MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

E & T Engineering Test No : 5

Section A : Computer Organization and Architecture + Materials Science

Q.1 (a) Solution:

Given, data:

$$\text{Data transfer rate } (R) = 100 \text{ MBps}$$
$$= 100 \times 10^6 \text{ bytes/sec}$$
$$\text{Word size } (W) = 32 \text{ bits} = 4 \text{ bytes}$$
$$\text{Transfer block size } (n) = 8 \text{ words}$$
$$= 8 \times 4$$
$$= 32 \text{ bytes}$$
$$\text{Machine cycle time } (T_{cyc}) = 5 \text{ ns}$$
$$= 5 \times 10^{-9} \text{ s}$$

(i) Percentage of CPU time consumed by the DMA operation

$$= \left(\frac{Y}{X+Y}\right) \times 100$$

where Y is data transfer time

and X is preparation time of data

To calculate preparation time of data (i.e., X) = ?

From 100 MBps, we need to prepare 8 word data

$$\text{Data size} = 8 \text{ Words}$$
$$= 8 \times 32 \text{ bits}$$
$$= 8 \times 4\text{B}$$
$$= 32 \text{ B}$$

Now, we need to prepare 32 B from 100 MBps

100 MB ________ 1 sec

32 B ________ ?

$$\text{Preparation time } (X) = \frac{32B}{100 \text{ MB}} \text{sec}$$
$$= 0.32 \ \mu\text{sec}$$
$$= 320 \text{ nsec}$$
$$X = 320 \text{ nsec}$$

Y : Data size is 8 words

Therefore, 8 machine cycles are required

$$Y = 8 * 5 \text{ nsec}$$
$$= 40 \text{ nsec}$$

$$\% \text{ time CPU consumed} = \left(\frac{Y}{X+Y}\right)\times 100$$
$$= \frac{40n\,\text{sec}}{(320+40)n\,\text{sec}}\times 100 = 11.11\%$$

(ii) Percentage of time CPU remains Busy

$$= \left(\frac{X}{X+Y}\right)\times 100$$
$$= \left(\frac{320}{360}\right)\times 100 = 88.88\%$$

Q.1 (b) Solution:

64 KB cache with 128 byte lines contains

$$\frac{64 \text{ KB}}{128 \text{ B}} = 512 \text{ lines}$$

As cache is 2 way set-associative, it has

$$\frac{512}{2} = 256 \text{ sets}$$

$\therefore \quad m = \log_2 (256) = 8$ bits (Set offset)

Lines are 128 byte long, mean that $n = \log_2(128) = 7$ bits (Line offset)

Thus, $(m + n) =$ 15 bits of address are used to select a set and determine the byte within the line that an address point to.

$\therefore$ Tag field of each tag array entry is 32 – 15 = 17 bits long

Since it's a write-back cache, each line must include both a valid bit and a dirty bit. Adding 2 bits for dirty and valid bits, we get 19 bits per tag entry.

Storage in tag array = 19 × 512 lines = 9728 bits

Q.1 (c) Solution:

The Fibonacci series is the sequence where the next number is determined as the sum of the previous two numbers of the sequence.

C Program:

```
#include<stdio.h>
int main ()
{
unsigned int fib[100]; //Use integer array to store the fibonacci numbers
fib[0] = 1;
fib[1] = 1;
for (int i = 2; i < 100; i++)
{
fib[i] = fib[i – 1] + fib[i – 2]; //Generate first 100 Fibonacci numbers.
}
for (int i = 0; i < 100; i++)
{
printf ("%d\n", fib[i]); //Print the first 100 Fibonacci numbers
}
return 0;
}
```

Q.1 (d) Solution:

(i) For BCC structure, relation between atomic radius and lattice parameter is given as,

$$a = \frac{4r}{\sqrt{3}}$$

$$\therefore \quad a = \frac{4 \times 1.24}{1.732} = 2.86 \text{ Å}$$

We have, density, $\rho = \dfrac{nA}{a^3 \times N_A}$

where, n = number of atoms/unit cell = 2 for BCC structure, A = atomic weight and a^3 = volume of unit cell.

$$\rho = \frac{2 \times 50}{(2.86 \times 10^{-8})^3 \times 6.023 \times 10^{23}}$$

$$\therefore \quad \rho = 7.1 \text{g/cm}^3$$

(ii) Given, intercepts, $(3, 2, \infty)$

Reciprocals of intercepts $\left(\frac{1}{3}, \frac{1}{2}, 0\right)$

Converting to integer values, we have LCM = 6

$\therefore$ Miller indices, (2 3 0)

(iii) Given, a = 4 Å = 4×10^{-8} cm

Miller indices of the plane = $(h\ k\ l)$ = (3 2 1)

$$\text{interplanar spacing, } d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$= \frac{4 \times 10^{-8}}{\sqrt{3^2 + 2^2 + 1^2}}$$

$$d = 1.07 \times 10^{-8} \text{ cm}$$

(or) $$d = 1.07 \text{ Å}$$

Q.1 (e) Solution:

Given, Curie temperature, $T_c = 1043$ K

Magnetic moment per Fe atom = $2\ \mu_B$

given, $\mu_B = 9.27 \times 10^{-24}$ A-m^2

Lattice parameter of Fe (BCC-structure),

$$a = 0.286 \text{ nm}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$K_B = 1.38 \times 10^{-23} \text{ J/K}$$

For BCC structure,

Number of atoms per unit cell = 2

(i) Magnetic moment per atom,

$$m = 2\ \mu_B = 2 \times 9.27 \times 10^{-24}$$

$$m = 1.854 \times 10^{-23} \text{ A-m}^2$$

$$\text{unit cell volume, } V = a^3 = (0.286 \times 10^{-9})^3$$

$$\therefore \quad V = 2.34 \times 10^{-29} \text{ m}^3$$

$$\text{Atoms per } m^3, N = \frac{\text{Number of atoms per unit cell}}{\text{Volume of unit cell}} = \frac{2}{2.34 \times 10^{-29}}$$

$$\therefore \quad N = 8.55 \times 10^{28} \text{ atoms/m}^3$$

$\therefore$ Saturation magnetization,

$$M_s = N \times m$$

$$M_s = 8.55 \times 10^{28} \times 1.854 \times 10^{-23}$$

$$M_s = 1.58 \times 10^6 \text{ A/m}$$

(ii) From Curie-Weiss theory,

$$\text{Curie-constant, } C = \frac{\mu_0 N m^2}{3K_B}$$

$$C = \frac{4\pi \times 10^{-7} \times 8.55 \times 10^{28} \times (1.854 \times 10^{-23})^2}{3 \times 1.38 \times 10^{-23}}$$

$$\therefore \quad C \approx 0.9$$

(iii) We know that,

Weiss field constant,

$$\lambda = \frac{3K_B T_C}{\mu_0 N m^2} \qquad [\because T_C = C\lambda]$$

$$\lambda = \frac{3\times1.38\times10^{-23}\times1043}{4\pi\times10^{-7}\times8.55\times10^{28}\times(1.854\times10^{-23})^2}$$

$$\therefore \quad \lambda = 1170$$

Q.2 (a) Solution:

(i) In Shortest Remaining Time First (SRTF) scheduling algorithm, the process with the least time left to finish is executed first. The running process will continue until it finishes or a new process with a shorter remaining time arrives.

GANTT chart:

P_1	P_2	P_4	P_2	P_6	P_5	P_3	P_1

0 1 3 4 6 7 9 12 17

Ready Queue:

Time	Processes in Ready state (Remaining Burst Time)
0	$P_1(6)$
1	$P_1(5)$, $P_2(4)$
2	$P_1(5)$, $P_2(3)$, $P_3(3)$
3	$P_1(5)$, $P_2(2)$, $P_3(3)$, $P_4(1)$
4	$P_1(5)$, $P_2(2)$, $P_3(3)$, $P_5(2)$
5	$P_1(5)$, $P_2(1)$, $P_3(3)$, $P_5(2)$, $P_6(1)$
6	$P_1(5)$, $P_3(3)$, $P_5(2)$, $P_6(1)$

Process	Arrival Time (AT)	Burst Time (BT)	Completion Time (CT)	Turn around time (TAT = CT – AT)	Waiting Time WT = (TAT – BT)
P_1	0	6	17	17	11
P_2	1	4	6	5	1
P_3	2	3	12	10	7
P_4	3	1	4	1	0
P_5	4	2	9	5	3
P_6	5	1	7	2	1

The average waiting time is calculated as below:

$$\text{Average waiting time} = \frac{\text{Sum of waiting time of all processes}}{\text{Number of processes}}$$

$$= \frac{(11+1+7+0+3+1)}{6} = \frac{23}{6} = 3.833 \text{ ms}$$

(ii)

Concurrency	Parallelism
1. Concurrency is the task of running and managing the multiple computations at the same time.	1. While parallelism is the task of running multiple computations simultaneously.
2. Concurrency is achieved through the interleaving operation of processes on the central processing unit (CPU) or in other words by the context switching.	2. While it is achieved through multiple central processing units (CPUs).
3. Concurrency can be done by using a single processing unit.	3. While this can't be done by using a single processing unit. It needs multiple processing units.
4. Concurrency increases the amount of work finished at a time.	4. While it improves the throughput and computational speed of the system.
5. Concurrency deals lot of things simultaneously.	5. While it do lot of things simultaneously.
6. Concurrency is the non-deterministic control flow approach.	6. While it is deterministic control flow approach.
7. In concurrency, debugging is very hard.	7. While in parallelism, debugging is also hard but simple than concurrency.

Q.2 (b) Solution:

(i) Given, demagnetization curve,

$$B = 1.1 - 4.4 \times 10^{-6} H^2 \quad \text{...(i)}$$

1. At coercive field H_C, magnetic flux density becomes zero.

i.e., $B = 0$

$$\therefore \quad 1.1 - 4.4\times10^{-6} H_C^2 = 0$$

$$4.4\times10^{-6} H_C^2 = 1.1$$

$$H_C^2 = \frac{1.1}{4.4\times10^{-6}}$$

$$\therefore \quad H_C = 500 \text{ kA/m}$$

2. Maximum energy product $(BH)_{max}$

From equation (i),

$$B = (1.1 - 4.4 \times 10^{-6} H^2)$$

$$\therefore \quad BH = 1.1H - 4.4 \times 10^{-6} H^3$$

For maximum value of BH i.e, $(BH)_{max}$

$$\frac{d(BH)}{dH} = 0$$

$$\frac{d}{dH}\left[1.1H - 4.4 \times 10^{-6} H^3\right] = 0$$

$$1.1 - 4.4 \times 10^{-6} \times (3)H^2 = 0$$

$$1.1 - 1.32 \times 10^{-5} H^2 = 0$$

$$1.32 \times 10^{-5} H^2 = 1.1$$

$$H^2 = \frac{1.1}{1.32 \times 10^{-5}}$$

$$\therefore \quad H = 288.7 \text{ kA/m}$$

$$\therefore \quad B = 1.1 - 4.4 \times 10^{-6} (288.7)^2$$

$$= 1.1 - 0.3667$$

$$B = 0.733 \text{ Wb/m}^2$$

$$\therefore \quad (BH)_{max} = 0.733 \times 288.7$$

$$(BH)_{max} = 2.12 \times 10^2 \text{ kJ/m}^3$$

(ii) Given, A long narrow rod has an atomic density,

$$N = 5 \times 10^{28} \text{ m}^{-3}$$

Atomic polarizability, $\alpha = 10^{-40}$ F-m^2

Applied axial field, $E = 1$ V/m

The Lorentz internal field is given by

$$E_i = \frac{E}{1 - \dfrac{N\alpha}{3E_0}}$$

where $\epsilon_0 = 8.854 \times 10^{-12}$ F/m

$$\therefore \quad E_i = \frac{1}{1 - \dfrac{5 \times 10^{28} \times 10^{-40}}{3 \times 8.854 \times 10^{-12}}}$$

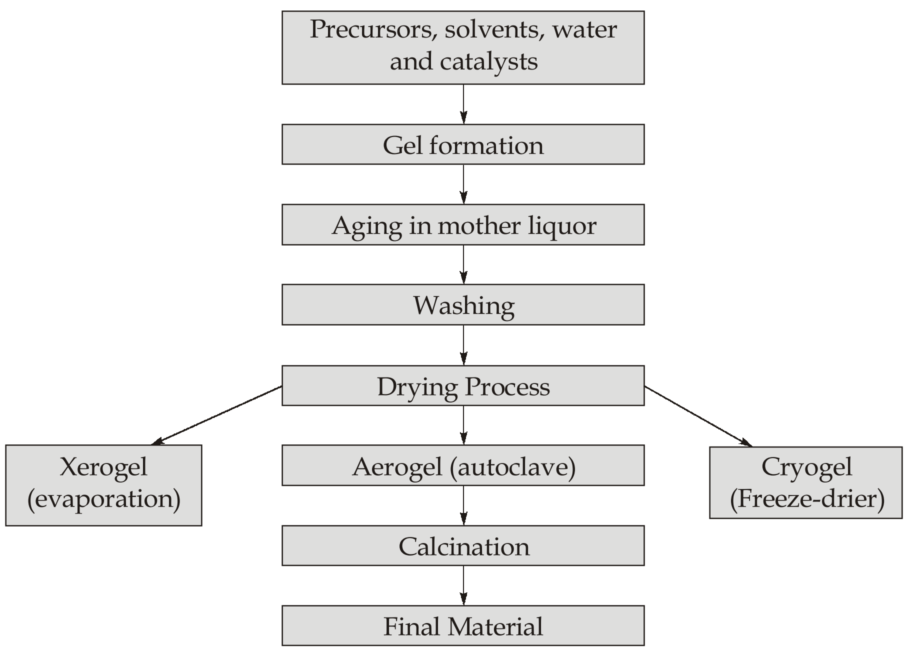
$$= \frac{1}{1-0.188}$$

$$E_i = \frac{1}{0.812} \approx 1.23 \text{ V/m}$$

Q.2 (c) Solution:

(i) The sol-gel method is a versatile process used for synthesis of various nanostructures, especially metal oxide nanopartcles. In sol-gel process, a colloidal suspension (sol) and gelation of the sol is formed from the hydrolysis and polymerization reaction of the precursor, which are usually inorganic metal salts or metal organic compounds such as metal alkoxides.

A flow chart for complete sol-gel process is shown below.



The steps in the synthesis of sol-gel process are:

1. Formation of different stable solutions of the alkoxide or solvated metal precursor (the sol).

2. Gelation resulting from the formation of oxide-or alochol-bridged network (the gel) by a polycondensation reaction that results in a dramatic increase in the viscosity of the solution.

3. Aging of the gel (syneresis) during which polycondensation reactions continues until the gel transforms into a solid mass, accompanied by contraction of the gel network and expulsion of solvent from gel pores.

4. Drying of the gel when water and other volatile liquids are removed from the gel network. The drying process itself has been broken into 4 distinct steps.
   (i) the constant rate period.
   (ii) the critical point.
   (iii) the falling rate period.
   (iv) the second falling rate period.

   If isolated by thermal evaporation, the resulting monolith is termed as Xerogel. If the solvent (such as water) is extracted under supecritical or near supercritical conditions, the product is an aerogel.
5. Dehydration, during which surface-bound M-OH groups are removed, thereby stabilizing the gel against rehydration.
6. Densification and decomposition of the gels at high temperature ($T > 800°C$). The pores of the gel network are collapsed and remaining organic species are volatilized.

(ii)
- Scientific challenges in nano science and nano technology includes the developement of nano particles with novel mechanical properties.
- Many of the mechanical properties of material are modified at the nanoscale. Hardness, elastic modulus, fracture toughness and fatigue strength are some properties that are modified.
- Tougher and harder cutting tools can be developed using nanomaterials. The material used for this purpose is titanium carbide. Such tools are much harder, more wear-resistant and also last longer than their conventional counter-part.
- Automobiles with greater fuel efficiency can be manufactured using nanomaterials. Since nanomaterials are stronger, harder, much more wear resistant and erosion resistant, they could be coated on spark plugs and other engine parts.
- Ceramics made of nanocrystals are observed to be ductile in nature. They can be pressed and sintered into various shapes at significantly lower temperatures.
- Nanocrystalline materials provide better thermal insulation.

Q.3 (a) Solution:

(i) Ferromagnetic materials which have wide applications in electrical engineering has a disadvantage that they have low electrical resistivity. The laminations used for electrical machines have resistivity of about 14×10^{-4} ohm-m and the highest value obtainable in ferromagnetic alloys is less than 10^{-2} ohm-m. This disadvantage of the ferromagnetic materials limits their application in the high frequency alternating current applications, and results in high eddy current losses and poor magnetic utilization of materials occur in sheets even at low frequencies. Ferrites on the other hand, with useful magnetic properties have d.c. resistivity of many orders of magnitude higher than that of Iron and are used for frequencies upto microwave in transformer cores.

In ferrimagnetic substances, the magnetic moment of adjacent atoms are aligned in opposite, but the moments are not equal so that there is a net magnetic moment, i.e. if the net magnetization of magnetic sublattice is not zero, the material exhibits a net magnetic moment, however it is less than in ferromagnetic materials. This moment disappears above a curie temperature T_c, analogous to neel temperature at which thermal energy randomizes the individual magnetic moments and the materials becomes paramagnetic.

The general electric and magnetic characteristics of ferrites are:

- Very high resistivity, generally more than 10^5 ohm-cm.
- Microwave dielectric constant of the order of 10 - 12.
- Extremely low dielectric loss.
- High permeability.
- A saturation magnetization which is appreciable but noticeably smaller than that of ferromagnetic materials and low coercive field.
- A curie temperature which varies from 100° C to several hundred °C.

(ii) A nanoparticle of Si can be made by laser evaporation of a Si substrate in the region of a helium gas pulse. The beam of neutral clusters is photolyzed by a UV laser producing ionized clusters whose mass to charge ratio is then measured in a mass spectrometer. The most striking property of nanoparticles made of semiconducting elements is the pronounced changes in their optical properties compared to those of bulk material.

In a bulk semiconductor a bound electron-hole pair, called an exciton, can be produced by a photon having an energy greater than that of the bandgap of material. The photon excites an electron from the filled band to the unfilled band above. Because of the coulomb attraction between the positive hole and the negative electron, a bound pair, called an exciton is formed that can move through the lattice. The separation between the hole and the electron is many lattice parameters.

The existence of the exciton has a strong influence on the electronic properties of the semiconductor and its optical absorption. The exciton can be modeled as a hydrogen-like atom and has energy levels with relative spacings analogous to the energy levels of the hydrogen atom but lower actual energies. Light induced transitions between these hydrogen like energy levels produce a series of optical absorptions.

When the size of the nanoparticle becomes smaller than or comparable to the radius of the orbit of the electron-hole pair, there are two situations, called weak-confinement and the strong-confinement regimes. In the weak regime, the particle radius is larger than the radius of the electron-hole pair, but the range of motion of the exciton is limited, which causes the blue shift of the absorption spectrum. When the radius of the particle is smaller than the orbit radius of the electron-hole pair, the motion of the electron and the hole become independent and the exciton does not exist. The hole and electron have their own set of energy levels. Here there is also a blue shift, and the emergence of a new set of absorption lines. The lowest energy absorption region, referred to as the absorption edge, is shifted to higher energy as the particle size decreases. Since the absorption edge is due to the band gap, this means that the band gap increases as particle size decreases. Also, the intensity of the absorption increases as the particle size is reduced. The higher energy peaks are associated with the exciton and they shift to higher energies with the decrease in particle size. Hence, as the particle size is reduced, the hole and the electron are forced closer together, and the separation between the energy levels changes.

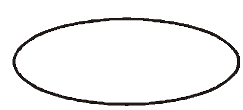
Q.3 (b) Solution:

(i) **1. Entity:** An entity is referred to as a real-world object. It can be a name, place, object, class, etc. These are represented by a rectangle in an ER diagram.

Entity set →

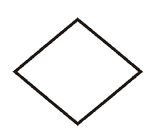
2. Attributes: An attribute can be defined as the description of the entity. These are represented by Ellipse in an ER Diagram. It can be Age, Roll Number, or Marks for a student.

Attributes →



3. Relationship: Relationships are used to define relations or association among different entities. In ER diagram, the relationship type is represented by a diamond connecting the entities with lines.

Relationship →



4. Domain: A domain represents the set of all possible values that an attribute can hold in a database. It defines the type, format etc. for the values associated with an attribute. For example, if an attribute is 'age', the domain would be a set of non-negative integers.

(ii)

$$\text{Cache size} = 64 \text{ KB}$$

$$\text{Cache block size} = 32\text{B}$$

$$\text{Number of cache lines} = \frac{\text{Cache size}}{\text{Cache block size}} = \frac{2^6 \times 2^{10}}{2^5} = 2^{11}$$

Since the cache is 8-way set associative $\Rightarrow$ Number of sets = $\dfrac{2^{11}}{2^3} = 2^8$

Hence, 28 bit physical address format would be as given below,

Tag	Set Index	Block offset
15	8	5
= (28 – 8 – 5) = 15 bits	Number of sets = 2^8 Thus, 8 bits required	Cache block size = 32 B Thus, 5 bits required

The cache controller maintains the tag information for each cache block, which also includes 2 valid bits, 2 update bits and 3 replacement bits.

Hence, Tag space in the line = 15 + 2 + 2 + 3 = 22 bits

$$\text{Tag directory size} = \text{Number of lines} \times \text{Tag space}$$

$$= 2^{11} \times 22$$

$$= 22 \times 2^{11} \text{ bits}$$

Q.3 (c) Solution:

In dielectric materials, all electrons are bound; the only motion possible in the presence of an electric field is a minute displacement of positive and negative charges in opposite directions. The displacement is usually small compared to atomic dimensions. A dielectric in which this charge displacement takes place is said to be polarized, and its molecules are said to possess induced dipole moment. These dipoles produce their own field, which adds to that of external fields. In addition to displacing the positive and negative charges, an applied electric field can also polarize a dielectric by orienting molecules that possess a permanent dipole moment.

The four basic *polarization mechanism* are:

(a) Electronic polarization: It is due to the displacement of the positively charged nucleus and the negative electrons of an atom in opposite directions.

(b) Ionic Polarization: It occurs due to relative displacements between positive and negative ions in an ionic crystal.

(c) Orientational polarization: This occurs due to the permanent dipole moment in a material. When an electric field is applied, the dipole moment vectors try to move along the direction of external electric field.

(d) Interfacial or space charge polarization: Interfacial or space charge polarization occurs when there is an accumulation of charge at an interface between two materials or between two regions within a material because of an external field. This can occur when there is a compound dielectric, or when there are two electrodes connected to a dielectric material.

For the given material, polarization is given by,

$$\vec{P} = \epsilon_0 (\epsilon_r - 1)\vec{E}$$

Given, $$\epsilon_r = 2.4$$

and $$\vec{E} = 10^4 \vec{a}_x \text{ V/m}$$

$$\vec{P} = 8.85 \times 10^{-12} \times (2.4 - 1)10^4 \vec{a}_x$$

$$= 8.85 \times 10^{-12} \times 1.4 \times 10^4$$

$$= 12.39 \times 10^{-8} \hat{a}_x \text{C/m}^2$$

Now for individual dipole moment,

$$\vec{P} = N\vec{p}$$

where N is the number of dipoles per unit volume

$$12.39 \times 10^{-8} \vec{a}_x = 3.2 \times 10^{19} \vec{p}$$

Dipole moment, $$\vec{p} = \frac{12.39 \times 10^{-8} \vec{a}_x}{3.2 \times 10^{19}} = 3.872 \times 10^{-27} \vec{a}_x \text{C-m}$$

Q.4 (a) Solution:

(i) Step 1 : $(AC)^+ = ADBEC$

Since (AC)+ contains all attributes, AC is a candidate key.

$$(BC)^+ = (ABCDE)$$

Since (BC)+ contains all attributes, BC is also a candidate key.

$$\{AC, BC\} = \text{Candidate keys}$$

Step 2 : Prime attributes : attributes part of candidate key : $\{A, B, C\}$

Non-prime attributes : $\{D, E\}$

Step 3 : The relation is in 1st normal form as a RDBMs does not allow multi-value/ composite attributes.

The relation is not in 2nd normal form as $A \rightarrow D$ is partial dependency, i.e., A which is a subset of candidate key AC is determining a non-prime attribute D and 2nd normal form does not allow partial dependency. So, the highest normal form is 1st normal form.

(ii) 1.

$$\begin{aligned}\text{Capacity of a track} &= 512 \text{ bytes} \times 1024 \text{ sectors} = 0.5 \text{ MB/track}\\ \text{Capacity of a platter} &= 0.5 \text{ MB/track} \times 4096 \text{ tracks/platter}\\ &= 2048 \text{ MB/platter}\\ &= 4 \text{ GB/platter}\\ \text{Capacity of a link} &= 20 \text{ platter} \times 4 \text{ GB/platter} = 80 \text{ GB}\end{aligned}$$

$\therefore$ The disk has 80 GB capacity.

2.

$$\begin{aligned}\text{The size of a disk} &= 2048 \text{ bytes} \times 4096 \text{ sectors/track}\\ &\quad \times 8192 \text{ sectors/track} \times 2 \text{ platters/disk}\\ &= 2^{37} = 2^{7} \times 2^{30} = 128 \text{ GB per disk}\end{aligned}$$

$\therefore$ No. of disks for > 512 GB = 4.

Q.4 (b) Solution:

The effective magnetic field of ferromagnetic system is,

$$H_{\text{eff}} = H + \lambda M$$

For spontaneous magnetization, external field

$$H = 0$$

$$\therefore \quad H_{\text{eff}} = \lambda M$$

Magnetization is given by,

$$M = Ng\mu_B J B_J(x)$$

where, $B_J(x) = \tanh\left[\dfrac{gJ\mu_B B}{K_B T}\right]$ is the Brillouin function, N is the number of atoms per unit volume and g is the Landé g-factor.

$$\therefore \quad M = Ng\mu_B J \tanh\left[\frac{gJ\mu_B B}{K_B T}\right]$$

On comparing,

$$x = \frac{g\mu_B J\lambda M}{K_B T}$$

and for the given spin $\frac{1}{2}$, $J = \frac{1}{2}$, Brillouin function simplifies to

$$B_J(x) = B_{1/2}(x) = \tanh x$$

∴ $M = N\mu_B \tanh x$ [∵ g is normally considered to be 2]

Near Curie temperature, T_C where magnetization is small.

i.e., $x << 1$

By using series expansion,

$$\tanh x \approx x - \frac{x^3}{3}$$

∴ $$M = N\mu_B\left(x - \frac{x^3}{3}\right)$$

Substituting x, we get

∴ $$M = N\mu_B\left[\frac{\mu_B\lambda M}{K_B T} - \frac{1}{3}\left(\frac{\mu_B\lambda M}{K_B T}\right)^3\right]$$

Dividing both sides by M:

$$1 = \frac{N\mu_B^2\lambda}{K_B T} - \frac{N\mu_B^4\lambda^3}{3K_B^3 T^3}M^2 \quad \text{...(i)}$$

At Curie temperature ($M \to 0$),

$$\frac{N\mu_B^2\lambda}{K_B T_C} = 1$$

Hence, $$\frac{N\mu_B^2\lambda}{K_B T} = \frac{T_C}{T}$$

Substituting in equation (i), we get

$$1 = \frac{T_C}{T} - AM^2$$

where, $$A = \frac{N\mu_B^4\lambda^3}{3K_B^3 T^3}$$

∴ $$AM^2 = \frac{T_C}{T} - 1 = \frac{T_C - T}{T}$$

∴ $$M^2 \propto (T_C - T)$$

(or) $$M \propto (T_C - T)^{1/2}$$

Q.4 (c) Solution:

Given data :Cache hit ratio, $(H_C) = 0.95$;

Main memory hit ratio, $(H_M) = 0.99$

Cache access time, $(T_C) = 5$ ns ;

SSD access time = 5 μs ; $T_{SSD} = 5000$ ns

Let Main memory access time $(T_M) = x$

Formula for effective memory access time

$$T_{\text{eff}} = H_C T_C + (1 - H_C)H_M(T_C + T_M) + (1 - H_C)(1 - H_M)(T_C + T_M + T_{\text{SSD}})$$

Since given effective memory access time = 20 ns

So,

$$\begin{aligned} 20 \text{ ns} &= 0.95 \times 5 + 0.05 \times 0.99 \times (5 + x) + 0.05 \times 0.01 \\ &\quad \times (5 + x + 5000) \\ 20 &= 4.75 + 0.0495(5 + x) + 0.0005(5005 + x) \\ 20 - 4.75 &= 0.0495x + 0.2475 + 2.5025 + 0.0005x \\ 15.25 &= 2.75 + 0.05x \\ 15.25 - 2.75 &= 0.05x \\ 12.5 &= 0.05x \\ x &= \frac{12.5}{0.05} = 250 \text{ ns} \end{aligned}$$

Capacity of SSD

Given cost here is cost per KB.

So,

$$\begin{aligned} 22000 &= \frac{2^{20}}{2^{10}} \times 1 + \frac{2^{27}}{2^{10}} \times 0.1 + \frac{2^x}{2^{10}} \times 0.001 \\ 22000 &= 1024 + 13107.2 + 2^x \times 0.001 \\ 7868.8 &= 2^x \times 0.001 \\ 2^x &= \frac{7868.8}{0.001} = 7868800 \end{aligned}$$

On taking $\log_2$ both sides we get,

$$x = 23$$

So, Size of SSD $= 2^{23} \times 2^{10} = 2^{33}B = 8$ GB

Section B : Electronic Devices & Circuits-1 + Advanced Communications + Analog & Digital Communication Systems-2

Q.5 (a) Solution:

Given,

$$\text{Temperature}, T = 293 \text{ K}$$
$$\text{Resistivity}, \rho_{Ge} = 1\ \Omega\text{-cm}$$
$$\phi_{Au} = 5.1 \text{ eV}$$
$$\chi_{Ge} = 4.0 \text{ eV}$$
$$\mu_n = 3900 \text{ cm}^2/\text{V-sec}$$
$$N_C = 1.98 \times 10^{15} \times T^{3/2} \text{ cm}^{-3}$$

At $T = 293$ K,

$$N_C = 1.98 \times 10^{15} \times (293)^{3/2}$$
$$N_C = 1.98 \times 10^{15} \times 5.02 \times 10^3$$
$$N_C = 9.94 \times 10^{18} \text{ cm}^{-3}$$

For non-degenerate n-type Ge:

$$n = N_C e^{-(E_C - E_F)/KT}$$

but $n \cong N_D$. Thus,

$$N_D = N_C e^{-(E_C - E_F)/KT}$$

The resistivity of Ge is given by,

$$\rho_{Ge} = \frac{1}{q\mu_n N_D}$$

$$\therefore \quad N_D = \frac{1}{q\mu_n \rho_{Ge}}$$

$$= \frac{1}{(1.6\times10^{-19}\times3900\times1)}$$

$$\therefore \quad N_D \simeq 1.6 \times 10^{15} \text{ cm}^{-3}$$

$$\therefore \quad E_C - E_F = kT \ln\left(\frac{N_C}{N_D}\right)$$

$$= \frac{1.38\times10^{-23}\times293}{1.6\times10^{-19}} \ln\left(\frac{9.94\times10^{18}}{1.6\times10^{15}}\right)$$

$$= (0.0253)\ln(6.2 \times 10^3)$$

$\therefore \quad E_C - E_F \simeq 0.22$ eV

Ideal Schottky barrier height,

$$\phi_{Bn} = \phi_M - \chi_{Ge}$$

$$\phi_{Bn} = 5.1 - 4.0 = 1.1 \text{ eV}$$

The effective barrier height (from metal fermilevel to semiconductor fermilevel)

$$\phi_B = \phi_{Bn} - (E_C - E_F)$$

$$= 1.1 - 0.22$$

$$\phi_B \simeq 0.88 \text{ eV}$$

$\therefore$ Height of the potential barrier for the Au-n-Ge Schottky contact ≈ 0.88 eV.

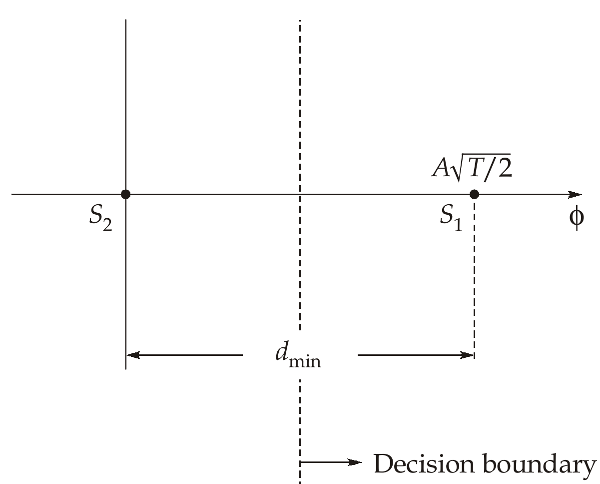
Q.5 (b) Solution:

A binary ASK signal can be expressed as the signal set $\begin{Bmatrix} S_1 = A\cos\omega_c t \\ S_2 = 0 \end{Bmatrix}$ in the symbol interval $0 < t < T$.

Using a single basis function $f(t) = \sqrt{\frac{2}{T}}\cos\omega_c t$,

we can write $\quad S_1(t) = A\sqrt{\frac{T}{2}}\,\phi(t), \quad S_2(t) = 0$

Constellation Diagram for BASK:



$$d_{min} = A\sqrt{\frac{T}{2}} \qquad \Rightarrow d_{min}^2 = \frac{A^2}{2}T$$

Average bit energy, $E_b \quad = \frac{1}{2}\left(\frac{A^2T}{2} + 0\right) = \frac{A^2T}{4}$

$$P_e = Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2T}{4N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Note that the vector length of S_1 is the square of energy of signal S_1 which is another advantage of using signal constellation.

For binary BPSK, the two signals can be represented as

$S_1(t) = A\cos\omega_c t$ and

$S_2(t) = -A\cos\omega_c t$ in the symbol interval $0 < t < T$, Expressing them in terms of the basis function,

$$\phi(t) = \sqrt{\frac{2}{T}}\cos\omega_c t$$

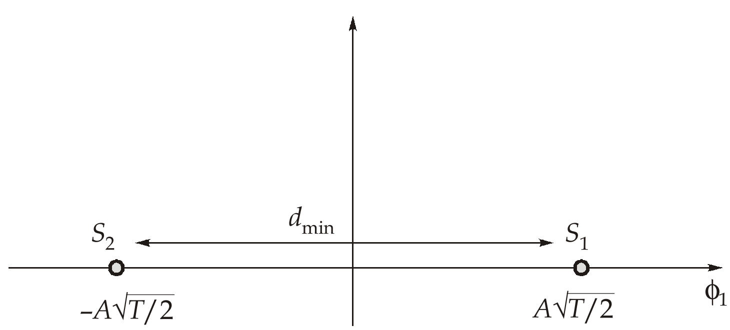
We can write,

$$S_1(t) = A\sqrt{\frac{T}{2}}\,\phi(t)$$

$$S_2(t) = -A\sqrt{\frac{T}{2}}\,\phi(t)$$

From the constellation diagram,

$$d_{\min}^2 = \left(2A\sqrt{\frac{T}{2}}\right)^2 = 2A^2T^2$$



We have,

$$E_b = \frac{A^2T}{2}$$

$$P_e = Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right) = Q\left(\sqrt{\frac{2A^2T}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2T}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

For binary BFSK orthogonal scheme, the two signal sets are orthogonal to each other

$$S_1(t) = A\cos 2\pi f_1 t$$

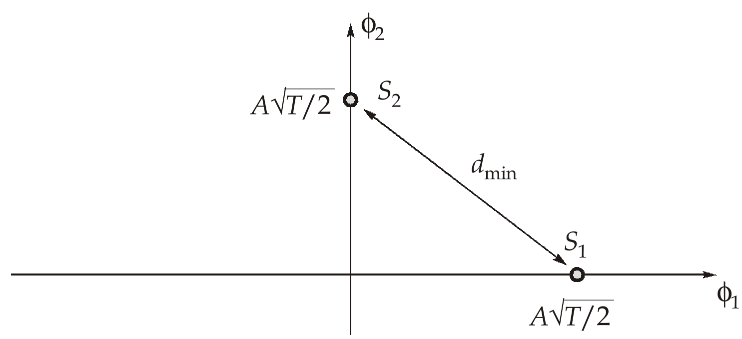
$$S_2(t) = A\cos 2\pi f_2 t$$

We need therefore two basis function to represent this system

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos\omega_1(t)$$

and
$$\phi_2(t) = \sqrt{\frac{2}{T}}\cos\omega_2(t)$$

So that
$$S_1(t) = A\sqrt{\frac{T}{2}}\,\phi_1(t); \qquad S_2 = A\sqrt{\frac{T}{2}}\,\phi_2(t)$$



$$d_{min}^2 = \sqrt{\left(A\sqrt{T/2}\right)^2 + \left(A\sqrt{T/2}\right)^2} = A^2T$$

We have,
$$E_b = \frac{E_{b1}+E_{b2}}{2} = \frac{A^2T}{2}$$

$$P_e = Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2T}{2N_0}}\right) = Q\left(\frac{E_b}{N_0}\right)$$

Q.5 (c) Solution:

(i) With an average holding time of 50ms per call, Number of calls handled per hour without delay

$$= \frac{3600}{50\times10^{-3}} = 72000$$

The delay is 10 ms = 10×10^{-3} sec

Number of calls handled per hour with delay = $\dfrac{3600}{(50+10)\times10^{-3}} = 60,000$

(ii) We know that

Mean waiting time, $\overline{T} = \dfrac{Ah}{1-A}$, where A is the Traffic Intensity and h is the holding time.

$$\text{Given,} \quad \overline{T} = 10 \times 10^{-3} \text{ sec}$$

$$10 \times 10^{-3} = \frac{Ah}{1-A}$$

$$\therefore \quad A = \frac{10\times10^{-3}}{(h+10\times10^{-3})}$$

$$\text{We have,} \quad A = \frac{Ch}{T}$$

$$\text{where} \quad T = 1 \text{ hour} = 3600 \text{ secs}$$

$$C = \text{Average number of calls in time } T = 18000 \text{ calls}$$

$$\therefore \quad \frac{10\times10^{-3}}{h+10\times10^{-3}} = \frac{18000h}{3600} = 5h$$

$$(10 \times 10^{-3}) = (5h)(h + 10 \times 10^{-3})$$

$$1 = (5h)(100h + 1)$$

$$1 = 500h^2 + 5h$$

On solving, we get $h_1 = 40$ msec

$h_2 = -50$ msec $= -$ve (not possible)

Thus, the maximum permissible average holding time is 40 msec.

Q.5 (d) Solution:

Under the condition of zero electric field ($E = 0$) and steady state $(\partial p_n / \partial t = 0)$, the one-dimensional minority carrier diffusion equation is:

$$D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} = 0$$

The general solution of the differential equation is:

$$\Delta p(x) = Ae^{-x/L_p} + Be^{x/L_p} \quad \text{...(i)}$$

where L_p is the diffusion length given by:

$$L_p = \sqrt{D_p \tau_p}$$

Given: $D_p = 10\ cm^2/s$, $\tau_p = 10^{-6}$ s

$$L_p = \sqrt{10\times10^{-6}} = \sqrt{10^{-5}} = 3.16\times10^{-3}\text{cm}$$

Applying the boundary conditions:

- At $x \to \infty$, the excess carrier concentration must decay to zero (equilibrium), so

$$\Delta p(\infty) = 0$$

Substituting in equation (i), we get $B = 0$.

- At $x = 0$, the concentration is given as the excess carriers at the surface. Thus,

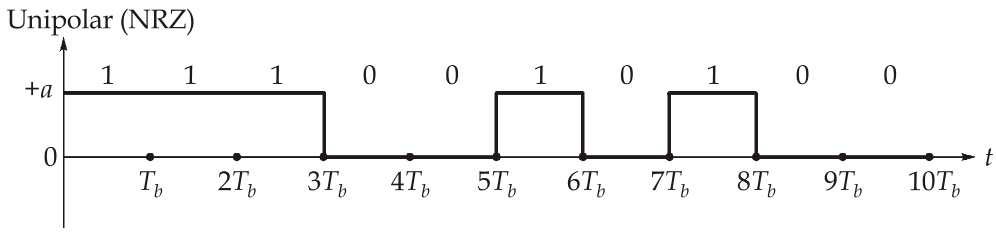
$$\Delta p(\infty) = A = 10^{14}\ cm^{-3}$$

Therefore, we get the steady-state excess carrier concentration as a function of distance from the surface ($x = 0$) of the semiconductor as,

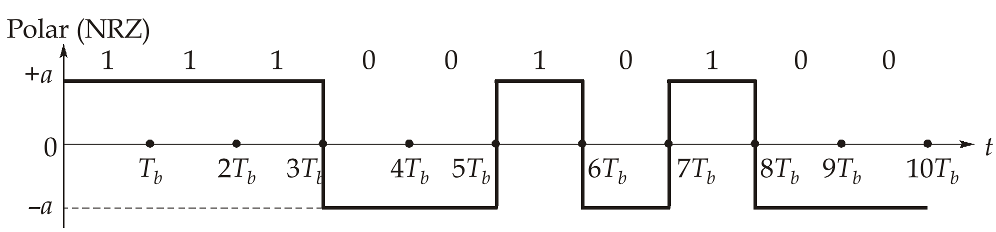
$$\Delta p(x) = Ae^{-x/L_p} = 10^{14}\exp\left(-\frac{x}{3.16\times10^{-3}}\right)\text{cm}^{-3}$$

Q.5 (e) Solution:

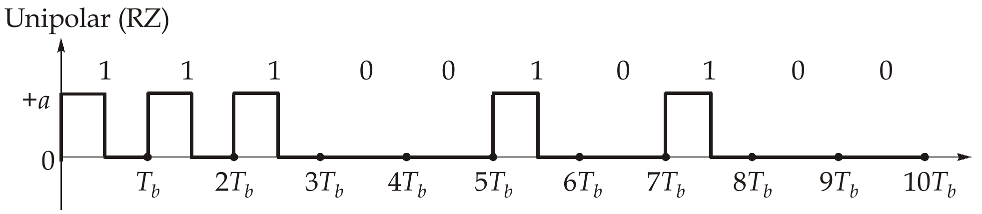
(i) Unipolar Nonreturn-to-Zero (NRZ): Binary '1' is represented by a high voltage (+V) for the full bit duration, and binary '0' is represented by 0 V.



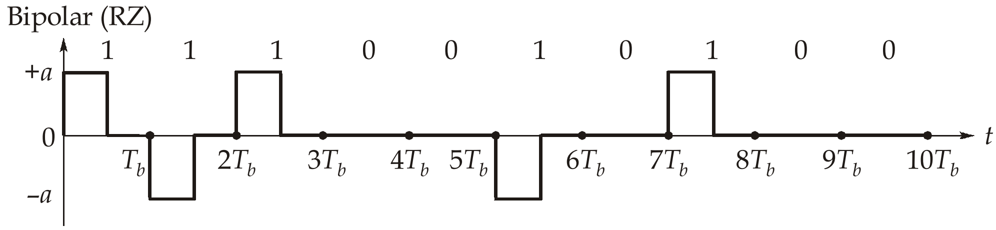
(ii) Polar Nonreturn-to-Zero (NRZ): Binary '1' is represented by (+V) and binary '0' is represented by (-V) for the full duration.



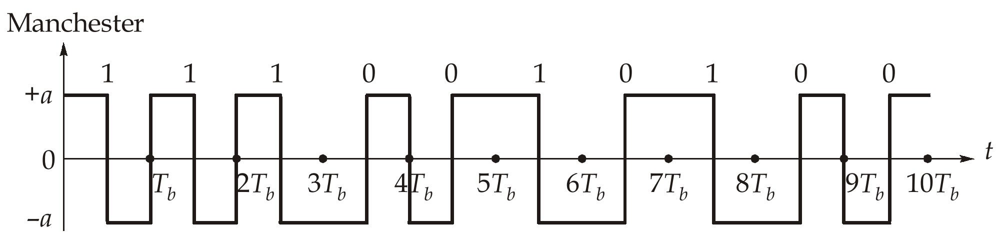
(iii) Unipolar Return-to-Zero (RZ): Binary '1' transitions from +V to 0 V halfway through the bit interval. Binary '0' remains at 0 V for the entire interval.



(iv) Bipolar Return-to-Zero: Binary '0' is 0 V. Binary '1's alternate between +V and –V, but each pulse only lasts for half the bit duration before returning to zero.



(v) Manchester Code: A 0 to 1 transition at the bit's centre denotes a logic 0 while a 1 to 0 transition denotes a logic 1



Q.6 (a) Solution:

Given, $m_p^* = 10\, m_n^*$; $E_g = 1.5$ eV; $T = 300$ K; $n_i = 1 \times 10^5/\text{cm}^3$

(i) For intrinsic semiconductor,

$$n = p$$

$$N_C e^{-(E_c - E_{Fi})/KT} = N_V e^{-(E_{Fi} - E_V)/KT}$$, where E_{Fi} is the intrinsic Fermi level

$$\frac{N_C}{N_V} = e^{\left\{\frac{-E_{Fi} + E_V + E_C - E_{Fi}}{KT}\right\}}$$

$$\frac{N_C}{N_V} = e^{\left(\frac{-2E_{Fi} + E_C + E_V}{KT}\right)}$$

Take ln on both sides,

$$KT \ln\left(\frac{N_C}{N_V}\right) = -2E_{Fi} + E_C + E_V$$

$$2E_{Fi} = (E_C + E_V) - KT \ln\left(\frac{N_C}{N_V}\right)$$

$$E_{Fi} = \frac{(E_C + E_V)}{2} - \frac{KT}{2}\ln\left(\frac{N_C}{N_V}\right)$$

where $\frac{E_C + E_V}{2}$ = Mid of band gap,

$$N_C \propto \left(\frac{2\pi kTm_n^*}{h^2}\right)^{3/2} \text{ and } N_V \propto \left(\frac{2\pi kTm_p^*}{h^2}\right)^{3/2}$$

$$E_{Fi} = \left(\frac{E_C + E_V}{2}\right) - \frac{KT}{2}\ln\left(\frac{m_n^*}{m_p^*}\right)^{3/2}$$

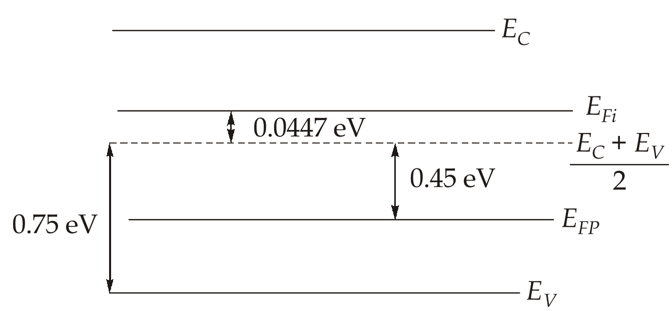
$$= \left(\frac{E_C + E_V}{2}\right) - \frac{3KT}{4}\ln\left(\frac{m_n^*}{10m_n^*}\right)$$

$$= \left(\frac{E_C + E_V}{2}\right) - \frac{3}{4}\times 0.0259\ln\left(\frac{1}{10}\right)$$

$$E_{Fi} = \left(\frac{E_C + E_V}{2}\right) + 0.0447 \text{ eV}$$

Thus, intrinsic Fermi level is 0.0447 eV above the center of the bandgap.

(ii)



Impurity atoms are to be added so that

$$\left(\frac{E_C + E_V}{2}\right) - E_{Fp} = 0.45 \text{ eV}$$

1. Since the fermi level has to be moved closer to the valence band, thus p-type, i.e., Acceptor impurities are added.
2. From the figure,

$$E_{Fi} - E_{Fp} = 0.0447 + 0.45$$
$$= 0.4947 \text{ eV}$$

We know, $$E_{Fi} - E_{Fp} = KT\ln\left(\frac{p}{n_i}\right)$$

$$p = n_i e^{\left(\frac{E_{Fi} - E_{Fp}}{KT}\right)}$$

$$p = 10^5 e^{\left[\frac{0.4947}{0.0259}\right]}$$

∴ Acceptor impurity concentration,

$$N_A = p = 1.97 \times 10^{13}/\text{cm}^3$$

Q.6 (b) Solution:

TCP/IP stands for Transmission Control Protocol/Internet Protocol. It has four layers as mentioned below:

Application layer
Transport layer
Network layer/Internet layer
Network Access layer

1. **Application Layer:** This is the top-most layer of TCP/IP protocol suite. This layer includes applications or processes that use transport layer services to deliver data to destination computer application layer. The application layer has various protocols that applications use to communicate with the transport layer. Some of the popular application layer protocols are:
   - HTTP (Hypertext transfer protocol)
   - FTP (File transfer protocol)
   - SMTP (Simple mail transfer protocol)
   - SNMP (Simple network management protocol) etc.
2. **Transport Layer:** This layer provides backbone to data flow between two hosts. There are many protocols that work at this layer but the two most commonly used protocols at transport layer are TCP and UPD.

   TCP is used where reliable connection is required while UDP is used in case of unreliable connections.

   TCP divides the data (coming from application layer) into proper sized chunks and then passes these chunks on the network. It acknowledges received packets, waits for the acknowledgment of the packets it sent and sets timeout to resend the packets if acknowledgments are not received in time.

   UDP provides a comparatively simpler but unreliable service by sending packets from one host to another. UDP does not take any extra measures to ensure that the data sent is received by the target host or not.
3. **Network layer:** This layer is also known as internet layer. The main purpose of this layer is to organize or handle movements of data on network, i.e., routing of data

over the network. The main protocol used at this layer is IP, while ICMP and IGMP are also used at this layer.

4. **Data link layer:** This layer is also known as network interface layer. This layer normally consist of device drivers and the network interface card attached to the system. Both the device drivers and the network interface card take care of the communication details with the media being used to transfer the data over the network. In most of the cases, this media is in the form of cables. Some of the famous protocols that are used at this layer include ARP (Address Resolution Protocol), PPP (Point to point protocol) etc.

Address used at different layers:

1. Application layer : Application-specific address (i.e., domain name)
2. Transport layer: Port Address
3. Internet Layer: IP Address (IPv4 or IPv6 address)
4. Network Access Layer: Physical Address (e.g. MAC address)

Q.6 (c) Solution:

(i) α for companded PCM using $\mu = 255$

$$\alpha_{PCM} = 4.77 - 20\log_{10}(\ln(1 + 255))$$

$$\alpha_{PCM} \approx 4.77 - 20\log_{10}(5.54)$$

$$\approx 4.77 - 14.87$$

$$\alpha_{PCM} = -10.1 \text{ dB}$$

$$\therefore \quad (SNR)_{PCM} = -10.1 + 6n$$

We have, $(SNR)_{DPCM} = \alpha_{DPCM} + 6n$ (where $-3 < \alpha < 15$)

Improvement in SNR offered by DPCM:

$$\Delta_{SNR} = \alpha_{DPCM} - (-10.1)$$

$$= \alpha_{DPCM} + 10.1$$

Taking the DPCM gain ($\alpha \approx -3$) or the maximum (15 dB), the improvement typically ranges from 7.1 dB to 25.1 dB.

(ii) Given $(SNR)_{DPCM} = (SNR)_{PCM}$

$$\alpha_{DPCM} + 6n_{DPCM} = \alpha_{PCM} + 6n_{PCM}$$

$$6(n_{PCM} - n_{DPCM}) = \alpha_{DPCM} - \alpha_{PCM}$$

$$\Delta n = \frac{\alpha_{DPCM} - (-10.1)}{6} = \frac{\alpha_{DPCM} + 10.1}{6}$$

If we choose α_{DPCM} = –3 dB, then reduction in the number of bits per sample required by DPCM to achieve the same SN as compared to PCM is

$$\Delta n = \frac{3+10.1}{6} = 1.18$$

It represents a saving of about 1 bits/sample due to the use of DPCM.

If α_{DPCM} = 15 dB, then reduction in the number of bits per sample required by DPCM to achieve the same SN as compared to PCM is

$$\Delta n = \frac{10.1+15}{6} = 4.18$$

It represents a saving of about 4 bits/sample due to the use of DPCM.

Q.7 (a) Solution:

(i) From the figure, it is clear that diode is reverse biased, because there is a deficit of minority carrier in the quasineutral region immediately adjacent to the depletion region. Forward biasing of the diode causes a build-up or storage of excess minority carriers in the quasineutral regions immediately adjacent to the depletion region.

(ii) Low-level injection does prevail. As required for low-level injection, excess minority carrier concentration is much less than the majority carrier concentration i.e.

$\Delta p_n << n_n$ and $\Delta n_p << p_p$ everywhere inside the quasineutral regions.

(iii) Since we have low level injection,

$$N_A = p_p = 10^{14}/\text{cm}^3$$
$$N_D = n_n = 10^{15}/\text{cm}^3$$

(iv) From the law of junction,

$$p_n(0) = p_{n0}\, e^{V_A/V_T} \qquad ...(1)$$

where $p_n(0) \rightarrow$ hole concentration at the edge of depletion region on N-side.

$p_{n0} \rightarrow$ hole concentration on N-side at equilibrium

∴ From the figure,

$$p_n(0) = 10^2/\text{cm}^3$$
$$p_{n0} = 10^5/\text{cm}^3$$

From equation (1)

$$p_n(0) = p_{n0}\, e^{V_A/V_T}$$

$$10^2 = 10^5 e^{V_A/V_T}$$

$$V_A = V_T \ln\left[\frac{10^2}{10^5}\right]$$

where, $V_T = 0.026$ volt

$$V_A = 0.026 \ln\left[\frac{10^2}{10^5}\right]$$

$$V_A = -0.18 \text{ volt}$$

(v) Built in potential, $$V_{bi} = V_T \ln\left[\frac{N_A N_D}{n_i^2}\right] \quad ...(2)$$

From the figure, $N_A = 10^{14}/\text{cm}^3$

$N_D = 10^{15}/\text{cm}^3$

and also $n_i^2 = n(\infty) \times p(\infty)$

$= 10^{15} \times 10^5 = 10^{14} \times 10^6 = 10^{20}$

$\therefore$ $n_i = 10^{10}/\text{cm}^3$

Now from equation (2),

$$V_{bi} = 0.026 \ln\left[\frac{10^{14} \times 10^{15}}{(10^{10})^2}\right]$$

$$V_{bi} = 0.538 \text{ volt}$$

Q.7 (b) Solution:

(i) Overall Noise Figure (F_{eq}) is given as:

$$F_{eq} = F_a + \frac{F_f - 1}{G_a} + \frac{F_m - 1}{G_a G_f}$$

Given: G_a = 10 dB = 10; G_f = –1 dB = $10^{-0.1}$ = 0.79; G_m = –3 dB = $10^{-0.3}$ = 0.5

F_a = 2 dB = $10^{0.2}$ = 1.58; F_f = 1 dB = $10^{0.1}$ = 1.25; F_m = 4 dB = $10^{0.4}$ = 2.51

$$\therefore \quad F_{eq} = 1.58 + \frac{1.25 - 1}{10} + \frac{2.51 - 1}{10 \times 0.79} = 1.79$$

(ii) Equivalent Noise Temperature (T_{eq}) will be;

$$T_{eq} = T_0(F_a - 1) + \frac{T_0(F_f - 1)}{G_a} + \frac{T_0(F_m - 1)}{G_a \cdot G_f}$$

$$= 290(1.58 - 1) + \frac{290(1.25 - 1)}{10} + \frac{290(2.51 - 1)}{10 \times 0.79}$$

$$T_{eq} = 230.88 \text{ K}$$

(iii) Overall Gain $= G_a G_f G_m$

$$= 10 \times 0.79 \times 0.5 = 3.95$$

(iv) Output noise power assuming input noise power from the feeding antenna at 150 K temperature and IF bandwidth of 10 MHz can be calculated as:

Output noise power, $P_0 = kT_e' BGF$

where, $T_e' = T_{\text{Ant}} + T_e = 150 \text{ K} + 230.8 \text{ K} = 380.8 \text{ K}$

$$\therefore \quad P_0 = 1.38 \times 10^{-23} \times 380.8 \times 10 \times 10^6 \times 3.95 \times 1.79$$

$$P_0 = 0.37 \times 10^{-12} \text{ Watt}$$

$$P_0 = 0.37 \text{ pWatt}$$

(v) We have taken bandwidth as 10 MHz from point (iv) of the question.

Input power if we require minimum signal to noise ratio of 20 dB i.e.

$$(S/N)_{i \min} = 20 \text{ dB}$$

$$(S/N)_{i \min} = 100$$

$$S_{i(\min)} = N_i \times 100 = \text{kTB} \times 100$$

$$S_{i(\min)} = 1.38 \times 10^{-23} \times 290 \times 10 \times 10^6 \times 100$$

$$S_{i(\min)} = 4.002 \text{ Pico Watt}$$

(vi) Given, characteristic Impedance = 150 Ω = R

$$S_{i(\min)} = P = \frac{V^2}{2R}$$

$$V = \sqrt{2P \times R} = \sqrt{2 \times 4.002 \times 10^{-12} \times 150}$$

$$V = 34.64 \times 10^{-6} \text{ Volt}$$

Minimum Signal Voltage is 34.64 μ volt.

Q.7 (c) Solution:

(i) **First Group :** Each customer needs 256 address. Therefore, 8 bits are needed to define each host. The prefix length is 32 – 8 = 24 bits.

The addresses are :

1st customer = 190. 100. 0.0/24 – 190.100.0.255/24

2nd customer = 190.100.1.0/24 – 190.100.1.255/24

⋮ ⋮ ⋮

64th customer = 190.100.63.0/24 – 190.100.63.255/24

Total address = 64 × 256 = 16,384.

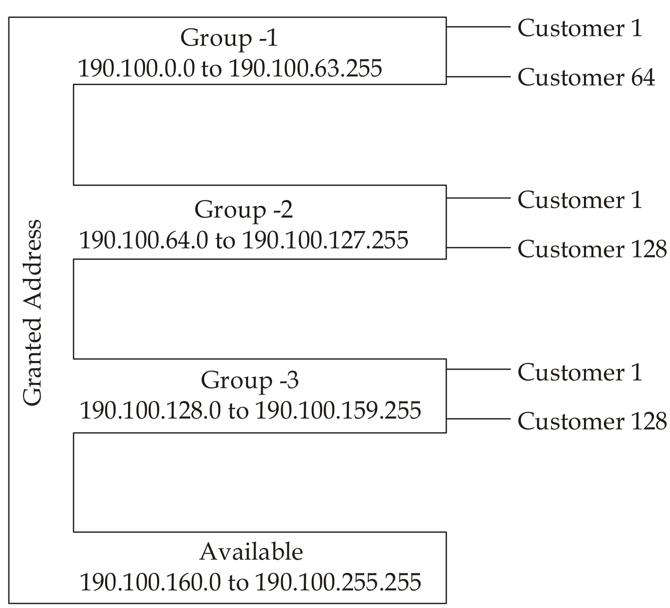
(ii) **Second Group :** For the group, each customer needs 128 addresses. The suffix length is 7 ($\because 2^7 = 128$). The prefix length is then 32 – 7 = 25. The addresses are :

1^{st} customer = 190. 100. 64.0/25 – 190.100.64.127/25
2^{nd} customer = 190.100.64.128/25 – 190.100.64.255/25
⋮ ⋮ ⋮
128^{th} customer = 190.100.127.128/25 – 190.100.127.255/25
Total address = 128 × 128 = 16,384

(iii) **Third Group :** For this group, each customer needs 64 address. This means the suffix length is 6 ($2^6 = 64$). The prefix length is then 32 – 6 = 26. The addresses are

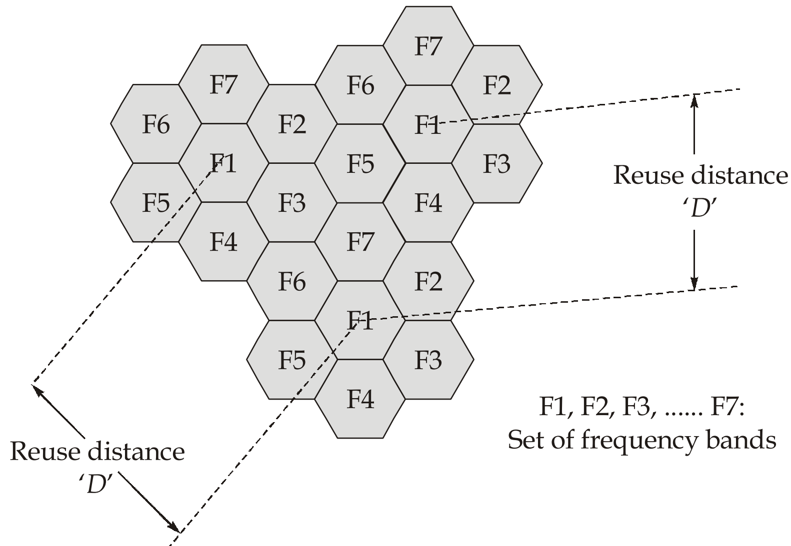
1^{st} customer = 190. 100. 128.0/26 – 190.100.128.63/26
2^{nd} customer = 190.100.128.64/26 – 190.100.128.127/26
⋮ ⋮ ⋮
128^{th} customer = 190.100.159.192/26 – 190.100.159.255/26
Total address = 128 × 64 = 8192

Hence,
No. of addresses granted to the ISP = 65536
No. of allocated address by the ISP = 16384 + 16384 + 8192 = 40960
No. of available addresses = 65536 – 40960 = 24576



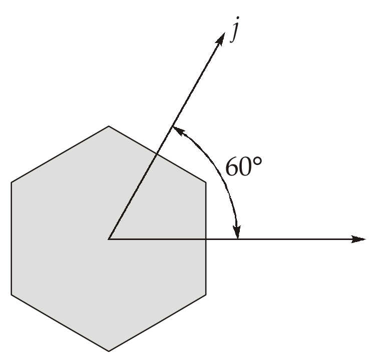
Q.8 (a) Solution:

(i) • In a cellular system, the frequency space allocated is insufficient. For a 7 cell cluster arrangement, the allocation of frequencies into seven sets is required. The same frequency band or channel used in a cell can be reused in another cluster i.e., same frequency can be used for multiple simultaneous conversations in different cells with cells are separated by sufficient distance to avoid cochannel interference. This is referred to as frequency reuse.



Frequency reuse for 7 cell cluster

- Frequency reuse is the process of using the same set of frequencies to more than one cell.
- However, frequency reuse depends on various factors such as transmitter power of base station, antenna gain and height, distance between cells. The distance between the two cells using the same frequency is known as reuse distance, is denoted by D.A typical cluster of seven cells showing frequency reuse pattern and reuse distance is as shown above.



- Frequency reuse distance is decided by cluster size 'N'. In hexagonal cell pattern the cluster size (number of cells per cluster) is given by,

$$N = i^2 + ij + j^2$$

where N represents the cluster size.

i represents the number of cells to be transversed along direction *i* rom center of cell.

j represents number of cells in direction 60° to the direction of *i*.

Substituting different values of *i* and *j* (non-negative integers), we get

N = 1, 3, 4, 7, 9, 12, 13 ....

Most popular values of *N* are 4 and 7.

- Due to hexagonal geometry, there are six equidistant neighbours and each neighbor is separated by multiples of 60°.

Frequency Reuse Factor

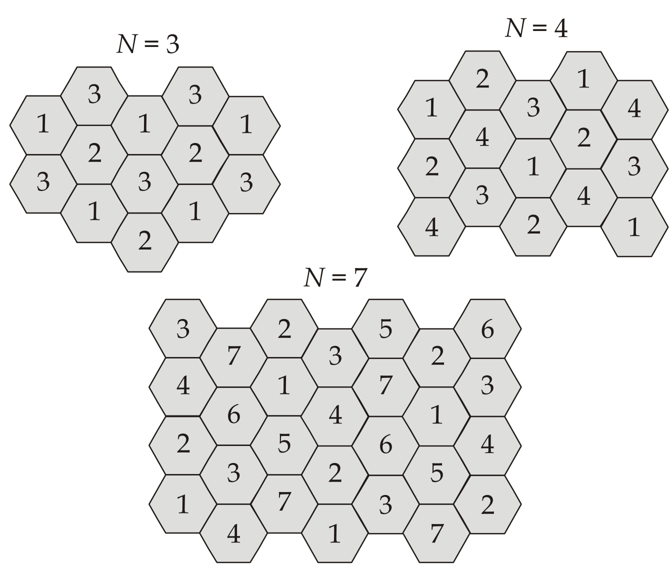
- The relationship between frequency reuse distance '*D*', cell radius '*R*' and number of cells per cluster '*N*' is represented by,

$$D = \sqrt{3N}\, R$$

The ratio $\frac{D}{R}$ is known as reuse factor.

$$\frac{D}{R} = \sqrt{3N}$$

- Figure below shows various reuse patterns.



Frequency reuse patterns for
$N = 3$, $N = 4$ and $N = 7$

- If the system is not properly designed with respect to the number of cells in a cluster, topographic cell distribution and channel assignment, then it will experience excessive interference between the channels in different cells which use the same carrier frequencies.

(ii) Given data:

- Bit rate (R_b) = 8000 bits/sec
- Available bandwidth (BW) = 5000 Hz

1. For M-ary QAM using rectangular pulses, the bandwidth (BW) defined by the first null is given by:

$$(\text{BW}) = 2R_s$$

where R_s is the symbol rate

and $$R_s = \frac{R_b}{\log_2 M}$$

Hence, $$(\text{BW})_{\text{M-ary QAM}} = \frac{2R_b}{\log_2 M}$$

$\Rightarrow$ $$5000 = 2 * \frac{8000}{\log_2 M}$$

$$M = (2)^{3.2} \approx 9.18$$

Since M must be an integer power of 2. Hence , the minimum value of M is 2^4 i.e.,

$$M = 16 \text{ (i.e., 16-QAM)}$$

For ideal Nyquist pulses, the bandwidth is given as:

$$\text{BW} = R_s$$

$$\text{BW} = \frac{R_b}{\log_2 M}$$

$$M = (2)^{1.6} \approx 3.03$$

The minimum integer power of 2 greater than 3.03 is 2^2.

Therefore, $M = 4$ (QPSK)

2. For QPSK, $M = 4$

Bandwidth of M-ary PSK with Raised cosine filter is given by:

$$\text{BW} = R_s(1 + \alpha)$$

where $$R_s = \frac{R_b}{\log_2 M} = \frac{8000}{\log_2 4} = 4000$$

Given the available bandwidth B = 5000 Hz, we have

$$5000 = 4000(1 + \alpha)$$

$$1 + \alpha = \frac{5}{4}$$

Thus, a maximum roll-off factor of 0.25 is supported by the channel.

$$\alpha = 0.25$$

Q.8 (b) Solution:

We know that, the drift-diffusion equation provides the Electron current density as the sum of drift electron current density and diffusion electron current density given as,

$$J_n(x) = n(x)q\mu_n E(x) + qD_n \frac{dn(x)}{dx} \quad \text{...(i)}$$

where, diffusion constant, $D_n = \mu_n \dfrac{kT}{q}$

The electric field is given by the slope of the band edge. Thus, the Electric field, $E(x)$ in terms of Energy level is given as,

$$E(x) = \frac{1}{q}\frac{dE_i(x)}{dx} = \frac{1}{q}\frac{E_c(x)}{dx} = \frac{1}{q}\frac{E_v(x)}{dx}$$

and
$$n(x) = n_i \exp\left[\frac{E_{Fn}(x)}{kT} - \frac{E_i(x)}{kT}\right] \quad \text{...(ii)}$$

Differentiating w.r.t 'x',

$$\frac{dn(x)}{dx} = n_i \frac{d}{dx}\left[e^{\frac{E_{Fn}(x)}{kT}} \cdot e^{\frac{-E_i(x)}{kT}}\right]$$

$$= n_i\left[e^{\frac{E_{Fn}(x)}{kT}} \cdot \frac{-dE_i(x)}{dx} \cdot \frac{1}{kT} e^{\frac{-E_i(x)}{kT}} + e^{\frac{-E_i(x)}{kT}} \cdot \frac{1}{kT}\frac{dE_{Fn}(x)}{dx} e^{\frac{E_{Fn}(x)}{kT}}\right]$$

$$\left[\because \frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}\right]$$

$$= \frac{n_i}{kT} \cdot \frac{-dE_i(x)}{dx}\left[e^{\frac{E_{Fn}(x)}{kT}} \cdot e^{\frac{-E_i(x)}{kT}}\right] + \frac{n_i}{kT}\frac{dE_{Fn}(x)}{dx}\left[e^{\frac{E_{Fn}(x)}{kT}} \cdot e^{\frac{-E_i(x)}{kT}}\right]$$

$$= \frac{n_i}{kT}\frac{-dE_i(x)}{dx}\left[e^{\frac{E_{Fn}(x) - E_i(x)}{kT}}\right] + \frac{n_i}{kT} \cdot \frac{dE_{Fn}(x)}{dx}\left[e^{\frac{E_{Fn}(x) - E_i(x)}{kT}}\right]$$

Using equation (ii), we can write

$$\frac{dn(x)}{dx} = \frac{n(x)}{kT}\frac{-dE_i(x)}{dx} + \frac{n(x)}{kT}\frac{dE_{Fn}(x)}{dx}$$

Substituting in equation (i), we get

$$J_n(x) = n(x)q\mu_n\left[\frac{1}{q}\frac{dE_i(x)}{dx}\right] + q\mu_n\frac{kT}{q}\left[\frac{n(x)}{kT}\cdot\frac{-dE_i(x)}{dx} + \frac{n(x)}{kT}\frac{dE_{Fn}(x)}{dx}\right]$$

$$\therefore \quad J_n(x) = n(x)q\mu_n\cdot\frac{d[E_{Fn}(x)/q]}{dx}$$

$$J_n(x) = \sigma_n(x)\frac{d[E_{Fn}(x)/q]}{dx} \qquad [\because \sigma = nq\mu]$$

Similarly,

hole current density, $J_p(x) = p(x)q\mu_p E(x) - qD_p\frac{dp(x)}{dx}$ ...(iii)

where $D_p = \mu_p\frac{kT}{q}$; $\quad E(x) = \frac{1}{q}\frac{dE_i(x)}{dx}$

$$p(x) = n_i \exp\left[\frac{E_i(x) - E_{Fp}(x)}{kT}\right] \qquad \text{...(iv)}$$

Differentiating w.r.t 'x' we get

$$\frac{dp(x)}{dx} = n_i\frac{d}{dx}\left[e^{\frac{E_i(x)}{kT}}\cdot e^{\frac{-E_{Fp}(x)}{kT}}\right]$$

$$= n_i\left[e^{\frac{E_i(x)}{kT}}\frac{-1}{kT}\frac{dE_{Fp}(x)}{dx}\cdot e^{\frac{-E_{Fp}(x)}{kT}}\right] + n_i\left[e^{\frac{-E_{Fp}(x)}{kT}}\frac{1}{kT}\frac{dE_i(x)}{dx}\cdot e^{\frac{E_i(x)}{kT}}\right]$$

$$= \frac{n_i}{kT}\frac{-dE_{Fp}(x)}{dx}\left[e^{\frac{E_i(x)-E_{Fp}(x)}{kT}}\right] + \frac{n_i}{kT}\frac{dE_i(x)}{dx}\left[e^{\frac{E_i(x)-E_{Fp}(x)}{kT}}\right]$$

$$\therefore \quad \frac{dp(x)}{dx} = \frac{p(x)}{kT}\frac{-dE_{Fp}(x)}{dx} + \frac{p(x)}{kT}\frac{dE_i(x)}{dx}$$

Substituting in equation (iii), we get

$$J_p(x) = p(x)q\mu_p\left[\frac{1}{q}\frac{dE_i(x)}{dx}\right] - \left[q\mu_p\cdot\frac{kT}{q}\left(\frac{p(x)}{kT}\cdot\frac{-dE_{Fp}(x)}{dx} + \frac{p(x)}{kT}\cdot\frac{dE_i(x)}{dx}\right)\right]$$

$$= p(x)q\mu_p\left[\frac{1}{q}\frac{dE_i(x)}{dx}\right] + p(x)q\mu_p\frac{d(E_{Fp}(x)/q)}{dx} - p(x)q\mu_p\frac{dE_q(x)}{dx}\frac{1}{q}$$

$$\therefore \quad J_p(x) = p(x)q\mu_p \frac{d(E_{Fp}(x)/q)}{dx}$$

We know that, $\sigma_p(x) = p(x)q\mu_p$

$$\therefore \quad J_p(x) = \sigma_p(x)\frac{d\left(\frac{E_{Fp}(x)}{q}\right)}{dx}$$

Q.8 (c) Solution:

(i) In a constellation diagram, the square of distance of a signalling point (symbol) from the origin gives the symbol energy. For constellation C_1

$$s_1 = (2a, 2a)$$

$$d_{s1} = \sqrt{(2a)^2 + (2a)^2} = a\sqrt{8}$$

$$E_{s1} = d_{s1}^2 = 8a^2$$

$$s_2 = (2a, 0)$$

$$d_{s2} = \sqrt{(2a)^2 + (0)^2} = 2a$$

$$E_{s2} = d_{s2}^2 = 4a^2$$

$$s_3 = (2a, -2a)$$

$$d_{s3} = \sqrt{(2a)^2 + (-2a)^2} = a\sqrt{8}$$

$$E_{s3} = d_{s3}^2 = 8a^2$$

$$s_4 = (0, -2a)$$

$$d_{s4} = \sqrt{(0)^2 + (-2a)^2} = 2a$$

$$E_{s4} = d_{s4}^2 = 4a^2$$

$$s_5 = (-2a, -2a)$$

$$d_{s5} = \sqrt{(-2a)^2 + (-2a)^2} = a\sqrt{8}$$

$$E_{s5} = d_{s5}^2 = 8a^2$$

$$s_6 = (-2a, 0)$$

$$d_{s6} = \sqrt{(-2a)^2 + (0)^2} = 2a$$

$$E_{s6} = d_{s6}^2 = 4a^2$$

$$s_7 = (-2a, 2a)$$

$$d_{s7} = \sqrt{(-2a)^2 + (2a)^2} = a\sqrt{8}$$

$$E_{s7} = d_{s7}^2 = 8a^2$$

$$s_8 = (0, 2a)$$

$$d_{s8} = \sqrt{(0)^2 + (2a)^2} = 2a$$

$$E_{s8} = d_{s8}^2 = 4a^2$$

Thus, the average symbol energy for constellation 1 is given by

$$(E_{\text{avg}})_{C_1} = \frac{E_{s1} + E_{s2} + E_{s3} + E_{s4} + E_{s5} + E_{s6} + E_{s7} + E_{s8}}{8}$$

$$= \frac{4 \times 8a^2 + 4 \times 4a^2}{8} = 6a^2$$

For constellation C_2,

$$s_1 = \left(2a, \sqrt{3a^2}\right)$$

$$d_{s1} = \sqrt{(2a)^2 + (\sqrt{3a^2})^2} = a\sqrt{7}$$

$$E_{s1} = d_{s1}^2 = 7a^2$$

$$s_2 = (a, 0)$$

$$d_{s2} = \sqrt{a^2 + (0)^2} = a$$

$$E_{s2} = d_{s2}^2 = a^2$$

$$s_3 = (2a, -\sqrt{3a^2})$$

$$d_{s3} = \sqrt{(2a)^2 + (-\sqrt{3a})^2} = a\sqrt{7}$$

$$E_{s3} = 7a^2$$

$$s_4 = (0, -\sqrt{3a^2})$$

$$d_{s4} = \sqrt{(0)^2 + (-\sqrt{3a^2})^2} = \sqrt{3a^2}$$

$$E_{s4} = d_{s4}^2 = 3a^2$$

$$s_5 = (-2a, -\sqrt{3a^2})$$

$$d_{s5} = \sqrt{(-2a)^2 + (-\sqrt{3a^2})^2} = a\sqrt{7}$$

$$E_{s5} = 7a^2$$

$$s_6 = (-a, 0)$$

$$d_{s6} = \sqrt{(-a)^2 + (0)^2} = a$$

$$E_{s6} = a^2$$

$$s_7 = (-2a, \sqrt{3a^2})$$

$$d_{s7} = \sqrt{(-2a)^2 + (\sqrt{3a^2})^2} = a\sqrt{7}$$

$$E_{s7} = 7a^2$$

$$s_8 = (0, \sqrt{3a^2})$$

$$d_{s8} = \sqrt{(0)^2 + (\sqrt{3a^2})^2} = \sqrt{3a^2}$$

$$E_{s8} = d_{s8}^2 = 3a^2$$

Thus, the average symbol energy for constellation 2 is given by

$$(E_{avg})_{C_2} = \frac{4\times 7a^2 + 2\times 3a^2 + 2\times a^2}{8} = 4.5a^2$$

(ii) The probability of error for a digital modulation scheme in an Additive White Gaussian Noise (AWGN) channel depends on the minimum Euclidean distance (d_{min}) between the signalling points. For constellation C_1 and C_2 it is given that distance between two adjacent signalling points is $2a$. Hence, $d_{min} = 2a$ for both C_1 and C_2. Since d_{min} is same, thus probability of symbol error for C_1 and C_2 will be same.

(iii) To transmit these signalling points, constellation-1 requires average energy (i.e., $E_{avg} = 6a^2$) while for constellation-2, average energy required to transmit the signalling points equals to $4.5a^2$.

Hence, constellation-2 requires less average energy to transmit the signalling point.

Hence, C_2 is more energy efficient or power efficient $\left(\text{as } P = \frac{E}{T}\right)$

$$\text{Gain} = 10\log_{10}\left(\frac{E_{\text{avg1}}}{E_{\text{avg2}}}\right)$$

$$= 10\log_{10}\left(\frac{6a^2}{4.5a^2}\right) = 10\log_{10} 1.33$$

$$= 1.23 \text{ dB}$$

Hence, constellation-2 is more power efficient than constellation-1 by 1.23 dB.

❍❍❍❍