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Detailed Solutions

ESE-2026 Mains Test Series

Electrical Engineering Test No : 5

Section A : Basic Electronics Engineering + Analog Electronics + Electrical Materials

Q.1 (a) Solution:

In a dielectric, the displacement flux density

$$D = \epsilon_0 E + P \quad \text{...(i)}$$

where E is the electric field strength and P is the polarization.

But, $$D = \epsilon_0 \epsilon_r E \quad \text{...(ii)}$$

From equation (i) and (ii) we have,

So, $$\epsilon_0 \epsilon_r E = \epsilon_0 E + P$$

or $$P = \epsilon_0 (\epsilon_r - 1) E$$

But polarization (P) = $N\alpha E$

where N is the number of dipoles per unit volume and α is called polarizability.

So, $$\epsilon_0 (\epsilon_r - 1) E = N\alpha E$$

or $$N\alpha = \epsilon_0 (\epsilon_r - 1) \quad \text{...(iii)}$$

⇒ This is the desired relationship between the polarizability and permittivity.

In a cubic crystal, the internal field seen by an atom is

$$E_i = E + \frac{\gamma P}{\epsilon_0}, \quad \text{where } \gamma = \frac{1}{3}$$

we get, $$E_i = E + \frac{P}{3\epsilon_0}$$

or $$E_i = E + \frac{\epsilon_0 (\epsilon_r - 1) E}{3\epsilon_0}$$

or $$E_i = E + \frac{\in_r E}{3} - \frac{E}{3} = \left(\frac{\in_r + 2}{3}\right)E$$

and $$P = N\alpha E_i = N\alpha\left(\frac{\in_r + 2}{3}\right)E = \in_0 (\in_r - 1)E$$

or, $$\frac{N\alpha}{3\in_0} = \frac{\in_r - 1}{\in_r + 2} \quad \text{...(iv)}$$

This equation (iv) is the Clausius-Mossotti equation.

Q.1 (b) (i) Solution:

Hall measurements are made on a *P*- type semiconductor bar of 500 μm wide and 20 μm thick. The hall contact *A* and *B* are displaced by 2 μm,

Current, $I = 3\text{ mA}$

V_{AB} when magnetic field is pointing out of the plane is 3.2 mV and –2.8 mV when magnetic field direction is revered,

The voltage measured is the Hall voltage plus the ohmic drop

The sign of V_H changes with the magnetic field, but not the ohmic voltage

$$V_H = \frac{V_{H1} - V_{H2}}{2} = \frac{3.2 - (-2.8)}{2} = 3\text{ mV}$$

$\Rightarrow$ ohmic drop $= 3.2 - 3 = 0.2\text{ mV}$

From the equation of hall coefficient,

$$P_0 = \frac{I_x B_z}{qtV_{AB}}$$

$$= \frac{(3\times10^{-3}\text{A})(10^{-4}\text{Wb/cm}^2)}{q(20\times10^{-4}\text{cm})(3\times10^{-3}\text{V})} = 3.125 \times 10^{17}/\text{cm}^3$$

$$J = \sigma\varepsilon$$

$$\Rightarrow \quad J = \frac{\varepsilon}{\rho}$$

$$\Rightarrow \quad \rho = \frac{\varepsilon}{J} = \frac{0.2\text{ mV}/2\,\mu\text{m}}{3\text{ mA}/(500\,\mu\text{m}\times20\,\mu\text{m})} = 0.033\,\Omega\text{ - cm}$$

$$\rho = \frac{1}{q\mu_p p_0}$$

$$\Rightarrow \quad \mu_p = \frac{1}{qp_0 p} = \frac{1}{(1.6\times10^{-19})(0.033)(3.125\times10^{17})} = 600\text{ cm/V.s}$$

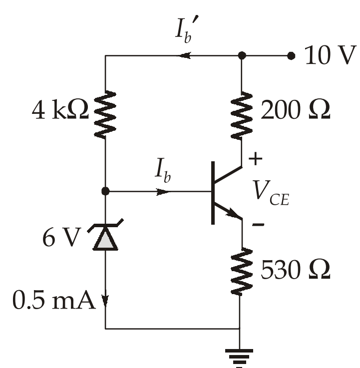
Q.1 (b) (ii) Solution:

Given,
$$N_d = 10^{17}$$
$$\mu_n = 700 \text{ cm}^2/\text{V.S}$$
$$\sigma = q\,\mu_n\, n \text{ and } n \approx N_d$$
$$\Rightarrow \quad \sigma = 1.6 \times 10^{-19} \times 700 \times 10^{17}$$
$$= 11.2/\Omega\text{-cm}$$
$$\rho = \frac{1}{\sigma} = \frac{1}{11.2} = 0.09\ \Omega\text{-cm}$$

Hall coefficient,
$$R_H = \frac{1}{qn} = \frac{-1}{1.6\times10^{-19}\times10^{17}} = -62.5 \text{ cm}^2/\text{C}$$

Hall voltage,
$$V_H = \frac{I_x B_z R_H}{t} = \frac{(1\times10^{-3})(10^{-5})(-62.5)}{10^{-2}} = -62.5\ \mu\text{V}$$

Q.1 (c) Solution:



The current through 4 Ω resistor is
$$I_b' = \frac{(10-6)\text{ V}}{4\text{ k}\Omega} = 1 \text{ mA}$$

Now, 0.5 mA of current passes through zener diode hence effective base current is
$$I_b = 0.5 \text{ mA}$$

Apply KVL to the base circuit,
$$V_Z - V_{BE} - I_E R_E = 0$$
$$6 - 0.7 - (\beta + 1)I_b \times 530 = 0$$
$$5.3 - (\beta + 1)\, 0.5 \times 10^{-3} \times 530 = 0$$
$$5.3 - 0.265\,(\beta + 1) = 0$$
$$\therefore \quad \beta + 1 = \frac{5.3}{0.265} = 20;$$
$$\therefore \quad \beta = 19$$

Now, apply KVL to the collector to emitter circuit

$$V_{CC} - I_C R_C - V_{CE} - I_E R_E = 0$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E$$

$$V_{CE} = 10 - I_C\, 200 - 530\, I_E$$

$$= 10 - \beta\, I_b\, 200 - 530 \times (\beta + 1)\, I_b$$

$$= 10 - 19 \times 0.5 \times 10^{-3} \times 200 \; - 530 \times 20 \times 0.5 \times 10^{-3}$$

$$= 10 - 1.9 - 5.3$$

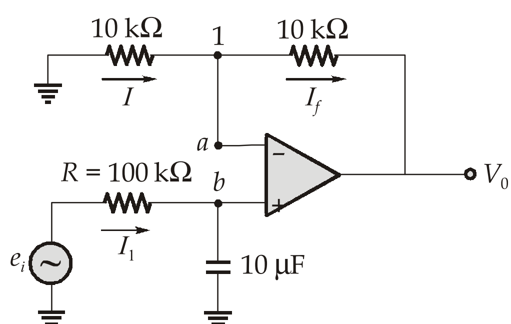
$$= 2.8 \text{ volts}$$

Q.1 (d) Solution:

Top-down Approach : In top-down approach, nano-scale objects are made by processing larger objects in size. Integrated circuit fabrication is an example for top down nanotechnology. For nano-material synthesis ball-milling is an important top-down approach where macro crystalline structures are broken down to nano-crystalline structures but original integrity of the material is retained. Sometimes this method is used to prepare nanostructured metal oxides by chemical reaction between two constituents during crushing. The crystallites are allowed to react with each other by the supply of kinetic energy during milling process to form the required nanostructured oxide.

Bottom-up Approach : Bottom-up approach in nanotechnology is making larger nanostructures from smaller building blocks such as atoms and molecules, therefore, very important for nano-fabrication. The bottom-up non-lithographic approach of nano-material synthesis is not completely proven in manufacturing yet, but has a great potential to become important alternative to lithographic process. Examples of bottom-up technique are self assembly of nanomaterials solgel technology, electrodeposition, physical and chemical vapour deposition (PVD and CVD), epitaxial growth, laser ablation etc.

Q.1 (e) Solution:



KCL at node-1

$$\frac{0-V_a}{10} = \frac{V_a - V_0}{10}$$

$$\Rightarrow \quad V_0 = 2V_a \quad \text{...(i)}$$

KCL at node b,

$$\frac{e_i - V_b}{R} = \frac{V_b - 0}{1/sC}$$

$$\Rightarrow \quad V_b = \left(\frac{e_i}{1+sCR}\right)$$

$$\Rightarrow \quad V_0 = \frac{2e_i}{1+sCR} \quad (\because V_b = V_a, \text{ By virtual ground})$$

$$V_0 = e_i\left(\frac{2}{1+sCR}\right)$$

$\therefore$ transfer function is

$$H(s) = \frac{2}{1+sCR}$$

$$|H(s)| = \frac{2}{\sqrt{1+\omega^2C^2R^2}}$$

for $\omega = 1$, $C = 10\ \mu\text{F}$ and $R = 100$ K

$$\omega CR = 1 \times 10 \times 10^{-6} \times 100 \times 10^3 = 1$$

$$= \frac{2}{\sqrt{1+1}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\angle H(s) = -\tan^{-1}\omega CR$$

$$= -\tan^{-1}(1) = 45°$$

$$\therefore \quad H(s) = \sqrt{2}\angle -45°$$

$$\Rightarrow \quad V_0 = \sqrt{2}\sin\left(t - \frac{\pi}{4}\right) \text{ Volts}$$

Q.2 (a) Solution:

(i) **Given:** Dielectric constant, $\epsilon_r = 5.5$; Magnetic susceptibility, $\chi_m = -2.17 \times 10^{-5}$

The magnetic susceptibility is related to the relative permeability of the material as

$$\chi_m = \mu_r - 1$$

$$\Rightarrow \quad -2.17 \times 10^{-5} = \mu_r - 1$$

$$\Rightarrow \quad \mu_r = 1 - 2.17 \times 10^{-5} = 0.9999783$$

The velocity of light in a material is given as

$$V = \frac{1}{\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_o \mu_r \in_o \in_r}} = \frac{1}{\sqrt{\mu_o \in_o}} \times \frac{1}{\sqrt{\mu_r \in_r}}$$

$$\Rightarrow \quad V = \frac{C}{\sqrt{\mu_r \in_r}}$$

where, $C = \frac{1}{\sqrt{\mu_o \in_o}}$ = Velocity of light in free space

Substituting the values,

we get, $$V = \frac{3 \times 10^8}{\sqrt{5.5 \times 0.9999783}} = 1.279 \times 10^8 \text{ m/s}$$

(ii) Given: Current through coil of wire,

$$I = 10 \text{ A};$$

Length of coil, $l = 0.20$ m

Number of turns in coil, $N = 200$

1. **Magnitude of magnetic field strength, H**

   Using Ampere's circuited law,

$$\oint H \cdot dl = NI$$

$$\Rightarrow \quad H \cdot l = NI$$

$$\Rightarrow \quad H = \frac{NI}{l}$$

   Substituting the values,

   we get, $$H = \frac{200 \times 10}{0.2} = 10000 \text{ A/m}$$

2. **Flux density *B* if coil is in vacuum:**

   The flux density *B* is given as,

$$B = \mu H, \quad \text{where } \mu = \text{Permeability}$$

   In vacuum, $\mu = \mu_o = 4\pi \times 10^{-7}$ H/m

   $\therefore$ $B = \mu_o H = 4\pi \times 10^{-7} \times 10000$

$$B = 12.57 \text{ mWb/m}^2$$

3. **Flux density inside a bar of titanium:**

   For titanium, susceptibility $\chi_m = 1.81 \times 10^{-4}$

   The relative permeability of titanium is obtained from magnetic susceptibility as

$$\chi_m = \mu_r - 1$$

$$\Rightarrow \quad \mu_r = \chi_m + 1 = 1.000181$$

The flux density inside a bar of titanium is given as

$$B = \mu H = \mu_o \mu_r H$$

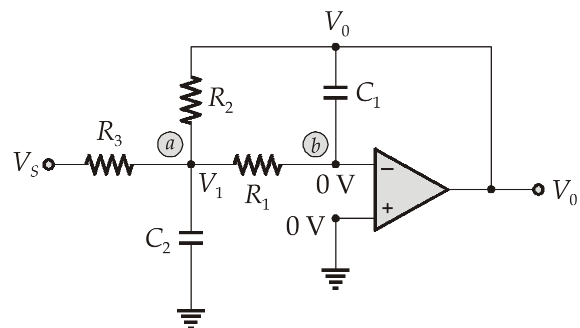
$$B = 4\pi \times 10^{-7} \times 1.000181 \times 10000 = 12.5686 \text{ mWb/m}^2$$

4. **Magnetization (M):**

Magnetization, $M = \chi_m \cdot H = 1.81 \times 10^{-4} \times 10000 = 1.81 \text{ A/m}$

Q.2 (b) Solution:

Given circuit is



Apply Nodal equation at node 'a'

$$\frac{V_1 - V_s}{R_3} + \frac{V_1 - 0}{R_1} + \frac{V_1 - V_0}{R_2} + \frac{V_1 - 0}{1/sC_2} = 0 \quad \text{...(i)}$$

Observe that current flowing through R_1 is equal to current flowing through C_1 (Because of high input impedance of op amp no current flows through the op amp).

Apply nodal at node 'b'

$$\therefore \quad \frac{0 - V_1}{R_1} + \frac{0 - V_0}{1/sC_1} = 0$$

$$\frac{V_1}{R_1} = -V_0 sC_1$$

$$\Rightarrow \quad V_1 = -V_0 sR_1C_1 \quad \text{...(ii)}$$

Substitute (ii) in (i),

$$\frac{(-V_0sR_1C_1) - V_s}{R_3} + \frac{(-V_0sR_1C_1)}{R_1} + \frac{(-V_0sR_1C_1) - V_0}{R_2} + \frac{(-V_0sR_1C_1)}{1/sC_2} = 0$$

$$V_0\left[\frac{-sR_1C_1}{R_3} - \frac{sR_1C_1}{R_1} - \frac{(1 + sR_1C_1)}{R_2} - s^2R_1C_1C_2\right] = \frac{V_s}{R_3}$$

$$V_0\left[\frac{s^2R_1R_2R_3C_1C_2 + sR_1R_2C_1 + sR_1R_3C_1 + sR_2R_3C_1 + R_3}{R_2R_3}\right] = \frac{-V_s}{R_3}$$

$$\frac{V_0}{V_s} = \left[\frac{-R_2}{s^2 R_1R_2R_3C_1C_2 + sC_1(R_1R_2 + R_2R_3 + R_3R_1) + R_3}\right]$$

The characteristic equation is

$$s_2R_1R_2R_3C_1C_2 + sC_1(R_1R_2 + R_2R_3 + R_3R_1) + R_3 = 0$$

$$s^2 + s\left[\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1R_2R_3C_2}\right]s + \frac{1}{R_1R_2C_1C_2} = 0$$

By comparing with 2nd order characteristics equation

$$s^2 + 2\xi\omega_n S + \omega_n^2 = 0$$

Natural frequency, $\omega_n = \dfrac{1}{\sqrt{R_1R_2C_1C_2}}$

Damping factor, $\xi = \dfrac{(R_1R_2 + R_2R_3 + R_3R_1)}{R_1R_2R_3 \times 2\omega_n}$

Substitute ω_n value in ξ

$$\xi = \frac{\sqrt{R_1R_2C_1C_2}\,(R_1R_2 + R_2R_3 + R_3R_1)}{2R_1R_2R_3C_2}$$

$$= \frac{1}{2}\sqrt{\frac{R_1R_2C_1}{C_2}}\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

For 2nd order equation (for given characteristics equation) the cut-off frequency is

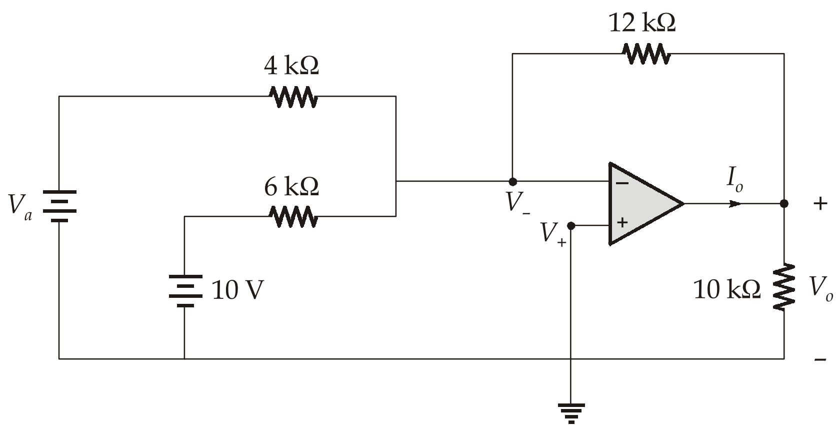
$$\omega_L = \omega_n(1 - 2\xi^2) + \left[(1 - 2\xi^2) + \sqrt{(1 - 2\xi^2)^2 + 1}\right]^{1/2}$$

Where, $\omega_n = \dfrac{1}{\sqrt{R_1R_2C_1C_2}}$

$$\xi = \frac{1}{2}\sqrt{\frac{R_1R_2C_1}{C_2}}\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

Q.2 (c) Solution:

(i) For $V_a = 4$ V



Due to virtual ground,

$$V_+ = V_- = 0 \text{ V}$$

Apply KCL at node V_-

$$\frac{V_- - 4}{4k} + \frac{V_- - 10}{6k} + \frac{V_- - V_o}{12k} = 0$$

$$V_o = \frac{12}{4}(-4) - \frac{12}{6} \times 10$$

$$V_o = -32 \text{ V}$$

Current I_o : Apply KCL at node V_o

$$\frac{v_o}{10k} + \frac{v_o - V_-}{12k} = I_o$$

$$I_o = \left[\frac{-32}{10} + \frac{-32}{12}\right] \text{mA}$$

$$V_o = -32 \text{ V}$$

$$I_o = -5.867 \text{ mA}$$

(ii) For linear operation,

$$-12 < v_o < 12 \text{ V}$$

Apply KCL at node V_-

$$\frac{V_- - V_a}{4k} + \frac{V_- - (+10)}{6k} + \frac{V_- - V_o}{12k} = 0$$

$$-3v_a - 20 = v_o$$

$$-12 < -3v_a - 20 < 12$$

$$-3v_a - 20 > -12$$

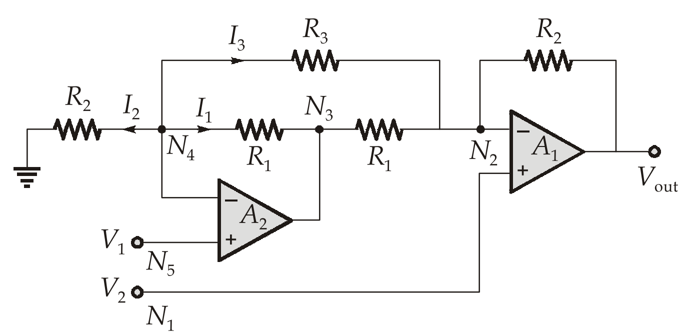
$$v_a < \frac{-8}{3} \Rightarrow v_a < -2.667$$

$$-3v_a - 20 < 12$$

$$v_a > -\left(\frac{20+12}{3}\right) \Rightarrow v_a > -10.667$$

Range : $-10.667 < v_a < -2.667$

Q.3 (a) (i) Solution:



For amplifier A_1, node N_1 and N_2 have same potential.

Hence, $V_{N1} = V_{N2} = V_2$

Similarly, for amplifier A_2, the potential at node N_4 and N_5 is same,

$$V_{N5} = V_{N4} = V_1$$

$$I_2 = \frac{V_{N4}}{R_2} = \frac{V_1}{R_2}$$

$$I_1 = \frac{V_{N4} - V_{N3}}{R_1} = \frac{V_1 - V_{N3}}{R_1}$$

$$I_3 = \frac{V_1 - V_2}{R_3}$$

Applying KCL at node N_4, we get,

$$I_1 + I_2 + I_3 = 0$$

Similarly, applying KCL at node N_2 and neglecting input current of op-amps, we get,

$$\frac{V_{N2} - V_{N4}}{R_3} + \frac{V_{N2} - V_{N3}}{R_1} + \frac{V_{N2} - V_{out}}{R_2} = 0$$

$$\frac{V_2 - V_1}{R_3} + \frac{V_2 - V_{N3}}{R_1} + \frac{V_2 - V_{out}}{R_2} = 0$$

$$V_2\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{V_1}{R_3} - \frac{V_{out}}{R_2} = \frac{V_{N3}}{R_1}$$

Equating the expression for V_{N3}/R_1, we get

$$V_1\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{V_2}{R_3} = V_2\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{V_1}{R_3} - \frac{V_{out}}{R_2}$$

$$\frac{V_1[R_2R_3 + R_1R_3 + R_1R_2] - R_1R_2V_2}{R_1R_2R_3} = \frac{V_2[R_1R_2 + R_2R_3 + R_1R_3] - R_1R_2V_1 - R_1R_3V_{out}}{R_1R_2R_3}$$

$$V_1(R_2R_3 + R_1R_3 + 2R_1R_2) - V_2[R_2R_3 + R_1R_3 + 2R_1R_2] = -R_1R_3\, V_{out}$$

$$R_1R_3\, V_{out} = (V_2 - V_1)\,(R_2R_3 + R_1R_3 + 2R_1R_2)$$

$$\frac{V_{out}}{V_2 - V_1} = 1 + \frac{R_2}{R_1} + 2\frac{R_2}{R_3}$$

Q.3 (a) (ii) Solution:

Matthiessen's Rule: An empirical rule which states that the total resistivity of a crystalline metallic specimen is the sum of the resistivity due to thermal agitation of the metal ions of the lattice and the resistivity due to the presence of imperfections in the crystal. The total resistivity is, thus given by

$$\rho_{total} = \rho_{thermal} + \rho_{impurity} + \rho_{def}$$

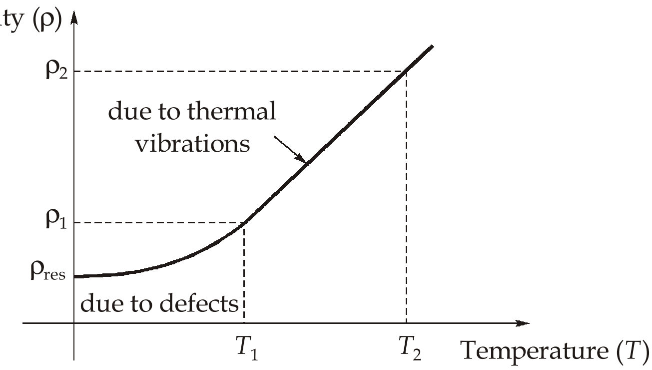
where $\rho_{thermal}$ is the resistivity due to temperature, $\rho_{impurity}$ is the resistivity due to impurities and ρ_{def} is the resistivity due to defects in the crystal.

Factors affecting resistivity of metals:

1. **Temperature:** The rise in the temperature of the metal increases thermal agitation of the metallic ions. This restricts the free movement of electrons and reduces the mean free path, thus reducing the conductivity of the metal i.e. increases the resistivity of the metal. The resistivity of a metal increases linearly with the temperature as shown below in the graph. We have,

$$\rho = \rho_0\,(1 + \alpha\Delta T)$$

where α = Temperature coefficient of resistivity and ΔT is the change in temperature. We have,



The resistivity at low temperature exists due to defects in the micro structure of a pure metal and increases linearly with the increase in temperature.

2. **Alloying:** The addition of alloying element to a pure metal increases the lattice imperfections. As a result of this, the resistivity of an alloy increases. In general, the resistivity of an alloy is given by the relation

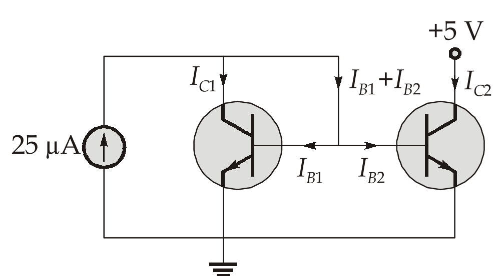
$$\rho_{alloy} = \rho_{metal} + x\rho_i$$

where ρ_{metal} = resistivity of the pure metal, ρ_i = resistivity of the impurity and x is the amount of impurity added in the metal.

3. **Cold work:** Mechanical distortion of the crystal structure increases the resistivity of a metal because the localized strains interfere with electron movement. For example, strain hardening results in higher resistivity than annealed samples of the same metal.

4. **Age hardening:** The electrical resistivity of a metal increases due to age hardening. During this process, the crystal lattice undergoes some distortion due to which the movement of electrons is reduced.

Q.3 (b) (i) Solution:



Both transistors are in the forward active region.

So, $$25\ \mu A = I_{C1} + I_{B1} + I_{B2}$$

Since the transistor are identical and have the same V_{BE},

$$I_{C2} = I_{C1}, \quad I_{B1} = I_{B2}$$

So, $$25\ \mu A = I_{C1} + 2I_{B1} = (2 + \beta)\, I_{B1}$$

$$I_{C2} = \beta I_{B2} = \beta I_{B1} = \left(\frac{\beta}{\beta+2}\right)\times 25\,\mu A$$

$$= \frac{25}{27}\times 25 = 23.15\,\mu A$$

So, $$I = I_{C2} = 23.15\,\mu A$$

Q.3 (b) (ii) Solution:

Given : $V_{Th} = 0.9$ V, $\mu_n C_{ox} = 40\ \mu A/V^2$

Both the transistors are operating in saturation mode as drain is shorted to gate.

So, $$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{Th})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{Th})^2$$

$$V_{GS1} = 6 - V_o$$

$$V_{GS2} = V_o$$

$$60(6 - V_o - 0.9)^2 = 25(V_o - 0.9)^2$$

$$12(5.1 - V_o)^2 = 5(V_o - 0.9)^2$$

$$12(26.01 - 10.2V_o + V_o^2) = 5(V_o^2 - 1.8V_o + 0.81)$$

$$7V_o^2 - 113.4V_o + 308.07 = 0$$

On solving,

$$V_o = 12.75\text{ V}, 3.45\text{ V}$$

$$V_o = 3.45\text{ V}$$

Q.3 (b) (iii) Solution:

Given data:

Density of Cu (ρ) = 8.96 gcm^{-3},

Atomic weight (M_{at}) = 63.5

and Avogadro's number (N_A) = 6×10^{23}

By assuming each Cu atom donates one free electron, we can find the concentration of electrons as,

$$n = \frac{\rho N_A}{M_{at}} = \frac{8.96\times 6\times 10^{23}}{63.5} = 8.5\times 10^{22}\text{ cm}^{-3}$$

$$= 8.5 \times 10^{28}\text{ m}^{-3}$$

Fermi energy at 0°K is, $$E_{F0} = \left(\frac{h^2}{8m_e}\right)\left(\frac{3n}{\pi}\right)^{2/3}$$

$$= \left(\frac{(6.62\times10^{-34})^2}{8\times9.11\times10^{-31}}\right)\left(\frac{3\times8.5\times10^{28}}{\pi}\right)^{2/3}$$

$$= 1.13 \times 10^{-18} = 7 \text{ eV}$$

$$\frac{1}{2}m_e v^2 = \frac{3}{5}E_{F0}$$

Here v is the rms speed, which will be close to the mean speed.

So, $$v = \sqrt{\frac{6E_{F0}}{5m_e}} = \sqrt{\frac{6\times1.13\times10^{-18}}{5\times9.11\times10^{-31}}} \text{ m/sec} = 1.22\times10^6 \text{ m/sec}$$

Q.3 (c) Solution:

(i) Here, $$\varepsilon_r' = 3$$

$$\omega = 2\pi f$$

$$= 2\times3.14\times1.5\times10^6$$

$$= 9.42\times10^6 \text{ Hz}$$

The loss tangent, $$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'} = 3.8\times10^{-4}$$

$$\varepsilon_r'' = \varepsilon_r' \tan\delta$$

$$= 3\times3.8\times10^{-4} = 11.4\times10^{-4}$$

$$R_P = \frac{d}{\omega\varepsilon_r''\varepsilon_0 A}$$

$$= \frac{0.3\times10^{-3}}{9.42\times10^6\times8.854\times10^{-12}\times11.4\times10^{-4}\times25\times10^{-4}}$$

$$= 1.2621\times10^6\ \Omega$$

$$C_P = \frac{\varepsilon_0\varepsilon_r' A}{d}$$

$$= \frac{8.854\times10^{-12}\times3\times25\times10^{-4}}{0.3\times10^{-3}} = 221.35 \text{ pF}$$

(ii) $$R_s = \frac{R_P}{\omega^2C_P^2R_P^2} \simeq \frac{X_P^2}{R_P} = \frac{1}{\omega^2C_P^2R_P}$$

$$= \frac{1}{(9.42\times10^6)^2\times(221.35)^2\times10^{-24}\times1.2621\times10^6}$$

$$= 0.18224\ \Omega$$

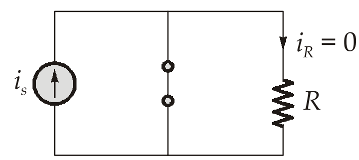
$$C_s^2 = \frac{1}{\omega^2 R_s R_P} = \frac{1}{\omega\sqrt{R_s R_P}}$$

$$C_s = \frac{1}{\omega\sqrt{R_s R_P}} = \frac{1}{\sqrt{(9.42\times10^6)^2\times0.18224\times1.2621\times10^6}}$$

$$= 221.35 \text{ pF}$$

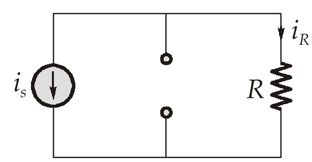
Q.4 (a) (i) Solution:

In +ve half cycle : Diode is ON



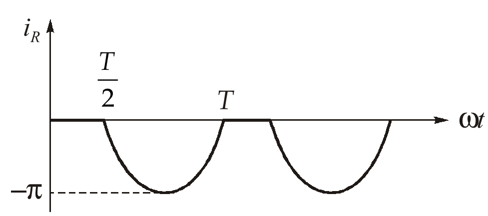
In –ve half cycle : Didoe is OFF,

$$i_R = i_s = -\pi \sin 300\pi t \text{ mA}$$



Average current through the resistor R,

$$i_R = \frac{\text{Peak value}}{\pi} = \frac{-\pi}{\pi} = -1 \text{ mA}$$

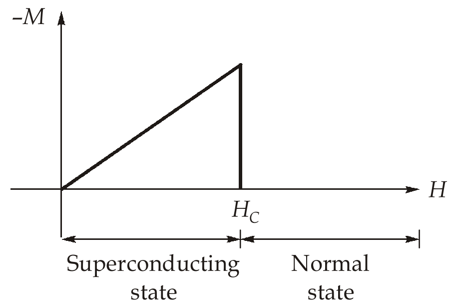


Magnitude of average current through the resistor R,

$$i_R = 1 \text{ mA}$$

Q.4 (a) (ii) Solution:

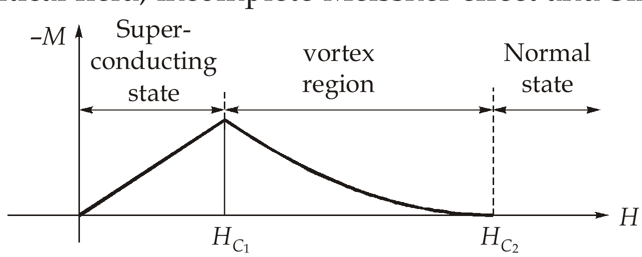
Depending upon the magnetic properties, the superconductors are classified as type-I and type- II. Those materials which shows the perfect diamagnetism upto the critical field, H_C and go to the normal state abruptly are known as type-I superconductors. The critical field of these superconductors are low therefore they are not suitable for high field application.



Magnetization characteristics of type-I superconductors

Eg.: Lead, Indium etc.

On the other hand the type-II superconductors are characterized by high transition temperature, high critical field, incomplete Meissner effect and Silsbee's rule.



Magnetization characteristics of type-II superconductors

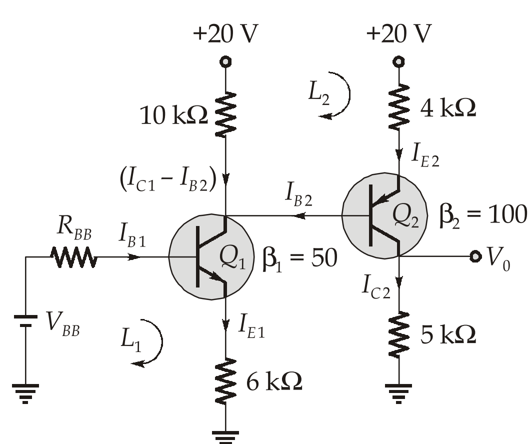
As shown in above characteristics even though the Meissner state breaks down at small magnetic field H_{C_1}, the superconducting state remains to a higher magnetic field H_{C_2}.

These critical fields are lower and upper critical fields respectively. Hence these materials are suitable for applications at high field.

Ex: Nb_3Sn.

Q.4 (b) Solution:

Given circuit can be redrawn,



$$V_{BB} = \frac{20\times100}{100+200} = 6.67 \text{ V or } \frac{20}{3} \text{ V}$$

$$R_{BB} = \frac{100\times200}{100+200} = 66.667 \text{ k}\Omega \text{ or } \frac{200}{3} \text{ k}\Omega$$

By applying KVL in loop-1,

$$6.67 - \frac{200}{3} k\, I_{B1} - 0.7 - 6k\, I_{E1} = 0$$

$$5.97 - \frac{200}{3} k\, I_{B1} - 6k(\beta_1 + 1) I_{B1} = 0$$

$$5.97 - \frac{200}{3} k\, I_{B1} - 6k(50 + 1) I_{B1} = 0$$

$$I_{B1} = \frac{5.97}{372666.67} = 16.02\ \mu\text{A}$$

$\therefore$

$$I_{C1} = \beta_1 I_{B1}$$

$$= 50 \times 16.02 \text{ mA} = 0.801 \text{ mA}$$

$$I_{E1} = I_{C1} + I_{B1}$$

$$= 0.801 \text{ mA} + 16.02\ \mu\text{A} = 0.817 \text{ mA}$$

Applying KVL in loop 2,

$$-20 + 4kI_{E2} + 0.7 - (I_{C1} - I_{B2})\, 10\, kI_{B2} + 20 = 0$$

$$4kI_{E2} + 0.7 - 10\, kI_{C1} + 10\, kI_{B2} = 0$$

$$4k(100 + 1)I_{B2} + 0.7 - 10\, kI_{C1} + 10\, kI_{B2} = 0$$

$$4k(101)\, I_{B2} + 0.7 - 10\, k(I_{C1}) + 10\, kI_{B2} = 0$$

$$404\, kI_{B2} + 0.7 - 10\, kI_{C1} + 10\, kI_{B2} = 0$$

$$414\, kI_{B2} + 0.7 - 10[0.801] = 0$$

$$414\, kI_{B2} = 7.31$$

$$I_{B2} = \frac{7.31}{414\times10^3} = 17.66\ \mu\text{A}$$

$$I_{C2} = 100 \times 17.66\ \mu\text{A}$$

$$= 1.766 \text{ mA}$$

Given, $R_L = 5 \text{ k}\Omega$

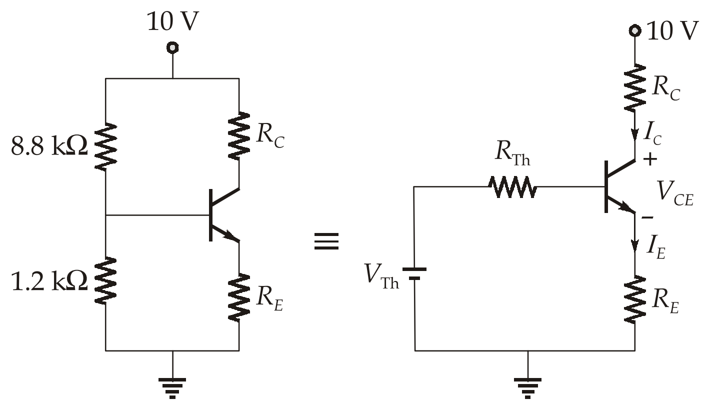
$$\text{Output voltage} = I_{C2} \times 5 \times 10^3$$

$$= 1.766 \times 10^{-3} \times 5 \times 10^3$$

$$= 8.83 \text{ V}$$

Q.4 (c) Solution:

D.C. analysis,



where,
$$V_{Th} = \frac{10 \times 1.2}{1.2 + 8.8} = 1.2 \text{ V}$$

$$R_{Th} = \frac{1.2 \times 8.8}{1.2 + 8.8} = 1.056 \text{ V}$$

∵ β is very large,

$$I_B \approx 0 \quad \text{[No voltage drop across } R_{Th}\text{]}$$
$$I_C = I_E$$

Apply KVL in input loop,

$$V_{Th} = I_B R_{Th} + V_{BE} + I_E R_E$$
$$1.2 = 0 + 0.7 + I_C R_E$$
$$I_C = \frac{0.5}{R_E} \quad \text{...(i)}$$

Apply KVL in output loop,

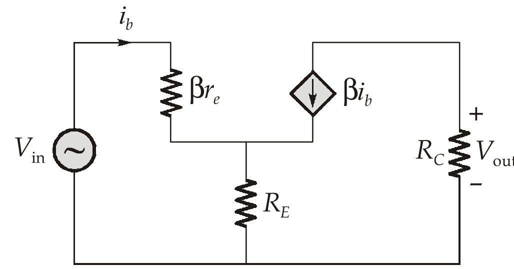
$$-10 + I_C R_C + \frac{10}{2} + I_E R_E = 0$$
$$5 = I_C(R_C + R_E)$$

From equation (i),

$$5 = \frac{0.5}{R_E}[R_C + R_E]$$

$$\Rightarrow \quad \frac{R_C}{R_E} = 9 \quad \text{...(ii)}$$

AC equivalent small is signal re model,



$$V_{out} = -\beta\, i_b R_C$$

$$V_{in} = i_b\beta r_e + (i_b + \beta i_b)\, R_E = i_b[\beta r_e + (1 + \beta)\, R_E$$

$\because$ $$1 + \beta \approx \beta$$

$$\cong \beta i_b[r_e + R_E]$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{\beta i_b R_C}{\beta i_b[r_e + R_E]} = \frac{R_C}{r_e + R_E}$$

$\because$ $r_e << R_E$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_C}{R_E}$$

From equation (ii),

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_C}{R_E} = 9$$

Section B : Electrical Machines-1 + Power Systems-2

Q.5 (a) Solution:

(i) As we know,

Average output voltage, $V_o = \frac{3V_{ml}}{\pi}\cos\alpha - \Delta V_{do}$ (for 6 pulse rectifier)

$$\Delta V_{do} = -V_o + \frac{3V_{ml}}{\pi}\cos\alpha$$

$$= -400 + \frac{3\times 400\sqrt{2}}{\pi}\cos 15^\circ$$

$$\Delta V_o = 121.78 \text{ kV}$$

Now, ΔV_{do} = Reduction in output voltage due to commutation = $R_{cr}I_o$

$$R_{cr} = \frac{121.78\times 10^3}{1000} = 121.78\ \Omega$$

(ii) Now,
$$\Delta V_{do} = \frac{V_{do}}{2}[\cos\alpha - \cos(\alpha+\mu)]$$
$$121.78 = \frac{3\times 400\sqrt{2}}{2\pi}[\cos(15°) - \cos(15+\mu)]$$
$$\mu = 44°$$

(iii) Now, as dc current is constant.

$$\therefore \quad I_o = \text{Constant}$$
$$\Delta V_{do} = 6fL_sI_o$$
$$= R_{cr}I_o$$
$$= \text{Constant}$$
$$121.78 = \frac{3\times 200\sqrt{2}}{2\pi}[\cos(15°) - \cos(15°+\mu)]$$

Overlap angle, $\mu = 71.32°$

Q.5 (b) Solution:

$$\text{Total losses} = \left(\frac{1}{0.9} - 1\right)\times 45\times 10^3$$
$$P_L = 5000 \text{ W}$$
$$P_L = P_{scu} + P_{rcu} + P_{si} + P_m$$
$$\therefore \quad P_{scu} = P_{rcu} = P_{si} = P_i$$
$$\therefore \quad P_L = 3P_i + P_m$$
$$\text{No load losses } (P_{NL}) = \text{Stator iron loss } (P_{si}) + \text{Mechanical loss } (P_m)$$
$$\therefore \quad P_m = \frac{1}{3}P_{NL}$$
$$\therefore \quad 3P_m = P_i + P_m$$
$$P_i = 2P_m$$
$$P_L = 3P_i + P_m$$
$$= 6P_m + P_m$$
$$5000 = 7P_m$$
$$P_m = \frac{5000}{7}\text{W}$$
$$P_i = \frac{10000}{7}\text{W}$$

$$P_{\text{air gap}} = P_g = P_{\text{in}} - (P_i + P_{scu})$$
$$= 50000 - \frac{20000}{7} = \frac{330000}{7}$$
$$P_{rcu} = P_i = \frac{10000}{7}$$
$$s = \frac{P_{rcu}}{P_g} = \frac{10000}{33000}$$
$$s = 0.0303 \text{ pu}$$

Q.5 (c) Solution:

Given,
$$Z_{\text{Bus}} = j\begin{bmatrix} 0.1397 & 0.1103 & 0.1250 \\ 0.1103 & 0.1397 & 0.1250 \\ 0.1250 & 0.1250 & 0.1750 \end{bmatrix}$$

(i)
$$I^f = \frac{V_r^o}{Z_{rr} + Z^f}$$
$$I^f = \frac{V_3^o}{Z_{33}} = \frac{1}{j0.175} = -j5.71$$

(ii)
$$V^f_i = V_i^o - \frac{Z_{ir}}{Z_{rr} + Z_f} V_r^o$$

Now,
$$V^f_1 = \left(1 - \frac{Z_{13}}{Z_{33}}\right) = 1 - \frac{0.125}{0.175} = 0.286$$

and
$$V^f_2 = \left(1 - \frac{Z_{23}}{Z_{33}}\right) = 0.286$$

These two voltages are equal because of the symmetry of the given power network.

(iii)
$$I^f_{ij} = Y_{ij}(V^f_i - V^f_j)$$
$$I^f_{12} = \frac{1}{j0.1}(0.286 - 0.286) = 0$$

and
$$I^f_{13} = I^f_{31} = \frac{1}{j0.1}(0.286 - 0)$$
$$= -j2.86$$
$$I^f_{23} = Y_{23}[V_2^f - V_3^f]$$

$$= \frac{1}{j0.125}\left[\frac{2}{7} - 0\right] = -j2.86 \text{ p.u.}$$

(iv)

$$I^f_{G1} = \frac{E'_{G1} - V^f_1}{jX'_{iG} + jX_T}$$

But $E'_{G1} = 1$ pu (prefault no load)

$\therefore$

$$1^f_{G1} = \frac{1 - 0.286}{j0.2 + j0.05} = -j2.86$$

Similarly, $I^f_{G2} = -j2.86$

Q.5 (d) Solution:

Per phase motor input current :

$$I_1 = \frac{500}{\sqrt{3} \times 3.3 \times 0.7 \times 0.86} = 145.32 \text{ A}$$

$$\vec{I}_1 = 145.32\angle{-\cos^{-1}}(0.7)$$

$$\vec{I}_1 = (101.72 - j103.76)\text{A}$$

$$|I'_1| = \frac{\text{Re}(I_1)}{0.9} = \frac{101.72}{0.9} = 113.02 \text{ A}$$

$$\vec{I}'_1 = 113.02(-0.9 - j0.436) = 101.72 - j49.28$$

Relative component of phase current supplied by capacitor :

$$I_c = \text{Im}(I_1) - \text{Im}(I'_1)$$

$$= 103.76 - 49.28 = 54.48 \text{ A}$$

$$X_Y = \frac{X_\Delta}{3} = \frac{1}{3 \times 2\pi fC}$$

Per phase capacitor voltage $= \dfrac{3300}{\sqrt{3}}$ V

Per phase capacitor current = 54.48 A

$\therefore$ Per phase capacity reactance,

$$X_Y = \frac{3300}{\sqrt{3} \times 54.48} = \frac{1}{3 \times 2\pi fC}$$

$$C = \frac{\sqrt{3} \times 54.48}{3300 \times 2\pi \times 50 \times 3}$$

$$C = 30.34\ \mu\text{F in each phase of } \Delta$$

$\because$ Each capacitor is rated 420 V, therefore, $\frac{3300}{420}$ = 8 capacitors will have to be connected in series in each phase of Δ.

Capacitance of individual capacitor

$$= 8 \times 30.34 = 242.72\ \mu F$$

Let the resistance of the distribution circuit be R ohms per phase. Then, the power lost in the distribution circuit without capacitor bank is given by

$$P_1 = 3(145.32)^2 R \text{ Watts}$$

Power loss after capacitor bank installation

$$P_2 = 3(113.02)^2 R \text{ Watts}$$

$$\% \text{ saving} = \frac{3(145.32)^2 R - 3(113.02)^2 R}{3(145.32)^2 R} \times 100$$

$$\% \text{ saving} = 39.51\%$$

Q.5 (e) Solution:

For 50 Hz, 16 V, current I = 30 A at 0.2 lagging

So, $$Z = \frac{\bar{V}}{\bar{I}} = \frac{16}{30\angle -\cos^{-1} 0.2}$$

$$Z = (0.1067 + j0.523) \text{ ohm}$$

For frequency of 25 Hz,

$$X' = \left(\frac{f'}{f}\right) \times X$$

$$X' = \left(\frac{25}{50}\right) \times 0.523 = 0.2615 \text{ ohm}$$

So, Current at V_s = 16 V

$$\text{Current } I' = \frac{V_s}{Z'} = \frac{16\angle 0^\circ}{(0.1067 + j0.2615)}$$

$$I' = 56.65\angle -67.80^\circ \text{ Amp}$$

So, Short current I' = 56.65 Amp

Power factor $\cos\phi = \cos(67.80^\circ)$

$$= 0.3778 \text{ (lagging)}$$

Q.6 (a) Solution:

(i)

$$Z_f = \frac{\left(\frac{0.04}{0.04} + j0.2\right)(j9.8)}{1 + j(9.8 + 02)} = \frac{-1.96 + j9.8}{1 + j10} \times \frac{1 - j10}{1 - j10}$$

$$R_f + jX_f = \frac{96.04 + j29.4}{101} = (0.951 + j0.291)\Omega$$

$$r_1 + jx_1 = 0.02 + j0.2$$

∴ Total input impedance,

$$Z = 0.971 + j0.491 = 1.088\angle 26.824°$$

Stator current, $$I_1 = \frac{400}{\sqrt{3} \times 1.088} = 212.27 \text{ A}$$

Air-gap power, $$P_g = 3I_1^2 R_f = 3(212.27)^2 \times 0.951$$

$$= 128552.1 \text{ W}$$

$$\text{Total stator } I^2R \text{ loss} = 3(212.27)^2 \times 0.02 = 2703.5 \text{ W}$$

$$\text{Total rotor } I^2R \text{ loss} = sP_g = 0.04 \times 128552.1 = 5142.1 \text{ W}$$

$$\text{Total losses} = \left(\frac{1}{n} - 1\right)\text{output} = \left(\frac{1}{0.93} - 1\right) \times 150000$$

$$= 11290.3 \text{ W}$$

Total rotational and core losses

$$= 11290.3 - (2703.5 + 5142.1) = 3444.7 \text{ W}$$

(ii) When slip s = –0.04,

$$Z_f = \frac{\left(-\frac{0.04}{0.04} + j0.2\right)(j9.8)}{-1 + j10} = \frac{-1.96 - j9.8}{-1 + j10} \times \frac{-1 - j10}{-1 - j10}$$

$$R_f + jX_f = -\frac{96.04 + j29.4}{101} = -0.951 + j0.291$$

$$r_1 + jx_1 = 0.02 + j0.2$$

$$\text{Total impedance, } Z = -0.931 + j0.491 = 1.0525\angle 152.2°$$

$$I_1 = \frac{400}{\sqrt{3}} \times \frac{1}{1.0525\angle 152.2°} = 219.43\angle -152.2°$$

∴ Stator output current = $219.43\angle 27.8°$

Power factor at machine terminals

$$= \cos 27.8° = 0.8846$$

1. Electric power output $= \sqrt{3} \times 400 \times 219.43 \times 0.8846$

$$= 134478 \text{ W}$$

2. Regarding 134478 W as output power, the induction generator works at a leading pf of 0.8846.

3. Air-gap power $= 3(219.43)^2 \times 0.951 = 137370.6$ W

Total stator i^2R loss $= 3\,(219.43)^2 \times 0.02 = 2889$ W

Total rotor i^2R loss $= sPg = 0.04 \times 137370.6 = 5494.8$ W

Total rotational and core losses = 3444.7 W

Total losses = 11828.5 W

Mechanical power input to drive the induction generator

= Power output + total losses

= 134,478 + 11828.5 = 146306.5 W

Efficiency of induction generator

$$= \frac{134478}{146306.5} \times 100 = 91.92\%$$

Q.6 (b) Solution:

(i) From test 1 and 2, leakage reactance referred to primary are:

$$x_{12} = \frac{500}{\sqrt{3} \times 100} = 2.887\ \Omega$$

$$x_{13} = \frac{600}{\sqrt{3} \times 100} = 3.464\ \Omega$$

Test -3, Referred to secondary,

$$x_{23} = \frac{100}{\sqrt{3} \times 200} = 0.289\ \Omega$$

Referred to primary,

$$x_{23} = 0.289\left(\frac{11}{3.3}\right)^2 = 3.208\ \Omega$$

$$x_1 = \frac{1}{2}(x_{12} + x_{13} - x_{23}) = 1.572\ \Omega$$

$$x_2 = \frac{1}{2}(x_{12} + x_{23} - x_{13}) = 1.316\ \Omega$$

$$x_3 = \frac{1}{2}(x_{13} + x_{23} - x_{12}) = 1.893\ \Omega$$

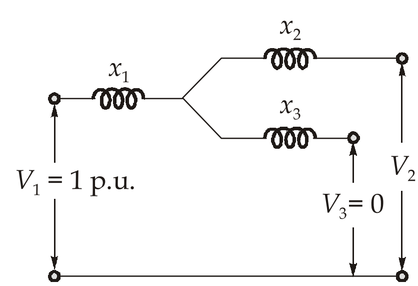
$$Z_{b1} = \frac{V_{b1}}{I_{b1}} = \frac{\left(\frac{11}{\sqrt{3}}\right)}{\left(\frac{6}{11\sqrt{3}}\right)} = 20.17\ \Omega$$

$$x_1 = \frac{1.572}{20.17} = 0.078 \text{ p.u.}$$

$$x_2 = \frac{1.316}{20.17} = 0.065 \text{ p.u.}$$

$$x_3 = \frac{1.893}{20.17} = 0.094 \text{ p.u.}$$

(ii)



$$I_{SC} = \frac{V_1}{x_1 + x_3} = \frac{1}{j(0.078 + 0.094)}$$

$$= 5.814\angle{-90°} \text{ p.u.}$$

$$V_2 = I_{sc}\, x_3$$

$$= 5.814\angle{-90°} \times j0.094$$

$$= 0.546 \text{ p.u.}$$

$$V_2 = 0.546 \times 3.3 = 1.803 \text{ kV}$$

Short circuit current in primary winding,

$$= I_{sc} \times I_{b1}$$

$$= 5.814 \times \frac{6000}{\sqrt{3} \times 11} = 1830.94 \text{ A}$$

Short circuit current in tertiary winding,

$$= 1830.94\left(\frac{11000/\sqrt{3}}{400}\right) = 29070 \text{ A} = 29.1 \text{ kA}$$

Q.6 (c) Solution:

For 1200 MW load period it is necessary to run both units

$$\frac{dC_1}{dP_1} = 52.8 + 11 \times 10^{-3} P_1 = \lambda$$

$$\frac{dC_2}{dP_2} = 15 + 0.1 P_2 = \lambda$$

Let the loads be P_1 and $1200 - P_1$. For economic loading incremental costs should be equal

$$52.8 + 11 \times 10^{-3} P_1 = 15 + 0.1(1200 - P_1)$$

The solution is,

$$P_1 = 740.54 \text{ MW},$$

$$P_2 = 459.46 \text{ MW}$$

For 900 MW load also, it is necessary to run both units.

Let loads be P_1 and $(900 - P_1)$. Then

$$52.8 + 11 \times 10^{-3} P_1 = 15 + 0.1(900 - P_1)$$

The solution is,

$$P_1 = 470.27 \text{ MW}$$

and $$P_2 = 429.73 \text{ MW}$$

For the 500 MW load, let both units be run. Then loads are P_1 and $(500 - P_1)$, we get

$$52.8 + 11 \times 10^{-3} P_1 = 15 + 0.1(500 - P_1)$$

The result is,

$$P_1 = 109.9 \text{ MW},$$

$$P_2 = 390.1 \text{ MW}$$

However the minimum load on any unit is 200 MW. Therefore

$$P_1 = 200 \text{ MW},$$

$$P_2 = 300 \text{ MW}$$

The operating cost for the 500 MW load period is

$$C_1 = 7700 + 52.8 \times 200 + 5.5 \times 10^{-3} \times 200^2$$

$$= \text{Rs } 18480 \text{ per hour}$$

$$C_2 = 2500 + 15 \times 300 + 0.05 \times 300^2$$

$$= \text{Rs } 11500 \text{ per hour}$$

Total operating cost for the 10 hour period of 500 MW is 10 ($C_1 + C_2$) and equals Rs 299800.

The other option is to run only one unit during 500 MW load period. It is easy to see that it is cheaper to run unit 2. Then the cost for 10 hour period is

Rs 10[2500 + 15(500) + 0.05(500)^2] = Rs 225000

Add the cost of shutting down and starting unit i.e. Rs 1000

Therefore, total cost for 500 MW load duration is Rs 225000 + Rs 1000 or Rs 226000. As compared to this, the cost of operation of both units operating is Rs 299800. Hence it is economical to run only unit 2 during 500 MW load. Hence the complete operating schedules is

1200 MW load: P_1 = 740.54 MW; P_2 = 459.46 MW

900 MW load: P_1 = 470.27 MW; P_2 = 429.73 MW

500 MW load :Shutdown unit 1, P_2 = 500 MW

Q.7 (a) Solution:

The power flow equation with voltages and admittances expressed in polar form is

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

The bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} 5\angle -53.13^\circ & 5\angle 126.87^\circ \\ 5\angle 126.87^\circ & 5\angle -53.13^\circ \end{bmatrix}$$

Substituting for admittances, the expression for real and reactive power at bus 2 becomes

$$P_2 = 5|V_2||V_1|\cos(126.87^\circ - \delta_2 + \delta_1) + 5|V_2|^2\cos(-53.13^\circ)$$

$$Q_2 = -5|V_2||V_1|\sin(126.87^\circ - \delta_2 + \delta_1) - 5|V_2|^2\sin(-53.13^\circ)$$

Partial derivatives of P_2 and Q_2 with respect to $|V_2|$ and δ_2 are

$$\frac{\partial P_2}{\partial \delta_2} = 5|V_2||V_1|\sin(126.87^\circ - \delta_2 + \delta_1)$$

$$\frac{\partial P_2}{\partial |V_2|} = 5|V_1|\cos(126.87° - \delta_2 + \delta_1) + 10|V_2|\cos(-53.13°)$$

$$\frac{\partial Q_2}{\partial \delta_2} = 5|V_2||V_1|\cos(126.87° - \delta_2 + \delta_1)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -5|V_1|\sin(126.87° - \delta_2 + \delta_1) - 10|V_2|\sin(-73.74°)$$

The load expressed in per units is

$$S_2^{sch} = -\frac{(100 + j50)}{100} = -1.0 - j0.5 \text{ p.u.}$$

The slack bus voltage is $V_1 = 1.0\angle 0$ p.u. starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, the power residuals are computed.

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -1.0 - 5[5\cos(126.87°) + 5\cos(-53.13°)]$$
$$= -1.0 \text{ p.u.}$$
$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -0.5 - [-5\sin(126.87°) + 5\sin(-53.13°)]$$
$$= -0.5 \text{ p.u.}$$

The elements of the Jacobian matrix at the initial estimate are:

$$J_1^{(0)} = 5(1)(1)\sin(126.87°) = 4$$

$$J_2^{(0)} = 5(1)\cos(126.87°) + 10(1)\cos(-53.13°) = 3$$

$$J_3^{(0)} = 5(1)(1)\cos(126.87°) = -3$$

$$J_4^{(0)} = -5(1)\sin(126.87°) - 10(1)\sin(-53.13°) = 4$$

The set of linear equations in first iteration becomes

$$\begin{bmatrix} -1.0 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus-2 in the first iteration is

$$\Delta\delta_2^{(0)} = -0.10$$

$$\delta_1^{(1)} = 0 + (-0.10) = -0.10 \text{ radian}$$

$$\Delta\left|V_2^{(0)}\right| = -0.2$$

$$\left|V_2^{(1)}\right| = 1 + (-0.2) = 0.8 \text{ p.u.}$$

For the second iteration, we have

$$\Delta P_2^{(1)} = P_2^{sch} - P_2^{(1)} = -1.0 - (-0.7875) = -0.2125 \text{ p.u.}$$

$$\Delta Q_2^{(1)} = Q_2^{sch} - Q_2^{(1)} = -0.5 - (-0.3844) = -0.1156 \text{ p.u.}$$

Also, computing the elements of the Jacobian matrix, the set of linear equations in the second iteration becomes,

$$\begin{bmatrix} -0.2125 \\ -0.1156 \end{bmatrix} = \begin{bmatrix} 2.9444 & 1.4157 \\ -2.7075 & 2.7195 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\left|V_2^{(1)}\right| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus-2 in the second iteration is

$$\Delta\delta_2^{(1)} = -0.0350$$

$$\delta_2^{(2)} = -0.1 + (-0.0350) = -0.135 \text{ radian}$$

$$\Delta\left|V_2^{(1)}\right| = -0.0773$$

$$\left|V_2^{(2)}\right| = 0.8 + (-0.0773) = 0.7227 \text{ pu.}$$

Q.7 (b) (i) Solution:

The choice between overhead and underground system depends upon a number of widely differing factors, depending upon the requirement as shown below:

1. **Public safety:** The underground system is more safe than overhead system because all distribution wiring is placed underground and there are little chances of any hazard.
2. **Initial cost:** The underground system is more expensive due to the high cost of trenching, conduits, cables, manholes and other special equipment. The initial cost of an underground system may be five to ten times than that of an overhead system.
3. **Flexibility:** The overhead system is much more flexibility than the underground system. In the latter case, manholes, duct lines etc., are permanently placed once

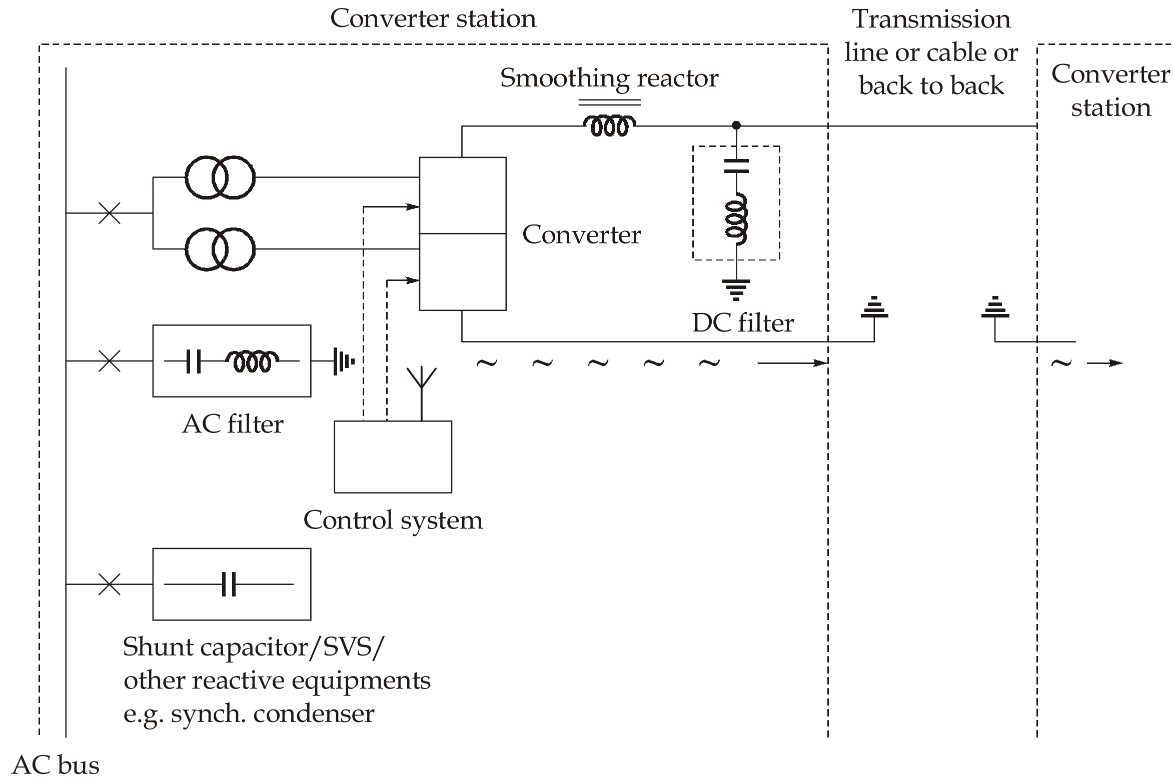
installed and the load expansion can only be met by laying new lines. However, on an overhead system, poles, wires, transformers etc., can be easily shifted to meet the changes in load conditions.

4. **Faults:** The chances of faults in underground system are very rare as the cables are laid underground and are generally provided with better insulation.
5. **Appearance:** The general appearance of an underground system is better as all the distribution lines are invisible. This factor is exerting considerable public pressure on electric supply companies to switch over to underground system.
6. **Fault location and repairs:** In general, there are little chances of faults in an underground system. However, if a fault does occur, it is difficult to locate and repair on this system. On an overhead system, the conductors are visible and easily accessible so that fault locations and repairs can be easily made.
7. **Current carrying capacity and voltage drop:** An overhead distribution conductor has a considerably higher current carrying capacity than an underground cable conductor of the same material and cross-section. On the other hand underground cable conductor has much lower inductive reactance than that of an overhead conductor because of closer spacing of conductors.
8. **Useful life:** The useful life of underground system is much longer than that of an overhead system. An overhead system may have a useful life of 25 years, whereas an underground system may have a useful life of more than 50 years.
9. **Maintenance cost:** The maintenance cost of underground system is very low as compared with that of overhead system because of less chances of faults and service interruptions from wind ice, lightning as well as from traffic hazards.
10. **Interference with communication circuits:** An overhead system causes electromagnetic interference with the telephone lines. The power line currents are superimposed on speech current, resulting in the potential of the communication channel being raised to an undesirable level. However, there is no such interference with the underground system.

Q.7 (b) (ii) Solution:

Structure of HVDC transmission:

HVDC transmission consists of two converter stations which are connected to each other by a DC cable or an overhead DC line. A typical arrangement of main components of an HVDC transmission is shown in figure.



Reactive power demand:

The requirement of reactive power at converter station is due to:

1. The control of HVDC converter (α, γ) which introduces a phase shift between the fundamentals of AC current and voltage.
2. The commutation process, in which the DC current is commutated from one valve to another, and which introduces further phase shift.

In addition to reactive power consumption by converters, converter transformers also consume reactive power. Considering normal values of α(rectifier) or γ(inverter), the reactive power demand usually is in the range of 50-60% of the transmitted active power.

The reactive power may be supplied from:

1. AC filters
2. Shunt capacitors (least costly)
3. AC network
4. Static compensators (SVS) (for fast voltage regulation)
5. Synchronous condensers (if AC network is weak)

While choosing reactive power generation equipment, one must consider both economic and technical aspects.

Q.7 (c) Solution:

(i) At the initial operating point,

Let $\vec{V}_1 = V_1\angle\delta_1$

$\vec{V}_2 = V_2\angle\delta_2$

$P_e = 1.0,\ V_1 = V_2 = 1.0,\ X_t + X = 0.5$

We get $\delta_1 - \delta_2 = \sin^{-1}(0.5) = 30°$

Let the current $\vec{I}$ in line and reference voltage is

$\vec{V}_2 = 1\angle 0°$ pu

$$\vec{I} = \frac{\vec{V}_1 - \vec{V}_2}{j(X_T + X)} = \frac{1\angle 30° - 1}{j0.5} = 1.0352\angle 15° \text{ pu}$$

$\vec{E}_b = \vec{V}_2 - j\vec{I}X_2$

$= 1 - (1.0352\angle 15°)(j0.1) = 1.032\angle -5.56°$ pu

but $\vec{E}_b = 1.032\angle 0°$ (given as reference phasor)

therefore, $\delta_2 = 5.56$ and $\delta_1 = 35.56°$

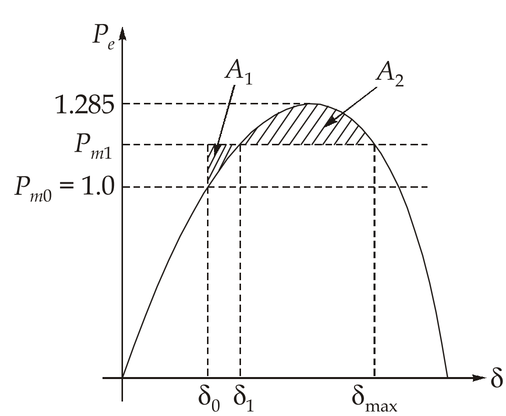
(shifting to E_b as reference phasor)

Hence, $\vec{E}_g = \vec{V}_1 + jX_g\vec{I} = 1.0\angle 35.56° + (j0.3)(1.0352\angle 15°)(1\angle 5.56°)$

$= 1.121\angle 51.1°$ pu

Therefore, $$P_{max} = \frac{1.121 \times 1.03}{0.9} = 1.285$$

Power angle curve,



$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_{max}\sin\delta)\cdot d\delta$$

$$= -P_{max}[\cos \delta_0 - \cos \delta_1] + P_{m1}[\delta_1 - \delta_0] \quad \text{...(i)}$$

$$A_2 = \int_{\delta_1}^{\pi-\delta_1} (P_{max} \sin \delta - P_{m1}) \cdot d\delta$$

$$= 2\, P_{max}.\cos \delta_1 - P_{m1}(\pi - 2\delta_1) \quad \text{...(ii)}$$

Equating (i) and (ii), we get

$$P_{m1}(\pi - \delta_1 - \delta_0) = P_{max}[\cos \delta_1 + \cos \delta_0]$$

$$\because \quad P_{m1} = P_{max} \sin \delta_1$$

$$\sin \delta_1(\pi - \delta_1 - \delta_0) = (\cos \delta_1 + \cos \delta_0)$$

$$\because \quad \delta_0 = \sin^{-1}\left(\frac{1}{1.285}\right) = 51.10° = 0.8918 \text{ rad}$$

$$\sin \delta_1(\pi - 0.8918 - \delta_1) = \cos \delta_1 + 0.6279$$

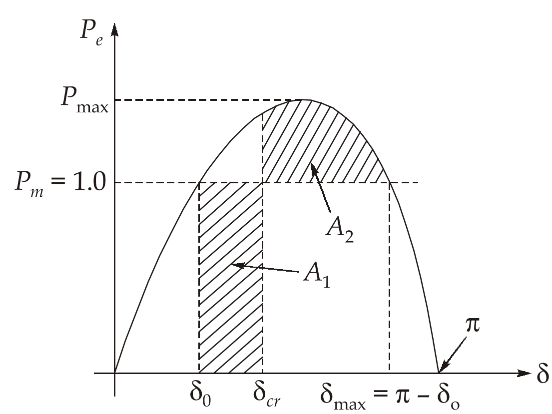
On solving above non-linear equation by hit and trial method,

$$\delta_1 \simeq 70.64°$$

Therefore, $P_{m1} = 1.285 \sin(70.64°) = 1.2123$ pu

Therefore, maximum step increase in mechanical power = 0.2123 pu.

(ii) Now 3-ϕ fault at generator terminals, therefore $P_e = 0$ during the fault.



$$A_1 = \int_{\delta_o}^{\delta_{cr}} (P_m - 0)d\delta = P_m(\delta_{cr} - \delta_o) \quad \text{...(i)}$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin \delta - P_m)d\delta$$

$$A_2 = P_{max}(\cos \delta_{cr} - \cos \delta_{max}) - P_m(\delta_{max} - \delta_{cr}) \quad \text{...(ii)}$$

Equating eqn. (i) and (ii),

$$P_m(\delta_{cr} - \delta_o) = P_{\max} \cos \delta_{cr} - P_{\max} \cos \delta_{\max} - P_m \delta_{\max} + P_m \delta_{cr}$$

$$-P_m \delta_o = P_{\max} \cos \delta_{cr} - P_{\max} \cos \delta_{\max} - P_m \delta_{\max}$$

where, $\delta_o = 51.1°$ or 0.8918 rad

$$P_{\max} = 1.285 \text{ pu}, P_m = 1.0, \delta_{\max} = \pi - 0.8918 = 2.249 \text{ rad}$$

$$-0.8918 = 1.285 \cos \delta_{cr} + 0.8062 - 2.249$$

On solving above equation,

$$\delta_{cr} = 64.60° \text{ or } 1.127 \text{ rad}$$

Now, critical clearing time

$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_o)}{\pi P_m f_s}} = \sqrt{\frac{2 \times 4 \times (1.127 - 0.8918)}{\pi \times 1 \times 50}}$$

$$t_{cr} = 0.1094$$

Q.8 (a) Solution:

(i) From the dc test,

$$R_1 = \frac{V_{DC}}{2I_{DC}} = \frac{13.6 \text{ V}}{2(28.0)\text{A}} = 0.243\ \Omega$$

From the no-load test,

$$I_{L,\text{av}} = \frac{8.12\text{A} + 8.20\text{A} + 8.18\text{A}}{3} = 8.17 \text{ A}$$

$$V_{f,nl} = \frac{208 \text{ V}}{\sqrt{3}} = 120 \text{ V}$$

Therefore,

$$|Z_{nl}| = \frac{120 \text{ V}}{8.17 \text{ A}} = 14.7\ \Omega = X_1 + X_M$$

When X_1 is known, X_M can be found, the stator copper losses are

$$P_{SCL} = 3I_1^2 R_1 = 3(8.17 \text{ A})^2(0.243\ \Omega) = 48.7 \text{ W}$$

Therefore, the no-load rotational losses are

$$P_{\text{rot}} = P_{\text{in, nl}} - P_{SCL,\text{ nl}}$$

$$= 420 \text{ W} - 48.7 \text{ W} = 371.3 \text{ W}$$

From the locked-rotor test,

$$I_{L,\text{ av}} = \frac{28.1\text{A} + 28.0\text{A} + 27.6\text{A}}{3} = 27.9 \text{ A}$$

The locked-rotor impedance is

$$|Z_{LR}| = \frac{V_\phi}{I_A} = \frac{V_T}{\sqrt{3}I_A} = \frac{25\text{ V}}{\sqrt{3}(27.9\text{A})} = 0.517\ \Omega$$

and the impedance angle θ is

$$\theta = \cos^{-1}\frac{P_{in}}{\sqrt{3}V_T I_L}$$

$$= \cos^{-1}\frac{920\text{ W}}{\sqrt{3}(25\text{ V})(27.9\text{A})} = \cos^{-1} 0.762 = 40.4°$$

Therefore, $R_{LR} = 0.517 \cos 40.4° = 0.394\ \Omega = R_1 + R_2$

Since, $R_1 = 0.243\ \Omega$

R_2 must be 0.151 Ω, the reactance at 15 Hz is

$$X_{LR}' = 0.517 \sin 40.4° = 0.335\ \Omega$$

The equivalent reactance at 60 Hz is

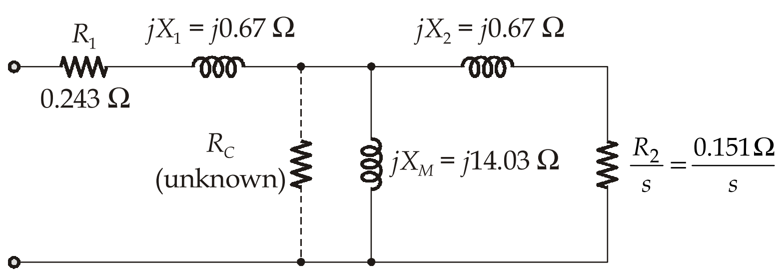
$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = \left(\frac{60\text{ Hz}}{15\text{ Hz}}\right) 0.335\ \Omega = 1.34\ \Omega$$

For design class A induction motors, this reactance is assumed to be divided equally between the rotor and stator, so

$$X_1 = X_2 = 0.67\ \Omega$$

$$X_M = |Z_{nl}| - X_1 = 14.7\ \Omega - 0.67\ \Omega = 14.03\ \Omega$$

The final per-phase equivalent circuit is shown in figure below



(ii) For this equivalent circuit, the Thevenin equivalents are found from equations

$$V_{Th} = 114.6\text{ V}$$

$$R_{Th} = 0.221\ \Omega$$

$$X_{Th} = 0.67\ \Omega$$

Therefore, the slip at the pullout torque is given by

$$s_{max} = \frac{R_2}{\sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}}$$

$$= \frac{0.151\,\Omega}{\sqrt{(0.243\,\Omega)^2 + (0.67\,\Omega + 0.67\,\Omega)^2}} = 0.111 = 11.1\%$$

The maximum torque of this motor is given by

$$\tau_{max} = \frac{3V_{Th}^2}{2\omega_{sync}\left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X^2)}\right]}$$

$$= \frac{3(114.6\,V)^2}{2(188.5\,rad/s)\left[0.221\,\Omega + \sqrt{(0.221\,\Omega)^2 + (0.67\,\Omega + 0.67\,\Omega)^2}\right]}$$

$$= 66.2\ Nm$$

Q.8 (b) Solution:

(i) For given power system:

Impedance between V_t and $V = j0.1 + \frac{j0.5}{2} = j0.35$ p.u.

Also let, $V_t = 1\angle\alpha$

$$\frac{1\times 1}{j0.35}\sin\alpha = 0.8$$

$$\alpha = 16.26°$$

$$I = \frac{1.0\angle 16.26° - 1\angle 0°}{0.35\angle 90°} = \frac{0.96 + j0.28 - 1.0}{j0.35}$$

$$= 0.8 + j0.1143 = 0.8081\angle 8.13°$$

$$E' = 1\angle 16.26° + 0.808\angle 8.13° \times 0.2\angle 90°$$

$$= 0.96 + j0.28 - 0.023 + j0.16$$

$$= 1.0352\angle 25.15°$$

$$P_e = \frac{1.0352\times 1}{0.35 + 0.20}\sin\delta = 1.882\sin\delta$$

(ii) Using given condition of power system,

$$P_{max}\sin\delta_0 = 0.6\,P_{max}$$

$$\delta_0 = 36.87°, \quad 0.6435 \text{ rad}$$

With increase in reactance of network by 400%,

$$r_1 = 0.25, \qquad r_2 = 0.8$$

Power reduces to factor by 0.20, after fault is cleared,

$$P_m = 0.8\ P_{\max} \sin\delta_{\max} \qquad \text{...(i)}$$

Also given, $$\frac{P_m}{P_{\max}} = 0.6$$

$$\sin\delta_{\max} = \frac{0.6}{0.8} = 0.75$$

$$\delta_{\max} = 180° - 48.59° = 131.41° = 2.294 \text{ rad}$$

$$\cos\delta_{cr} = \frac{(P_m / P_{\max})(\delta_{\max} - \delta_0) + r_2 \cos\delta_{\max} - r_1 \cos\delta_0}{r_2 - r_1}$$

$$\cos\delta_{cr} = \frac{(0.6)(2.294 - 0.6435) + 0.8\cos 131.4° - 0.25\cos 36.87°}{0.8 - 0.25}$$

$$= 0.475$$

$$\delta_{cr} = \cos^{-1} 0.475 = 61.64°$$

Q.8 (c) Solution:

No-load tests:

$$Z_{nl} = \frac{400}{\sqrt{3} \times 7.5} = 30.79\ \Omega$$

$$R_{nl} = Z_{nl} \cos\phi_{nl}$$

$$= 30.79 \times 0.135 = 4.157\ \Omega$$

$$X_{nl} = \sqrt{30.79^2 - 4.157^2} = 30.51\ \Omega$$

Blocked rotor test:

$$Z_{br} = \frac{150}{\sqrt{3} \times 35} = 2.474\ \Omega$$

$$R_{br} = 2.47 \times 0.44 = 1.09\ \Omega$$

$$X_{br} = \sqrt{2.474^2 - 1.09^2} = 2.22\ \Omega$$

$$\frac{x_1}{x_2'} = 2$$

$$\therefore \quad x_1 + x_2' = X_{br}$$

$$3x_2' = 2.22$$

$$x_2' = 0.74$$

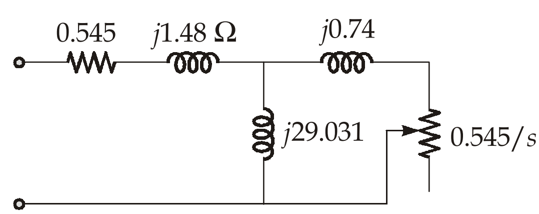
$$\therefore \quad x_1 = 2 \times 0.74 = 1.48\ \Omega$$

$$X_m = X_{nl} - x_1$$

$$= 30.511 - 1.48 = 29.031\ \Omega$$

It is given that stator and rotor copper losses are equal,

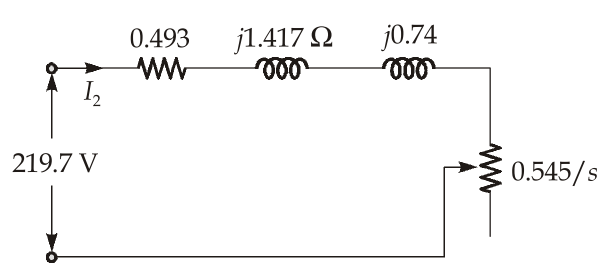
$$r_1 = r_2' = \frac{R_{br}}{2} = 0.545$$



$$(0.545 + j1.48)||(29.031j) = \frac{(0.545 + j1.48)(29.031j)}{0.545 + j1.48 + 29.031j} = 0.493 + 1.417j$$

$$V_e = 230.95 \times \frac{29.031j}{0.545 + j1.48 + 29.031j}$$

$$|V_e| = 219.71 \text{ Volt}$$



Slip, $$s = \frac{1000 - 960}{1000} = 0.04$$

$$I_2' = \frac{219.7}{0.493 + \frac{0.54}{0.04} + j(1.417 + j0.74)} = 15.51\angle{-8.76^\circ}$$

Mechanical power developed,

$$P_m = 3I_2'^2 r_2\left(\frac{1-s}{s}\right)$$

$$= 3\times 15.51^2\times 0.545\times\frac{0.96}{0.04} = 9440\text{ W}$$

$$\text{Rotational losses} = \sqrt{3}\times 400\times 7.5\times 0.135 - 3\times 7.5^2\times 0.545$$

$$= 609.5\text{ Watt}$$

(i) Net mechanical power output

$$= 9440 - 609.5 = 8830.5\text{ Watt}$$

(ii)

$$T_{sh} = \frac{P_{sh}}{\omega_r} = \frac{8830.5\times 60}{2\pi\times 960} = 87.83\text{ Nm}$$

(iii)

$$\text{Rotor ohmic losses} = 9440\frac{(0.04)}{0.96} = 393.33\text{ Watt}$$

$$\text{Stator ohmic losses} = 393.33\text{ W}$$

$$\text{Rotational losses} = 609.5\text{ W}$$

$$\text{Total losses in motor} = 393.3\times 2 + 609.5 = 1396.1\text{ W}$$

$$\therefore \quad \eta = \frac{8830.5}{8830.5+1396.1}\times 100 = 86.34\%$$

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