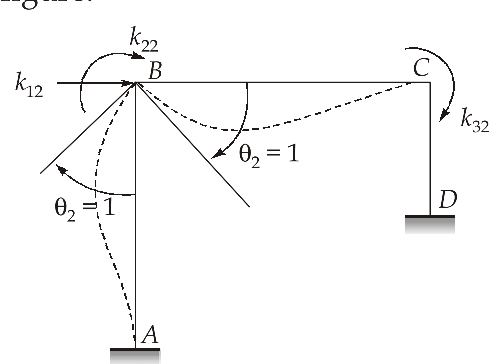
$$k_{11} = \frac{12E(4I)}{10^3} + \frac{12E(I)}{5^3} = 0.144EI$$

$$k_{21} = -\frac{6E(4I)}{10^2} = -0.24EI$$

$$k_{31} = -\frac{6E(I)}{5^2} = -0.24EI$$

- For 2nd column of stiffness matrix:

To generate the second column of the stiffness matrix, give a unit displacement at coordinate 3 as shown in figure.



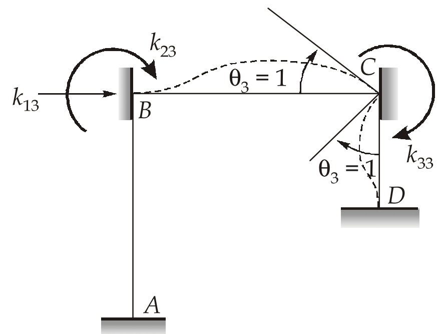
$$k_{12} = \frac{-3}{2}\left(\frac{M_{BA}}{l_{BA}}\right) = \frac{-3}{2}\frac{4E(4I)}{10 \times 10} = -0.24EI$$

$$k_{22} = \frac{4E(4I)}{10} + \frac{4E(4I)}{10} = 3.2EI$$

$$k_{23} = \frac{M_{BC}}{2} = \frac{1}{2} \times \frac{4E(4I)}{10} = 0.80EI$$

- For 3rd column of stiffness matrix

To generate the third column of the stiffness matrix, give a unit displacement at coordinate 3 as shown in figure.



$$k_{13} = \frac{-3}{2}\left(\frac{M_{CD}}{2}\right) = \frac{-3}{2}\frac{\left(\frac{4EI}{5}\right)}{5} = -0.24EI$$

$$k_{23} = \frac{1}{2}\left(\frac{4E(4I)}{10}\right) = 0.8EI$$

$$k_{33} = \frac{4E(4I)}{10} + \frac{4EI}{5} = 2.4EI$$

Hence, the required stiffness matrix $[k]$ is given by the equation

$$[k] = EI\begin{bmatrix} 0.144 & -0.240 & -0.240 \\ -0.240 & 3.200 & 0.800 \\ -0.240 & 0.800 & 2.400 \end{bmatrix}$$

4. (b) Solution:

Given data

Rate of interest, $i = 12\% = 0.12$

Scheme A

Initial cost, $P_A = 15$ lacs

Annual running cost, $C_A = 2$ lacs

Annual benefit, $B_A = 4.5$ lacs

Life, $n_A = 6$ years

Scheme B

Initial cost, $P_B = 25$ lacs

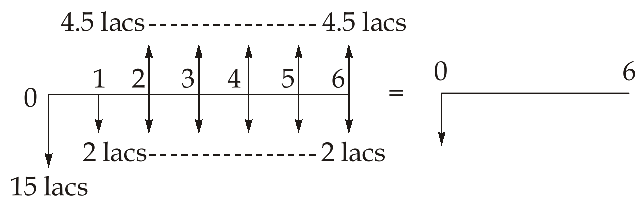
Annual running cost, C_B = 2.5 lacs

Annual benefit, B_B = 6.2 lacs

Life, n_B = 12 years

Common study period = *LCM* of 6 and 12 = 12 years

Present worth of Scheme *A* for 6 years



Cash flow diagram for 6 years

Present worth of running cost:

$$PV\ (C_A) = 2\left[\frac{(1+0.12)^6 - 1}{0.12(1+0.12)^6}\right]$$

$$PV(C_A) = 2 \times 4.111 = 8.222 \text{ lacs}$$

Present worth of benefits

Benefits occur from year 2 to year 6, that is 5 payments.

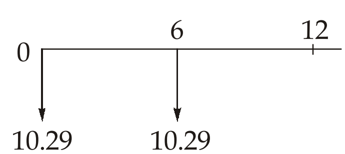
$$PV(B_A) = 4.5\left[\frac{(1+0.12)^5 - 1}{0.12(1+0.12)^5}\right] \times \frac{1}{(1+0.12)^2} = 12.932 \text{ lacs}$$

Net present worth for one life cycle of Scheme *A*:

$$P_A(6) = 15 + 8.222 - 12.932$$

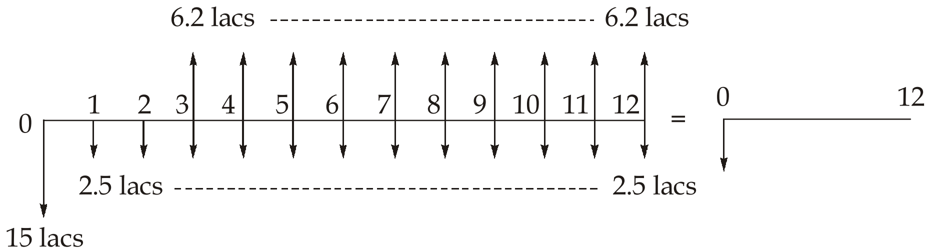
$$P_A(6) = 10.29 \text{ lacs}$$

Since the common period is 12 years, Scheme *A* will repeat after 6 years.



$$P_A(12) = 10.29 + \frac{10.29}{(1.12)^6} = 15.503 \text{ lacs}$$

Present worth of Scheme *B* for 12 years



Present worth of running cost:

$$PV(C_B) = 2.5\left[\frac{(1+0.12)^{12}-1}{0.12(1+0.12)^{12}}\right]$$

$$PV(C_B) = 2.5 \times 6.194 = 15.485 \text{ lacs}$$

Benefits occur from year 3 to year 12, that is 10 payments.

$$PV(B_B) = 6.2\left[\frac{(1+0.12)^{10}-1}{0.12(1+0.12)^{10}}\right]\times\frac{1}{(1.12)^3} = 24.935 \text{ lacs}$$

Net present worth for Scheme *B*:

$$P_B(12) = 25 + 15.485 - 24.935$$

$$P_B(12) = 15.55 \text{ lacs}$$

Comparison

$$P_A(12) = 15.503 \text{ lacs}$$

$$P_B(12) = 15.55 \text{ lacs}$$

Since the present worth of net cost of Scheme *A* is less than that of Scheme *B*, Scheme *A* is the most economical proposal.

4. (c) (i) Solution:

Given data

$$m = 5 \text{ kg}$$

$$k = 4500 \text{ N/m}$$

$$\frac{x_1}{x_2} = \frac{1.00}{0.85}$$

Natural frequency of the undamped system

$$w_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4500}{5}} = \sqrt{900} = 30 \text{ rad/s}$$

Logarithmic decrement

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \ln\left(\frac{1}{0.85}\right) = 0.1625$$

Damping ratio

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \approx 2\pi\xi$$

$$\Rightarrow \quad \xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$$

$$\Rightarrow \quad \xi = \frac{0.1625}{\sqrt{4\pi^2+0.1625^2}}$$

$$\Rightarrow \quad \xi = 0.02585$$

Damping coefficient

Critical damping coefficient

$$c_c = 2m\omega_n = 2 \times 5 \times 30 = 300 \text{ N-sec/m}$$

Actual damping coefficient

$$c = \xi c_c = 0.02585 \times 300$$

$$c = 7.757 \text{ N-sec/m}$$

Damped natural frequency

$$\omega_d = \omega_n\sqrt{1-\xi^2} \approx \omega_n$$

$$\Rightarrow \quad \omega_d = 30\sqrt{1-(0.02585)^2}$$

$$\Rightarrow \quad \omega_d = 29.98 \text{ rad/s}$$

4. (c) (ii) Solution:

The output of a power shovel during excavation is not fixed; it varies depending on the material being handled, site conditions, machine characteristics, and operational efficiency. The important factors are discussed below.

1. **Class of material:** The type of earth being excavated has a direct impact on output. Soft and loose materials like moist loam or sandy clay are easy to dig and fill the dipper quickly, resulting in higher output. On the other hand, hard, compact soils or poorly blasted rock require more effort and time for excavation, which reduces productivity.

2. **Depth of cut:** The depth at which the shovel is operating plays an important role in its efficiency. If the cut is too shallow, the dipper does not get filled properly. At an optimum depth, the dipper fills efficiently and output is maximum. However, if the depth becomes excessive, the cycle time increases, leading to a reduction in output.
3. **Angle of swing:** The angle of swing refers to the horizontal angle through which the shovel rotates from digging to dumping. A larger swing angle increases the time required for one cycle, thereby reducing output. Keeping the swing angle as small as possible improves efficiency. In fact, reducing the swing angle from 90° to 60° can increase output significantly.
4. **Job conditions:** Site conditions greatly influence the working efficiency of a shovel. In ideal situations, such as a large open excavation with a firm, level, and well-drained surface, the shovel operates at maximum efficiency. However, poor conditions like uneven ground, congested working space, or bad haul roads can slow down operations and reduce output.
5. **Management conditions:** Good management practices are essential for maintaining high productivity. Regular lubrication, timely replacement of worn-out parts, and proper maintenance reduce breakdowns. Keeping the working area clean and organized helps in reducing unnecessary delays. Proper supervision and motivated workers also contribute to better performance of the shovel.
6. **Size of hauling units:** The capacity of hauling units such as trucks should be compatible with the shovel size. If the truck size is too small or too large compared to the shovel, it leads to delays and inefficient operation. Proper matching ensures smooth loading and continuous operation, thereby improving output.
7. **Skill of operator:** The operator's skill has a significant influence on productivity. An experienced operator can maintain the optimum depth of cut, reduce the swing angle, and coordinate efficiently with hauling units. This reduces cycle time and increases output.
8. **Physical condition of the shovel:** The condition of the machine itself is equally important. A well-maintained shovel operates smoothly and gives higher output. In contrast, a poorly maintained machine suffers from frequent breakdowns and increased downtime, which adversely affects productivity.

Section B : Flow of fluids, hydraulic machines and hydro power-1 + Design of Concrete and Masonry Structures-2

5. (a) Solution:

Given data

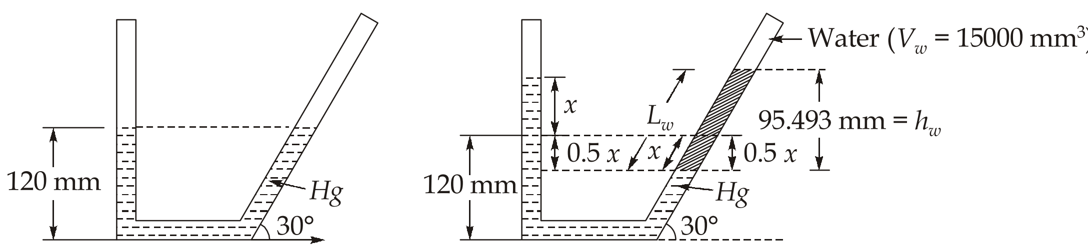
Diameter of tube, d = 10 mm (Radius r = 5 mm)

Angle of inclination of right leg, θ = 30° (From horizontal)

Volume of water added, V_w = 15.0 mL = 15,000 mm³

Specific gravity of mercury, SG_{Hg} = 13.6

Initial vertical height of mercury, h_{initial} = 120 mm



(a) Initial condition (b) Final condition

Length and Vertical Height of Water Column

The water occupies the inclined leg. The length of this column along the tube is

$$L_w = \frac{V_w}{\pi r^2} = \frac{15000}{\pi \times 5^2} = \frac{15000}{25\pi} \approx 190.986 \text{ mm}$$

The vertical height of the water column is

$$h_w = L_w \times \sin(30°) = 190.986 \times 0.5 = 95.493 \text{ mm}$$

Mercury Displacement

Let the mercury level in the vertical left leg rise by a vertical distance x. Due to the constant diameter of the tube, the mercury in the inclined right leg moves down by the same length along the tube.

$$\text{Vertical rise in left leg} = x$$

$$\text{Vertical drop in right leg} = x \times \sin(30°) = 0.5x$$

Total vertical change of mercury:

$$\Delta h_{Hg} = x + 0.5x = 1.5x$$

Pressure Equilibrium at the Interface

Equating pressures at the interface level in both legs:

$$P_{atm} + \rho_w g h_w = P_{atm} + \rho_{Hg} g(1.5x)$$

$$SG_w \times h_w = SG_{Hg} \times 1.5x$$

$$\Rightarrow \quad 1 \times 95.493 = 13.6 \times 1.5x$$

$$\Rightarrow \quad 95.493 = 20.4\,x$$

$$\Rightarrow \quad x = \frac{95.493}{20.4} = 4.681 \text{ mm}$$

Ultimate Vertical Height

Left leg (vertical): $\text{Height}_{\text{left}} = 120 + 4.681 = \mathbf{124.681\ mm}$

Right leg (inclined): Mercury level dropped vertically by

$$0.5x = 0.5 \times 4.681 = 2.341 \text{ mm}$$

$$\text{Mercury height in right leg} = 120 - 2.341 = 117.659 \text{ mm}$$

Adding vertical height of water:

$$\text{Height}_{\text{right}} = 117.659 + 95.493 = \mathbf{213.152\ mm}$$

Alternate solution

$$\because \quad x + x\sin\theta = \frac{(L_w \sin\theta)G_{\text{water}}}{G_{Hg}}$$

$$\Rightarrow \quad x(1 + \sin 30°) = \frac{190.986 \sin 30° \times 1}{13.6}$$

$$\Rightarrow \quad x = 4.681 \text{ mm}$$

Ultimate Vertical Height

Left leg (vertical): $\text{Height}_{\text{left}} = 120 + 4.681 = \mathbf{124.681\ mm}$

Right leg (inclined): Mercury level dropped vertically by

$$0.5x = 0.5 \times 4.681 = 2.341 \text{ mm}$$

$$\text{Mercury height in right leg} = 120 - 2.341 = 117.659 \text{ mm}$$

Adding vertical height of water:

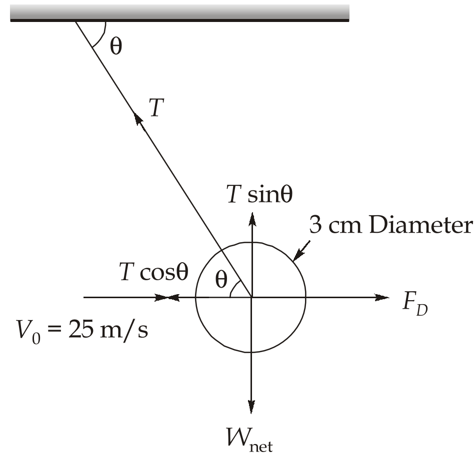
$$\text{Height}_{\text{right}} = 117.659 + 95.493 = \mathbf{213.152\ mm}$$

5. (b) Solution:

Given: Diameter of sphere, d = 3 cm = 0.03 m

Relative density of sphere, G = 2.5

Velocity of air, V_o = 25 m/sec.



For equilibrium of sphere.

$$T\cos\theta = F_D = C_D A\frac{\rho V_0^2}{2}$$

$$T\sin\theta = W_{net}$$

$$\therefore \qquad \tan\theta = \frac{W_{net}}{C_D A\rho V_0^2/2} \qquad ...(i)$$

Also,
$$R_e = \frac{V_0 D}{\nu} = \frac{25\times 0.03}{1.4\times 10^{-5}} = 53571.43 \simeq 53571$$

As $10^4 < Re < 3 \times 10^5$; $C_D = 0.5$

$$W_{net} = W = \gamma_{sphere}(\text{Volume}) = 2.5 \times 10^3 \times 9.81 \times \frac{4}{3}\pi\left(\frac{0.03}{2}\right)^2$$

$$= 0.347 \text{ N}$$

Now, drag force on sphere

$$F_D = \frac{1}{2}C_D\rho_{air}AV_o^2$$

$$\Rightarrow \qquad F_D = \frac{1}{2}\times 0.5\times 1.25\times\frac{\pi}{4}(0.03)^2(25)^2$$

$$\Rightarrow \qquad F_D = 0.138 \text{ N}$$

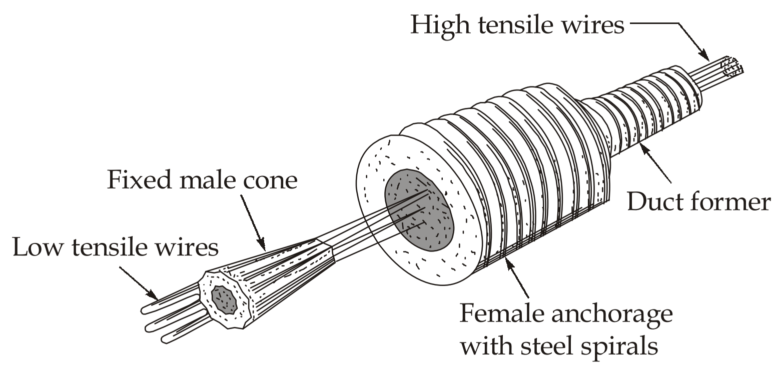
From equation (i),
$$\tan\theta = \frac{W_{net}}{F_D} = \frac{0.347}{0.138} = 2.514$$

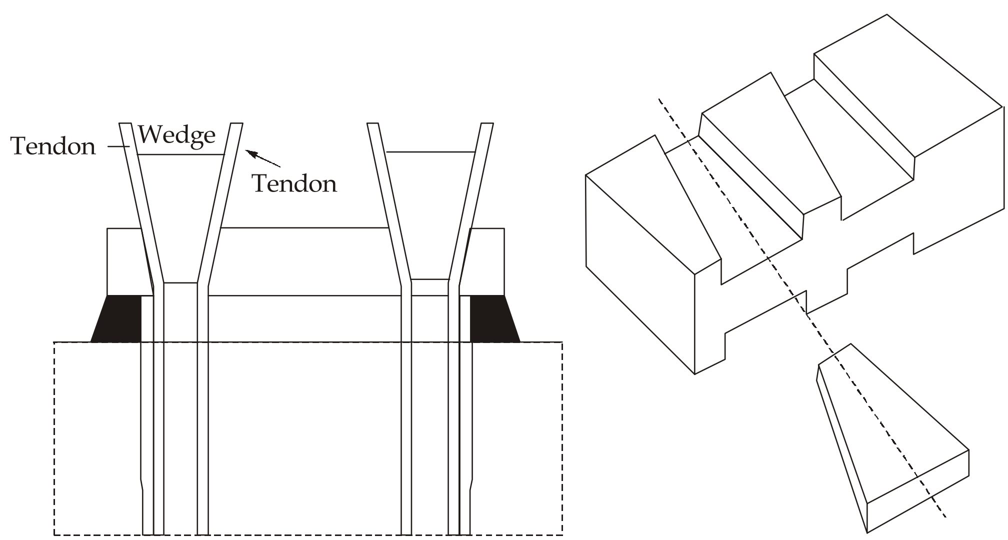
$$\theta = \tan^{-1}(2.514) = 68.31°$$

∴ Tension in the string,
$$T = \frac{W_{net}}{\sin\theta} = \frac{0.347}{\sin(68.31)°} = 0.3734 \text{ N}$$

5. **(c) Solution:**

Freyssinet system: Freyssinet system was the first to be introduced among the post-tensioning systems. High tension steel wires 5 mm to 8 mm in diameter and about 12 in numbers are arranged to form a group into a cable with a spiral spring inside. The spiral spring provides the means for a proper clearance between the wires and thus provides a channel which can be cement grouted. It further assists to transfer the reaction to concrete. The whole thing is enclosed in a thin metal steel.





The anchorage consists of a cylinder of ordinary good quality concrete and is provided with corrugations on the outside. It has a central conical hole and is provided with heavy hoop reinforcement. These cylinders are kept in the proper position and the conical plugs are pushed into the conical holes after cables are tightened. The central hole passing axially through the plug permits cement grout to be injected through it. In this way the space between the wires will be filled with the grout. This provides additional restraint against the slipping of the tendons.

Advantages of Freyssinet system:

- Securing the wires is not expensive.
- The desired stretching force is obtained quickly.
- The plugs may be left in the concrete and they do not project beyond the ends of the member.

Disadvantages of Freyssinet system:

- All the wires of a cable are stretched together. Hence the stresses in the wires may not be exactly the same.
- The greatest stretching force applied to a cable is from 250 kN to 500 kN. This may not be sufficient.
- The jacks used are heavy and expensive.

5. (d) Solution:

Given data

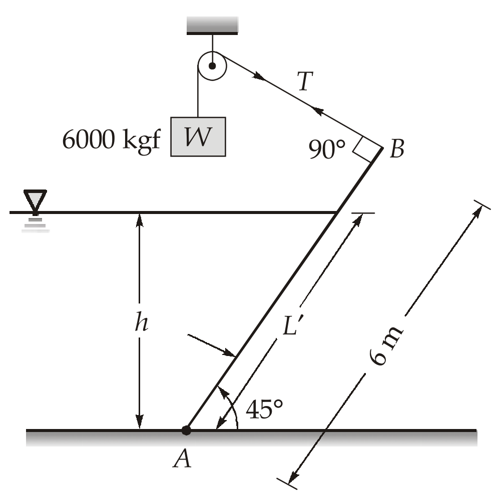
$$\text{Length of gate, } L = 6 \text{ m}$$

$$\text{Width of gate, } b = 3\text{m}$$

$$\text{Inclination angle, } \theta = 45° \quad \text{(from horizontal)}$$

$$\text{Counterweight, } W = 6000\ kgf = 6000 \times 9.81 = 58860 \text{ N}$$

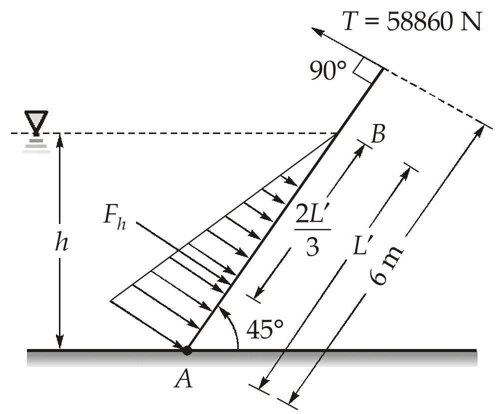
$$\text{Tension in string, } T = W = 58860 \text{ N}$$



The inclined length of the submerged portion of the gate is

$$L' = \frac{h}{\sin 45°} = \sqrt{2}h$$

Pressure diagram on gate, $T = 58860 \text{ N}$



Total hydrostatic force acting normal to the gate,

$$F_h = \text{Volume of pressure prism}$$

$$\Rightarrow \quad F_h = \frac{1}{2}(\rho g h)L' \times b$$

$$\Rightarrow \quad F_h = \frac{1}{2} \times 10^3 \times 9.81 \times h\sqrt{2}h \times 3$$

$$\Rightarrow \quad F_h = 20810.15\, h^2 \text{ (N)}$$

Taking moment about hinge (A)

$$\Sigma M_A = 0$$

$$\Rightarrow \quad T \times 6 = F_h \times \frac{L'}{3}$$

$$\Rightarrow \quad 58860 \times 6 = 20810.15\, h^2 \times \left(\frac{1}{3} \times \sqrt{2}h\right)$$

$$\Rightarrow \quad h^3 = 36$$

$$h = 3.30 \text{ m}$$

5. (e) Solution:

Given Data

Discharge, $Q = 6 \text{ m}^3/\text{s}$

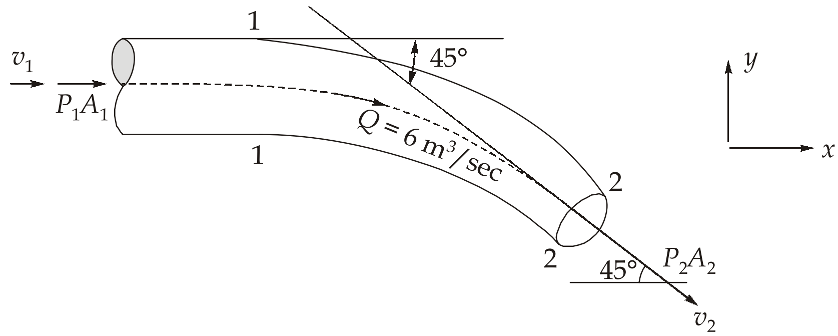
Area at inlet, $A_1 = 1.2 \text{ m}^2$

Area at outlet, $A_2 = 0.6 \text{ m}^2$

Pressure at inlet, $P_1 = 50 \text{ kN/m}^2$

Pressure at outlet, $P_2 = 25 \text{ kN/m}^2$

Angle of bend, $\theta = 45°$



At inlet (section 1):

$$V_1 = \frac{Q}{A_1} = \frac{6}{1.2} = 5 \text{ m/s}$$

At outlet (section 2):

$$V_2 = \frac{Q}{A_2} = \frac{6}{0.6} = 10 \text{ m/s}$$

Mass flow rate:

$$m = \rho Q = 1000 \times 16 = 6000 \text{ kg/s}$$

Applying the momentum equation in the horizontal plane.

In the x-direction:

$$-F_x + P_1A_1 - P_2A_2 \cos 45° = \rho Q(V_2 \cos 45° - V_1)$$

$$-F_x + (50 \times 10^3 \times 1.2) - (25 \times 10^3 \times 0.6 \times \cos 45°) = 6000 \times (10 \times \cos 45° - 5)$$

$$F_x = 36.97 \text{ kN } (\leftarrow)$$

In the y-direction:

$$-F_y + P_2A_2 \sin 45° = \rho Q[-V_2 \sin 45 - 0]$$

$$-F_y + 25 \times 10^3 \times 0.6 \times \sin 45 = -6000 \times 10 \sin 45$$

$$F_y = 53.03 \text{ kN } (\downarrow)$$

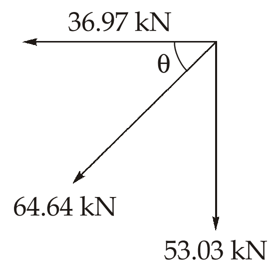
The force required to hold the bend is equal and opposite to the force exerted by the fluid.

Magnitude of resultant force:

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{36.97^2 + 53.03^2}$$

$$F_R = 64.64 \text{ kN}$$

Direction of resultant force:



$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{53.03}{36.97}\right) = 55.12°$$

The magnitude of the force required to hold the bend is 64.64 kN, acting at an angle of 55.12° with the x-axis.

6. (a) Solution:

Given data

Total head difference, $H = 12$ m

Length of pipe 1, $L_1 = 300$ m

Length of pipe 2, $L_2 = 170$ m

Length of pipe 3, $L_3 = 210$ m

Diameter of pipe 1, $D_1 = 0.30$ m

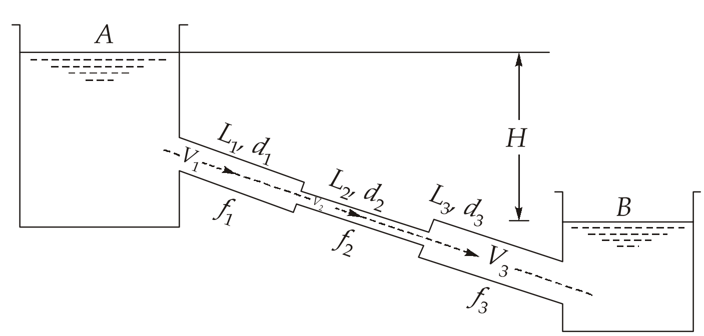
Diameter of pipe 2, $D_2 = 0.20$ m

Diameter of pipe 3, $D_3 = 0.40$ m

Coefficient of friction for pipe 1, $f_1 = 0.005$

Coefficient of friction for pipe 2, $f_2 = 0.0052$

Coefficient of friction for pipe 3, $f_3 = 0.0048$



Let the discharge through the pipes be Q. Since the pipes are connected in series, the discharge is the same through all pipes. From continuity,

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

The velocities in terms of discharge are

$$V_1 = \frac{Q}{\frac{\pi}{4}(0.3)^2}, V_2 = \frac{Q}{\frac{\pi}{4}(0.2)^2}, V_3 = \frac{Q}{\frac{\pi}{4}(0.4)^2}$$

Expressing velocities in terms of V_2,

$$V_1 = V_2\left(\frac{D_2}{D_1}\right)^2 = V_2\left(\frac{0.2}{0.3}\right)^2 = 0.444\ V_2$$

$$V_3 = V_2\left(\frac{D_2}{D_3}\right)^2 = V_2\left(\frac{0.2}{0.4}\right)^2 = 0.25\ V_2$$

(i) Considering minor losses

The total head loss is equal to the sum of entrance loss, friction losses, contraction loss, expansion loss, and exit loss. Therefore,

$$12 = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2gD_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gD_2} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2gD_3} + \frac{V_3^2}{2g}$$

Substituting $V_1 = 0.444\ V_2$ and $V_3 = 0.25\ V_2$,

$$12 = \frac{V_2^2}{2g}\left[0.5(0.444)^2 + \frac{4(0.005)(300)(0.444)^2}{0.3} + 0.5 + \frac{4(0.0052)(170)}{0.2} + (1-0.25)^2 + \frac{4(0.0048)(210)(0.25)^2}{0.4} + (0.25)^2\right]$$

$\Rightarrow$

$$12 = \frac{V_2^2}{2\times 9.81}\times 23.476$$

$$V_2 = 3.167 \text{ m/s}$$

The discharge is

$$Q = A_2V_2 = \frac{\pi}{4}(0.2)^2(3.167) = 0.0995 \text{ m}^3/\text{s}$$

(ii) Neglecting minor losses

When minor losses are neglected, only friction losses are considered. Hence,

$$12 = \frac{4f_1L_1V_1^2}{2gD_1} + \frac{4f_2L_2V_2^2}{2gD_2} + \frac{4f_3L_3V_3^2}{2gD_3}$$

Substituting $V_1 = 0.444V_2$ and $V_3 = 0.25V_2$,

$$12 = \frac{V_2^2}{2g}\left(\frac{4\times0.005\times300\times0.444^2}{0.3}+\frac{4\times0.0052\times170}{0.2}+\frac{4\times0.0048\times210\times0.25^2}{0.4}\right)$$

$$\Rightarrow \qquad 12 = \frac{V_2^2}{2\times9.81}\times22.253$$

$$\Rightarrow \qquad V_2 = 3.253 \text{ m/s}$$

The discharge is

$$Q = \frac{\pi}{4}(0.2)^2\,(3.253) = 0.102 \text{ m}^3\text{/s}$$

$$\Rightarrow \qquad \%\text{ error in discharge} = \frac{0.102-0.0995}{0.0995}\times100$$

$$= 2.512\%$$

6. (b) (i) Solution:

Thrust line or C-line: The thrust line in a prestressed beam is the locus of points through which the resultant compressive force acts at different sections along the span.

In a prestressed member, the internal force at any section consists of:

- A direct compressive force P due to prestressing
- A bending moment M due to external loads

These two combine to form a single resultant compressive force acting at some distance from the neutral axis. The line joining the positions of this resultant force along the length of the beam is called the thrust line or C-line (pressure line).

Mathematically, the distance of the thrust line from the neutral axis at any section is

$$y = e+\frac{M}{P}$$

where,

e = eccentricity of prestressing force from neutral axis

M = bending moment due to external loads at that section

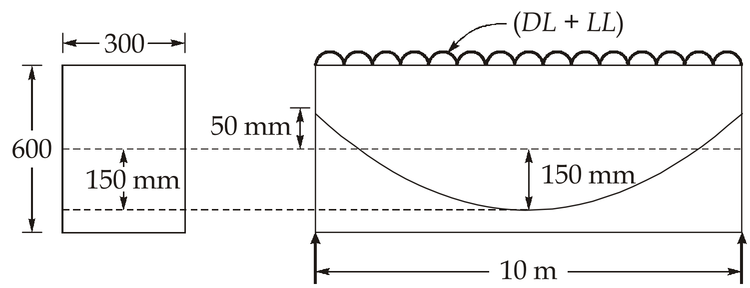
P = prestressing force

Physical meaning:

- If the thrust line lies within the kern (middle third for rectangular section), the entire section remains in compression.
- If it moves outside the kern, tensile stresses develop.
- The thrust line represents the actual line of action of the resultant compressive force after considering both prestress and applied loads.

6. (b) (ii) Solution:

Given data



$$\text{Width of beam } (b) = 300 \text{ mm}$$
$$\text{Depth of beam } (D) = 600 \text{ mm}$$
$$\text{Span } (L) = 10 \text{ m}$$
$$\text{Prestressing force } (P) = 1000 \text{ kN}$$
$$\text{Eccentricity at supports } (e_1) = 50 \text{ mm (above neutral axis)}$$
$$\text{Eccentricity at mid-span } (e_2) = 150 \text{ mm (below neutral axis)}$$
$$\text{Live load } (W_{LL}) = 10 \text{ kN/m}$$
$$\text{Concrete density} = 24 \text{ kN/m}^3$$

Self-weight and Total Load

$$W_{DL} = b \times D \times \text{density} = 0.3 \times 0.6 \times 24 = 4.32 \text{ kN/m}$$
$$w = W_{DL} + W_{LL} = 4.32 + 10 = 14.32 \text{ kN/m}$$

Shift of C-line with respect to cable profile at section x-x.

$$a_{xx} = \frac{M_{xx}}{P}$$

$$\Rightarrow \quad a_{xx} = \frac{\frac{wx}{2}(L-x)}{P} = \frac{\frac{14.32x}{2}(10-x)}{1000}$$

$$\Rightarrow \quad a_{xx} = 7.16 \times 10^{-3} x\,(10-x) \text{ meter}$$

$$a_{xx} = 7.16x(10-x) \text{ mm}$$

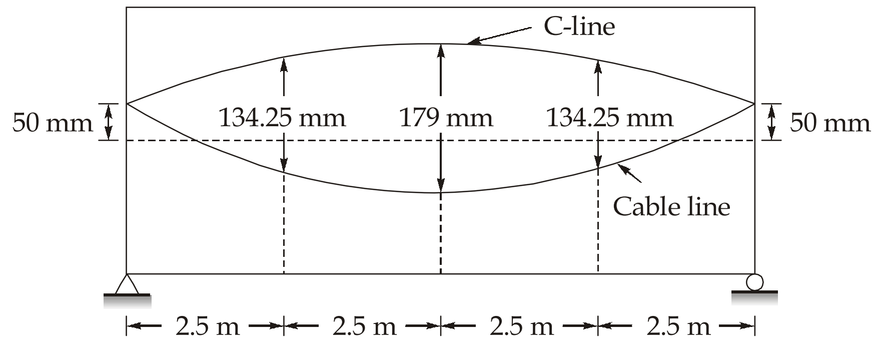
at support, $x = 0$, $a_{(x = 0\text{ m})} = 0$ mm

at $x = 2.5$ m, $a_{(x = 2.5\text{ m})} = 134.25$ mm

at $x = 5$ m, $a_{(x = 5\text{ m})} = 179$ mm

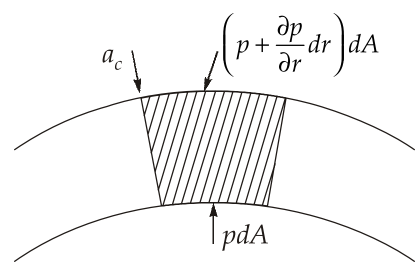
at $x = 7.5$ m, $a_{(x = 7.5\text{ m})} = 134.25$ mm

at $x = 10$ m, $a_{(x = 10\text{ m})} = 0$ mm



6. (c) (i) Solution:

Let us assume a differential fluid element at a distance r from the centre, having a thickness of dr and mass dm.



Force analysis:

$$\left(p+\frac{\partial p}{\partial r}dr\right)dA - pdA = dma_c \quad \text{...(i)}$$

where a_c is centripetal acceleration

$$\therefore \quad \frac{\partial p}{\partial r}drdA = \rho dAdr\omega^2 r \quad \text{...(ii)}$$

[$\because d_m = \rho\ dAdr,\ ac = \omega^2 r$]

$$\Rightarrow \quad \frac{\partial p}{\partial r} = \rho\omega^2 r \quad \text{...(iii)}$$

As we know, $p = f(r, z)$

$$\Rightarrow \quad dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz$$

$$\Rightarrow \quad dp = \rho\omega^2 rdr - \rho gdz \quad \text{...(iv)} \left\{\frac{\partial p}{\partial r} = \rho\omega^2 r, \frac{\partial p}{\partial z} = -\rho g\right\}$$

Equation of isobars in forced vortex motion,

So from (iv), $dp = 0$ [$\because$ At isobars pressure change is zero]

$$\Rightarrow \qquad \omega^2 r dr = g dz$$

Since, ω is constant in forced vortex motion

$$\therefore \qquad \omega^2 \int_{r=0}^{r=r} r.dr = g\int_{z=0}^{z=h} dz$$

$$\Rightarrow \qquad \frac{\omega^2 r^2}{2} = gh$$

$$\Rightarrow \qquad h = \frac{w^2 r^2}{2g} \qquad \text{...(v)}$$

$$\therefore \qquad h \propto r^2$$

Take a differential element of width '*dr*' at a distance '*r*' from centre. The height '*h*' of *t*

Volume of paraboloid:

$$dV = \pi r^2 dh$$

From (v),
$$r^2 = 2g\frac{h}{\omega^2}$$

$$\therefore \qquad \int dV = \frac{2\pi g}{\omega^2}\int_0^H h.dh$$

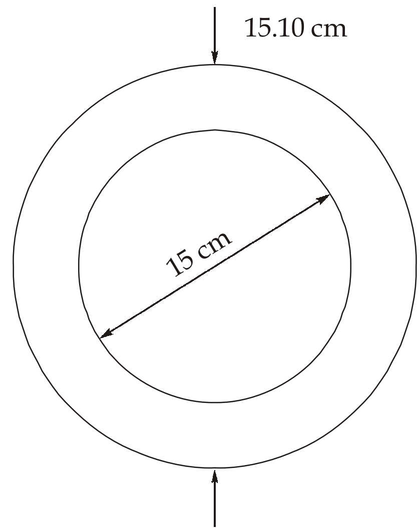
$$\Rightarrow \qquad V = \frac{2\pi g}{\omega^2}\left[\frac{h^2}{2}\right]_0^H = \frac{\pi H^2 g}{\omega^2}$$

$$= \pi\frac{\omega^2 R^2}{2g}H\frac{g}{\omega^2} \qquad \left[H = \frac{\omega^2 R^2}{2g}\right]$$

$$\therefore \qquad V = \frac{\pi R^2 H}{2}$$

$$\therefore \qquad \text{Volume of paraboloid} = \frac{1}{2}\times \text{Vol. circumscribing cylinder}$$ Hence proved.

6. (c) (ii) Solution:



Given data: Torque = 12 Nm, $r = \frac{15}{2} = 7.5\,\text{cm}$

N = 100 rpm, l = 25 cm

Given linear velocity profile within the thin oil film and thus viscous shear stress is,

$$\tau = \frac{\mu du}{dy} = \mu \times \frac{u}{t}$$

Viscous resistance or viscous force

= Shear stress × Area

$$= \mu \times \frac{u}{t} \times (2\pi r l)$$

Viscous torque = Viscous force × Torque arm

The torque arm equals the radius of the cylinder which is rotating.

$\therefore$ Viscous torque, $$T = \mu \times \frac{u}{t} \times (2\pi r l) \times r \qquad \text{...(i)}$$

$$t = \frac{15.1 - 15}{2} = 0.05\,\text{cm} = 0.0005\,\text{m}$$

$$r = \frac{15}{2} = 7.5\,\text{cm} = 0.075\,\text{m}$$

$$u = \frac{2\pi r N}{60} = \frac{2 \times \pi \times 0.075 \times 100}{60} = 0.785\,\text{m/s}$$

Using eq. (i) $$12 = \mu \times \frac{0.785}{0.0005} \times (2\pi \times 0.075 \times 0.25) \times 0.075$$

$\Rightarrow$ $$\mu = 0.865 \text{ Ns/m}^2$$

$\Rightarrow$ $$\mu = 8.65 \text{ poise}$$

7. (a) (i) Solution:

Given Data

Head over orifice, $H = 6$ m

Diameter of orifice, $d = 150$ mm $= 0.15$ m

Area of orifice,

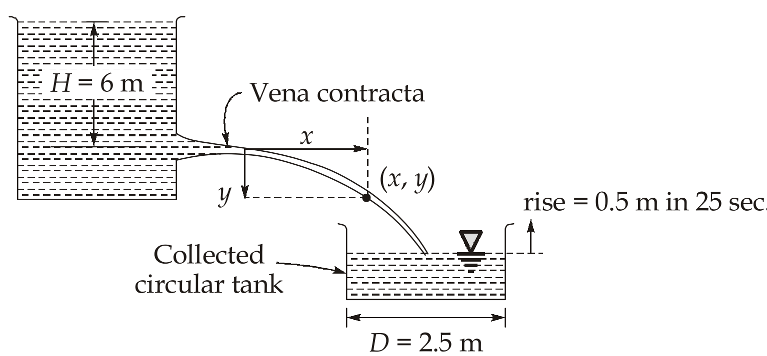
Diameter of measuring tank, $D = 2.5$ m

Rise of water level in tank, $h = 0.5$ m

Time of collection, $t = 25\ s$

Horizontal distance of jet, $x = 120$ cm $= 1.2$ m

Vertical distance of jet, $y = 6.5$ cm $= 0.065$ m



The actual discharge is obtained from the volume of water collected in the measuring tank per unit time.

$$Q_{act} = \frac{\text{Volume collected}}{\text{Time}}$$

$\Rightarrow$ $$Q_{act} = \frac{\frac{\pi}{4}D^2 h}{t}$$

$\Rightarrow$ $$Q_{act} = \frac{\frac{\pi}{4}(2.5)^2 \times 0.5}{25}$$

$\Rightarrow$ $$Q_{act} = 0.09818 \text{ m}^3/\text{s}$$

The theoretical discharge through the orifice is

$$Q_{\text{th}} = a\sqrt{2gH}$$

$$\Rightarrow \quad Q_{\text{th}} = \frac{\pi}{4}(0.15)^2 \times \sqrt{2 \times 9.81 \times 6}$$

$$\Rightarrow \quad Q_{\text{th}} = 0.1917 \text{ m}^3/\text{s}$$

The coefficient of discharge is

$$C_d = \frac{Q_{\text{act}}}{Q_{\text{th}}}$$

$$\Rightarrow \quad C_d = \frac{0.09818}{0.1917}$$

$$\Rightarrow \quad C_d = 0.512$$

The coefficient of velocity is determined using the jet trajectory equation,

$$x = v_a$$

where, v_a = Actual velocity

$$y = \frac{1}{2}gt^2$$

$$\therefore \quad \frac{x^2}{y} = \frac{v_a^2}{\frac{1}{2}gt^2} = \frac{2v_a^2}{g}$$

Actual velocity, $v_a = \sqrt{\dfrac{gx^2}{2y}}$

Theoritical velocity, $v_{th} = \sqrt{2gH}$

Now, coefficient of velocity, $c_v = \dfrac{v_a}{v_{th}} = \sqrt{\dfrac{gx^2}{2y \times 2gH}}$

$$\Rightarrow \quad C_v = \sqrt{\frac{x^2}{4yH}}$$

$$\Rightarrow \quad C_v = \sqrt{\frac{(1.2)^2}{4 \times 0.065 \times 6}}$$

$$\Rightarrow \quad C_v = 0.961$$

The coefficient of contraction is obtained from the relation

$$C_d = C_c C_v$$

$$\Rightarrow \quad C_c = \frac{C_d}{C_v} = \frac{0.512}{0.961}$$

$$C_c = 0.533$$

The coefficient of discharge is $C_d = 0.512$, the coefficient of velocity is $C_v = 0.961$, and the coefficient of contraction is $C_c = 0.533$.

7. (a) (ii) Solution:

Given data

Velocity profile: $$\frac{u}{u_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

The displacement thickness is defined as:

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy$$

Substitute the given velocity profile and introduce $\eta = \frac{y}{\delta}(dy = \delta\, d\eta)$:

$$\Rightarrow \quad \delta^* = \int_0^1 \left[1 - \left(2\eta - \eta^2\right)\right] \delta\, d\eta$$

$$\Rightarrow \quad \delta^* = \delta\left[\eta - \eta^2 + \frac{\eta^3}{3}\right]_0^1$$

$$\Rightarrow \quad \delta^* = \delta\left(1 - 1 + \frac{1}{3}\right)$$

$$\Rightarrow \quad \delta^* = \delta \times \frac{1}{3} \approx 0.333\,\delta$$

Ratio of displacement thickness to boundary layer thickness:

$$\Rightarrow \quad \frac{\delta^*}{\delta} = 0.333$$

Momentum thickness:

$$\theta = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$$

$$\Rightarrow \quad \theta = \int_0^1 \left(2\eta - \eta^2\right)\left(1 - 2\eta + \eta^2\right) \delta\, d\eta$$

$$\Rightarrow \quad \theta = \delta\int_0^1 \left(2\eta - \eta^2 - 4\eta^2 + 2\eta^3 + 2\eta^3 - \eta^4\right) d\eta$$

$$\Rightarrow \quad \theta = \delta\int_0^1 \left(2\eta - 5\eta^2 + 4\eta^3 - \eta^4\right) d\eta$$

$$\Rightarrow \quad \theta = \delta\left[\eta^2 - \frac{5\eta^3}{3} + \frac{4\eta^4}{4} - \frac{\eta^5}{5}\right]_0^1$$

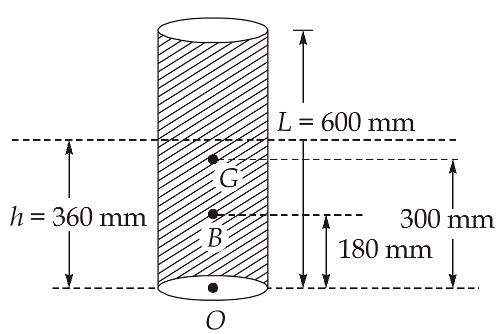
$$\Rightarrow \quad \theta = \delta\left[1 - \frac{5}{3} + 1 - \frac{1}{5}\right]$$

$$\theta = \frac{2\delta}{15}$$

7. (b) (i) Solution:

Given data

$$D = 400 \text{ mm}$$
$$R = 200 \text{ mm}$$
$$L = 600 \text{ mm}$$
$$S_s = 0.6$$
$$\rho_w = 1000 \text{ kg/m}^3$$
$$S_w = 1$$



For a floating body, weight of body equals buoyant force.

$$\text{Weight of cylinder} = \frac{\pi D^2}{4} \times L \times S_s \times \rho_w g$$

$$\text{Buoyant force} = \frac{\pi D^2}{4} \times h \times S_w \times \rho_w g$$

For floating $\quad w = F_B$

$$\Rightarrow \quad h = L \times \frac{S_s}{S_w}$$

$$\Rightarrow \quad h = 600 \times 0.6$$

$$\Rightarrow \quad h = 360 \text{ mm}$$

Distance of center of gravity from base,

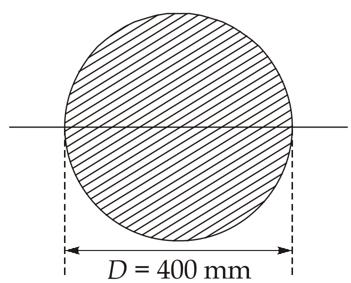
$$OG = \frac{L}{2} = \frac{600}{2} = 300 \text{ mm}$$

Distance of center of buoyancy from base,

$$OB = \frac{h}{2} = \frac{360}{2} = 180 \text{ mm}$$

$$BG = OG - OB = 300 - 180 = 120 \text{ mm}$$

Moment of inertia of circular water-line area,



$$I = \frac{\pi D^4}{64} = \frac{\pi \times 400^4}{64}$$

$$\Rightarrow \quad I = 1256.637 \times 10^6 \text{ mm}^4$$

Volume of displaced water,

$$V = \frac{\pi D^2}{4} \times h$$

$$\Rightarrow \quad V = \frac{\pi \times 400^2}{4} \times 360$$

$$\Rightarrow \quad V = 45238934.212 \text{ mm}^3$$

Distance BM is

$$BM = \frac{I}{V}$$

$$\Rightarrow \quad BM = \frac{1256.637 \times 10^6}{45238934.212}$$

$$\Rightarrow \qquad BM = 27.778 \text{ mm}$$

Metacentric height,

$$GM = BM - BG$$

$$\Rightarrow \qquad GM = 27.778 - 120$$

$$\Rightarrow \qquad GM = -92.222 \text{ mm}$$

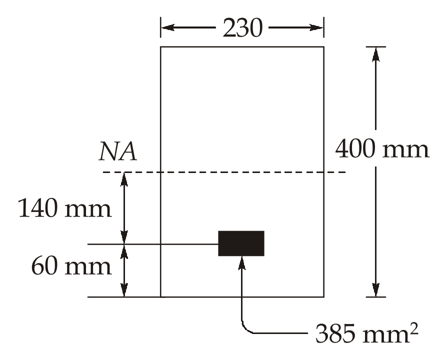
Since the metacentric height is negative, the metacenter lies below the center of gravity and the equilibrium is unstable.

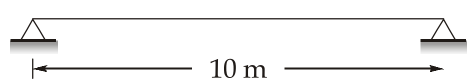
7. (b) (ii) Solution:

Given data

$$E_s = 2.1 \times 10^5 \text{ N/mm}^2$$

$$E_c = 5000\sqrt{f_{ck}} = 5000\sqrt{50} = 35355.339 \text{ N/mm}^2$$





$$p_0 = 1200 \text{ N/mm}^2$$

$$e = \frac{D}{2} - 60 = 140 \text{ mm}$$

$$A = b \times D = 230 \times 400 = 92000 \text{ mm}^2$$

$$I = \frac{bD^3}{12} = \frac{230 \times 400^3}{12} = 1.2267 \times 10^9 \text{ mm}^4$$

$$m = \frac{E_s}{E_c} = \frac{2.1 \times 10^5}{35355.339} = 5.94$$

Short-term losses:

(a) Friction loss, (assuming jacking from one end):

$$\Delta p = p_o(kx + \mu\theta)$$

Since $\theta = 0°, x = L$

$$\Delta p = p_o \times (k \times L) = 1200 \times (0.0025 \times 10) = 30 \text{ N/mm}^2$$

(b) Anchorage slip loss,

$$\Delta p = \frac{\Delta L}{L} \times E_s = \frac{2}{10000} \times 2.1 \times 10^5 = 42 \text{ N/mm}^2$$

Total short-term loss,

$$\Delta p = 30 + 42 = 72 \text{ N/mm}^2$$

Effective prestress after short-term losses:

$$p_e = 1200 - 72 = 1128 \text{ N/mm}^2$$

Effective prestressing force, p_e

$$P_{eff} = p_e \times A_s = 1128 \times 385 = 434280 \text{ N}$$

Stress in concrete at the level of steel:

$$f_c = \frac{p_{eff}}{A} + \frac{P_{eff} \times e^2}{I} = \frac{434280}{92000} + \frac{434280 \times 140^2}{1.2267 \times 10^9} = 4.72 + 6.94 = 11.66 \text{ N/mm}^2$$

Long-term losses:

Creep loss,

$$= \phi \times m \times f_c = 1.6 \times 5.94 \times 11.66 = 110.814 \text{ N/mm}^2$$

Shrinkage loss,

$$= \varepsilon_{sh} \times E_s = 0.0003 \times 2.1 \times 10^5 = 63 \text{ N/mm}^2$$

Relaxation loss,

$$= 0.045 \times \sigma_{pe} = 0.045 \times 1128 = 50.76 \text{ N/mm}^2$$

Total loss and percentage loss:

$$\sigma_{total} = 72 + 110.814 + 63 + 50.76 = 296.574 \text{ N/mm}^2$$

Percentage loss,

$$\%\text{Loss} = \frac{\sigma_{total}}{\sigma_{pi}} \times 100 = \frac{296.574}{1200} \times 100 = 24.715\%$$

The final percentage loss of prestress is 24.715%.

7. (c) Solution:

Given Data:

$$\text{Clear span } (l_x) = 4 \text{ m}$$

$$\text{Longer span } (l_y) = 10 \text{ m}$$

$$\text{Support width } (w) = 350 \text{ mm} = 0.35 \text{ m}$$

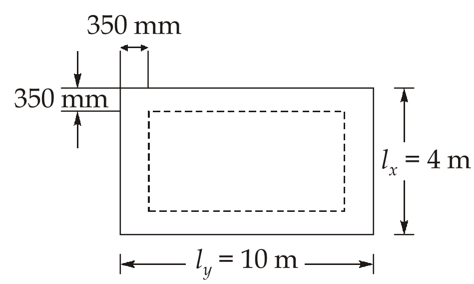
$$\text{Live load } (LL) = 3 \text{ kN/m}^2$$

$$\text{Floor finish } (FF) = 0.75 \text{ kN/m}^2$$

Characteristic strength of concrete (f_{ck}) = 25 N/mm^2

Characteristic strength of steel (f_y) = 415 N/mm^2

$$\text{Unit weight of RCC} = 25 \text{ kN/m}^3$$



Type of Slab

$$\text{Ratio of spans} = \frac{l_y}{l_x} = \frac{10}{4} = 2.5$$

Since the ratio is greater than 2, the slab is designed as a one-way slab.

Step 1: Preliminary Depth and Effective Span

For a simply supported slab, the basic $\frac{\text{span}}{\text{depth}}$ ratio is 20.

Assuming a modification factor (MF) of 1.2 for tension reinforcement:

$$\text{Required } d \approx \frac{4000}{20 \times 1.2} = 166.67 \text{ mm}$$

Let us assume an overall depth (D) of 200 mm.

Using 20 mm clear cover and 10 mm diameter bars:

$$\text{Effective depth } (d) = 200 - 20 - \frac{10}{2} = 175 \text{ mm}$$

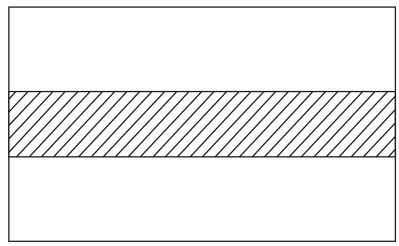
Effective span (l_{eff}) is the lesser of:

$$l_c + d = 4 + 0.175 = 4.175 \text{ m}$$

$$l_c + w = 4 + 0.35 = 4.35 \text{ m}$$

$$\therefore \quad l_{eff} = 4.175 \text{ m}$$

Step 2: Load Calculation (per meter width)



$$\text{Self-weight of slab} = 0.2 \times 25 \times 1 = 5 \text{ kN/m}$$

$$\text{Floor finish} = 0.75 \times 1 = 0.75 \text{ kN/m}$$

$$\text{Live load} = 3 \times 1 = 3 \text{ kN/m}$$

$$\text{Total load } (w) = 5 + 0.75 + 3 = 8.75 \text{ kN/m}$$

$$\text{Factored load } (w_u) = 1.5 \times 8.75 = 13.125 \text{ kN/m}$$

Step 3: Bending Moment and Shear Force

$$\text{Ultimate Moment } (M_u) = \frac{w_u \times l_{eff}^2}{8} = \frac{13.125 \times 4.175^2}{8} = 28.598 \text{ kNm}$$

$$\text{Ultimate Shear } (V_u) = \frac{w_u \times l_{eff}}{2} = \frac{13.125 \times 4.175}{2} = 27.398 \text{ kN}$$

Step 4: Check for Depth

For Fe-415,

$$M_{u,\lim} = 0.138 \times f_{ck} \times b \times d^2$$

$$28.598 \times 10^6 = 0.138 \times 25 \times 1000\, d^2$$

$$d_{req} = 91.01 \text{ mm} < 175 \text{ mm}$$

Step 5: Main reinforcement

$$M_u = 0.87 f_y A_{st} \left(d - \frac{f_y A_{st}}{f_{ck} b} \right)$$

$$28.598 \times 10^6 = 0.87 \times 415 \times A_{st} \times \left(175 - \frac{415 \times A_{st}}{25 \times 1000} \right)$$

Solving the quadratic equation for A_{st}:

$$A_{st} = 478.435 \text{ mm}^2$$

Using 10 mm dia bars (area = 78.54 mm^2):

$$\text{Spacing} = \frac{78.54 \times 1000}{478.435} = 164.16 \text{ mm}$$

Provide 10 mm dia bars @ 160 mmc/c.

$$A_{st,\text{ prov}} = \frac{1000 \times 78.54}{160} = 490.875 \text{ mm}^2$$

Step 6: Distribution Reinforcement

$$A_{st,\text{ dist}} = 0.12\% \text{ of bD} = 0.0012 \times 1000 \times 200 = 240 \text{ mm}^2$$

Using 8 mm dia bars (area = 50.265 mm^2):

$$\text{Spacing} = \frac{50.265 \times 1000}{240} = 209.438 \text{ mm}$$

Provide 8 mm dia bars @ 200 mm c/c.

Step 7: Check for Shear

$$\text{Nominal shear stress } (\tau_v) = \frac{V_u}{bd} = \frac{27398}{1000 \times 175} = 0.157 \text{ N/mm}^2$$

$$\text{Percentage steel } (p_t) = \frac{100 \times 490.875}{1000 \times 175} = 0.281\%$$

From provided table,

$$\text{For } p_t = 0.25, \tau_c = 0.36 \text{ N/mm}^2$$

$$\text{For } p_t = 0.50, \tau_c = 0.49 \text{ N/mm}^2$$

By interpolation:

$$\tau_c = 0.36 + \frac{0.49 - 0.36}{0.5 - 0.25} \times (0.281 - 0.25) = 0.376 \text{ N/mm}^2$$

Since $\tau_v < \tau_c$

The slab is safe in shear.

Step 8: Check for Deflection

$$\text{Service stress } (f_s) = 0.58 \times f_y \times \frac{A_{st,req}}{A_{st,prov}} = 0.58 \times 415 \times \frac{478.435}{490.875}$$

$$= 234.582 \text{ N/mm}^2$$

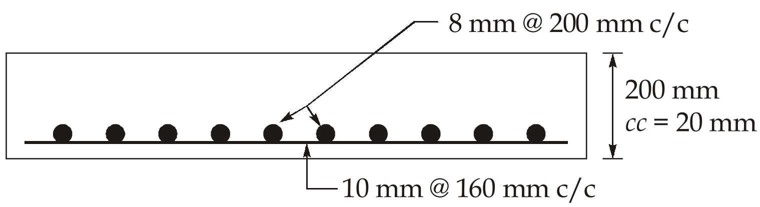
From the provided chart, for $p_t = 0.281\%$ and $f_s \approx 234.582$,

$$MF \approx 1.5$$

$$\text{Allowable } \frac{span}{d} = 20 \times 1.5 = 30$$

$$\text{Actual } \frac{span}{d} = \frac{4175}{175} = 23.857$$

Since Actual < Allowable, the slab is safe in deflection.



8. (a) Solution:

Given data

$$D = 2 \text{ m}$$

$$A = \frac{\pi D^2}{4} = \frac{\pi(2)^2}{4} = 3.141 \text{ m}^2$$

$$d = 0.15 \text{ m}$$

$$a = \frac{\pi d^2}{4} = \frac{\pi(0.15)^2}{4} = 0.0177 \text{ m}^2$$

$$Q_{\text{in}} = 0.15 \text{ m}^3/\text{s}$$

$$C_d = 0.62$$

$$h_1 = 1 \text{ m}$$

$$h_2 = 3 \text{ m}$$

$$g = 9.81 \text{ m/s}^2$$

Discharge through orifice at head h is

$$Q_{\text{out}} = C_d a\sqrt{2gh}$$

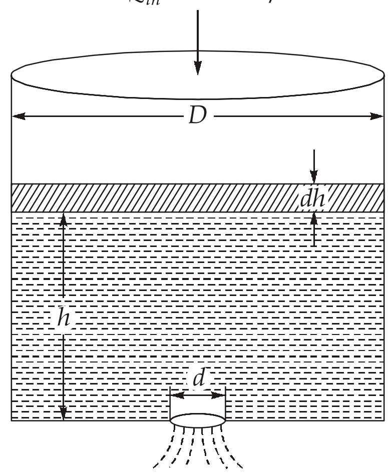
Let

$$K = C_d a\sqrt{2g}$$

$$\Rightarrow \quad K = 0.62 \times \frac{\pi(0.15)^2}{4} \times \sqrt{2 \times 9.81}$$

$$\Rightarrow \quad K = 0.04853$$

Net rate of rise of water level is obtained from continuity equation ..



$$A\frac{dh}{dt} = Q_{in} - K\sqrt{h}$$

$$dt = \frac{A}{Q_{in} - K\sqrt{h}}dh$$

Total time required is

$$T = \int_{h_1}^{h_2} \frac{A}{Q_{in} - K\sqrt{h}}dh$$

Let

$$u = \sqrt{h}$$

$$h = u^2$$

$$dh = 2\,u\,du$$

When $h = 1, u = 1$

$$h = 3, u = \sqrt{3}$$

Thus, $T = \int_{1}^{\sqrt{3}} \frac{A \times 2u}{Q_{in} - Ku}du$

$\Rightarrow$ $T = \int_{1}^{\sqrt{3}} \frac{\frac{\pi(2)^2}{4} \times 2u}{0.15 - 0.04853 \times u}du$

Integrating, $T = 77.682$ sec

The time required for the water level to rise from 1 m to 3 m is 77.682 seconds.

8. (b) Solution:

Given Data:

Effective short span (L_x) = 3.5 m

Effective long span (L_y) = 5.25 m

Aspect ratio (L_y/L_x) = 1.5

Total design load (w_u) = 25 kN/m^2

Effective depth (d) = 150 mm

Diameter of bars (ϕ) = 10 mm

Area of one 10 mm bar (a_{st}) = 78.54 mm^2

Moment Coefficients and Design Moments

For an interior panel with L_y/L_x = 1.5, the coefficients (α) are taken from Table 26 of IS 456:

Location	Short Span Coefficient (α_x)	Long Span Coefficient (α_y)
Negative (at continuous edge)	0.053	0.032
Positive (at mid-span)	0.041	0.024

Short Span Moments $(M_u = \alpha_x \times w \times L_x^2)$

$$M_{ux(-)} = -0.053 \times 25 \times 3.5^2 = -16.231 \text{ kNm}$$

$$M_{ux(+)} = 0.041 \times 25 \times 3.5^2 = 12.556 \text{ kNm}$$

Long Span Moments $(M_u = a_y \times w \times L_x^2)$

$$M_{uy\ (-)} = -0.032 \times 25 \times 3.5^2 = -9.8 \text{ kNm}$$

$$M_{uy\ (+)} = 0.024 \times 25 \times 3.5^2 = 7.35 \text{ kNm}$$

Reinforcement Calculations (Short Span)

Negative Reinforcement (Top):

$$\frac{M_u}{bd^2} = \frac{16.231 \times 10^6}{1000 \times 150^2} = 0.721 \text{ N/mm}^2$$

From Table 3- by linear interpolation, p_t = 0.207%

$$A_{st} = \frac{0.207 \times 1000 \times 150}{100} = 310.5 \text{ mm}^2$$

$$\text{Spacing} = \frac{78.54 \times 1000}{310.5} = 252.947 \text{ mm}$$

Provide 10 mm ϕ bars @ 250 mm c/c at Top.

Positive Reinforcement (Bottom):

$$\frac{M_u}{bd^2} = \frac{12.556\times10^6}{1000\times150^2} = 0.558 \text{ N/mm}^2$$

From Table 3, p_t = 0.158%

$$A_{st} = \frac{0.158\times1000\times150}{100} = 237 \text{ mm}^2$$

$$\text{Spacing} = \frac{78.54\times1000}{237} = 331.392 \text{ mm}$$

$$\text{Limit} = 300 \text{ mm}$$

Provide 10 mm ϕ bars @ 300 mm c/c at Bottom.

Reinforcement Calculations (Long Span)

Negative Reinforcement (Top):

$$\frac{M_u}{bd^2} = \frac{9.8\times10^6}{1000\times150^2} = 0.436 \text{ N/mm}^2$$

From Table 3, p_t = 0.123%

$$A_{st} = \frac{0.123\times1000\times150}{100} = 184.5 \text{ mm}^2$$

$$\text{Spacing} = \frac{78.54\times1000}{184.5} = 425.691 \text{ mm}$$

$$\text{Limit} = 300 \text{ mm}$$

Provide 10 mm ϕ bars @ 300 mm c/c at Top.

Positive Reinforcement (Bottom):

$$\frac{M_u}{bd^2} = \frac{7.35\times10^6}{1000\times150^2} = 0.327 \text{ N/mm}^2$$

From table 3 – required r/f is 0.099 % which is less than minimum r/f of 0.12 percent, so Provide min R/f.

$$\text{Check Minimum } p_t = 0.12\% \text{ for Fe 415:}$$

$$A_{st} = \frac{0.12\times1000\times150}{100} = 180 \text{ mm}^2$$

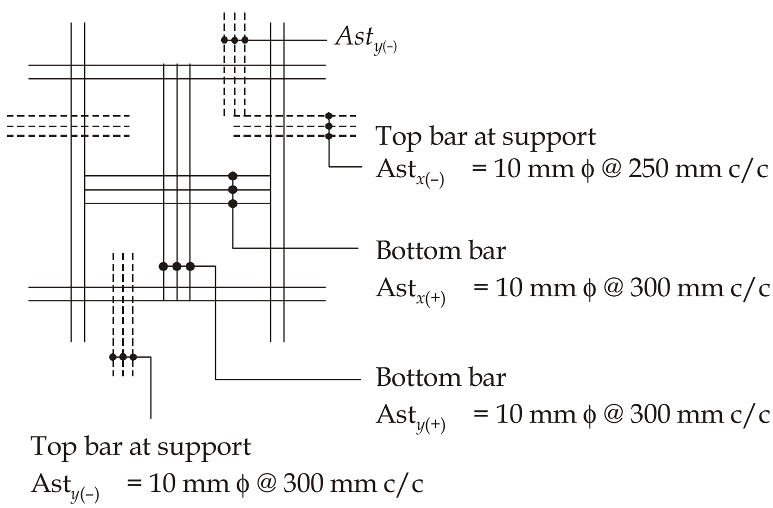
$$\text{Spacing} = \frac{78.54\times1000}{180} = 436.333 \text{ mm}$$

Limit = 300 mm.

Provide 10 mm ϕ bars @ 300 mm c/c at Bottom.

Summary of Main Reinforcement

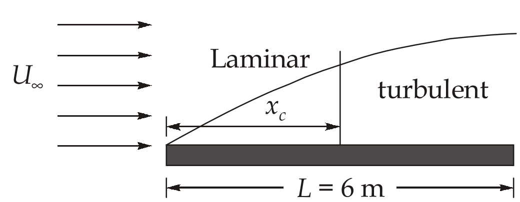
Span	Moment Type	Provided Spacing (mm)	Vertical Position
Short (L_x)	Negative	250	Top (over support)
Short (L_x)	Positive	300	Bottom (mid-span)
Long (L_y)	Negative	300	Top (over support)
Long (L_y)	Positive	300	Bottom (mid-span)



8. (c) Solution:

Given data

$$L = 6 \text{ m}$$
$$b = 1.5 \text{ m}$$
$$U = 6 \text{ m/s}$$
$$\rho = 1.205 \text{ kg/m}^3$$
$$\mu = 1.81 \times 10^{-5} \text{ Pa-s}$$
$$Re_c = 5 \times 10^5$$



The critical Reynolds number is given by

$$Re_c = \frac{\rho U x_c}{\mu}$$

$$\Rightarrow \quad 5 \times 10^5 = \frac{1.205 \times 6 \times x_c}{1.81 \times 10^{-5}}$$

$$\Rightarrow \quad x_c = \frac{5 \times 10^5 \times 1.81 \times 10^{-5}}{1.205 \times 6}$$

$$\Rightarrow \quad x_c = 1.252 \text{ m}$$

For the laminar portion from 0 to x_c, the average drag coefficient is

$$C_{D,\text{lam}} = \frac{1.328}{\sqrt{\text{Re}_c}} = \frac{1.328}{\sqrt{5 \times 10^5}}$$

$$\Rightarrow \quad C_{D,\text{lam}} = 0.001878$$

Laminar drag force is

$$F_{D,\text{ lam}} = C_{f,\text{ lam}} \times \frac{1}{2}\rho U^2 \times (x_c \times b)$$

$$\Rightarrow \quad F_{D,\text{ lam}} = 0.001878 \times 0.5 \times 1.205 \times 36 \times (1.252 \times 1.5)$$

$$\Rightarrow \quad F_{D,\text{ lam}} = 0.0765 \text{ N}$$

Reynolds number at the trailing edge is

$$Re_L = \frac{\rho U L}{\mu}$$

$$\Rightarrow \quad Re_L = \frac{1.205 \times 6 \times 6}{1.81 \times 10^{-5}}$$

$$\Rightarrow \quad Re_L = 2396685.083$$

Average turbulent drag coefficient over full length is

$$C_{D,\text{ turb},\text{ }L} = \frac{0.074}{(\text{Re}_L)^{1/5}}$$

$$\Rightarrow \quad C_{D,\text{ turb},\text{ }L} = \frac{0.074}{(2396685.083)^{0.2}}$$

$$\Rightarrow \quad C_{D,\text{ turb},\text{ }L} = 0.0039202$$

Total turbulent drag over full length is

$$F_{D,\ turb,\ L} = C_{f,\ turb,L} \times \frac{1}{2}\rho U^2 \times (L \times b)$$

$\Rightarrow$ $$F_{D,\ turb,\ L} = 0.0039202 \times 0.5 \times 1.205 \times 36 \times (6 \times 1.5)$$

$\Rightarrow$ $$F_{D,\ turb,\ L} = 0.765\ \text{N}$$

Average turbulent drag coefficient over transition length is

$$F_{D,\ turb,\ xc} = \frac{0.074}{(\text{Re}_c)^{1/5}}$$

$\Rightarrow$ $$F_{D,\ turb,\ xc} = \frac{0.074}{(5 \times 10^5)^{0.2}}$$

$\Rightarrow$ $$F_{D,\ turb,\ xc} = 0.0053634$$

Turbulent drag over transition length is

$$F_{D,\ turb,\ xc} = C_{f,\ turb,xc} \times \frac{1}{2}\rho U^2 \times (x_c \times b)$$

$\Rightarrow$ $$F_{D,\ turb,\ xc} = 0.0053634 \times 0.5 \times 1.205 \times 36 \times (1.252 \times 1.5)$$

$\Rightarrow$ $$F_{D,\ turb,\ xc} = 0.219\ \text{N}$$

Total drag force is $$F_D = F_{D,\ lam} + (F_{D,\ turb,\ L} - F_{D,\ turb,\ xc})$$

$\Rightarrow$ $$F_D = 0.0765 + (0.765 - 0.219)$$

$\Rightarrow$ $$F_D = 0.6225\ \text{N}$$