MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

Mechanical Engineering Test No : 5

Section A : Production Engineering & Material Science

Section B : Theory of Machines-1 & Fluid Mechanics & Turbo Machinery-2

Section A : Production Engineering & Material Science

1. (a) Solution:

Given : B = 200 mm; h_0 = 8 mm; R = 250 mm; k = 380 MPa; μ = 0.15

$$h_f = (1 - 0.25)\,8 = 6 \text{ mm}$$

Angle subtended at the roll centre,

$$\theta_i = \sqrt{\frac{h_i - h_f}{R}} = \sqrt{\frac{8-6}{250}} = 0.0894 \text{ rad} = 5.125°$$

Location of neutral plane, $$\theta_n = \sqrt{\frac{h_f}{R}} \tan\left[\frac{H_n}{2}\sqrt{\frac{h_f}{R}}\right]$$

$$H_n = \frac{1}{2}\left[\frac{1}{\mu}\ln\left(\frac{h_f}{h_i}\right) + H_i\right]$$

and $$H_i = 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left[\sqrt{\frac{R}{h_f}}\theta_i\right]$$

$$\Rightarrow \quad H_i = 2\sqrt{\frac{250}{6}}\tan^{-1}\left[\sqrt{\frac{250}{6}} \times 0.0894\right]$$

$$\Rightarrow \quad H_i = 6.76$$

$$\therefore \quad H_n = \frac{1}{2}\left[\frac{1}{0.15}\ln\left(\frac{6}{8}\right) + 6.76\right] = 2.42$$

and
$$\theta_n = \sqrt{\frac{6}{250}}\tan\left[\frac{2.42}{2}\sqrt{\frac{6}{250}}\right] = 0.0294$$

or
$$\theta_n = 1.684°$$

Thus final strip thickness = 6 mm

Angle subtended at roll centre = 5.125°

Location of neutral plane = 1.684°

1. (b) Solution:

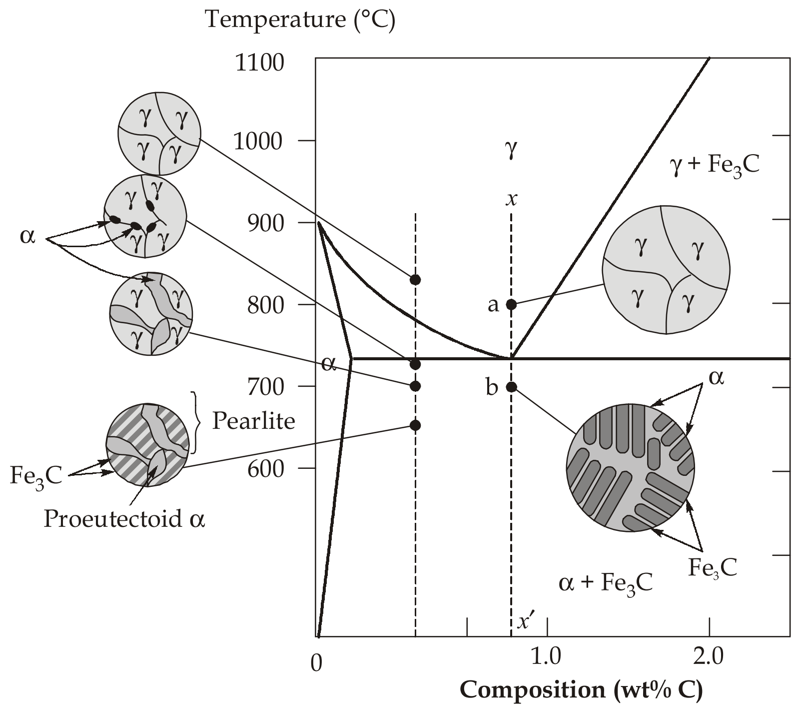
The properties of steel are primarily governed by its microstructure, which varies with carbon content and temperature understanding the microstructures of different phases is essential for controlling strength, hardness, and ductility in engineering applications.

(i) Ferrite : Ferrite (α-iron) is a low temperature phase with BCC crystal structure and very limited carbon stability (0.022%). This phase is relatively soft, may be made magnetic at temperatures below 768°C. [Curie temperature]

(ii) Austenite : Austenite (γ-iron) exists at higher temperatures and has an FCC crystal structure, allowing greater carbon solubility (2.14 wt%). Phase transformations involving austenite are very important in the heat treatment of steels.

(iii) Cementite : Cementite (Fe_3C) is an intermetallic compound containing 6.67 wt% carbon. Mechanically, cementite is very hard and brittle, the strength of some steels is greatly enhanced by its presence.

(iv) Pearlite : Pearlite is a two phase lamellar structure consisting of alternating layers of ferrite and cementite. It forms by eutectoid transformation of austenite at 727°C and 0.76%C, providing a desirable combination of strength and ductility.



1. (c) Solution:

Given : $V_p = 72 - \frac{I}{80}$, $V_a = 32 + 4L_a$; $L_{a_1} = 0.5$ cm, $L_{a_2} = 1.5$ cm

For stable arc, $$72 - \frac{I}{80} = 32 + 4L_a$$

$$I = 80(40 - 4L_a)$$

Hence, $$I_1 = 80(40 - 4L_{a1}) = 80(40 - 2) = 3040\text{A}$$

$$I_2 = 80(40 - 4L_{a2}) = 80(40 - 6) = 2720\text{A}$$

∴ The welding current range is 2720A to 3040A.

The power P is given by

$$P = VI = 80(32 + 4L_a)(40 - 4L_a) \text{ Volt-amp.}$$

For maximum power,

$$\frac{dP}{dL_a} = 0$$

$$\Rightarrow \quad 4(40 - 4L_a) - 4(32 + 4L_a) = 0$$

$$\Rightarrow \quad (L_a)_{opt} = 1 \text{ cm}$$

So, the optimum arc length is 1 cm and the maximum power of the arc is

$$P_{max} = 80(32 + 4 \times 1)(40 - 4 \times 1)$$
$$= 103.68 \text{ kVA}$$

1. (d) Solution:

Given : slab dimensions : 25 cm × 15 cm × 5 cm

Let riser with diameter, d and height = h

For side riser, $$A = \pi dh + 2 \times \frac{\pi}{4} d^2$$

and Volume, $$V = \frac{\pi}{4} d^2 h$$

$$\Rightarrow \quad h = \frac{4V}{\pi d^2}$$

$$\therefore \quad A = \pi d \frac{4V}{\pi d^2} + 2 \times \frac{\pi}{4} d^2 = \frac{4V}{d} + \frac{\pi}{2} d^2$$

For most optimum cylindrical riser, the surface area should be minimum for a given volume.

$$\therefore \quad \frac{\partial A}{\partial d} = 0$$

$$\Rightarrow \quad \frac{-4V}{d^2} + \pi d = 0$$

$$\Rightarrow \quad d^3 = \frac{4V}{\pi}$$

$$\therefore \quad d^3 = \frac{4}{\pi} \times \frac{\pi}{4} d^2 h = d^2 h$$

$$\Rightarrow \quad h = d$$

[Condition for minimum surface area for a given value of volume]

Now, the minimum volume of the riser

$$V_r = 3 \times \text{Shrinkage volume}$$
$$= 3 \times 0.065 \times 25 \times 15 \times 5 = 365.625 \text{ cm}^3$$

or $$h = d = \left(\frac{4V_r}{\pi}\right)^{1/3} = 7.75 \text{ cm}$$

Now, $$\left(\frac{A}{V}\right)_r = \frac{6}{d} = \frac{6}{7.75} = 0.774 \text{ cm}^{-1}$$

and $$\left(\frac{A}{V}\right)_c = \frac{2\times[25\times15+25\times5+15\times5]}{25\times15\times5} = 0.613 \text{ cm}^{-1}$$

So, here $$\left(\frac{A}{V}\right)_r > \left(\frac{A}{V}\right)_c$$

But riser should not have a shorter solidification time than casting.

Or $$\left(\frac{A}{V}\right)_r \le \left(\frac{A}{V}\right)_c$$

$$\Rightarrow \quad \frac{6}{d} \le 0.613$$

$$\Rightarrow \quad d \ge 9.78 \text{ cm}$$

For this value of diameter,

$$V_r = \frac{\pi}{4}d^2 \cdot d = \frac{\pi}{4}(9.78)^3 = 734.69 \text{ cm}^3$$

Hence, the volume of riser is greater than minimum volume (365.625 cm^3) required.

$$\therefore \quad d_{\text{riser}} = h_{\text{riser}} = 9.78 \text{ cm}$$

1. (e) Solution:

Atomic packing fraction is given by,

$$\text{APF} = \frac{\text{N}_{\text{avg}} \times V_{\text{molecule}}}{V_{\text{unit cell}}}$$

In BCC crystal structure, atoms are located at all eight corners and a single atom at the cube centre.

$$\therefore \quad N_{\text{avg}} = \frac{1}{8}\times8+1 = 2$$

Centre and corner atoms touch one another along cube diagonals, and unit cell length a and atomic radius are related as

$$4R = \sqrt{3}a \text{ or } a = \frac{4R}{\sqrt{3}}$$

$$\text{A.P.F.} = \frac{2\times\frac{4}{3}\pi R^3}{a^3}$$

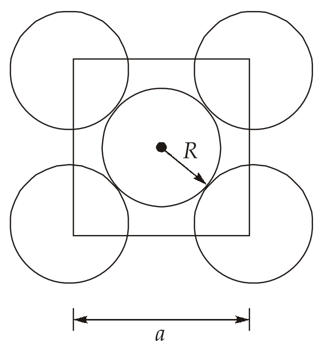
$$\therefore \quad \text{APF} = \frac{2\times\frac{4}{3}\pi R^3}{\left(\frac{4R}{\sqrt{3}}\right)^3} = 0.68$$

$$\text{Planar density } (0\ 1\ 0) = \frac{\text{Number of atoms}}{\text{Area of plane}}$$

For FCC structure,

$$4R = \sqrt{2}a$$

$$\Rightarrow \qquad a = 2\sqrt{2}R$$



Number of atoms centered on plane (0 1 0) = $\frac{1}{4} \times 4 + 1 = 2$

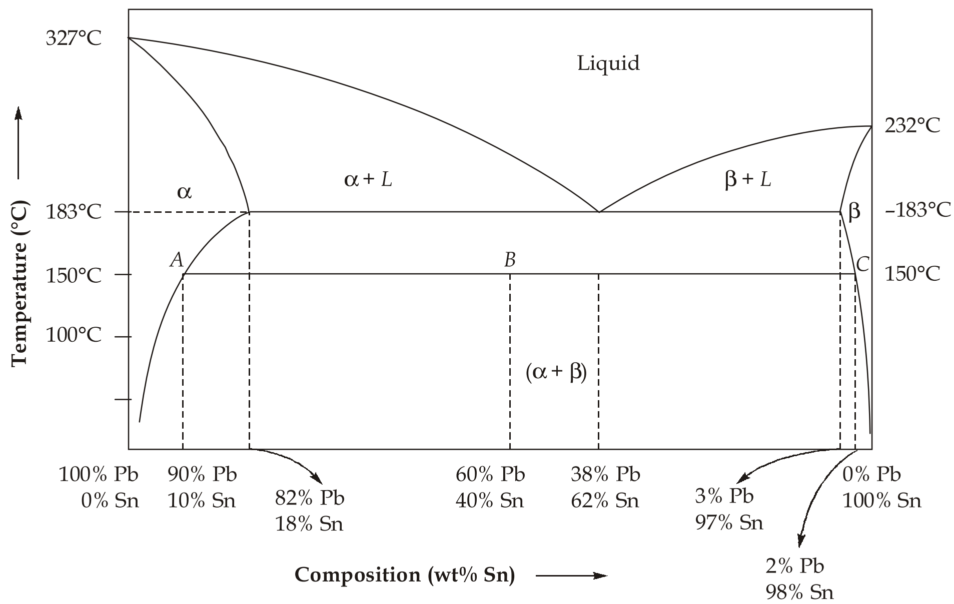
$$\therefore \qquad \text{Planar density} = \frac{2}{a^2} = \frac{2}{\left(2\sqrt{2} \times 0.128 \times 10^{-6}\right)^2}$$

$$= 15.26 \times 10^{12} \text{ atoms/mm}^2$$

Hence, 15.26×10^{12} atoms per mm^2 are these on the (0 1 0) plane of copper (Cu).

Q.2 (a) Solution:

The lead tin phase diagram is shown in figure.



(i) For relative amount of α and β phase present in terms of mass fraction:

Mass fractions can be computed in terms of weight fractions

$$W_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha} = \frac{98-40}{98-10} = 0.6591 \simeq 0.66$$

$$W_\beta = \frac{C_\beta - C_\alpha}{C_\beta - C_\alpha} = \frac{40-10}{98-10} = 0.3409 \simeq 0.34$$

Mass fraction of α and β phases are 66% and 34% respectively.

(ii) For relative amount of α and β phase present in terms of volume fraction

For calculation of volume fraction of α and β phase, first we have to calculate the density of each phase.

$$\text{Density of } \alpha\text{-phase, } \rho_\alpha = \frac{100}{\dfrac{C_{Pb(\alpha)}}{\rho_{Pb}} + \dfrac{C_{Sn(\alpha)}}{\rho_{Sn}}}$$

$$\rho_\alpha = \frac{100}{\dfrac{90}{11.23} + \dfrac{10}{7.24}} = 10.643 \text{ g/cm}^3$$

$$\text{Density of } \beta\text{-phase, } \rho_\beta = \frac{100}{\dfrac{C_{Pb(\beta)}}{\rho_{Pb}} + \dfrac{C_{Sn(\beta)}}{\rho_{Sn}}}$$

$$\rho_\beta = \frac{100}{\frac{2}{11.23} + \frac{98}{7.24}} = 7.292 \text{ g/cm}^3$$

Now,

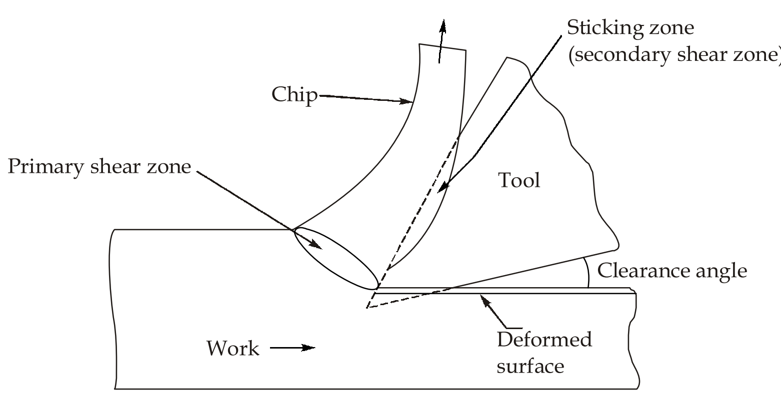
$$\text{Volume fraction of } \alpha\text{-phase, } V_\alpha = \frac{\frac{W_\alpha}{\rho_\alpha}}{\frac{W_\alpha}{\rho_\alpha} + \frac{W_\beta}{\rho_\beta}} = \frac{\frac{0.66}{10.643}}{\frac{0.66}{10.643} + \frac{0.34}{7.292}} = 0.5708$$

$$\text{Volume fraction of } \beta\text{-phase, } V_\beta = \frac{\frac{W_\beta}{\rho_\beta}}{\frac{W_\alpha}{\rho_\alpha} + \frac{W_\beta}{\rho_\beta}} = \frac{\frac{0.34}{7.292}}{\frac{0.66}{10.643} + \frac{0.34}{7.292}} = 0.4292$$

Volume fraction of α and β phases present is 57.08% and 42.92% respectively.

2. (b) (i) Solution:

Mechanism of chip formation : The uncut layer deforms into a chip after it goes through a severe plastic deformation in the primary shear zone. Just after its formation, the chip flows over the rake surface of the tool and the strong adhesion between the tool and the newly formed chip surface results in some sticking. Thus, the chip material at this surface (and the adjacent layers) undergoes a further plastic deformation since, despite the sticking, it flows. This zone is referred to as the secondary shear zone.



(a)

- Under suitable conditions, the machining operation is smooth and stable and produces continuous ribbon-like chips. As a result, the surface produced is smooth and the power consumption is not unnecessarily high.

- At moderate speeds, high temperature causes material to stick to the cutting edge forming Built-up Edge (BUE); it grows and breaks off, sticking to the surface and producing rough finish. At higher speeds or with cutting fluid, BUE disappears.
- At very low speeds or when machining brittle materials, fracture occurs intermittently during shearing, producing discontinuous chips and a rough surface.

2. (b) (ii) Solution:

Given : h_{max} = 16 mm; R = 0.5 mm; TCT = 2.7 min

Tool life equation, $Vf^{0.18}T^{0.24} = 24$

For turning,
$$h_{max} = \frac{f^2}{8R}$$

$$\Rightarrow \quad 16 \times 10^{-3} = \frac{f^2}{8 \times 0.5}$$

$$\Rightarrow \quad f = 0.253 \text{ mm/rev}$$

For maximum production rate, optimum tool life

$$T_{opt} = \frac{1-n}{n}(TCT)$$

Here,
$$n = 0.24$$

$$T_{opt} = \frac{1-0.24}{0.24}(2.7) = 8.55 \text{ minutes}$$

From tool life equation,

$$V_{opt}f^{0.18}T_{opt}^{0.24} = 24$$

$$V_{opt} = \frac{24}{(0.253)^{0.18}(8.55)^{0.24}} = 18.36 \text{ m/min}$$

Thus, the most productive cutting speed is 18.36 m/min.

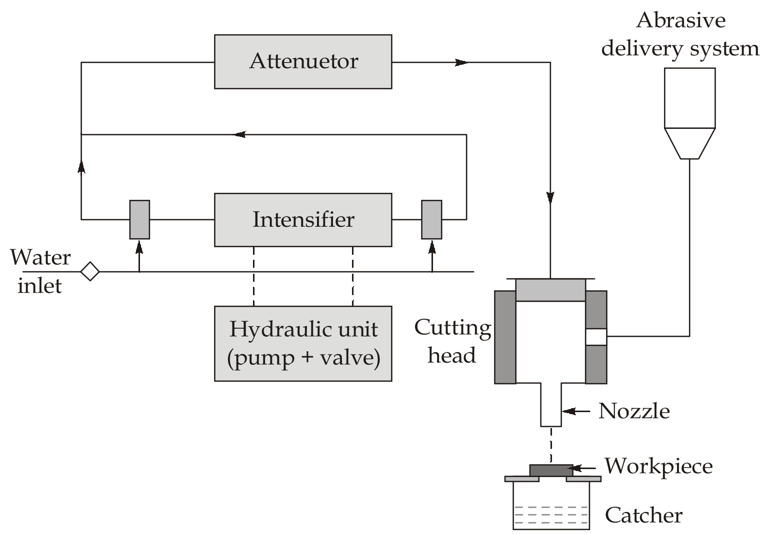
2. (c) Solution:

AWJM is a process that uses a very high speed water jet (supersonic Mach number $\simeq$ 2.5), which is mixed with abrasives to cut any material without affecting work material and environment.

Greatest advantage of AWJM is that it cuts material without any heating effect, so the work material is free from thermal and mechanical distortions.

Equipments:

1. An intensifier pump to provide high pressure water.
2. The abrasive delivery system.
3. A cutting head for producing abrasive water jet.
4. A computer controlled manipulator to provide desired motion to cutting head.
5. A catcher that dissipates the remaining jet energy.



$$\text{Pressure at Nozzle} = 1300 - 4000 \text{ bar}$$

$$\text{Nozzle dia} = 0.18 - 0.40 \text{ mm}$$

Process parameters:

1. Jet velocity
2. Feed rate
3. Abrasive size
4. Work material and its thickenss.

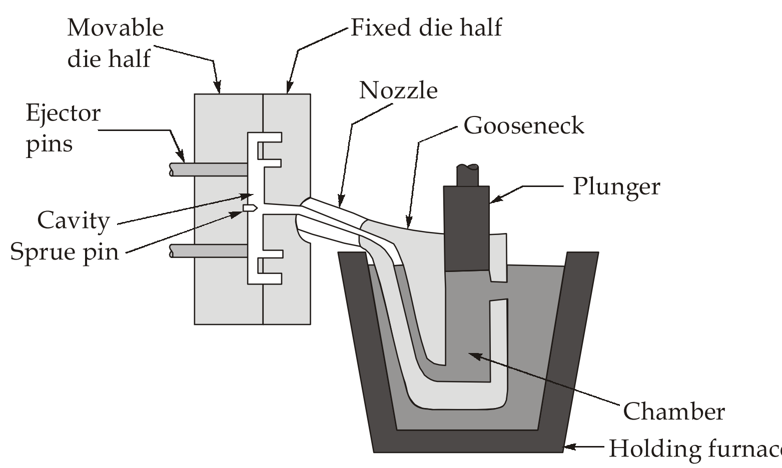
Advantages:

1. Cuts any material irrespective of hardness.
2. Low cost than other machining processes.
3. No heat affected zone.
4. Scrap material left can be reused.
5. Water jet cuts with very little force so it is possible to cut thin walled parts.

Applications: The main applications of the process is, in machining inaccessible areas like inside surface of bottle, cleaning metallic moulds, producing very fine holes in the tool for the purpose of lubrication etc.

3. (a) (i) Solution:

Hot Chamber Die Casting : In a hot chamber die casting machine, as shown in the figure below, a gooseneck is used for pumping the liquid metal into the die cavity. The gooseneck is submerged in the holding furnace containing the molten metal. The gooseneck is made of grey, alloy or ductile iron, or of cast steel. A plunger made of alloy cast iron and which is hydraulically operated, moves up in the gooseneck to uncover the entry port for the entry of liquid metal into the gooseneck. The plunger can then develop the necessary pressure for forcing the metal into the die cavity. A nozzle at the end of the gooseneck is kept in the close contact with the sprue located in the cover die.



Hot chamber die casting process

Procedure : The operating sequence of the hot chamber process starts with the closing of the die, when the plunger is in the highest position in the gooseneck, thus facilitating the filling of the gooseneck by the liquid metal. The plunger then starts moving down to force the metal in the gooseneck to be injected into the die cavity. The metal is then held at the same pressure till it is solidified. The die is opened, and any cores, if present are also retracted. The plunger then moves back returning the unused liquid metal to the gooseneck. The casting which is in the ejector die is now ejected and at the same time the plunger uncovers the filling hole, letting the liquid metal from the furnace to enter the gooseneck.

3. (a) (ii) Solution:

Given, Basic size = 60 mm; $\frac{H7}{m6}$ fit

As the basic size lies in 50 – 80 mm

So, $D_1 = 50$ mm, $D_2 = 80$ mm

$$\text{Geometric mean diameter, } D = \sqrt{D_1 \times D_2} = \sqrt{50 \times 80} = 63.245 \text{ mm}$$

$$\therefore \qquad \text{Tolerance grade, } i = 0.45\sqrt[3]{D} + 0.001D, \text{ microns}$$

$$= 0.45\sqrt[3]{63.245} + 0.001 \times 63.245$$

$$= 1.856\ \mu\text{m}$$

Now, for shaft m_6, fundamental deviation

$$\text{F.D. for shaft} = +(\text{IT7} - \text{IT6})$$

$$= 16i - 10i$$

$$= 6i = 11.1368\ \mu\text{m}$$

$$= 0.011 \text{ mm}$$

For hole H7, Tolerance $= 16i = 0.029$ mm

For hole m6, Tolerance $= 10i = 0.019$ mm

Dimension,

Hole, Lower limit = Basic size = 60 mm [$\because$ This is a hole basis system]

Higher limit = Lower limit + Tolerance

= 60.029 mm

or $60^{+0.029}_{+0.00}$ mm

Shaft, Lower limit = Basic size + F.D.

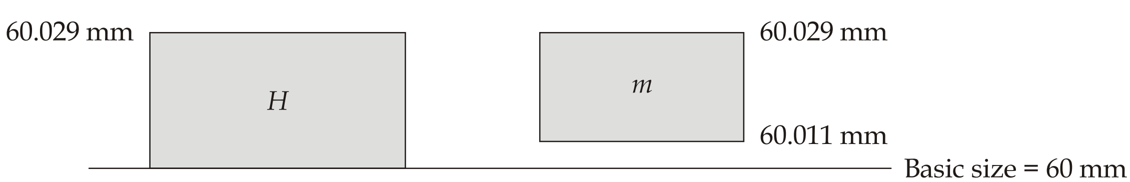
= 60.011 mm

Upper limit = Lower limit + Tolerance

= 60.011 + 0.019

= 60.030 mm

or $60^{+0.030}_{+0.011}$ mm

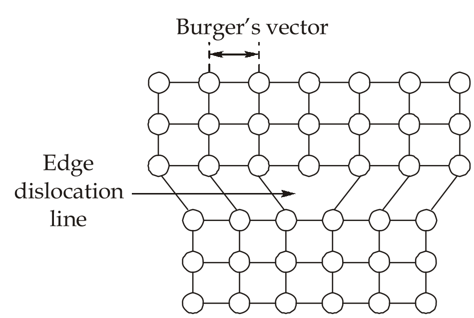


Hence, the type of fit is transition fit.

3. (b) Solution:

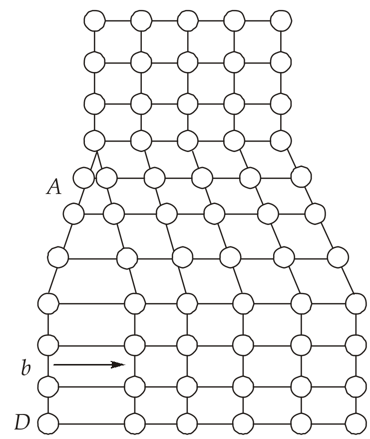
Linear defect: A dislocation is a linear or one dimensional defect around which some of the atoms are misaligned. There are following types of linear dislocation as given below:

i. **Edge dislocation**: In an edge dislocation, an extra portion of a plane of atoms, or half-plane appears and the edge of which terminates within the crystal. It is a linear defect that centers around the line that is defined along the end of the extra half plane of atoms. This is sometimes termed as dislocation line, which for the edge dislocation as shown in figure, is perpendicular to the plane of the page. Within the region around the dislocation line there is some localized lattice distortion. The atoms above the dislocation line are squeezed together and those below are pulled apart. This is reflected in the slight curvature for the vertical planes of atoms as they bend around this extra half plane. The magnitude of this dislocation decreases with distance away from the dislocation line. An edge dislocation may also be formed by an extra half plane of atoms that is included in the bottom portion of the crystal.



Edge dislocation

ii. **Screw dislocation**: Screw dislocation is formed by shear stress that is applied to produce the distortion. The upper front region of the crystal is shifted one atomic distance to the right relative to the bottom portion. The atomic distortion associated with a screw dislocation is also linear and along a dislocation line. The screw dislocation derivers its name from the spiral or helical path or ramp that is traced around the dislocation line by the atomic planes of atoms.



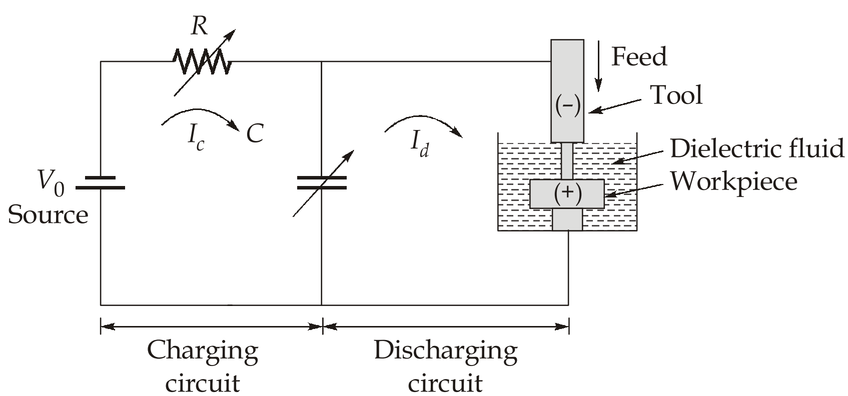
Screw dislocation

iii. Mixed dislocation: Most dislocations found in crystalline materials are probably neither pure edge nor pure screw, but exhibit components of both types, these are termed as mixed dislocations.

The nature of a dislocation is defined by the relative orientations of dislocation line and Burger's vector. For an edge, they are perpendicular, whereas for a screw, they are parallel. They are neither parallel nor perpendicular for a mixed dislocation. Also, even though a dislocation changes direction and nature within a crystal (e.g. from edge to mixed to screw), the Burger's vector will be the same at all points along its line.

3. (c) Solution:

Electric discharge machining : A typical RC type EDM circuit consist of a DC supply V_0, a series resistance R, a capacitor C, and a spark gap connected across the capacitor.



Current flowing through the charging circuit

$$I_c = \frac{V_0 - V}{R} = C\frac{dV}{dt}$$

$\Rightarrow$ $$\frac{dV}{(V_0 - V)} = \frac{dt}{RC}$$

Integrating,

$\Rightarrow$ $$-\ln(V_0 - V) = \frac{t}{RC} + c_1$$

For c_1, $$V = 0 \text{ at } t = 0$$

$\therefore$ $$c_1 = -\ln(V_0)$$

$$-\ln(V_0 - V) = \frac{t}{RC} - \ln V_0$$

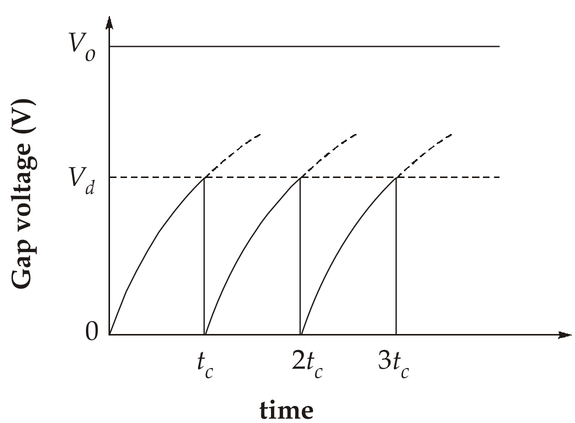
$$\ln\left(\frac{V_0 - V}{V_0}\right) = -\frac{t}{RC}$$

$$V = V_0[1 - e^{-\frac{t}{RC}}]$$

The frequency of sparking is given by

$$f = \frac{1}{t_c}$$

where, t_c denotes the charging time or the time required for the gap voltage to reach discharge voltage (V_d). The energy released per spark is given by



$$E = \frac{1}{2}CV_d^2,$$ then the average power delivered is given by

$$P_{av} = \frac{E}{\text{Cycle time}} = \frac{E}{t_c}$$

and $$V_d = V_0\left(1 - e^{-t/RC}\right)$$

$$\therefore \qquad P_{av} = \frac{1}{2}\frac{C}{t_c}V_0^2\left[1 - e^{-t_c/RC}\right]^2$$

$$= \frac{1}{2}\frac{C}{t_c}V_0^2\left[1 - e^{\tau}\right]^2$$

where, $\tau = \dfrac{t_c}{RC}$

For maximum power delivery,

$$\left.\frac{\partial P_{av}}{\partial \tau}\right|_{\tau=\tau_{opt}} = 0$$

$$\Rightarrow \qquad \left(2\tau_{opt} + 1\right)\left(e^{-\tau}\right) = 1$$

$$\Rightarrow \qquad \tau_{opt} = \left(\frac{t_c}{RC}\right)_{opt} = 1.26$$

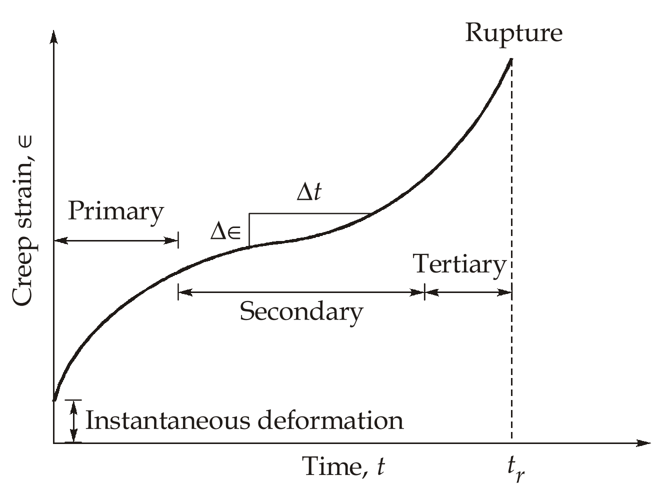
Thus, $\left(\dfrac{V_d}{V_0}\right)_{opt} = 1 - e^{-\tau_{opt}} = 1 - e^{-1.26}$

or $\left(\dfrac{V_d}{V_0}\right)_{opt} = 0.72$

Thus, for maximum power delivery, the discharge voltage should be 72% of the supply voltage.

4. (a) (i) Solution:

Typical creep curve of strain versus time at constant stress at constant elevated temperature.



Primary or transient creep

The deformation rate decrease with time due to work hardening.

Secondary or steady-state creep

It exhibits a constant creep rate. This stage represents the right balance between work hardening and recovery. This is stage of creep of longest duration.

Tertiary creep:

It is characterized by an accelerated creep rate and ends in material rupture. It's associated with both high stresses and temperature.

4. (a) (ii) Solution:

Cathodic Protection

- One of the most effective means of corrosion prevention.
- It may completely stop corrosion in some situations.
- Oxidation or corrosion reaction for a metal *M*.

$$M \rightarrow M^{n+} + ne^-$$

- Cathodic protection simply involves supplying, from an external source, electrons to the metal to be protected, making it a cathode; the reaction above is thus forced in reverse (or reduction) direction.
- Cathodic protection is useful in preventing corrosion of water heaters, underground tanks and pipes, and marine equipment.

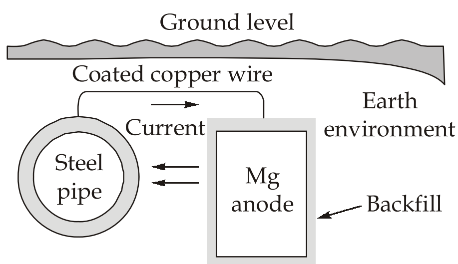
Sacrificial Anode Method:

- This method of cathodic protection employs a galvanic couple.
- The metal to be protected is electrically connected to another metal that is more reactive in the particular environment.
- The latter experiences oxidation, and, upon giving up electrons, protects the first metal from corrosion.
- Magnesium and zinc are commonly used as they lies at the anodic end of galvanic series.
- This form of galvanic protection, used for structures buried in the ground.

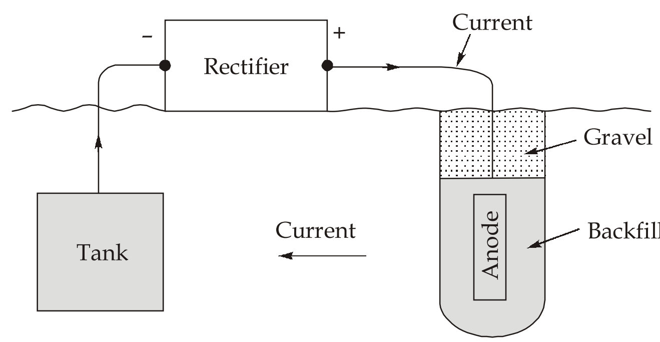
Impressed Current Method

- In this technique, the source of electrons is an external dc power source.
- The negative terminal of the power source is connected to the structure to be prevented.

- The other terminal is joined to an inert anode (often graphite); high-conductivity back fill material provides good electrical contact between the anode and surrounding.
- A current path exists between the cathode and anode completing the electrical circuit.



Magnesium sacrificial anode method



Cathodic prevention of an underground tank using an impressed current

4. (b) (i) Solution:

CNC is superior to conventional manufacturing because of the programmability. The reasons are as follows:

- The non-productive time can be reduced in NC machine tools:
  1. by reducing the number of set-ups,
  2. by reducing set-up time,
  3. by reducing workpiece-handling time, and
  4. by reducing tool-changing time.

These makes NC machines highly productive.

- In the conventional machine tools, precision is largely determined by the human skill. NC machines, because of automation and the absence of interrelated human factors, provide much higher precision.
- Since the part program takes care of the geometry generated, the need for expensive jigs and fixtures is reduced or eliminated, depending upon the part geometry. Even if the fixture is to be used, it would be very simple compared to a conventional machine tool.
- CNC machining centres can perform a variety of machining operations that have to be carried out on several conventional machine tools, thus reducing the machine tools on the shop floor. This saves the floor space and also results in the less lead time in manufacture. This results in the overall reduction in production costs.

4. (b) (ii) Solution:

Given : $I = 950$ A

The material removal rate is given by,

$$Q = \frac{eI}{F\rho} \text{ cm}^3/\text{sec} \qquad [F \text{ is faraday constant}] \qquad ...(i)$$

The density of alloy(ρ) can be expressed in the form

$$\frac{1}{\rho} = \sum_i \left(\frac{x_i}{\rho_i}\right)$$

$$= \frac{0.76}{8.9} + \frac{0.195}{7.19} + \frac{0.025}{4.51} + \frac{0.01}{2.67} + \frac{0.01}{7.86}$$

$$\Rightarrow \quad \rho = 8.125 \text{ g/cm}^3$$

And also, $$\frac{1}{e} = \sum_i \left(\frac{x_i z_i}{A_i}\right)$$

$$= \frac{0.76 \times 2}{58.71} + \frac{0.195 \times 2}{51.99} + \frac{0.025 \times 3}{47.9} + \frac{0.01 \times 3}{26.97} + \frac{0.01 \times 2}{55.85}$$

$$\Rightarrow \quad e = 27.452$$

Substituting all the values in equation (i), we get

$$Q = \frac{27.452 \times 950}{96500 \times 8.125}$$

$$Q = 0.03326 \text{ cm}^3/\text{sec}$$

or $$Q = 0.03326 \times 60 = 1.996 \text{ cm}^3/\text{min}$$

4. (c) Solution:

Given : $r = 0.35$; $\alpha = 10°$; $d = 2.5$ mm; $f = 0.10$ mm/rev.; $F_c = 650$ N; $F_t = 1250$ N;
$V = 150$ m/min

Shear plane angle, ϕ

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} = \frac{0.35\cos 10°}{1 - 0.35 \times \sin 10°}$$

$$\Rightarrow \quad \phi = \tan^{-1}(0.367) = 20.15°$$

The shear force along the shear plane,

$$F_s = F_c \cos\phi - F_t \sin\phi$$

$$= 650 \cos(20.15°) - 1250 \sin(20.15°)$$

$$\Rightarrow \quad F_s = 179.62 \text{ N}$$

The normal force on shear plane,

$$F_n = 1250\cos(20.15°) + 650\sin(20.15°)$$
$$= 1397.4 \text{ N}$$

The area of shear plane, $A_s = \dfrac{bt}{\sin\phi} = \dfrac{fd}{\sin\phi} = \dfrac{2.5\times 0.1}{\sin(20.15°)} = 0.726 \text{ mm}^2$

Normal force on rake face, $N = F_c\cos\alpha - F_t\sin\alpha$

$$= 650\cos(10°) - 1250\sin(10°)$$

$\Rightarrow$ $N = 423.065 \text{ N}$

Friction force along rake surface

$$F = F_c\sin\alpha + F_t\cos\alpha$$
$$F = 650\sin 10 + 1250\cos 10$$
$$F = 1343.88 \text{ N}$$

The coefficient of friction at tool chip interface

$$\mu = \frac{F}{N} = 3.1765$$

The friction angle, $\beta = \tan^{-1}(\mu) = 72.53°$

The shear velocity, $V_s = \dfrac{V\cos\alpha}{\cos(\phi-\alpha)} = \dfrac{150\cos 10°}{\cos(20.15-10)} = 150.07 \text{ m/min}$

The chip velocity, $V_c = \dfrac{V\sin\phi}{\cos(\phi-\alpha)} = 52.49 \text{ m/min}$

(i) The workdone by friction

$$W_f = F\times V_C = 1343.88\times 52.5 = 70540.26 \text{ Nm/min}$$

or $W_f = 1175.67 \text{ J/s}$

(ii) The workdone in shear,

$$W_s = F_sV_s = 179.62\times 150.07 = 26955.57 \text{ Nm/min}$$

or $W_s = 449.26 \text{ J/s}$

(iii) The total workdone,

$$W = F_c\cdot V = 650\times 150 = 97500 \text{ Nm/min}$$

or $W = 1625 \text{ J/s}$

(iv) The shear stress,

$$\tau = \frac{F_s}{A_s} = \frac{179.62}{0.726} = 247.4 \text{ MPa}$$

(v) The shear strain,

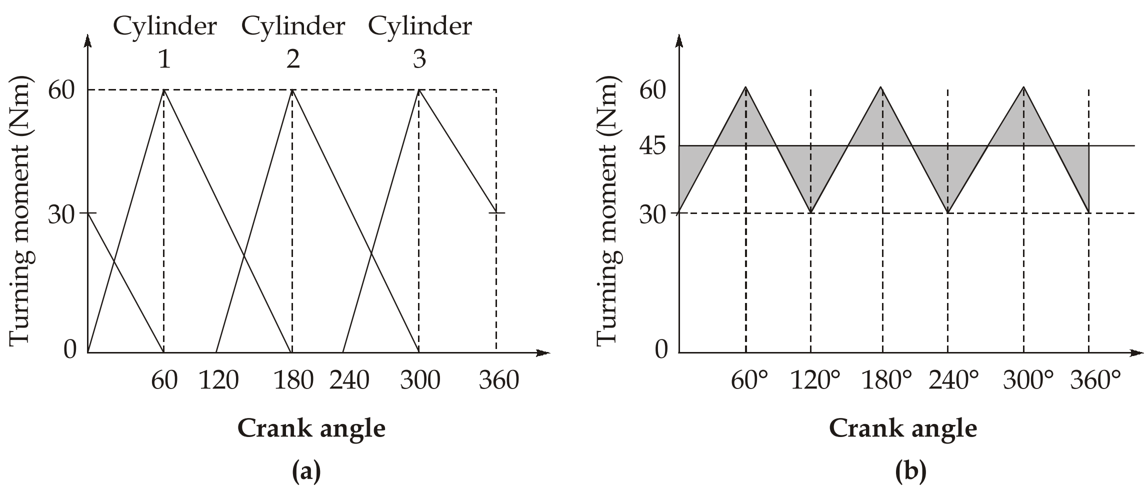
$$\gamma = \frac{V_s}{V \sin\phi} = \cot\phi + \tan(\phi - \alpha)$$

$$= \cot(20.15°) + \tan(10.15°)$$

$$\Rightarrow \quad \gamma = 2.904$$

Section B : Theory of Machines-1 + Fluid Mechanics & Turbo Machinery-2

5. (a) Solution:



(i) Work done/cycle = Area of three triangles

$$= 3 \times \left[\frac{1}{2} \times 60 \times \pi\right] = 90\pi$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 450}{60} = 15\pi \text{ rad/s}$$

$$\text{Mean torque} = \frac{\text{Work done/cycle}}{\text{Angle turned}} = \frac{90\pi}{2\pi} = 45 \text{ Nm}$$

$$\text{Power, } P = T\omega = 45 \times 15\pi = 2120.57 \text{ W} = 2.12 \text{ kW}$$

(ii) As the area above or below the mean torque line is the maximum fluctuation of energy

$$e_{max} = \frac{60\pi}{180} \times (60 - 45) \times \frac{1}{2} = 2.5\pi \text{ Nm}$$

Coefficient of fluctuation of speed,

$$C_s = \frac{e_{max}}{I\omega^2}$$

$$C_s = \frac{2.5\pi}{12 \times 0.078^2 \times (15\pi)^2} = 0.0484$$

$$C_s = 4.84\%$$

(iii) Coefficient of fluctuation of energy

$$C_E = \frac{\text{Maximum fluctiation of energy}}{\text{Work done per cycle}}$$

$$C_E = \frac{2.5\pi}{90\pi} = 0.0278$$

(iv) Maximum fluctuation of torque = 60 – 45 = 15 Nm

$$\Delta T = 15 \text{ Nm}$$

$$I\alpha = mk^2\alpha = 15$$

$$\alpha = \frac{15}{12 \times 0.078^2} = 205.45 \text{ rad/s}^2$$

5. (b) Solution:

1. Propulsive efficiency : The utilization of the gross thrust may be considered in terms of the propulsive efficiency.

Propulsive efficiency is the measure of the effectiveness with which the kinetic energy imparted to the fluid is the difference between kinetic energy at exit and the kinetic energy at inlet and is called propulsive power.

2. Propeller efficiency : The propeller produces thrust power by accelerating the air. The propeller itself is driven by engine. The efficiency of the propeller is defined as the ratio of the thrust power to the shaft power.

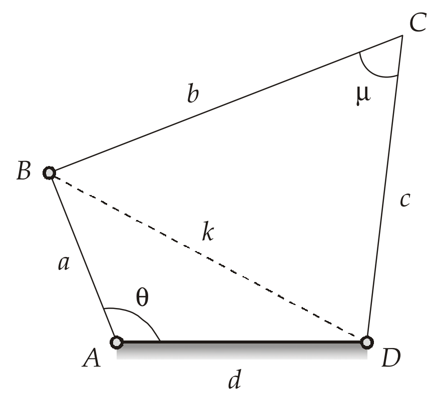
$$\text{Propeller efficiency} = \frac{\text{Thrust power}}{\text{Shaft power}}$$

3. Overall efficiency of a propulsive system : The performance of the propulsion system is normally evaluated in terms of overall efficiency, η_0 is defined as the ratio of rate at which useful propulsion work is done to the rate at which energy is supplied to the system.

4. Specific thrust : The thrust per kg of air flow is known as specific thrust or specific impulse.

5. (c) (i) Solution:

The angle μ between the output link and the coupler in a four bar mechanism is known as transmission angle.



If the link *AB* is the input link, then the force applied to the output link is transmitted through coupler *BC*.

Applying cosine law to triangles *ABD* and *BCD*,

$$a^2 + d^2 - 2ad\cos\theta = k^2 \quad \text{...(i)}$$

$$b^2 + c^2 - 2bc\cos\mu = k^2 \quad \text{...(ii)}$$

From equation (i) and (ii),

$$a^2 + d^2 - 2ad\cos\theta = b^2 + c^2 - 2bc\cos\mu$$

$$a^2 + d^2 - 2ad\cos\theta - b^2 - c^2 + 2bc\cos\mu = 0$$

Differentiating the above equation with respect to θ

$$2ad\sin\theta - 2bc\sin\mu\frac{d\mu}{d\theta} = 0$$

$$\frac{d\mu}{d\theta} = \frac{ad\sin\theta}{bc\sin\mu}$$

Thus, if $\frac{d\mu}{d\theta}$ is to be zero, the term $ad\sin\theta$ has to be zero for which θ is either 0° and 180°.

It can be seen that μ is maximum when θ is 180° and minimum when θ is 0°.

5. (c) (ii) Solution:

Effort of a Governor :

The effort of the governor is the mean force acting on the sleeve to raise or lower it for a given change of speed. At constant speed, the governor is in equilibrium and the resultant force acting on the sleeve is zero.

However, when the speed of the governor increases or decreases, a force is exerted on the sleeve which tends to move it. When the sleeve occupies a new steady position, the resultant force acting on it again becomes zero.

If the force acting at the sleeve changes gradually from zero (when the governor is in the equilibrium position) to a value E for an increased speed of the governor, the mean force or the effort is $\frac{E}{2}$.

Power of a Governor :

The power of a governor is the work done at the sleeve for a given percentage change of speed, i.e., it is the product of the effort and the displacement of the sleeve.

For a Porter governor, having all equal arms which intersect on the axis or pivoted at points equidistant from the spindle axis,

$$\text{Power} = \frac{E}{2}(2 \times \text{Height of governor})$$

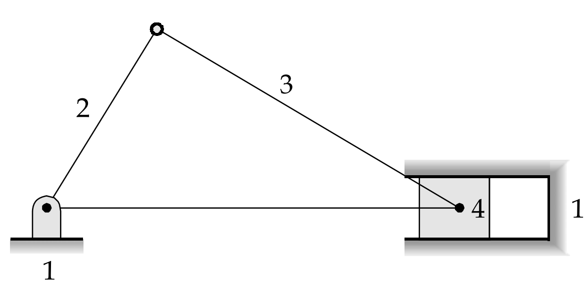
5. **(d) Solution:**

Inversion of Mechanism : When one of the links is fixed in a kinematic chain, it is called a mechanism. We can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanism by fixing different links in a kinematic chain, is known as inversion of the mechanism.

It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to the fixed link) may be changed drastically.

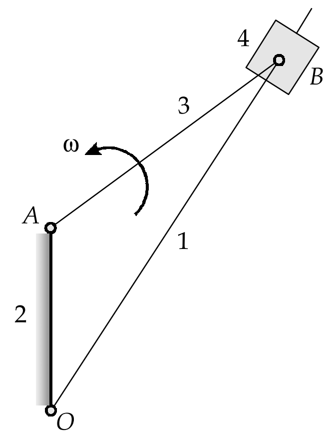
First inversion of Slider-Crank Mechanism:

1. Reciprocating engine
2. Reciprocating compressor



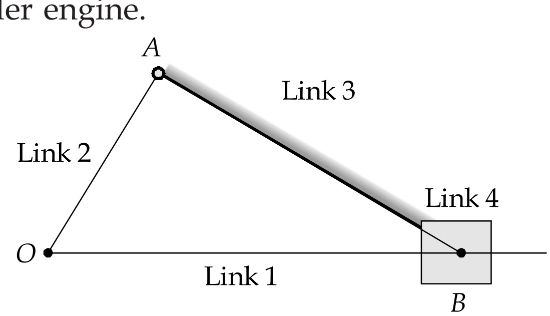
Second inversion of Slider-Crank Mechanism:

1. Whitworth quick-return mechanism
2. Rotary engine



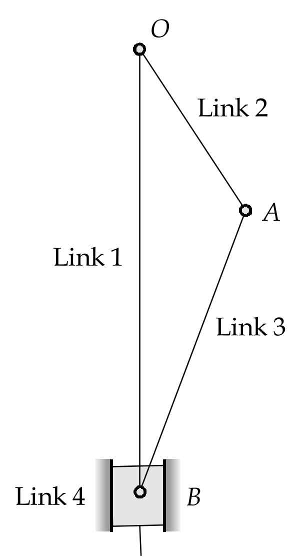
Third inversion of Slider-Crank Mechanism: (Connecting rod is fixed)

1. Crank and slotted lever quick return mechanism.
2. Oscillating cylinder engine.



Fourth inversion of Slider-Crank Mechanism: (Slider is fixed)

1. Hand pump



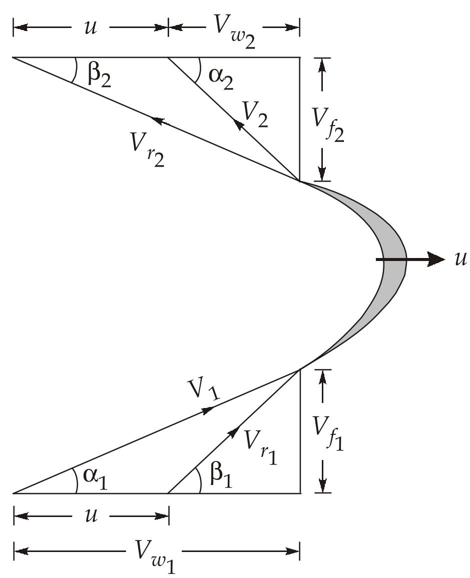
Oscillating cylinder engine: The link 4 is made in the form of a cylinder and a piston is fixed to the end of the link 1. The piston reciprocates inside the cylinder pivoted to the fixed link 3. The arrangement is known as oscillating cylinder engine, in which as the piston reciprocates in the oscillating cylinder, the crank rotates.

Whitworth Quick-return mechanism: It is a mechanism used in work shops to cut metals. The forward stroke takes a little longer and cuts the metal whereas the return stroke is idle and takes a shorter period.

5. (e) Solution:

Given : V_1 = 900 m/s; u = 410 m/s; α_1 = 18°; $\dot{m}$ = 0.75 kg/s

$\because$ Blades are symmetrical = $\beta_1 = \beta_2$



$$V_{f1} = V_1 \sin\alpha_1 = V_{r1} \sin\beta_1 \quad \text{...(i)}$$

$$V_{r1} \cos\beta_1 = V_1 \cos\alpha_1 - u \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\tan\beta_1 = \frac{V_1 \sin\alpha_1}{V_1 \cos\alpha_1 - u}$$

$$\beta_1 = \tan^{-1}\left[\frac{900 \sin 18°}{900 \cos 18° - 410}\right]$$

$$\beta_1 = 31.94°$$

From equation (i),

$$V_{r1}\sin 31.94° = 900 \times \sin 18°$$

$$V_{r1} = 525.56 \text{ m/s} = V_{r2} \qquad [\because \text{ No friction}]$$

$$V_{f1} = V_{r1}\sin\beta_1 = 525.56 \times \sin 31.94°$$

$$V_{f1} = 278.037 \text{ m/s}$$

$$V_{w1} = V_1\cos\alpha_1$$

$$V_{w1} = 900\cos 18° = 855.95 \text{ m/s}$$

$$V_{w2} = V_{r2}\cos\beta_2 - u$$

$$= 525.56\cos 31.94 - 410$$

$$= 35.99 \text{ m/s}$$

$$\Delta V_w = V_{w1} + V_{w2}$$

$$\Delta V_w = 891.94 \text{ m/s}$$

$$V_{f2} = V_{r2}\sin\beta_2$$

$$= 525.56\sin 31.94°$$

$$= 278.037 \text{ m/s}$$

$$\Delta V_a = V_{f1} - V_{f2} = 278.037 - 278.037 = 0$$

Tangential thrust, $F_t = \dot{m}\Delta V_w$

$$F_t = 0.75 \times 891.94 \text{ N} = 668.955 \text{ N}$$

Diagram power, $W_D = F_t \times u = 668.955 \times 410 = 274.271 \text{ kW}$

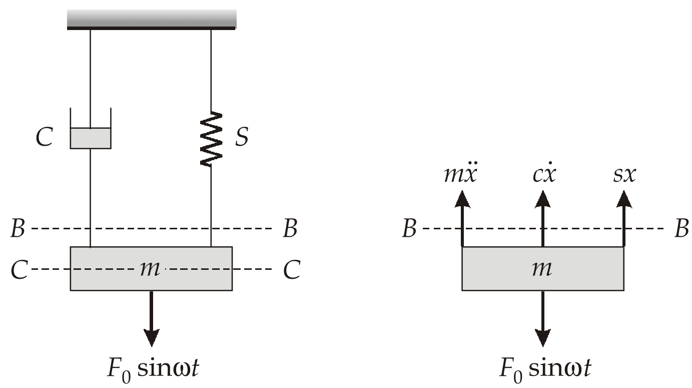
Diagram efficiency, $\eta_D = \dfrac{W_D}{\frac{1}{2}\dot{m}V_1^2 \times 10^{-3}}$

$$\eta_D = \frac{274.271 \times 2}{0.75 \times 900^2 \times 10^{-3}}$$

$$\eta_D = 0.9029 \text{ or } 90.29\%$$

Axial thrust, $F_a = \dot{m}\Delta V_a = 0.75 \times 0 = 0 \text{ N}$

6. (a) Solution:



A mass m is attached to a helical spring and is suspended from a fixed support as before. Damping is also provided in the system with a dashpot.

Before the mass is set in motion, let B-B be the static equilibrium position under the weight of the mass. Now, if the mass is subjected to an oscillating force $F = F_0 \sin\omega t$, the forces acting on the mass at any instant will be

- Impressed oscillating force,

$$F = F_0 \sin\omega t \quad \text{(downwards)}$$

- Inertia force $= m\ddot{x}$ (upwards)
- Damping force $= c\dot{x}$ (upwards)
- Spring force $= sx$ (upwards)

Thus the equation of motion will be

$$m\ddot{x} + c\dot{x} + sx - F_0 \sin\omega t = 0$$

or

$$m\ddot{x} + c\dot{x} + sx = F_0 \sin\omega t$$

Complete solution of this equation consists of two parts, the complementary function (CF) and the particular integral (PI).

$$\text{CF} = Xe^{\xi\omega_n t}\sin(\omega_d t + \phi_1)$$

[ω_d is damped frequency of oscillation]

To obtain the PI, let $\frac{c}{m} = a,\ \frac{s}{m} = b$ and $\frac{F_0}{m} = d$

Then, using the operator D, the equation becomes

$$(D^2 + aD + b)x = d \sin\omega t$$

$$\text{PI} = \frac{d\sin\omega t}{D^2 + aD + b} = \frac{d\sin\omega t}{-\omega^2 + aD + b}$$

$$= \frac{1}{(b-\omega^2)+aD} \times \frac{(b-\omega^2)-aD}{(b-\omega^2)-aD} d\sin\omega t$$

$$= d\left[\frac{\sin\omega t(b-\omega^2)-aD\sin\omega t}{(b-\omega^2)^2-a^2D^2}\right]$$

$$= d\left[\frac{\sin\omega t(b-\omega^2)-a\omega\cos\omega t}{(b-\omega^2)^2-(a\omega)^2}\right]$$

Take $(b - \omega^2) = R\cos\phi$ and $a\omega = R\sin\phi$

Constant R and ϕ are given by

$$R = \sqrt{(b-\omega^2)^2+(a\omega)^2} \text{ and } \phi = \tan^{-1}\frac{a}{b-\omega^2}$$

$$\text{PI} = \frac{dR(\sin\omega t\cos\phi - \cos\omega t\sin\phi)}{(b-\omega^2)^2+(a\omega)^2}$$

$$= \frac{d\sqrt{(b-\omega^2)^2+(a\omega)^2}}{(b-\omega^2)^2+(a\omega)^2}\sin(\omega t-\phi)$$

$$= \frac{d}{\sqrt{(b-\omega^2)^2+(a\omega)^2}}\sin(\omega t-\phi)$$

$$= \frac{F_0/m}{\sqrt{\left(\frac{s}{m}-\omega^2\right)^2+\left(\frac{c}{m}\omega\right)^2}}\sin(\omega t-\phi)$$

$$= \frac{F_0}{\sqrt{\left(s-m\omega^2\right)^2+(c\omega)^2}}\sin(\omega t-\phi)$$

$$x = \text{CF} + \text{PI}$$

$$Xe^{-\xi\omega_n t}\sin\left(\omega_d t-\phi_1\right)+\frac{F_0}{\sqrt{\left(s-m\omega^2\right)^2+(c\omega)^2}}\sin(\omega t)$$

The damped-free vibrations represented by the first part (CF) becomes negligible with time as $e^{-\infty} = 0$. The steady-state response of the system is then given by the second part PI.

The amplitude of the steady-state response is given by

$$A = \frac{F_0}{\sqrt{(s - m\omega^2)^2 + (c\omega)^2}}$$

$$= \frac{F_0 / s}{\sqrt{\left(1 - \frac{m\omega^2}{s}\right)^2 + \left(\frac{c}{s}\omega\right)^2}} = \frac{F_0 / s}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

The equation is in the dimensionless form and is more convenient for analysis. It may be noted that the numerator $\frac{F_0}{s}$ is the static deflection of the spring of stiffness s under a force F_0. The frequency of the steady-state forced vibration is the same as that of the impressed vibrations. ϕ is the phase lag for the displacement relative to the velocity vector.

$$\tan\phi = \frac{a\omega}{b - \omega^2} = \frac{\frac{c}{m}\omega}{\frac{s}{m} - \omega^2} = \frac{c\omega}{s - m\omega^2} = \frac{2\xi\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

The particular solution of the equation of motion can also be obtained graphically as follows:

Assume that the displacement of the vibrating mass under the action of the applied simple harmonic force $F_0 \sin \omega t$ is also simple harmonic and lags by an amount ϕ. Then

$$x = A \sin (\omega t - \phi)$$

and

$$\dot{x} = \omega A \cos(\omega t - \phi) = \omega A \sin\left[\frac{\pi}{2} + (\omega t - \phi)\right]$$

$$\ddot{x} = -\omega^2 A \sin(\omega t - \phi)$$

where A is the amplitude of vibrations.

Substituting these values in equation

$$m\ddot{x} + c\dot{x} + sx = F_0 \sin \omega t$$

$$-m\omega^2 A \sin(\omega t - \phi) + c\omega A \sin\left[\frac{\pi}{2} + (\omega t - \phi)\right] + sA \sin(\omega t - \phi) - F_0 \sin \omega t = 0$$

$$F_0 \sin \omega t + m\omega^2 A \sin(\omega t - \phi) + c\omega A \sin\left[\frac{\pi}{2} + (\omega t - \phi)\right] - sA \sin(\omega t - \phi) = 0$$

The forces and the vector sum of the same have been shown in figure. In triangle *abc*.

$$\sqrt{(sA - m\omega^2 A)^2 + (c\omega A)^2} = F_0$$

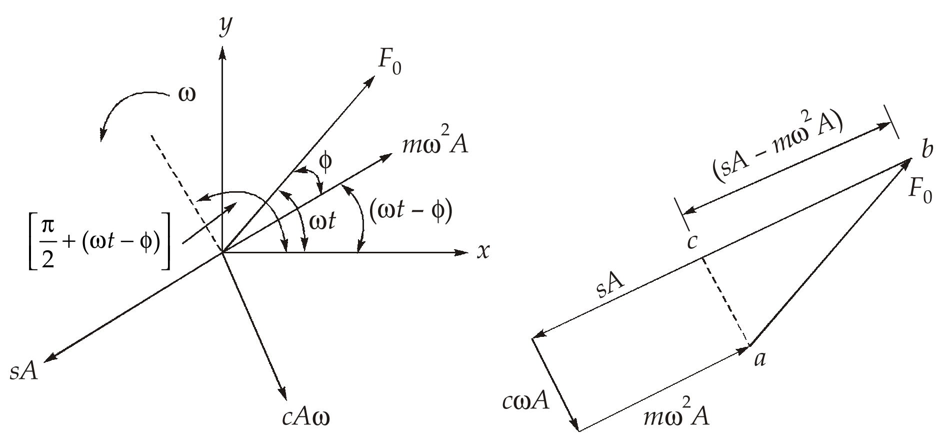
or $$A\sqrt{(s - m\omega^2)^2 + (c\omega)^2} = F_0$$

or $$A = \frac{F_0}{\sqrt{(s - m\omega^2)^2 + (c\omega)^2}}$$

[Amplitude of steady state vibration]

and $$\tan\phi = \frac{c\omega}{s - m\omega^2}$$

The vectors as shown in the figure are fixed relative to one another and rotate with angular velocity ω.

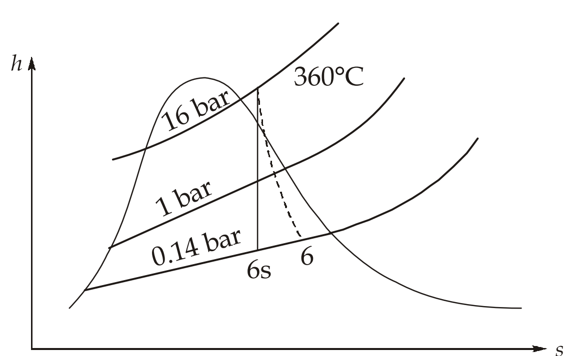


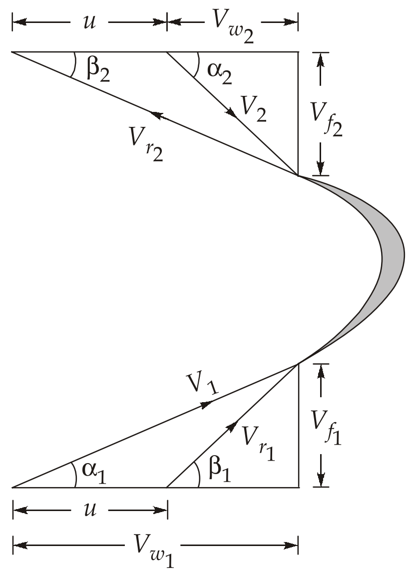
6. (b) Solution:

Given data : Number of stage = 20; 50% reaction, Power = 15 MW; T_1 = 360°C; P_1 = 16 bar;

η_{stage} = 0.8; RHF = 1.03; Velocity ratio = 0.72; $h_b = \frac{1}{2}D_m$, α_1 = 19°

$$\eta_{overall} = \eta_{stage} \times \text{RHF}$$
$$= 0.8 \times 1.03 = 0.824$$





From steam table, at 16 bar, 360°C

$$h_1 = 3167.8 \text{ kJ/kg}$$

$$s_i = 7.1061 \text{ kJ/kgK}$$

at 0.14 bar,

$$h_f = 219.99 \text{ kJ/kg}$$

$$h_{fg} = 2375.8 \text{ kJ/kg}$$

$$s_f = 0.73664 \text{ kJ/kgK}$$

$$s_{fg} = 7.2945 \text{ kJ/kgK}$$

$$s_1 = s_{6s}$$

$$7.1061 = 0.73664 + x(7.2945)$$

$$x = 0.8731$$

$$h_{6s} = h_f + xh_{fg}$$

$$= 219.99 + 0.8731 \times 2375.8$$

$$= 2294.506 \text{ kJ/kg}$$

$$\Delta h_6 = h_1 - h_{6s}$$

$$= 3167.8 - 2294.506$$

$$= 873.29 \text{ kJ/kg}$$

$$(\Delta h)_{actual} = 0.824 \times 873.29$$

$$= 719.59 \text{ kJ/kg}$$

(i) Power = 15 MW

$$\dot{m}(\Delta h)_{actual} = 15 \times 10^3$$

$$\dot{m} \times 719.59 = 15 \times 10^3$$

$$\dot{m} = 20.845 \text{ kg/s}$$

(ii)

$$\frac{u}{V_1} = 0.72$$

$$V_1 = V_{r_2}, V_2 = V_{r_1} \quad \text{[50\% reaction]}$$

$$\alpha_1 = \beta_2$$

$$\alpha_2 = \beta_1$$

$$\Delta V_w = V_1 \cos\alpha_1 + V_2 \cos\alpha_2$$

$$\Delta V_w = V_1 \cos\alpha_1 + V_{r_2} \cos\beta_1$$

$$\Delta V_w = 2V_1 \cos\alpha_1 - u$$

Work done per stage = Enthalpy drop per stage

$$(\Delta V_w) \cdot u = \frac{(\Delta h)_{actual}}{n}$$

$$(2V_1\cos\alpha_1 - u)u = \frac{719.59}{20}$$

$$(2V_1 \cos 19 - 0.72V_1) \times 0.72V_1 = \frac{719.59 \times 10^3}{20}$$

$$V_1 = 206.574 \text{ m/s}$$

$$u = 0.72 \times 206.574$$

$$u = 148.73 \text{ m/s}$$

Volume flow rate per sec at 1 bar = $\pi D_m h_b \times V_f$

$$Q = \pi D_m \frac{D_m}{12} \times V_1 \sin\alpha_1$$

$$Q = \frac{\pi}{12} \times 206.57 \times \sin 19° D_m^2$$

$$Q = 17.606 \ D_m^2$$

at 1 bar, $v_g = 1.6939 \text{ m}^3\text{/kg}$

$$\dot{m} = \frac{\dot{V}}{v_g}$$

$$\dot{m} = \frac{17.606 D_m^2}{1.6939}$$

$$D_m^2 = 216.658$$

$$D_m = 1.416 \text{ m}$$

$$u = \frac{\pi D_m N}{60} = 148.73$$

$$N = \frac{148.73 \times 60}{\pi \times 1.416}$$

$$N = 2006.02 \text{ rpm}$$ **Ans.**

6. (c) Solution:

Given data : $r_p = 4$; $\dot{m} = 3$ kg/s; $\eta_{poly} = 0.84$; $\Delta T_{stage} = 23$ K; $\alpha_1 = 18°$; $V_1 = 160$ m/s

$$\text{Work done factor, } \psi_s = 0.85$$

$$D_m = 16 \text{ cm}$$

$$T_0 = 293 \text{ K; } P_0 = 1 \text{ bar}$$

$$\eta_{poly} = \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{n}{n-1}\right)$$

$$\frac{n-1}{n} = \left(\frac{\gamma-1}{\gamma}\right) \times \frac{1}{\eta_{poly}}$$

$$\frac{n-1}{n} = \frac{1.4-1}{1.4} \times \frac{1}{0.84}$$

$$n = 1.515$$

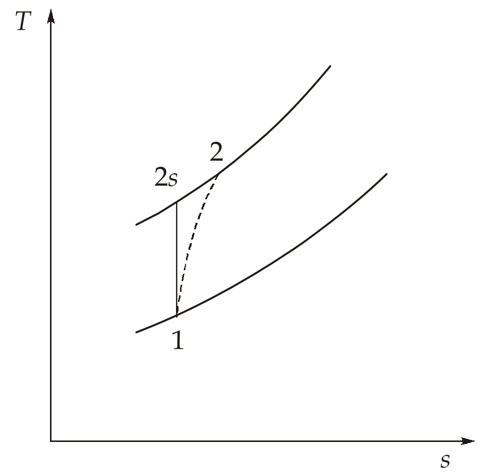
$$T_{02} = T_{01}\left(r_p\right)^{\frac{n-1}{n}} = 293 \times (4)^{\frac{0.515}{1.515}}$$

$$T_{02} = 469.384 \text{ K}$$

$$\text{Number of stages} = \frac{\text{Temperature rise across}}{\text{Total temperature rise per stage}}$$

$$\text{Number of stages} = \frac{469.384 - 293}{23} = 7.67 \simeq 8$$

$$\text{Number of stages} = 8$$ **Ans.**



$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$\eta_c = \frac{r_p^{\frac{\gamma-1}{\gamma}} - 1}{r_p^{\frac{n-1}{n}} - 1} = \frac{(4)^{\frac{0.4}{1.4}} - 1}{(4)^{\frac{0.515}{1.515}} - 1} = 0.8073$$

$$(\Delta T)_{\text{stage}} = \frac{469.384 - 293}{8} = 22.048 \text{ K}$$

Let, T_{s_1} be the temperature after stage 1 and r_1 be the pressure ratio of first stage.

$$T_{s_1} = 293 + 22.048 = 315.048 \text{ K}$$

T_{s_i} — temperature after stage 1, if compression is isentropic

$$\eta_c = \frac{T_{s_i} - T_1}{T_{s_1} - T_1}$$

$$0.8073 = \frac{T_{s_i} - 293}{315.048 - 293}$$

$$T_{s_i} = 310.80 \text{ K}$$

$$\frac{T_{s_i}}{T_1} = (r_1)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{310.80}{293} = (r_1)^{\frac{1.4-1}{1.4}}$$

$$r_1 = 1.2294$$ **Ans.**

Now, temperature at inlet to last stage be T_{ls}

$$\Delta T_s = T_2 - T_{ls}$$

$$22.064 = 469.515 - T_{ls}$$

$$T_{ls} = 447.451 \text{ K}$$

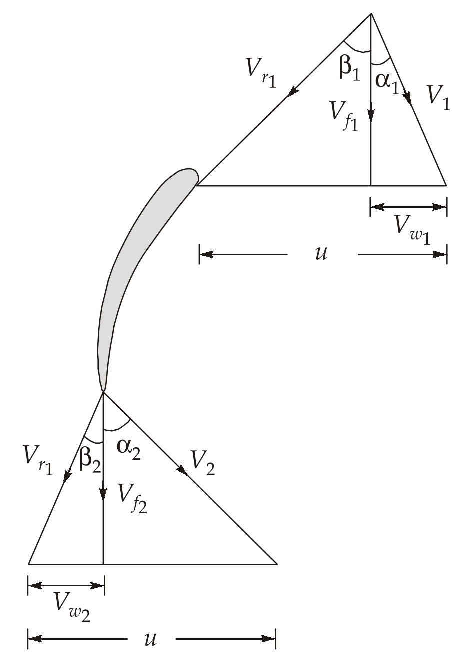
$$\eta_c = \frac{T_{2^\circ} - T_{ls}}{T_2 - T_{ls}}$$

$$0.8073 = \frac{T_{2^\circ} - 447.451}{469.515 - 447.451}$$

$$T_{2^\circ} = 465.263 \text{ K}$$

$$\frac{T_{2^\circ}}{T_{ls}} = (r_{ls})^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{465.263}{447.451} = (r_{ls})^{\frac{0.4}{1.4}}$$

$$r_{ls} = 1.1464$$ **Ans.**



For symmetrical blades, $R = 0.5$ [$\because$ 50% reaction turbine]

$$\alpha_1 = \beta_2 = 18°$$

$$\alpha_2 = \beta_1$$

$$u = V_{f_1}(\tan\beta_1 + \tan\beta_2)$$

$$\Delta V_w = V_{w_1} - V_{w_2} = V_{f_1}(\tan\beta_1 - \tan\beta_2)$$

$$V_{f_1} = V_1 \cos\alpha_1$$

$$= 160 \cos 18°$$

$$= 152.169 \text{ m/s}$$

$$\text{Work done per stage} = \psi u \Delta V_w$$

$$c_p \Delta T_s = \psi V_{f1}^2 \left(\tan^2\beta_1 - \tan^2\beta_2\right)$$

$$1005 \times 22.064 = 0.85 \times (152.169)^2 \times \left[\tan^2\beta_1 - \tan^2 18\right]$$

$$\beta_1 = 47.985°$$

$$u = 152.169 \times (\tan 47.985° + \tan 18°)$$

$$u = 218.356 \text{ m/s}$$

$$u = \frac{\pi D N}{60}$$

$$\frac{218.35 \times 60}{\pi \times 0.160} = N$$

$$\Rightarrow \quad N = 26064.43 \text{ rpm}$$

Pressure ratio across last stage = 1.1464

$$\text{Total pressure at inlet to last stage} = \frac{4}{1.1464} = 3.489 \text{ bar}$$

Total temperature at inlet to last stage = 447.451 K

$$\text{Static temperature,} \quad T_{st} = T_0 - \frac{V_1^2}{2c_p} = 447.451 - \frac{160^2}{2 \times 1005}$$

$$= 434.714 \text{ K}$$

$$P_{st} = \frac{P_0}{\left(\frac{T_o}{T_{st}}\right)^{\frac{\gamma}{\gamma-1}}} = \frac{3.489}{\left(\frac{447.451}{434.714}\right)^{\frac{1.4}{1-1.4}}} = 3.1535 \text{ bar}$$

$$\rho = \frac{P_{st}}{RT} = \frac{3.1535 \times 10^2}{0.287 \times 434.714} = 2.527 \text{ kg/m}^3$$

$$\dot{m} = \rho A V_1 = \rho(\pi D_m h) V_1$$

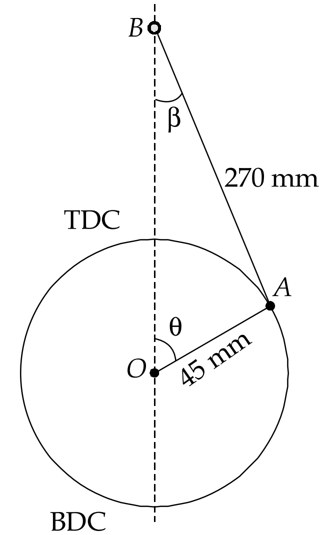
$$3 = 2.527 \times \pi \times 0.16 \times h \times 160$$

$$h = 0.01475 \text{ m}$$

Height of blade, h = 14.757 mm **Ans.**

7. (a) Solution:

Given: $r = 0.045$ m, $l = 0.27$ m, $N = 2000$ rpm, $P = 1250$ kPa, $m = 2.5$ kg, $d = 0.08$ m, $\theta = 30°$



Refer to figure, $$n = \frac{l}{r} = \frac{0.27}{0.045} = 6$$

$$w = \frac{2\pi \times 2000}{60} = 209.33 \text{ rad/s}$$

$$\sin\beta = \frac{\sin\theta}{n} = \frac{\sin 30°}{6}$$

or $$\beta = 4.78°$$

Force due to gas pressure, $$F_P = \frac{\pi}{4} d^2 \times P$$

$$= \frac{\pi}{4} \times (0.08)^2 \times 1250 \times 10^3 = 6280 \text{ N}$$

Inertia force, $$F_b = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$= 2.5 \times 0.045 \times 209.33^2 \times \left(\cos 30 + \frac{\cos 60}{6}\right)$$

$$= 4680.13 \text{ N}$$

(i) Net (effective) force on piston,

$$F = F_P - F_b + mg$$

$$= 6280 - 4680.13 + 2.5 \times 9.81$$

$$= 1624.39 \text{ N}$$ **Ans.**

(ii) Net load on the gudgeon pin = Force on connecting rod

$$F_C = \frac{F}{\cos\beta} = \frac{1624.39}{\cos 4.78} = 1630.05 \text{ N}$$ **Ans.**

(iii) Speed at which the gudgeon pin load is reversed in direction,

$$F = F_P - mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right) + mg$$

$$0 = 6280 - 2.5 \times 0.045 \times \omega^2 \left(\cos 30 + \frac{\cos 60}{6} \right) + 2.5 \times 9.81$$

$$0.1068\ \omega^2 = 6304.52 \text{ or } \omega = 242.96$$

$$\therefore \quad \frac{2\pi N}{60} = 242.96 \text{ or N} = 2321.3 \text{ rpm}$$ **Ans.**

7. (b) Solution:

$$V = \frac{0.02}{\frac{\pi}{4}(0.15)^2} = 1.132 \text{ m/s}$$

(i) For horizontal pipe from Hagen-Poiseuille equation, we have

$$p_1 - p_2 = \frac{32\mu VL}{D^2}$$

$$p_1 - p_2 = \frac{32 \times 12.066 \times 10^{-2} \times 1.132 \times 300}{(0.15)^2} = 58277.17 \text{ N/m}^2$$

$$\tau_0 = \left(-\frac{\partial p}{\partial x} \right) \frac{R}{2}$$

$$= \frac{58277.17}{300} \times \frac{0.15}{2 \times 2} = 7.285 \text{ N/m}^2$$

Power required to maintain the flow is

$$P = Q(p_1 - p_2)$$

$$= 0.02 \times 58277.17 = 1166 \text{ W} = 1.166 \text{ kW}$$

(ii)

1. For inclined pipe with flow in upward direction, we have

$$\rho g(h_1 - h_2) = \frac{32\mu VL}{D^2}$$

$$h_1 - h_2 = \frac{32 \times 12.066 \times 10^{-2} \times 1.132 \times 300}{9810 \times 0.82 \times (0.15)^2}$$

or $$\left(\frac{p_1}{\rho g} + 0 \right) - \left(\frac{p_2}{\rho g} + Z_2 \right) = 7.245$$

Since $$\frac{Z_2}{300} = \sin 15° = 0.25882;\ Z_2 = 77.646 \text{ m}$$

$$\therefore \quad (p_1 - p_2) = (9810 \times 0.82)(7.245 + 77.646)$$

$$= 682880 \text{ N/m2} = 682.880 \text{ kN/m}^2$$

$$\tau_0 = w\left(\frac{\partial h}{\partial x}\right)\frac{R}{2}$$

$$= \frac{(9810 \times 0.82) \times 7.245}{300} \times \frac{0.15}{2 \times 2} = 7.285 \text{ N/m}^2$$

Power required to maintain the flow is

$$P = Q(p_1 - p_2)$$

$$= 0.02 \times 682.880 = 13.657 \text{ W} = 13.657 \text{ kW}$$

2. For inclined pipe with flow in downward direction, we have

$$\rho g(h_1 - h_2) = \frac{32\mu VL}{D^2}$$

$$(h_1 - h_2) = \frac{32 \times 12.066 \times 10^{-2} \times 1.132 \times 300}{9810 \times 0.82 \times (0.15)^2} = 7.245 \text{ m}$$

or
$$\left(\frac{p_1}{\rho g} + Z_1\right) - \left(\frac{p_2}{\rho g} + 0\right) = 7.245$$

Since,
$$\frac{Z_1}{300} = \sin 15° = 0.2588;\ Z_1 = 77.646 \text{ m}$$

$\therefore$
$$(p_1 - p_2) = (9810 \times 0.82)(7.245 - 77.646)$$

$$= -566319 \text{ N/m}^2 = -566.319 \text{ kN/m}^2$$

i.e. in this case the pressure increases in the direction of flow, or there is positive pressure gradient.

$$\tau_0 = w\left(-\frac{\partial h}{\partial x}\right)\frac{R}{2}$$

$$= \frac{(9810 \times 0.82) \times 7.245}{300} \times \frac{0.15}{2 \times 2} = 7.285 \text{ N/m}^2$$

In this case the resistance to flow is compensated by the excessive downward slope of the pipe and hence no external power is required to maintain the flow. Moreover, in this case the flow will have to be regulated by means of a regulating value to maintain the given flow rate.

For pressure gradient along the pipe to be zero, $p_1 = p_2$.

Then from the two above noted cases of inclination of the pipe we have either, $-Z_2 = 7.245$ m, or $Z_1 = 7.245$ m, from which it may be concluded that point 1 is higher than point 2, so that the flow is in the downward direction. The slope required to be provided for the pipe in this case is given by

$$\sin\theta = \frac{Z_1}{300} = \frac{7.245}{300} = 0.0242$$

$$\therefore \quad \theta = 1°23'$$

7. (c)

Universal joint or Universal coupling/Hook's joint.

- This joint is basically used to connect shafts which are non parallel but coplanar (intersecting shaft).
- This joint gives variable velocities ratio i.e. driven shaft angular velocity is continuously changing with respect to the angle turned by driver shaft but mean velocity of driven shaft during one revolution is exactly same as the mean velocity of driving shaft during one revolution. $(\omega_1)_{avg} = (\omega_2)_{avg.}$
- The time taken by driver shaft to complete one revolution with constant angular velocity is exactly same as time taken by driven shaft to complete one revolution with variable angular velocity.

1-Driver

2-Driven

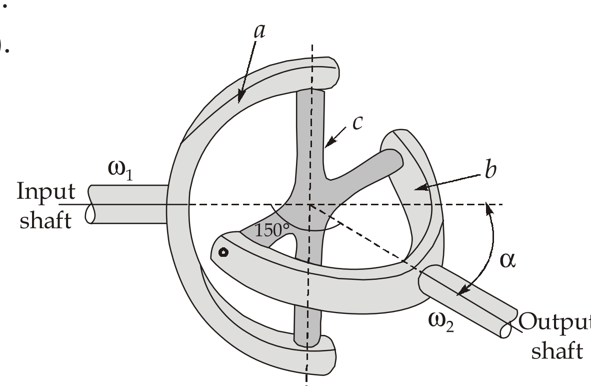
α - Angle between shafts.

ω_1 - Angular velocity of driver (constant).

ω_2 - Angular velocity of driven (variable).

θ - Angle turned by driver.

ϕ - Angle turned by driven.



Fundamental equation,

$$\tan\theta = \cos\alpha \tan\phi$$

$$\tan\phi = \frac{\tan\theta}{\cos\alpha}$$

Differentiating both side,

$$\cos\alpha \cdot \sec^2\phi \frac{d\phi}{dt} = \sec^2\theta \frac{d\theta}{dt} \qquad \because \left(\frac{d\phi/dt}{d\theta/dt}\right) = \left(\frac{\omega_2}{\omega_1}\right)$$

$$\frac{\omega_2}{\omega_1} = \frac{\sec^2\theta}{\cos\alpha \cdot \sec^2\phi} = \frac{1}{\cos^2\theta\cos\alpha\left(1+\tan^2\phi\right)} \qquad \because \left(\tan\phi = \frac{\tan\theta}{\cos\alpha}\right)$$

$$= \frac{1}{\cos^2\theta\cos\alpha\left[1+\frac{\tan^2\theta}{\cos^2\alpha}\right]}$$

$$= \frac{1}{\cos^2\theta\times\cos\alpha\left[1+\frac{\sin^2\theta}{\cos^2\theta\cdot\cos^2\alpha}\right]}$$

$$= \frac{\cos\alpha}{\cos^2\theta\cos^2\alpha+\sin^2\theta} = \frac{\cos\alpha}{\cos^2\theta\left(1-\sin^2\alpha\right)+\sin^2\theta}$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{\cos^2\theta-\cos^2\theta\sin^2\alpha+\sin^2\theta}$$

$$\omega_2 = \frac{\omega_1\cos\alpha}{1-\cos^2\theta\sin^2\alpha} \quad \text{... (i)}$$

As per given information, $\alpha = 180° - 150° = 30°$

ω_1 = Constant (uniform)

$N_1 = 120$ rpm

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi\times 120}{60} = 12.566 \text{ rad/s}$$

$T_{steady} = 135$ Nm

Flywheel mass, $m = 45$ kg

Radius of gyration, $k = 0.15$ m

Moment of inertia, $I_2 = mk^2$

$I_2 = 45 \times 0.15^2$

$I_2 = 1.0125$ kg.m^2

For angular acceleration of driven shaft to be maximum,

$$\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha} = \frac{2\times(\sin 30°)^2}{2-(\sin 30°)^2}$$

$\cos 2\theta = 0.285714$

$2\theta = 73.398°,\ 286.602°$

$\theta = 36.699°, 143.301°$

$$(\alpha_2) = \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin 2\theta}{\left(1-\cos^2\theta \sin^2\alpha\right)^2}$$

$$(\alpha_2)_{(\theta_1 = 36.699°)} = \frac{-(12.566)^2 \cos 30° (\sin 30°)^2 \sin(2\times 36.699°)}{\left[1-(\cos 36.699°)^2 \times (\sin 30°)^2\right]^2}$$

$$= \frac{-32.762}{0.7044} = -46.51 \text{ rad/s}^2$$

$$(\alpha_2)_{(\theta_2 = 143.301°)} = \frac{-(12.566)^2 \cos 30° (\sin 30°)^2 \sin(2\times 143.301°)}{\left(1-(\cos 143.301°)^2 (\sin 30°)^2\right)^2}$$

$$= \frac{+32.762}{0.7044} = 46.51 \text{ rad/s}^2$$

Inertia torque at driven at θ_1 $= I_2\alpha_2 = 1.0125 \times (-46.51)$

$= -47.098$ Nm

At θ_2, Inertia torque $= I_2\alpha_2 = 1.0125 \times (46.51)$

$= 47.092$ Nm

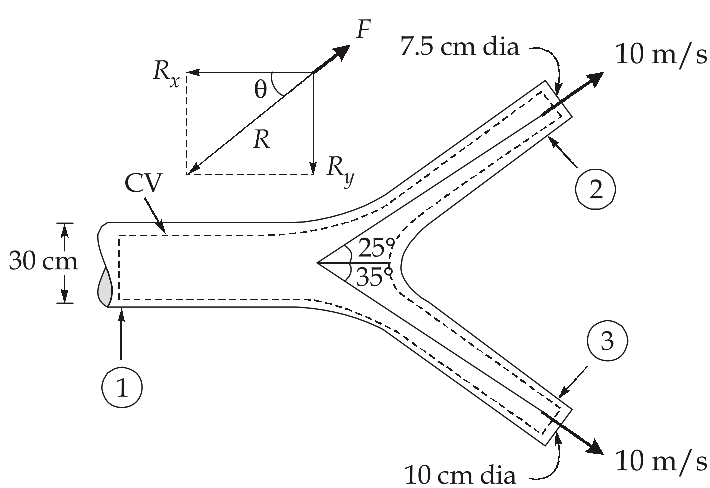
Maximum torque at driven,

$$(T_2)_{\text{max at } \theta = \theta_2} = (T_2)_{\text{steady}} + (T)_{\text{inertia torque}}$$

$= 135$ Nm $+ 47.092$ Nm

$= 182.092$ Nm

8. (a) Solution:



$$A_1 = \frac{\pi}{4}(0.3)^2 = 0.0706 \text{ m}^2$$

$$A_2 = \frac{\pi}{4}(0.075)^2 = 4.417 \times 10^{-3} \text{ m}^2$$

$$A_3 = \frac{\pi}{4} \times (0.10)^2 = 7.854 \times 10^{-3} \text{ m}^2$$

$$Q_1 = Q_2 + Q_3$$

$$Q_1 = A_1 V_1$$

$$Q_2 = A_2 V_2 = 4.417 \times 10^{-3} \times 10 = 4.417 \times 10^{-2} \text{ m}^3/\text{s}$$

$$Q_3 = A_3 V_3 = 7.854 \times 10^{-3} \times 10 = 7.854 \times 10^{-2} \text{ m}^3/\text{s}$$

$$V_1 = \frac{4.417 \times 10^{-2} + 7.854 \times 10^{-2}}{0.0706} = 1.738 \text{ m/s}$$

By applying Bernoulli's theorem to section 1 and 2

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1}$$

$$0 + \frac{(10)^2}{2 \times 9.81} = \frac{P_1}{\rho g} + \frac{(1.738)^2}{2 \times 9.81}$$

$$P_1 = 48.489 \text{ kPa [Gauge]}$$

Applying momentum equation in x-direction

$$P_1 A_1 - R_x = \rho[Q_2 V_2 \cos 25° + Q_3 V_3 \cos 35°] - \rho[Q_1 V_1]$$

$$48.489 \times 10^3 \times 0.0706 - R_x = 10^3 \times \begin{bmatrix} 0.04417 \times 10 \times \cos 25° + 0.07854 \times \\ 10 \times \cos 35 - 0.1227 \times 1.738 \end{bmatrix}$$

$$R_x = 2592.65 \text{ N}$$

By applying momentum equation in y-direction

$$0 - R_y = \rho(Q_2 V_2 \sin 25° - Q_3 V_3 \sin 35°)$$

$$-R_y = 10^3 \times [0.04417 \times 10 \times \sin 25° - 0.07854 \times 10 \times \sin 35°]$$

$$R_y = 263.82 \text{ N}$$

The reaction R on the fluid in the control volume is

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{2592.65^2 + 263.82^2}$$

$$R = 2606.03 \text{ N}$$

Inclination θ of R with $(-x)$ direction

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{263.82}{2592.6}\right) = 5.81°$$

8. (b)

Given : $m = 5$ kg; $e = 2$ mm; $l = 60$ cm; $E = 2 \times 10^{11}$ N/m²; $d = 20$ mm; $c = 60$ N-sec/m

(i) The static deflection, $\delta = \dfrac{WL^3}{48EI}$

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}(0.02)^4 = 7.854 \times 10^{-9}$$

$$\delta = \frac{(5 \times 9.81) \times (0.6)^3}{48 \times 2 \times 10^{11} \times 7.854 \times 10^{-9}} = 1.405 \times 10^{-4} \text{ m}$$

Critical speed, $\omega_c = \sqrt{\dfrac{g}{\delta}} = \sqrt{\dfrac{9.81}{1.405 \times 10^{-4}}} = 264.22$ rad/s

Angular speed of shaft, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 360}{60} = 37.699$ rad/s

Frequency ratio, $\beta = \dfrac{\omega}{\omega_c} = \dfrac{37.699}{264.22} = 0.1427$

Damping factor, $\xi = \dfrac{c}{2\sqrt{sm}} = \dfrac{c}{2m\sqrt{\omega_c^2}}$

$$= \frac{60}{2 \times 5 \times 264.22} = 0.0227$$

$$\frac{x}{e} = \frac{\beta^2}{\sqrt{(1-\beta^2)^2 + (2\varepsilon\beta)^2}}$$

$$\frac{x}{2} = \frac{(0.1427)^2}{\sqrt{\left[1-(0.1427)^2\right]^2 + (2 \times 0.1427 \times 0.0227)^2}}$$

$$x = 0.0416 \text{ mm}$$

For the dynamic load on the bearing

$$F_d = \sqrt{(\text{Spring force})^2 + (\text{Damping force})^2}$$

$$= \sqrt{(k_t x)^2 + (cwx)^2} = x\sqrt{(k_t)^2 + (cw)^2}$$

$$w_c = \sqrt{\frac{k_t}{m}}$$

$$\Rightarrow \quad k_t = w_c^2 \cdot m = 264.22^2 \times 5$$

$$k_t = 349.067 \text{ kN/m}$$

$$F_d = 0.0416 \times 10^{-3}\sqrt{(349.067)^2 + (60)^2 \times (37.70)^2} = 14.52$$

The dead load on the shaft, $w = mg = 5 \times 9.81 = 49.05$ N

Total maximum load on the shaft under the above condition

$$= 14.52 + 49.05 = 63.57 \text{ N}$$

By stress relation,

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\sigma = \frac{M}{I} \cdot y = \frac{F \cdot ld}{4\left(\frac{\pi}{64}d^4\right) \times 2} = \frac{8F \cdot l}{\pi d^3}$$

$$I = \frac{\pi}{64}d^4$$

$$y = \frac{d}{2}$$

$$\sigma_{max} = \frac{8F \cdot l}{\pi d^3} = \frac{8F \times 0.6}{\pi \times \left(20 \times 10^{-3}\right)^3}$$

$$= 12.141 \times 10^6 \text{ N/m}^2 = 12.141 \text{ MPa}$$

(ii) The power required to drive the shaft

$$P = \frac{2\pi NT}{60}$$

Damping torque, T = Damping force × x

$$= cwxx = cwx^2 = 60 \times (37.7) \times (0.0416 \times 10^{-3})^2$$

$$= 3.915 \times 10^{-6} \text{ Nm}$$

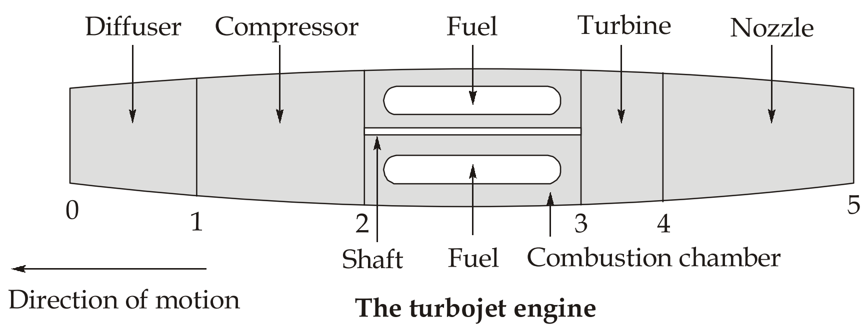
$$P = \frac{2\pi \times 360 \times 3.915 \times 10^{-6}}{60}$$

$$P = 1.47 \times 10^{-4} \text{ Watt}$$

8. **(c) Solution:**

The turbojet Engine:

The two pilotless air breathing engines described, viz., ramjet and pulsejet are simple in construction. However, their application is limited and, to date, they have not been used very extensively. The most common type of air breathing engine apart from turboprop is the turbojet engine. The important features are shown in figure.



The turbojet engine

This engine consists of the following components:

(i) a diffuser,

(ii) a mechanical compressor,

(iii) a combustion chamber,

(iv) a mechanical turbine

(v) an exhaust nozzle

The function of the diffuser is to convert the kinetic energy of the entering air into a static pressure rise which is achieved by the ram effect. After this air enters the mechanical compressor.

The compressor used in a turbojet can be either centrifugal type or axial flow type. The use of a particular type of compressor produces a high boost typical characteristics. The centrifugal compressor produces a higher pressure ratio of about 4 : 1 to 5 : 1 in a single stage and usually a doubled sided rotor is used to reduce the engine diameter. The turbojet using a centrifugal compressor has a short and sturdy appearance. The advantages of centrifugal compressor are high durability, ease of manufacture and low cost, and good operation under adverse circumstances such as icing and when sand and small foreign particles are inhaled in inlet duct.

The demand for increased power has led to the use of split or two-spool axial flow compressor. A very high pressure ratio of about 9 : 1 to 13 : 1 is obtained by using a high pressure and a low pressure rotor driven by separate shafts. The use of high pressure ratio gives very good specific fuel consumption (0.75 kg/kg thrust per hour) and use of two rotors allows greater efficiency because firstly, the high pressure rotor can be governed for speed and secondly, the low pressure rotor can be allowed to run at a speed giving maximum efficiency.

The turbojets having centrifugal compressor have about 20 per cent weight advantage over the axial flow turbojets. Thrust per unit weight is more for the first type while thrust per unit diameter is more the second type. The axial flow turbojets have about 6 to 8 per cent less specific fuel consumption. After the compressor air enters to the combustion chamber, the fuel nozzles feed fuel continuously and continuous combustion takes place at constant-pressure. The high pressure, high temperature gases then enter the turbine, where they expand to provide enough power output from the turbine.

The turbine is directly connected to the compressor and all the power developed by the turbine is absorbed by the compressor and the auxiliaries. The main function of the turbine is to provide power, to drive the compressors. After the gases leave the turbine they expand further in the exhaust nozzle, and are ejected into the atmosphere with a velocity greater than the flight velocity thereby producing thrust for propulsion.

Advantages of Turbojet

(i) The power-to-weight ratio of a turbojet is about 4 times that of a propeller system with a reciprocating engine.

(ii) It is simple, easy to maintain, and requires lower lubricating oil consumption. Furthermore, the complete absence of liquid cooling results in reduced the frontal area.

(iii) There is no limit to the power output that can be obtained from a turbojet, while piston engines have almost reached their peak power, and further increases will be at the cost of greater complexity, increased engine weight, and frontal area of the aircraft.

Disadvantages of Turbojet

(i) The fuel economy at low operational speeds is extremely poor.

(ii) It has low take-off thrust and hence, poor starting characteristics.