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Detailed Solutions

ESE-2026 Mains Test Series

Civil Engineering Test No : 5

Section A : Flow of Fluids, Hydraulic Machines and Hydro Power

1. (a) Solution:

Given: velocity vector, $\vec{V} = (3x^2 + 2y)\,\hat{i} + (-4xy + 2y^2 + 2zy)\,\hat{j} + \left(-\frac{5}{2}z^2 + 3xz - 4y^2z\right)\hat{k}$

Here,
$$u = 3x^2 + 2y$$
$$v = -4xy + 2y^2 + 2zy$$
$$w = -\frac{5}{2}z^2 + 3xz - 4y^2z$$

At (1, 1, 1):
$$u = 3(1)^2 + 2(1) = 5$$
$$v = -4(1)(1) + 2(1)^2 + 2(1)(1) = 0$$
$$w = -\frac{5}{2}(1)^2 + 3(1)(1) - 4(1)^2(1) = -3.5$$

Since the flow is steady, the local acceleration $\frac{d\vec{v}}{dt} = 0$. Therefore, total acceleration is purely convective and given by

$$\vec{a} = \frac{d\vec{V}}{dt}$$

The components are

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

First, evaluate the required partial derivatives at (1, 1, 1).

$$\frac{\partial u}{\partial x} = 6x = 6,\ \frac{\partial u}{\partial y} = 2, \frac{\partial u}{\partial z} = 0$$

$$\frac{\partial v}{\partial x} = -4y = -4, \frac{\partial v}{\partial y} = -4x + 4y + 2z = 2, \frac{\partial v}{\partial z} = 2y = 2$$

$$\frac{\partial w}{\partial x} = 3z = 3, \frac{\partial w}{\partial y} = -8yz = -8, \frac{\partial w}{\partial z} = -5z + 3x - 4y^2 = -6$$

Now, substitute into the acceleration components.

$$a_x = (5)(6) + (0)(2) + (-3.5)(0) = 30$$

$$z_y = (5)(-4) + (0)(2) + (-3.5)(2) = -20 - 7 = -27$$

$$a_z = (5)(3) + (0)(-8) + (-3.5)(-6) = 15 + 21 = 36$$

Therefore, the total acceleration vector is

$$\vec{a} = 30\hat{i} - 27\hat{j} + 36\hat{k}$$

1. (b) Solution:

Effect of Pressure Gradient on Boundary Layer Separation

Boundary layer separation is strongly influenced by the pressure gradient along the surface over which a fluid flows.

When a fluid flows over a surface, the velocity near the surface is reduced due to viscosity, forming a boundary layer. The behaviour of this boundary layer depends on the pressure gradient in the direction of flow.

1. Favourable Pressure Gradient $\left(\frac{dp}{dx} < 0\right)$

   In this case, pressure decreases in the direction of flow.

   The fluid particles are accelerated, and the boundary layer remains thin and stable.

   Separation is less likely to occur because the fluid near the surface has sufficient kinetic energy to overcome viscous resistance.

2. Adverse Pressure Gradient $\left(\frac{dp}{dx} > 0\right)$

   Here, pressure increases in the direction of flow.

   The fluid particles decelerate, and the velocity near the wall decreases further.

   If the adverse pressure gradient is strong enough, the velocity of fluid particles close to the surface becomes zero and may even reverse direction.

   This results in:

   - Zero wall shear stress
   - Flow reversal near the surface
   - Detachment of the boundary layer from the surface

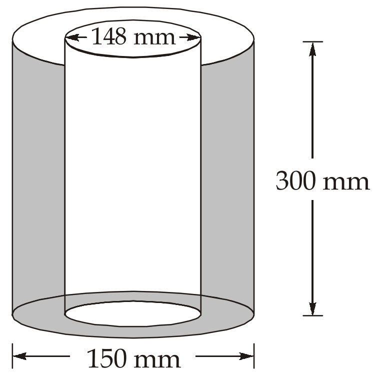
This phenomenon is called boundary layer separation. Thus, an adverse pressure gradient is the primary cause of boundary layer separation.

Methods to Prevent Boundary Layer Separation

Boundary layer separation can be delayed or prevented by increasing the energy of the fluid near the surface or by reducing the adverse pressure gradient. Common methods include:

1. **Streamlining of Body:** Designing smooth, gradual contours reduces the adverse pressure gradient and delays separation.
2. **Suction of Boundary Layer:** Removing low-energy fluid near the wall through small perforations helps maintain attachment of flow.
3. **Blowing or Injection of High-Energy Fluid:** Injecting high-velocity fluid into the boundary layer increases its kinetic energy and prevents reversal.
4. **Use of Guide Vanes or Slots (Slotted Wings):** These allow high-energy air from the upper surface to energize the boundary layer.
5. **Providing a Moving Surface:** Moving the surface in the direction of flow reduces relative velocity difference and delays separation.
6. **Turbulators or Roughness Elements:** Converting laminar flow to turbulent flow increases momentum transfer within the boundary layer, making it more resistant to separation.

1. (c) Solution:



Given

Height of cylinders:

$$H = 300 \text{ mm} = 0.3 \text{ m}$$

Diameter of outer cylinder:

$$d_o = 150 \text{ mm} = 0.15 \text{ m}$$

Diameter of inner cylinder:

$$d_1 = 148 \text{ mm} = 0.148 \text{ m}$$

Speed of outer cylinder:

$$N = 120 \text{ rpm}$$

Torque on inner cylinder:

$$T = 1.5 \text{ N-m}$$

Tangential velocity of the outer cylinder

$$V = \frac{\pi d_o N}{60}$$

$$\Rightarrow \quad V = \frac{\pi \times 0.15 \times 120}{60} = 0.942 \text{ m/s}$$

Thickness of liquid film

$$dy = \frac{d_o - d_i}{2} = \frac{0.15 - 0.148}{2}$$

$$\Rightarrow \quad dy = 0.001 \text{ m}$$

Shear stress in the liquid

From Newton's law of viscosity,

$$\tau = \mu \frac{dV}{dy} = \mu \frac{0.942}{0.001}$$

$$\Rightarrow \qquad \tau = 942\,\mu\ \text{N/m}^2$$

Viscous force on the inner cylinder

$$F = \tau A$$

$$\Rightarrow \qquad F = 942\,\mu \times \pi \times 0.148 \times 0.3$$

$$\Rightarrow \qquad F = 131.54\,\mu\text{N}$$

Torque on the inner cylinder,

$$T = F \times \frac{d_i}{2}$$

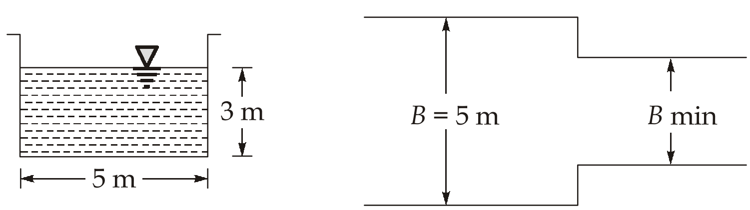
$$\Rightarrow \qquad 1.5 = 131.54\,\mu \times \frac{0.148}{2}$$

$$\Rightarrow \qquad 1.5 = 9.734\,\mu$$

$$\Rightarrow \qquad \mu = 0.154\ \text{Pa-s}$$

1. (d) Solution:

Given: $B = 5.0$ m, $y = 3.0$ m, $S_0 = 1/1200$, $N = 0.02$



(i) For minimum throat width

$$A = B + y = 5 \times 3 = 15\ \text{m}^2,$$

$$P = B + 2y = 5 + 2(3) = 11\ \text{m},$$

$$R = \frac{A}{P} = \frac{15}{11} = 1.364\ \text{m}.$$

From manning's equation,

$$v = \frac{1}{n} R^{2/3} S_0^{1/2} = \frac{1}{0.02}(1.364)^{2/3}\left(\frac{1}{1200}\right)^{1/2} = 1.775\ \text{m/s}$$

Discharge, $Q = Av = 15 \times 1.775 = 26.625\ \text{m}^3/\text{s}$

Specific energy, $E_1 = y + \frac{v^2}{2g} = 3 + \frac{(1.775)^2}{2(9.81)} = 3.161\ \text{m}$

For minimum throat width, $E_1 = E_c = \frac{3}{2}y_c$

$$y_c = \frac{2}{3}E_1 = \frac{2}{3}(3.161) = 2.107 \text{ m}$$

$\Rightarrow$Critical unit discharge $q_{max} = \sqrt{gy_c^3} = \sqrt{9.81(2.107)^3} = 9.58 \text{ m}^2/\text{s}$

Minimum throat width $B_{min} = \frac{Q}{q_{max}} = \frac{26.625}{9.58} = 2.779 \text{ m}$

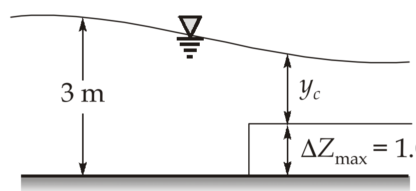
(ii) For maximum hump height,

Unit discharge in original channel, $q = \frac{Q}{B} = \frac{26.625}{5} = 5.325 \text{ m}^2/\text{s}$

Critical depth at hump $y_{c2} = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{5.325^2}{9.81}\right)^{1/3} = 1.424 \text{ m}$

Critical energy at hump $E_{c2} = \frac{3}{2}y_{c2} = \frac{3}{2}(1.424) = 2.138 \text{ m}$

Maximum hump height $\Delta Z_{max} = E_1 - E_2 = 3.161 - 2.138 = 1.023 \text{ m}$



1. **(e) Solution:**

Given:

Speed (N_1) = 210 r.p.m.,

Power (P_1) = 6200 kW,

Head (H_1) = 250 m,

Overall efficiency (η_o) = 85% or 0.85,

New head (H_2) = 160 m,

1. Calculation of Unit Quantities

Unit Speed (N_u):

$$N_u = \frac{N_1}{\sqrt{H_1}}$$

$$\Rightarrow \qquad N_u = \frac{210}{\sqrt{250}} = \frac{210}{15.811} = 13.282 \text{ r.p.m.}$$

Unit Power (P_u):

$$P_u = \frac{P_1}{H_1^{3/2}}$$

$$\Rightarrow \qquad P_u = \frac{6200}{250^{1.5}} = \frac{6200}{3952.847} = 1.568 \text{ kW}$$

Unit Discharge (Q_u):

First, calculate the actual discharge (Q_1) at 250 m head using the overall efficiency formula:

$$\eta_o = \frac{P_1 \times 1000}{\rho \times g \times Q_1 \times H_1}$$

$$\Rightarrow \qquad 0.85 = \frac{6200 \times 1000}{1000 \times 9.81 \times Q_1 \times 250}$$

$$\Rightarrow \qquad Q_1 = \frac{6200000}{0.85 \times 9.81 \times 1000 \times 250} = \frac{6200000}{2084625} = 2.974 \text{ m}^3/\text{s}$$

Now, find the unit discharge:

$$Q_u = \frac{Q_1}{\sqrt{H_1}}$$

$$\Rightarrow \qquad Q_u = \frac{2.974}{\sqrt{250}} = 0.188 \text{ m}^3/\text{s}$$

2. Performance at a Reduced Head of 160 m

New Speed (N_2):

$$N_2 = N_u \times \sqrt{H_2}$$

$$\Rightarrow \qquad N_2 = 13.282 \times \sqrt{160} = 13.282 \times 12.649 = 168.004 \text{ r.p.m.}$$

New Discharge (Q_2):

$$Q_2 = Q_u \times \sqrt{H_2}$$

$$\Rightarrow \qquad Q_2 = 0.188 \times \sqrt{160} = 0.188 \times 12.649 = 2.378 \text{ m}^3/\text{s}$$

New Power (P_2):

$$P_2 = P_u \times H_2^{1.5}$$

$$\Rightarrow \qquad P_2 = 1.568 \times (160)^{1.5} = 1.568 \times 2023.858 = 3173.409 \text{ kW}$$

2. (a) Solution:

The discharge through an orifice may be expressed as a functional relationship

$$Q = f(D, H, \rho, \mu, g)$$

or

$$f(Q, D, H, \rho, \mu, g) = 0$$

The total number of variables is

$$n = 6$$

The fundamental dimensions involved are mass M, length L, and time T,

Hence, $$m = 3$$

Therefore, the number of dimensionless π-terms is

$$n - m = 6 - 3 = 3$$

The repeating variables are chosen as ρ, D, and g.

The dimensions of the variables are

$$[Q] = L^3T^{-1}$$
$$[D] = L$$
$$[H] = L$$
$$[\rho] = ML^{-3}$$
$$[\mu] = ML^{-1}T^{-1}$$
$$[g] = LT^{-2}$$

First π-term:

$$\pi_1 = \rho^{a_1} D^{b_1} g^{c_1} Q$$

Substituting dimensions,

$$M^0L^0T^0 = \left(ML^{-3}\right)^{a_1} (L)^{b_1} \left(LT^{-2}\right)^{c_1} \left(L^3T^{-1}\right)$$

Equating powers of fundamental dimensions:

For mass M: $$a_1 = 0$$

For time T: $$-2c_1 - 1 = 0 \Rightarrow c_1 = -\frac{1}{2}$$

For length L: $$-3a_1 + b_1 + c_1 + 3 = 0$$
$$0 + b_1 - 0.5 + 3 = 0 \Rightarrow b_1 = -2.5$$

Hence,

$$\pi_1 = \frac{Q}{D^{2.5} g^{0.5}}$$

Second π-term: $$\pi_2 = \rho^{a_2} D^{b_2} g^{c_2} \mu$$

Substituting dimensions,

$$M_0 L_0 T_0 = (ML^{-3})^{a2} (L)^{b2} (LT^{-2})^{c2} (ML^{-1}T^{-1})$$

Equating powers:

For mass M: $$a_2 + 1 = 0 \Rightarrow a_2 = -1$$

For time T: $$-2c_2 - 1 = 0 \Rightarrow c_2 = -\frac{1}{2}$$

For length L:

$$-3a_2 + b_2 + c_2 - 1 = 0$$

$$3 + b_2 - 0.5 - 1 = 0 \Rightarrow b_2 = -1.5$$

Thus,

$$\pi_2 = \frac{\mu}{\rho D^{1.5} g^{0.5}}$$

Third π-term:

$$\pi_3 = \rho^{a_3} D^{b_3} g^{c_3} H$$

$$[M°L°T°] = [ML^{-3}]^{a_3} [L]^{b_3} [LT^{-2}]^{c_3} [L]$$

Power of m: $$a_3 = 0$$

Power of L: $$-3a_3 + b_3 + c_3 + 1 = 0$$

Power of T: $$-2c_3 = 0$$

$\Rightarrow$ $$c_3 = 0$$

Since both H and D have the dimension of length, the dimensionless combination is obtained directly as

$$\pi_3 = \frac{H}{D}$$

Hence, the functional relationship becomes

$$\pi_1 = \phi(\pi_2, \pi_3,)$$

or $$\frac{Q}{D^{2.5} g^{0.5}} = \phi\left(\frac{\mu}{\rho D^{1.5} g^{0.5}}, \frac{H}{D}\right)$$

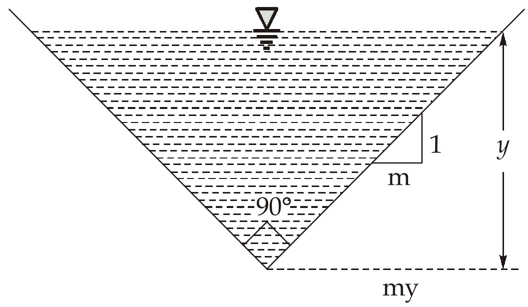
Rearranging, the expression for discharge is

$$Q = D^{2.5}\, g^{0.5} \phi\left(\frac{H}{D}, \frac{\mu}{\rho D^{1.5} g^{0.5}}\right)$$

2. (b) (i) Solution:

Given data

$\alpha = 90° \Rightarrow m = 1, y_1 = 0.05 \text{ m}, y_2 = 0.15 \text{ m}, g = 9.81 \text{ m/s}^2.$



Area of triangular channel: $A = my^2$

$\Rightarrow \quad A_1 = 1(0.05)^2 = 0.0025 \text{ m}^2$

$\Rightarrow \quad A_2 = 1(0.15)^2 = 0.0225 \text{ m}^2$

Centroid depth: $\bar{z} = \dfrac{y}{3}$

Hydrostatic term: $A\bar{z} = \dfrac{y^3}{3}$

Specific force balance:

$$\frac{Q^2}{gA_1} + A_1\bar{z}_1 = \frac{Q^2}{gA_2} + A_2\bar{z}_2$$

$$\Rightarrow \quad \frac{Q^2}{gA_1} + \frac{y_1^3}{3} = \frac{Q^2}{gA_2} + \frac{y_2^3}{3}$$

$$\Rightarrow \quad \frac{Q^2}{g}\left(\frac{1}{A_1} - \frac{1}{A_2}\right) = \frac{y_2^3 - y_1^3}{3}$$

$$\Rightarrow \quad \frac{Q^2}{9.81}\left(\frac{1}{0.0025} - \frac{1}{0.0225}\right) = \frac{0.15^3 - 0.05^3}{3}$$

$$\Rightarrow \quad \frac{Q^2}{9.81}(400 - 44.444) = \frac{0.003375 - 0.000125}{3}$$

$$\Rightarrow \quad \frac{Q^2}{9.81} \times 355.556 = 0.001083$$

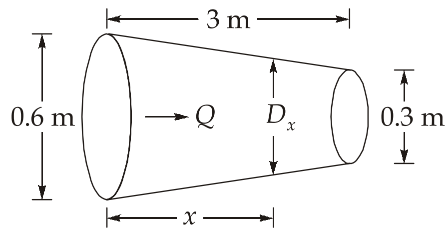
$$\Rightarrow \quad Q^2 = \frac{0.001083 \times 9.81}{355.556} = 0.000029881$$

$$\Rightarrow \quad Q = \sqrt{0.000029881} = 0.005466 \text{ m}^3/\text{s}$$

$$Q = 5.466 \; l/\text{sec}$$

2. (b) (ii) Solution:

$$Q = 40\ l/s = 0.040\ \text{m}^3/\text{sec}$$



Dia of pipe at a distance x,

$$D(x) = 0.6 - \frac{0.6-0.3}{3}x = 0.6 - 0.1\,x$$

Connective acceleration :

$$a_{con} = v\frac{dv}{dx} = \frac{Q}{A}\frac{d(Q/A)}{dx}$$

$$\Rightarrow \quad a_{con} = \frac{0.040}{\frac{\pi}{4}\times(0.6-0.1x)^2}\times 0.04\times\frac{d}{dx}\left[\frac{1}{\frac{\pi}{4}\times(0.6-0.1x)^2}\right]$$

$$\Rightarrow \quad a_{con} = \frac{0.040}{\frac{\pi}{4}\times(0.6-0.1x)^2}\times 0.04\times\frac{1}{\left(\frac{\pi}{4}\right)}\times\frac{(-2)\times(-0.1)}{(0.6-0.1x)^3}$$

at x = 1.5 m,

$$a_{con} = \frac{0.040^2}{\frac{\pi}{4}\times(0.6-0.1\times1.5)^2}\times\frac{0.2}{\frac{\pi}{4}\times(0.6-0.1\times1.5)^3}$$

$$(a_c)_{x\,=\,1.5\text{ m}} = 0.02811\ \text{m/sec}^2$$

Total acceleration:

at t = 20 sec,

$$Q = \frac{40+80}{2} = 60\ l/\text{sec}$$

Total accleration = (convective acceleration + local acceleration) at Q = 60 l/sec

$$\text{Total accleration} = 0.02811\times\left(\frac{60}{40}\right)^2 + \frac{dv}{dt}$$

$$\because \quad \frac{dv}{dt} = \frac{dQ}{Adt} = \frac{1}{\frac{\pi}{4}\times(0.6-0.1\times1.5)^2}\times\frac{0.080-0.040}{40}$$

$$= 0.0062876 \text{ m/sec}^2$$

$$\therefore \quad \text{Total acceleration} = 0.02811 \times \left(\frac{60}{40}\right)^2 + 0.0062876$$

$$= 0.06953 \text{ m/sec}^2$$

2. (c) Solution:

Given data

Head of reservoir A, $H_A = 50$ m

Head of reservoir B, $H_B = 48$ m

Length of pipe AD, $L_1 = 1200$ m

Diameter of pipe AD, $d_1 = 0.3$ m

Length of pipe DB, $L_2 = 600$ m

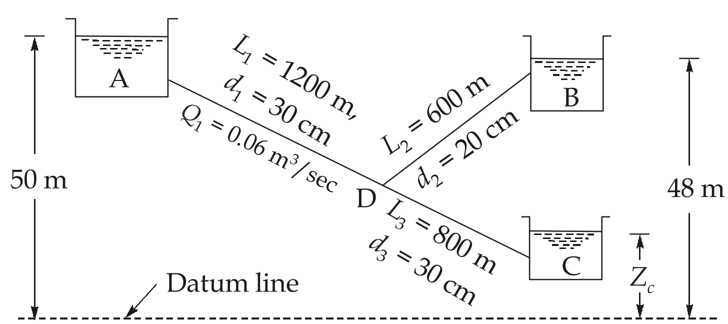
Diameter of pipe DB, $d_2 = 0.2$ m

Length of pipe DC, $L_3 = 800$ m

Diameter of pipe DC, $d_3 = 0.3$ m

Discharge from reservoir A, $Q_1 = 60\ L/s = 0.06\ \text{m}^3/\text{s}$

Friction factor, $f = 0.024$



The head loss in pipe AD is calculated using Darcy. Weisbach formula:

$$h_{f1} = \frac{fL_1Q_1^2}{12.1d_1^5}$$

$$\Rightarrow \quad h_{f1} = \frac{0.024 \times 1200 \times (0.06)^2}{12.1 \times (0.3)^5}$$

$$\Rightarrow \quad h_{f1} = \frac{0.10368}{0.029403} = 3.526 \text{ m}$$

Therefore, head at junction D is

$$H_D = H_A - h_{f1} = 50 - 3.526 = 46.474 \text{ m}$$

Since $H_B > H_D$, water flows from reservoir B to junction D.

The head loss in pipe DB is

$$h_{f2} = H_B - H_D = 48 - 46.474 = 1.526 \text{ m}$$

Using Darcy Weisbach for pipe DB:

$$h_{f1} = \frac{fL_2Q_2^2}{12.1d_2^5}$$

$$\Rightarrow \quad 1.526 = \frac{0.024 \times 600 \times Q_2^2}{12.1 \times (0.2)^5} = \frac{14.4Q_2^2}{0.003872}$$

$$\Rightarrow \quad Q_2^2 = \frac{1.526 \times 0.003872}{14.4} = 0.000410324$$

$$\Rightarrow \quad Q_2 = 0.020256 \text{ m}^3/\text{s} = 20.256 \text{ L/s}$$

At junction D, continuity gives

$$Q_1 + Q_2 = Q_3$$

$$0.06 + 0.020256 = Q_3$$

$$Q_3 = 0.080256 \text{ m}^3/\text{s} = 80.256 \text{ L/s}$$

The head loss in pipe DC is

$$h_{f3} = \frac{fL_3Q_3^2}{12.1d_3^5} = \frac{0.024 \times 800 \times (0.080256)^2}{12.1 \times (0.3)^5}$$

$$\Rightarrow \quad h_{f3} = \frac{19.2 \times 0.006441025}{0.029403}$$

$$\Rightarrow \quad h_{f3} = \frac{0.12366768}{0.029403} = 4.206 \text{ m}$$

Since flow is from D to C,

$$H_C = H_D - h_{f3}$$

$$H_C = 46.474 - 4.206 = 42.268 \text{ m}$$

Hence,

$$H_D = 46.474 \text{ m}, \; Q_2 = 20.256 \; L/s, \; Q_3 = 80.256 \; L/s, \; H_C = 42.268 \text{ m}$$

3. (a) (i) Solution:

Given data

$$\text{Geometric scale ratio } L_r = 1/40$$

$$\text{Width of model flume } B_m = 50 \text{ cm} = 0.5 \text{ m}$$

$$\text{Height of prototype } H_p = 20 \text{ m}$$

$$\text{Maximum head on prototype } h_p = 2 \text{ m}$$

$$\text{Discharge over model } Q_m = 10 \text{ litres/s} = 0.01 \text{ m}^3\text{/s}$$

$$\text{Negative pressure in model } P_m = 150 \text{ mm}$$

For geometrical similarity, all linear dimensions of the model and prototype are in the ratio $L_r = 1/40$.

The height of the model is

$$H_m = H_p \times L_r = 20 \times \frac{1}{40} = 0.5 \text{ m}$$

The head on the model is

$$h_m = h_p \times L_r = 2 \times \frac{1}{40} = 0.05 \text{ m}$$

Hence, the model height is 0.5 m and the head on the model is 0.05 m.

For flow over spillways, Froude similarity is applicable. Therefore, the discharge scale ratio is

$$Q_r = L_r^{2.5}$$

The prototype discharge corresponding to the given model discharge is

$$Q_p = \frac{Q_m}{L_r^{2.5}} = \frac{0.01}{(1/40)^{2.5}} = 0.01 \times 40^{2.5}$$

$$Q_p = 101.19 \text{ m}^3\text{/s}$$

The width of the prototype spillway is

$$B_p = \frac{B_m}{L_r} = \frac{0.5}{1/40} = 20 \text{ m}$$

The discharge per metre length of the prototype is

$$q_p = \frac{Q_p}{B_p} = \frac{101.19}{20} = 5.06 \text{ m}^3\text{/s per m}$$

Pressure head scales linearly with the length scale ratio. Therefore, the negative pressure in the prototype is

$$P_p = \frac{P_m}{L_r} = \frac{150}{1/40} = 6000 \text{ mm} = 6 \text{ m of water}$$

Since, cavitation generally becomes critical when negative pressure approaches the vapour pressure head of water (about 10.3 m of water), the negative pressure of 6 min the prototype is within safe limits.

Thus, the negative pressure condition in the prototype is practicable.

3. (a) (ii) Solution:

Given data

Diameter of inlet pipe $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

Area of inlet pipe $A_1 = \frac{\pi}{4}(0.3)^2 = 0.070686 \text{ m}^2$

Throat ratio $d_2/D_1 = 1/3$

Diameter of throat $d_2 = 100 \text{ mm} = 0.1 \text{ m}$

Area of throat $A_2 = \frac{\pi}{4}(0.1)^2 = 0.007854 \text{ m}^2$

Pressure at inlet $P_1 = 13.783 \text{ N/cm}^2 = 13.783 \times 10^4 \text{ N/m}^2$

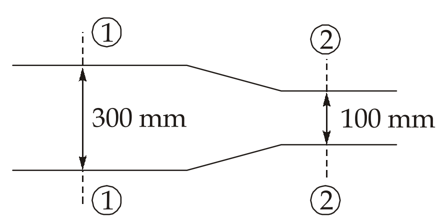
Vacuum at throat $h_{vac} = 37.5 \text{ cm of } Hg = 0.375 \text{ m of } Hg$

Specific gravity of mercury $= 13.6$

Density of water $\rho = 1000 \text{ kg/m}^3$

Acceleration due to gravity $g = 9.81 \text{ m/s}^2$

Coefficient of discharge $C_d = 1.0$ $(\because \text{ loss} = 0\%)$



The pressure head at the inlet is

$$h_1 = \frac{P_1}{\rho g} = \frac{13.783 \times 10^4}{1000 \times 9.81} = 14.05 \text{ m of water}$$

The pressure head at the throat corresponding to vacuum is

$$h_2 = -(13.6 \times 0.375) = -5.10 \text{ m of water}$$

The differential head causing flow is

$$h = h_1 - h_2 = 14.05 - (-5.10) = 19.15 \text{ m}$$

The discharge through a venturimeter is given by

$$Q = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

Substituting the values,

$$Q = \frac{1.0 \times 0.070686 \times 0.007854 \times \sqrt{2 \times 9.81 \times 19.15}}{\sqrt{(0.070686)^2 - (0.007854)^2}}$$

$$\Rightarrow \quad Q = \frac{0.00055517 \times 19.3836}{\sqrt{0.0049348}}$$

$$\Rightarrow \quad Q = 0.153 \text{ m}^3/\text{s}$$

3. (b) Solution:

Given data

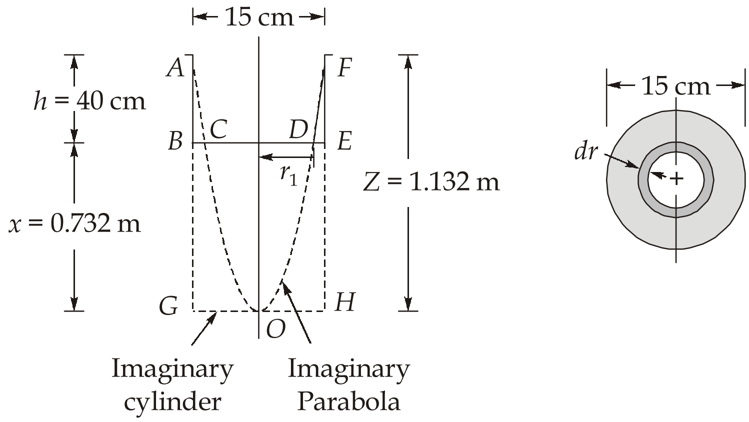
Diameter of vessel	$D = 15 \text{ cm} = 0.15 \text{ m}$
Radius of vessel	$R = 7.5 \text{ cm} = 0.075 \text{ m}$
Depth of vessel	$H = 40 \text{ cm} = 0.4 \text{ m}$
Speed of rotation	$N = 600 \text{ r.p.m.}$
Density of water	$\rho = 1000 \text{ kg/m}^3$
Acceleration due to gravity	$g = 9.81 \text{ m/s}^2$

The angular velocity of rotation is given by $\omega = \frac{2\pi N}{60}$. Substituting the values,

$$\omega = \frac{2\pi \times 600}{60} = 62.832 \text{ rad/s.}$$

The free surface of water takes the shape of a paraboloid of revolution. The rise of the free surface at the outer edge relative to the vertex is given by

$$z = \frac{62.832^2 \times 0.075^2}{2 \times 9.81} = 1.132 \text{ m}$$



Since the maximum height of parabola is greater than the depth of the vessel H = 0.4 m, water spills

out and the vertex of the paraboloid lies below the bottom of the vessel. The distance of the vertex below the bottom is

$$x = z - H = 1.132 - 0.4 = 0.732 \text{ m}$$

Using the property of the paraboloid,

$$\frac{z}{R^2} = \frac{x}{r_1^2}$$

$$\Rightarrow \quad r_1^2 = \frac{xR^2}{z} = \frac{0.732 \times 0.075^2}{1.132} = 0.003638$$

$$\Rightarrow \quad r_1 = 0.060 \text{ m}$$

The volume of the original cylindrical vessel is

$$V_{cyl} = \pi R^2 H = \pi \times 0.075^2 \times 0.4 = 0.007069 \text{ m}^3$$

The volume of the paraboloid segment spilled out is

$$V_{cyl} = \frac{\pi}{2}\left(R^2 z - r_1^2 x\right)$$

$$V_{parab} = \frac{\pi}{2}\left(0.075^2 \times 1.132 - 0.060^2 \times 0.732\right)$$

$$V_{parab} = 0.005863 \text{ m}^3$$

Hence, the volume of water left in the vessel is

$$V_{left} = 0.007069 - 0.005863 = 0.001206 \text{ m}^3$$

$$\text{Quantity of water left} = 1.206 \text{ litres}$$

The pressure force on the bottom is obtained by integrating the pressure over the wetted portion of the base from $r = r_1$ to $r = R$. The pressure at radius r is

$$p = \rho g\left(\frac{w^2r^2}{2g} - x\right)$$

The total force on the bottom is

$$F = \int_{r_1}^{R} \rho g\left(\frac{\omega^2 r^2}{2g} - 0.732\right) 2\pi r dr$$

$$\Rightarrow \quad F = 2\pi \times 1000 \times 9.81 \int_{0.060}^{0.075} \left(-0.733\, r + \frac{62.832^2 r^3}{2 \times 9.81}\right) dr$$

$$\Rightarrow \quad F = 2\pi \times 10^3 \times 9.81 \times \left[-\frac{0.732(0.075^2 - 0.06^2)}{2} + \frac{62.832^2}{2 \times 9.81} \times \frac{1}{4}(0.075^4 - 0.06^4)\right]$$

$$\Rightarrow \quad F = 12.24 \text{ N}$$

3. (c) Solution:

Given Data: Net head (H) = 8 m

Power produced (P) = 147.15 kW

Overall efficiency (h_o) = 70% or 0.7

Speed (N) = 200 r.p.m.

Peripheral velocity (u_1) = $0.30 \times \sqrt{2gH}$

Velocity of flow at inlet (V_{f1}) = $0.96 \times \sqrt{2gH}$

Hydraulic losses = 20% of H

Radial discharge at outlet

$$\Rightarrow \quad V_{w2} = 0$$

Acceleration due to gravity (g) = 9.81 m/s^2

Calculation of Velocities:

$$\sqrt{2gH} = \sqrt{2 \times 9.81 \times 8} = \sqrt{156.96} = 12.528 \text{ m/s}$$

Peripheral velocity (u_1):

$$u_1 = 0.30 \times 12.528 = 3.758 \text{ m/s}$$

Velocity of flow at inlet (V_{f1}):

$$(V_{f1}) = 0.96 \times 12.528 = 12.027 \text{ m/s}$$

Calculation of Hydraulic Efficiency (η_h):

Hydraulic losses are 20%, so the hydraulic head available is 80% of H:

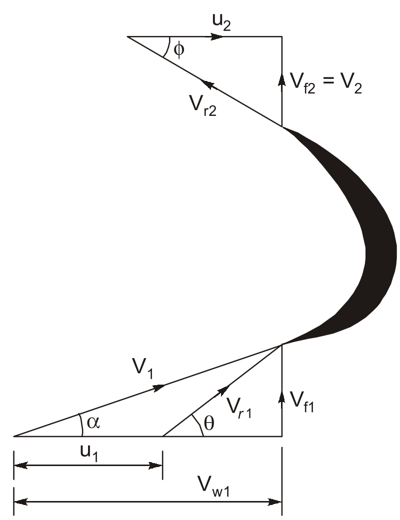
$$\eta_h = 100\% - 20\% = 80\% \text{ or } 0.8$$

For radial discharge:

$$\eta_h = \frac{V_{w1} \times u_1}{gH}$$

$$\Rightarrow \quad 0.8 = \frac{V_{w1} \times 3.758}{9.81 \times 8}$$

$$\Rightarrow \quad V_{w1} = \frac{0.8 \times 9.81 \times 8}{3.758} = 16.707 \text{ m/s}$$



Guide Blade Angle (α):

$$\tan \alpha = \frac{V_{f1}}{V_{w1}}$$

$$\Rightarrow \quad \tan \alpha = \frac{12.027}{16.707} = 0.72$$

$$\Rightarrow \quad \alpha = \tan^{-1}(0.72) = 35.75°$$

Wheel Vane Angle at Inlet (θ):

$$\tan \theta = \frac{V_{f_1}}{V_{w1} - u_1}$$

$$\Rightarrow \quad \tan \theta = \frac{12.027}{16.707 - 3.758} = \frac{12.027}{12.949} = 0.929$$

$$\Rightarrow \quad \theta = \tan^{-1}(0.929) = 42.89°$$

Diameter of the Wheel at Inlet (D_1):

$$\because \qquad u_1 = \frac{\pi \times D_1 \times N}{60}$$

$$\Rightarrow \qquad 3.758 = \frac{\pi \times D_1 \times 200}{60}$$

$$\Rightarrow \qquad D_1 = \frac{3.758 \times 60}{200 \times \pi} = 0.359 \text{ m}$$

Width of Wheel at Inlet (B_1):

First, find the discharge (Q) using overall efficiency:

$$\eta_0 = \frac{P}{\rho g Q H}$$

$$\Rightarrow \qquad 0.7 = \frac{147.15 \times 10^3}{1000 \times 9.81 \times Q \times 8}$$

$$\Rightarrow \qquad Q = \frac{147150}{0.7 \times 9.81 \times 8 \times 1000} = 2.678 \text{ m}^3/\text{s}$$

Using the discharge equation:

$$Q = \pi \times D_1 \times B_1 \times V_{f1}$$

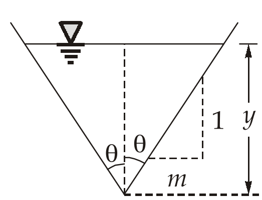
$$\Rightarrow \qquad 2.678 = \pi \times 0.359 \times B_1 \times 12.027$$

$$\Rightarrow \qquad B_1 = \frac{2.677}{\pi \times 0.359 \times 12.027} = 0.197 \text{ m}$$

4. (a) (i) Solution:

For a triangular channel section, if θ is the angle of inclination of each of the sloping sides with the vertical and y is the depth of flow, then the wetted area 'A' and wetted perimeter 'P' can be respectively given by

$$A = my^2 \qquad [\because \quad m = \tan\theta]$$



$$\Rightarrow \qquad A = y^2 \tan\theta$$

$$\Rightarrow \qquad y = \sqrt{A / \tan\theta} \qquad \text{...(i)}$$

$$\text{and} \qquad P = 2y \sec\theta \qquad \text{...(ii)}$$

Substituting value of y from (i) in (ii), we get

$$P = 2\sec\theta\sqrt{\frac{A}{\tan\theta}} \quad \text{...(iii)}$$

Assuming wetted area, A to be constant, equation (iii) can be differentiated with respect to θ and equated to zero for obtaining the condition for minimum wetted perimeter.

$$\therefore \quad \frac{dP}{d\theta} = 2\sqrt{A}\left[\frac{\sec\theta\tan\theta}{\sqrt{\tan\theta}} - \frac{\sec^3\theta}{2(\tan\theta)^{3/2}}\right] = 0$$

$$\Rightarrow \quad \sec\theta\,(2\tan^2\theta - \sec^2\theta) = 0 \quad [\because \sec\theta \neq 0]$$

$$\therefore \quad 2\tan^2\theta - \sec^2\theta = 0$$

$$\Rightarrow \quad \sqrt{2}\tan\theta = \sec\theta$$

$$\Rightarrow \quad \theta = 45°$$

$$\therefore \quad m = \tan\theta = 1$$

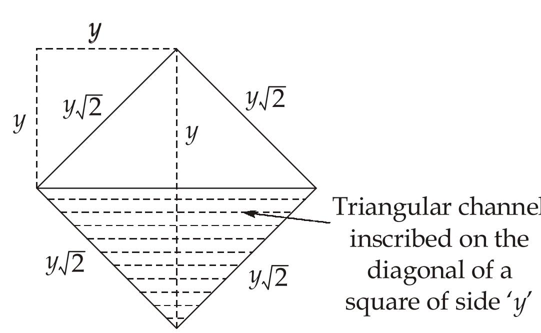
Hence a triangular channel will be most efficient when each of its sloping sides makes an angle of 45° with the vertical.

Hydraulic radius, $$R = \frac{A}{P} = \frac{y^2\tan\theta}{2y\sec\theta} \quad (\theta = 45°)$$

$$\Rightarrow \quad R = \frac{y^2\tan 45°}{2y\sec\theta}$$

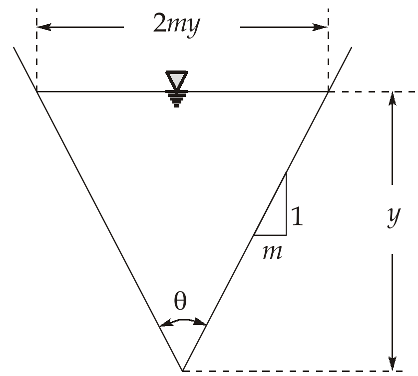
$$\Rightarrow \quad R = \frac{y}{2\sqrt{2}}$$

Thus it can be seen that the most efficient triangular channel section will be half the square inscribed on a diagonal and having equal sloping sides.



4. (a) (ii) Solution:

Triangular channel section is shown in figure below:



Area of cross-section, $A = \frac{1}{2} \times 2my \times y = my^2$

Top width of the section, $T = 2my$

Given y_1 and y_2 are the alternate depths in the channel and let be the discharge through the channel be Q.

∴ Specific energy, $E_1 = y_1 + \frac{V_1^2}{2g}$

$$E_2 = y_2 + \frac{V_2^2}{2g}$$

∴ $E_1 = y_1 + \frac{Q^2}{2gA_1^2}$ and $E_2 = y_2 + \frac{Q^2}{2gA_2^2}$

⇒ $E_1 = y_1 + \frac{Q^2}{2g(m^2 y_1^4)}$ and $E_2 = y_2 + \frac{Q^2}{2g(m^2 y_2^4)}$

We know that, $E_1 = E_2$

$$\Rightarrow \quad y_1 + \frac{Q^2}{2g(m^2 y_1^4)} = y_2 + \frac{Q^2}{2g(m^2 y_2^4)} \quad \text{...(1)}$$

Dividing above equation with y_1,

$$1 + \frac{Q^2}{2g(m^2 y_1^5)} = \frac{y_2}{y_1} + \frac{Q^2}{2g(m^2 y_2^4) y_1}$$

$$\Rightarrow \quad 1+\frac{Q^2}{2g\left(m^2y_1^5\right)} = \frac{y_2}{y_1}+\frac{Q^2}{2g\left(m^2y_2^5\right)}\left(\frac{y_2}{y_1}\right)$$

$$\Rightarrow \quad 1+\frac{Q^2}{2g\left(m^2y_1^5\right)} = \frac{y_2}{y_1}\left[1+\frac{Q^2}{2g\left(m^2y_2^5\right)}\right] \qquad ...(2)$$

Now

Froude number is given by $F = \frac{V}{\sqrt{gD}}$

where V = Velocity of flow

g = Acceleration due to gravity

D = Hydraulic depth

$$\therefore \quad F_1{}^2 = \frac{V_1^2}{gD_1} = \frac{V_1^2}{g\,A_1/T_1}$$

$$\Rightarrow \quad F_1{}^2 = \frac{\left(\frac{Q^2}{m^2y_1^4}\right)}{g\times\frac{my_1^2}{2my_1}} = \frac{2Q^2}{gm^2y_1^5} \qquad ...(3)$$

Similarly
$$F_2{}^2 = \frac{\left(\frac{Q^2}{m^2y_2^4}\right)}{g\times\frac{my_2^2}{2my_2}} = \frac{2Q^2}{gm^2y_2^5} \qquad ...(4)$$

From (3) and (4)

$$\frac{F_1^2}{F_2^2} = \frac{y_2^5}{y_1^5}$$

$$\Rightarrow \quad \frac{y_2}{y_1} = \left(\frac{F_1}{F_2}\right)^{2/5} \qquad ...(5)$$

Substituting (3) and (4) in (2)

$$1+\frac{F_1^2}{4} = \frac{y_2}{y_1}\left[1+\frac{F_2^2}{4}\right]$$

$$\Rightarrow \qquad \frac{y_2}{y_1} = \frac{1+\frac{F_1^2}{4}}{1+\frac{F_2^2}{4}} \qquad \text{...(6)}$$

From eq. (5) and (6)

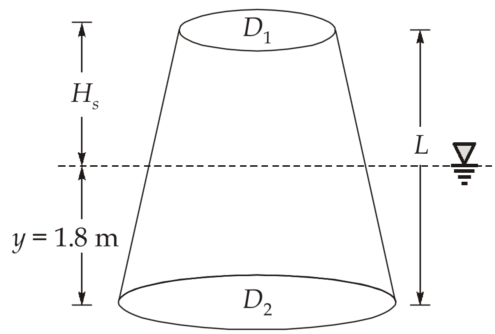
$$\left(\frac{F_1}{F_2}\right)^{2/5} = \frac{1+\frac{F_1^2}{4}}{1+\frac{F_2^2}{4}}$$

$$\Rightarrow \qquad \left(\frac{F_1}{F_2}\right)^{2/5} = \frac{4+F_1^2}{4+F_2^2}$$

$$\Rightarrow \qquad \left(\frac{F_1}{F_2}\right)^{2} = \left(\frac{4+F_1^2}{4+F_2^2}\right)^5$$

(Hence proved)

4. (b) (i) Solution:



Given Data: Inlet diameter (D_1) = 1.1 m

Outlet diameter (D_2) = 1.6 m

Velocity at outlet (V_2) = 2.2 m/s

Total length of draft-tube (L) = 7.5 m

Length immersed in water (y) = 1.8 m

Atmospheric pressure head (H_a) = 10.3 m

Friction loss (h_f) = $0.25 \times \frac{V_2^2}{2g}$

Acceleration due to gravity (g) = 9.81 m/s^2

1. Pressure Head at Inlet

First, we find the height of the inlet above the tailrace surface (H_s):

$$H_s = L - y = 7.5 - 1.8 = 5.7 \text{ m}$$

Calculate the velocity at the inlet (V_1) using the continuity equation ($A_1V_1 = A_2V_2$):

$$V_1 = V_2 \times \left(\frac{D_2}{D_1}\right)^2 = 2.2\times\left(\frac{1.6}{1.1}\right)^2 = 2.2 \times 2.116 = 4.655 \text{ m/s}$$

Now, calculate the relevant velocity heads and friction loss:

Inlet velocity head:

$$\frac{V_2^2}{2g} = \frac{4.655^2}{2\times 9.81} = \frac{21.669}{19.62} = 1.104 \text{ m}$$

Outlet velocity head:

$$\frac{V_2^2}{2g} = \frac{2.2^2}{2\times 9.81} = \frac{4.84}{19.62} = 0.247 \text{ m}$$

Friction loss (h_f):

$$h_f = 0.25 \times 0.247 = 0.062 \text{ m}$$

Applying Bernoulli's equation to find the absolute pressure head at the inlet (H_1):

$$\frac{P_1}{\rho g} = H_a - H_S - \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f\right]$$

$$\Rightarrow \quad \frac{P_1}{\rho g} = 10.3 - 5.7 - [1.104 - 0.247 - 0.062]$$

$$\Rightarrow \quad \frac{P_1}{\rho g} = 4.6 - [0.795]$$

$$\Rightarrow \quad \frac{P_1}{\rho g} = 3.805 \text{ m (absolute)}$$

To express this as a vacuum head:

$$\text{Vacuum head} = H_a - \text{Absolute head} = 10.3 - 3.805 = 6.495 \text{ m}$$

2. Efficiency of the Draft-Tube (η_d)

The efficiency is the ratio of actual kinetic energy head converted into pressure energy head to the total kinetic energy head available at the inlet:

$$\eta_d = \frac{\left[\dfrac{V_1^2 - V_2^2}{2g}\right] - h_f}{\dfrac{V_1^2}{2g}} = \frac{[1.104 - 0.247] - 0.062}{1.104}$$

$$\Rightarrow \qquad \eta_d = \frac{0.795}{1.104} = 0.72$$

The efficiency of the draft-tube is 72%.

4. (b) (ii) Solution:

Given data: Width of river, $W = 45$ m

Initial depth of water, $d_1 = 3.5$ m

Mean velocity of flow, $v = 1.5$ m/s

Afflux = 1.2 m

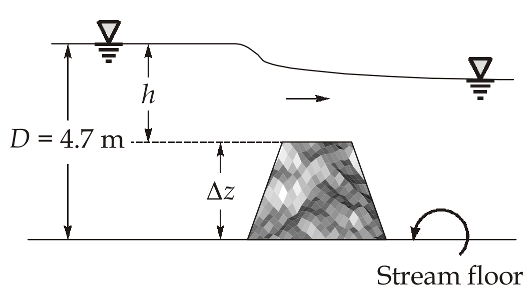
Coefficient of discharge, $C_d = 0.92$

Acceleration due to gravity, $g = 9.81$ m/s^2

The discharge flowing in the river is obtained from the product of the cross-sectional area and the mean velocity. The area of flow is 45 × 3.5 = 157.5 m^2. Therefore,

$$Q = Av = 157.5 \times 1.5 = 236.25 \text{ m}^3/\text{s}$$

The coefficient of discharge given is quite high compared to that of a sharp-crested weir, which usually lies between 0.62 and 0.65. Hence, the weir is considered to be a broadcrested weir and the corresponding discharge equation is used.



The afflux represents the rise in upstream water level due to the construction of the weir. Thus, the new upstream depth is

$$D = d_1 + \text{afflux} = 3.5 + 1.2 = 4.7 \text{ m}$$

The velocity of approach at this depth is

$$v_a = \frac{Q}{wD} = \frac{236.25}{45 \times 4.7} = 1.117 \text{ m/s}$$

The velocity head of approach is,

$$h_a = \frac{v_a^2}{2g} = \frac{(1.117)^2}{2 \times 9.81} = 0.064 \text{ m}$$

For a broad-crested weir including velocity of approach, the discharge equation is

$$Q = 1.7C_d L[(h + h_a)^{3/2} - h_a^{3/2}]$$

Substituting the known values,

$$236.25 = 1.7 \times 0.92 \times 45[(h + 0.064)^{3/2} - (0.064)^{3/2}]$$

$$\Rightarrow \quad (h + 0.064)^{3/2} = 3.373$$

$$\Rightarrow \quad h + 0.064 = (3.373)^{2/3} = 2.249 \text{ m}$$

$$\Rightarrow \quad h = 2.249 - 0.064 = 2.185 \text{ m}$$

The total upstream depth is equal to the sum of the height of the weir above the river bed and the head of water over the crest. Hence,

$$D = Z + h$$

$$\Rightarrow \quad 4.7 = Z + 2.185$$

$$\Rightarrow \quad Z = 2.515 \text{ m}$$

Therefore, the required height of the weir is approximately 2.515 m.

4. (c) Solution:

Given data

Outer diameter of impeller, $D_2 = 0.5$ m

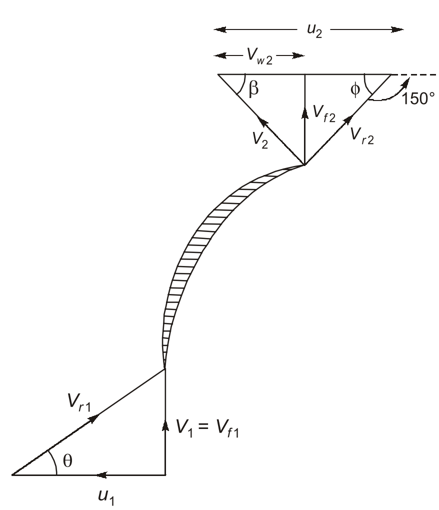
Outlet width, $B_2 = 0.04$ m

Speed of impeller, $N = 900$ r.p.m.

Manometric head, $H_m = 20$ m

Vane angle at outlet, $\phi = 30°$

Manometric efficiency, $\eta_{man} = 0.8$



The tangential velocity of the impeller at outlet is given by

$$u_2 = \frac{\pi D_2 N}{60}$$

$$\Rightarrow \quad u_2 = \frac{\pi \times 0.5 \times 900}{60} = 23.56 \text{ m/s}$$

From the definition of manometric efficiency,

$$\eta_{man} = \frac{gH_m}{V_{w2}u_2}$$

$$\Rightarrow \quad 0.8 = \frac{9.81 \times 20}{V_{w2} \times 23.56}$$

$$\Rightarrow \quad V_{w2} = \frac{196.2}{0.8 \times 23.56} = 10.41 \text{ m/s}$$

From the outlet velocity triangle,

$$\tan \phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$\Rightarrow \quad \tan 30° = \frac{V_{f2}}{23.56 - 10.41}$$

$$\Rightarrow \quad V_{f2} = 13.15 \times \tan 30° = 7.59 \text{ m/s}$$

The absolute velocity of water leaving the vane is

$$V_2 = \sqrt{V_{f2}^2 + V_{w2}^2}$$

$$\Rightarrow \quad V_2 = \sqrt{(7.59)^2 + (10.41)^2}$$

$$\Rightarrow \quad V_2 = 12.89 \text{ m/s}$$

The angle made by the absolute velocity at outlet with the direction of motion is

$$\tan \beta = \frac{V_{f2}}{V_{w2}}$$

$$\Rightarrow \quad \tan \beta = \frac{7.59}{10.41}$$

$$\Rightarrow \quad \beta = 36.1°$$

The discharge through the pump is,

$$Q = \pi D_2 B_2 V_{f2}$$

$$\Rightarrow \quad Q = \pi \times 0.5 \times 0.04 \times 7.59$$

$$\Rightarrow \quad Q = 0.477 \text{ m}^3\text{/s}$$

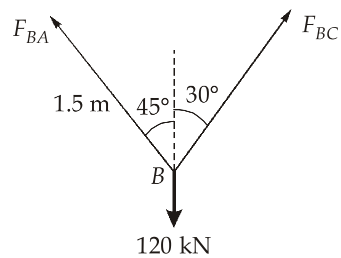
Section B : Design of Concrete and Masonry Structures-1 + Strength of Materials-2

5. (a) Solution:

Given: $L_{AB} = 1.5\text{ m}, L_{BC} = 2\text{ m}$

$d_{AB} = 20\text{ mm}, d_{BC} = 35\text{ mm}$

FBD at joint *B*,



From sine rule

$$\frac{F_{BA}}{\sin(180° - 30°)} = \frac{F_{BC}}{\sin(180° - 45°)} = \frac{120}{\sin(75°)}$$

$$\therefore \quad F_{BA} = \frac{120\sin(150°)}{\sin(75°)} = 62.12\text{ kN}$$

$$F_{BC} = \frac{120\sin(135°)}{\sin 75°} = 87.85\text{ kN}$$

Stress in wire *AB* :

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{62.12 \times 10^3}{\frac{\pi}{4}(20)^2}$$

$$\Rightarrow \quad \sigma_{AB} = 197.734\text{ MPa}$$

From graph, strain in wire *AB* is,

$$\in_{AB} = 0.01 + \frac{(0.03 - 0.01)}{(220 - 110)}(197.734 - 110)$$

$$\Rightarrow \quad \frac{\Delta_{AB}}{L_{AB}} = 2.595 \times 10^{-2}$$

$$\Rightarrow \quad \Delta_{AB} = 1.5 \times 2.595 \times 10^{-2}\text{ m} = 38.925\text{ mm}$$ **Ans.**

Stress in wire *BC* :

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{87.85 \times 10^3}{\frac{\pi}{4}(35)^2}$$

$\Rightarrow \qquad \sigma_{BC} = 91.309$ MPa

From graph, strain in wire *BC* is,

$$\epsilon_{BC} = \frac{0.01}{110} \times 91.309$$

$$\Rightarrow \qquad \frac{\Delta_{BC}}{L_{BC}} = 8.3 \times 10^{-3}$$

$$\Rightarrow \qquad \Delta_{BC} = 8.3 \times 10^{-3} \times 2000 = 16.60 \text{ mm}$$ **Ans.**

5. (b) Solution:

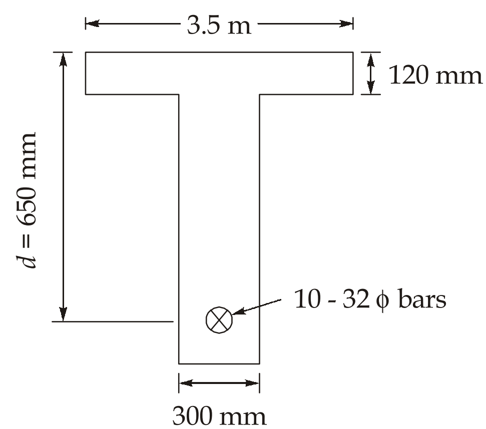
Given data

$$f_{ck} = 25 \text{ N/mm}^2, f_y = 500 \text{ N/mm}^2,$$

$$A_{st} = 10 \times \frac{\pi}{4} \times 32^2 = 8042.48 \text{ mm}^2,$$

$$l_0 = 8500 \text{ mm}$$

Width of flange, $b_f = 3.5$ m



Effective flange width:

$$b_f = \min\left(\frac{l_0}{6} + b_w + 6D_f, \text{actual width}\right)$$

$$\Rightarrow \qquad b_f = \min\begin{cases} \frac{8500}{6} + 300 + 6 \times 120 = 2436.67 \text{mm} \\ \text{actual width} = 3500 \text{ mm} \end{cases}$$

So, $b_f = 2436.67$ mm

Step 1: Assuming NA is in flange portion ($x_u \le D_f$)

$$0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$$

$$\Rightarrow \quad 0.36 \times 25 \times 2436.67 \times x_u = 0.87 \times 500 \times 8042.48$$

$$\Rightarrow \quad x_u = 159.53 \text{ mm}$$

Since $x_u > D_f$, neutral axis lies in the web. (So, assumption is wrong)

Step 2: Assuming $x_u > D_f$ and flange is uniformly stress $\left(D_f \le \frac{3}{7}x_u\right)$

Now, $$C_1 + C_2 = T$$

$$\Rightarrow \quad 0.36 f_{ck}\, b_w\, x_u + 0.45 f_{ck}\,(b_f - b_w)\, D_f = 0.87\, f_y A_{st}$$

$$\Rightarrow \quad 0.36 \times 25 \times 300\, x_u + 0.45 \times 25\,(2436.67 - 300) \times 120 = 0.87 \times 500 \times 8042.48$$

$$\Rightarrow \quad x_u = 227.4 \text{ mm}$$

$$\because \quad \frac{3}{7}x_u = 97.45 \text{ mm } (< D_f = 120 \text{ mm})$$

So, Assumption is wrong.

Step 3: Assuming NA is in web ($x_u > D_f$) and flange is not uniformly stressed $\left(D_f \ge \frac{3}{7}x_u\right)$

Now, $$C_1 + C_2 = T$$

$$y_f = 0.15x_u + 0.65D_f$$

$$\Rightarrow \quad 0.36\, f_{ck} b_w x_u + 0.45 f_{ck}(b_f - b_w) y_f = 0.87 f_y A_{st}$$

$$\Rightarrow \quad 0.36 \times 25 \times 300\, x_u + 0.45 \times 25\,(2436.67 - 300)\,(0.15x_u + 0.65 \times 120) = 0.87 \times 500 \times 8042.48$$

$$2700x_u + 24037.5(0.15\, x_u + 78) = 3498478.8$$

$$x_u = 257.48 \text{ mm} > (D_f = 120 \text{ mm})$$

$$\frac{3}{7}x_u = 110.35 \text{ mm}, (D_f = 120 \text{ mm})$$

So, $$\frac{3}{7}x_u < D_f$$

Flange not uniformly stressed, assumption is correct.

Check for limiting neutral axis:

$$x_{u,\lim} = 0.46d = 0.46 \times 650 = 299 \text{ mm}$$

$$x_u = 257.48 \text{ mm} < x_{u,\lim} \text{ (under-reinforced)}$$

Ultimate moment of resistance:

$$y_f = 0.15x_u + 0.65D_f = 116.62 \text{ mm}$$

Web contribution:

$$M_{u,\text{web}} = 0.36 f_{ck} b_w x_u (d - 0.42x_u)$$

$$\Rightarrow \quad M_{u,\text{web}} = 0.36 \times 25 \times 300 \times 257.48 \times (650 - 0.42 \times 257.48) \times 10^{-6}$$

$$\Rightarrow \quad M_{u,\text{web}} = 376.69 \text{ kNm}$$

Flange contribution:

$$M_{u,\text{flange}} = 0.45 f_{ck}(b_f - b_w) y_f (d - y_f/2)$$

$$\Rightarrow \quad M_{u,\text{flange}} = 0.45 \times 25 \times (2436.67 - 300) \times 116.62 \times (650 - 116.62/2) \times 10^{-6}$$

$$\Rightarrow \quad M_{u,\text{flange}} = 1658.66 \text{ kNm}$$

Total ultimate moment of resistance:

$$M_u = 376.69 + 1658.66 = 2035.35 \text{ kNm}$$

5. (c) Solution:

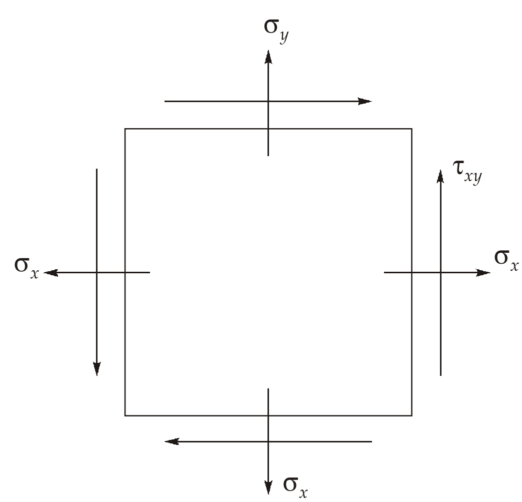
Given Stresses

Vertical tensile stress: $\sigma_y = 80 \text{ N/mm}^2$

Inclined tensile stress: $= 120 \text{ N/mm}^2$

Acting at 60° with the vertical face.

Normal stress in x-direction:



$$\sigma_x = 120 \sin 60°$$

$$\sigma_x = 120 \times \frac{\sqrt{3}}{2}$$

$$\sigma_x = 103.92 \text{ N/mm}^2 \text{ (tensile)}$$

Shear stress: $\tau_{xy} = 120 \sin 60°$

$$\tau_{xy} = 120 \times \frac{1}{2}$$

$$\tau_{xy} = 60 \text{ N/mm}^2$$

Vertical stress: $\sigma_y = 80 \text{ N/mm}^2$ (tensile)

Principal Stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \quad \sigma_{1,2} = \frac{103.92+80}{2} \pm \sqrt{\left(\frac{103.92-80}{2}\right)^2 + 60^2}$$

$$\Rightarrow \quad \sigma_{1,2} = 91.96 \pm \sqrt{(11.96)^2 + 3600}$$

$$\Rightarrow \quad \sigma_{1,2} = 91.96 \pm 61.18$$

Major Principal Stress

$$\sigma_1 = 153.14 \text{ N/mm}^2 \text{ (Tensile)}$$

Minor Principal Stress

$$\sigma_2 = 30.78 \text{ N/mm}^2 \text{ (Tensile)}$$

Location of Principal Planes

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\Rightarrow \quad \tan(2\theta_p) = \frac{2 \times 60}{103.92 - 80}$$

$$\Rightarrow \quad \tan(2\theta_p) = \frac{120}{23.92}$$

$$\Rightarrow \quad 2\theta_p = 78.72°$$

First principal plane:

$$\theta_{p1} = 39.36° \,(\circlearrowleft)$$

Second principal plane:

$$\theta_{p2} = 129.36° \,(\circlearrowleft)$$

Stress on principal plane

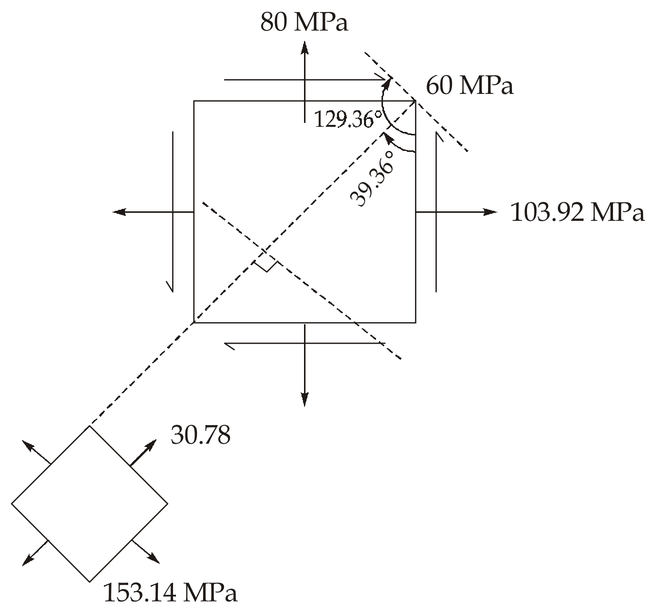
$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta_{p1} + \tau_{xy} \sin 2\theta_{p1}$$

$$\Rightarrow \quad \sigma_{\theta_1} = \frac{103.92+80}{2} + \frac{103.92-80}{2}\cos(2 \times 39.36) + 60 \times \sin(2 \times 39.36)$$

$$\Rightarrow \quad \sigma_{\theta_1} = 153.14 \text{ N/mm}^2 \text{ (tensile)}$$

Now, $$\sigma_{\theta_2} = \left(\frac{103.92+80}{2}\right) + \left(\frac{103.92-80}{2}\right)\cos(2 \times 129.36°) + 60\sin(2 \times 129.36°)$$

$$\Rightarrow \qquad \sigma_{\theta_2} = 30.78 \text{ N/mm}^2 \text{ (tensile)}$$



Checking, $\sigma_x + \sigma_y = \sigma_1 + \sigma_2 = \sigma_{\theta_1} + \sigma_{\theta_2}$

$$\Rightarrow \qquad 103.92 + 80 = 153.14 + 30.78$$

$$\Rightarrow \qquad 183.92 = 183.92 \qquad \text{(OK)}$$

Q.5 (d) Solution:

Given: Maximum allowable shear stress,

$$\tau_{max} = 80 \text{ N/mm}^2$$

Diameter ratio, $k = \dfrac{d}{D} = 0.5$ (since $D = 2d$)

Torque, $T = 13.5 \times 10^6$ Nmm

Bending Moment, $M = 10.125 \times 10^6$ Nmm

Equivalent Twisting Moment (T_e)

$$T_e = \sqrt{(M)^2 + (T)^2} = \sqrt{\left(10.125 \times 10^6\right)^2 + \left(13.5 \times 10^6\right)^2}$$

$$\Rightarrow \qquad T_e = \sqrt{102.515 \times 10^{12} + 182.25 \times 10^{12}}$$

$$\Rightarrow \qquad T_e = 16.875 \times 10^6 \text{ Nmm}$$

Determination of Diameters

Governing equation for a hollow shaft:

$$\because \qquad \frac{T_e}{J} = \frac{\tau_{max}}{R_{max}}$$

$$\Rightarrow \quad \frac{T_e}{\frac{\pi}{32}(D^4 - d^4)} = \frac{\tau_{max}}{\frac{D}{2}}$$

$$\Rightarrow \quad T_e = \frac{\pi}{16}\tau_{max}D^3(1-k^4)$$

$$\Rightarrow \quad 16.875 \times 10^6 = \frac{\pi}{16} \times 80 \times D^3(1-0.5^4)$$

$$\Rightarrow \quad D^3 = \frac{16.875 \times 10^6}{5\pi \times 0.9375}$$

$$\Rightarrow \quad D = 104.65 \text{ mm}$$

Internal diameter:

$$d = 0.5\,D = 0.5 \times 104.65$$

$$d = 52.32 \text{ mm}$$

5. (e) Solution:

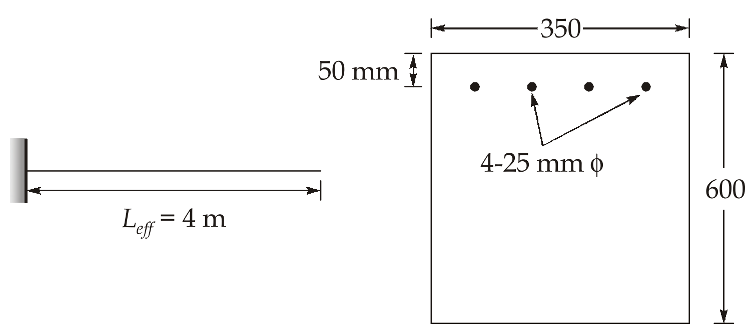
Given data

$$\text{Width } b = 350 \text{ mm}$$

$$\text{Overall depth } D = 600 \text{ mm}$$

$$\text{Effective depth } d = 600 - 50 = 550 \text{ mm}$$

$$\text{Span } L = 4.0 \text{ m} = 4000 \text{ mm}$$



$$\text{Area of steel at top } A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.5 \text{ mm}^2$$

$$\text{Ultimate shrinkage strain } \varepsilon_{cs} = 0.0005$$

Percentage of tension reinforcement:

$$p_t = \frac{100A_{st}}{bd} = \frac{100 \times 1963.5}{350 \times 550} = 1.02\%; \; p_c = 0$$

Shrinkage curvature constant:

$$k = 0.65 \times \frac{p_t - p_c}{\sqrt{p_t}}$$

$$\Rightarrow \quad k = 0.65 \times \frac{1.02}{\sqrt{1.02}} = 0.656 < 1 \quad \text{(OK)}$$

Curvature due to shrinkage:

$$\psi_{cs} = k \times \frac{\varepsilon_{cs}}{D} = 0.656 \times \frac{0.0005}{600} = 5.470 \times 10^{-7}\ \text{mm}^{-1}$$

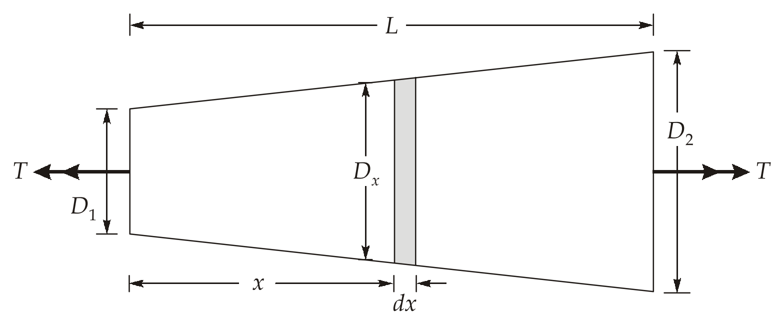
Shrinkage deflection at free end:

$$\delta_{cs} = k_s \times \psi_{cs} \times L^2 \quad \text{(for cantilever } k_s = 0.5)$$

$$\Rightarrow \quad \delta_{cs} = 0.5 \times (5.47 \times 10^{-7}) \times 4000^2$$

$$\Rightarrow \quad \delta_{cs} = 4.376\ \text{mm}$$

6. (a) (i) Solution:



Consider an element of length dx at a distance x from the smaller diameter end. The diameter of the element slice is,

$$D_x = D_1 + \frac{(D_2 - D_1)}{L} x = D_1 + kx$$

where $k = \left(\frac{D_2 - D_1}{L}\right)$

Corresponding polar moment of inertia of the shaft at the section under consideration.

$$J_x = \frac{\pi D_x^4}{32} = \frac{\pi (D_1 + kx)^4}{32}$$

The angle of twist over the length dx can be obtained from the relation,

$$\frac{T}{J_x} = \frac{G d\theta_x}{dx}$$

$$\Rightarrow \quad d\theta_x = \frac{T dx}{G J_x}$$

Therefore, $$d\theta_x = \frac{T}{G}\left[\frac{32}{\pi(D_1+kx)^4}\right]dx$$

Total angle of twist over the entire length is

$$\theta = \int_0^L d\theta_x = \frac{32T}{\pi G}\int_0^L \frac{dx}{(D_1+kx)^4}$$

$$\Rightarrow \quad \theta = \frac{32T}{\pi G}\left[\frac{-(D_1+kx)^{-3}}{3k}\right]_0^L$$

$$\Rightarrow \quad \theta = \frac{32T}{3\pi Gk}\left[-\frac{1}{(D_1+kL)^3}+\frac{1}{D_1^3}\right] = \frac{32T}{3\pi Gk}\left[-\frac{1}{D_2^3}+\frac{1}{D_1^3}\right]$$

$$\Rightarrow \quad \theta = \frac{32TL}{3\pi G(D_2-D_1)}\frac{(D_2^3-D_1^3)}{D_1^3D_2^3}$$

$$\Rightarrow \quad \theta = \frac{32TL}{3\pi G}\left(\frac{D_2^2+D_1^2+D_1D_2}{D_1^3D_2^3}\right)$$

6. (a) (ii) Solution:

Given: Shear force, $F = 20 \text{ kN} = 20 \times 10^3$ N

Since the section is symmetrical about x-x and y-y axes therefore, centre of the section will lie on the geometrical centroid of the section. For the purpose of moment of inertia and shear stress, the two semi-circular grooves may be assumed to be together and considered as one circular hole of 60 mm diameter.

Therefore moment of inertia of section about an axis passing through its centre of gravity and parallel to x-axis,

$$I = \left(\frac{80\times100^3}{12}\right) - \frac{\pi}{64}(60)^4 = 6.03\times10^6\,\text{mm}^4$$

We know that shear stress at the top and bottom edges (A and E) of the section is zero. Now let us find out the shear stress at B by considering the area between A and B.

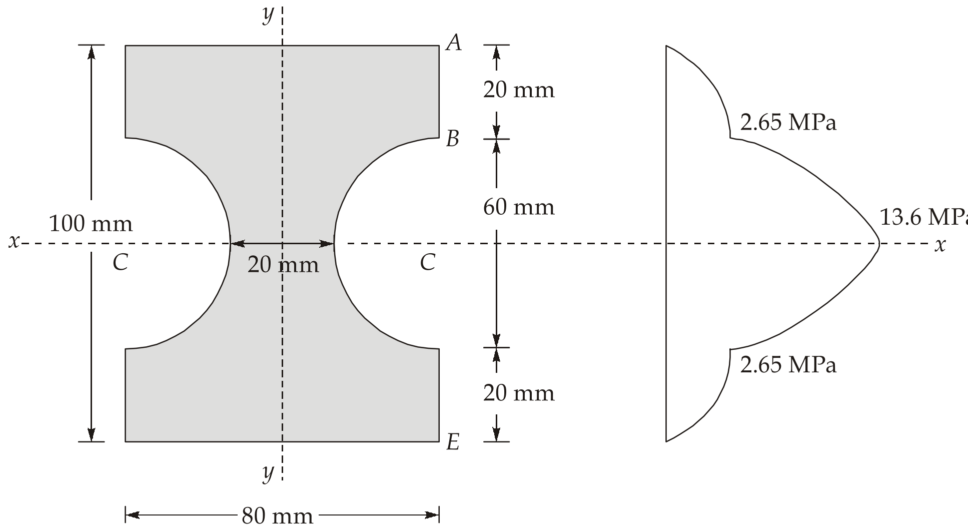
We know that area of the upper portion between A and B is

$$A = 80 \times 20 = 1600 \text{ mm}^2$$

$$\bar{y} = 30 + \frac{20}{2} = 40 \text{ mm}$$

and $B = 80$ mm

$\therefore$ Shear stress at B, $$\tau = \frac{FA\bar{y}}{IB} = \frac{20\times10^3\times1600\times40}{6.03\times10^6\times80} = 2.65 \text{ MPa}$$



Shear stress distribution diagram

Now let us find out the shear stress at neutral axis, where the shear stress is maximum. Consider the area above the neutral axis, we know that

$$A\bar{y} = [80\times50\times25]-\left[\frac{\pi}{2}(30)^2\times\frac{4\times30}{3\pi}\right]$$

$\Rightarrow$ $$A\bar{y} = 100000 - 18000 = 82000 \text{ mm}^3$$

and $$b = 20 \text{ mm}$$

$\therefore$ Maximum shear stress, $$\tau_{max} = \frac{F\times A\times\bar{y}}{I\times b} = \frac{20\times10^3\times82000}{6.03\times10^6\times20}$$

$$= 13.6 \text{ MPa}$$

6. (b) Solution:

Given data

Factored axial load P_u = 1800 kN

Grade of concrete f_{ck} = 25 N/mm^2

Grade of steel f_y = 415 N/mm^2

Unsupported length L = 3.5 m

Effective length L_{ef} = 0.80 × 3.5 = 2.8 m

Clear cover = 40 mm

Type of reinforcement is Helical

For a short axially loaded circular column with helical reinforcement, the design axial strength is increased by 5 percent. The governing expression is

$$P_u = 1.05 \times (0.4\, f_{ck}\, A_c + 0.67\, f_y A_{sc})$$

Assuming the area of longitudinal steel as 1 percent of the gross area,

$$A_{sc} = 0.01\, A_g,\ A_c = 0.99\, A_g$$

Substituting values,

$$\Rightarrow \quad 1800 \times 10^3 = 1.05 \times (0.4 \times 25 \times 0.99\, A_g + 0.67 \times 415 \times 0.01\, A_g)$$

$$\Rightarrow \quad A_g = 135190.70 \text{ mm}^2$$

For a circular section,

$$A_g = \frac{\pi}{4} \times D^2$$

$$D = \sqrt{\frac{4 \times 135190.70}{\pi}} = 414.886 \text{ mm}$$

$$\text{Adopt column diameter } D = 420 \text{ mm.}$$

The actual gross area provided is

$$A_g = \frac{\pi}{4} \times 420^2 = 138544.236 \text{ mm}^2$$

Slenderness ratio is checked as

$$\frac{L_{eff}}{D} = \frac{2800}{420} = 6.667 < 12$$

Hence the column is short.

Minimum eccentricity is

$$e_{min} = \frac{L}{500} + \frac{D}{30} = \frac{3500}{500} + \frac{420}{30} = 21 \text{ mm}$$

$$0.05\text{ D} = 21 \text{ mm}$$

Since $e_{min} \le 0.05D$, axial load design is valid.

Using the adopted diameter, the required area of longitudinal steel is obtained from

$$1800 \times 10^3 = 1.05 \times [0.4 \times 25 \times (138544.236 - A_{sc}) + 0.67 \times 415 \times A_{sc}]$$

$$\Rightarrow \quad A_{sc} = 1226.8 \text{ mm}^2$$

Adopt 8 bars of 16 mm diameter,

Percentage of steel provided is

$$= \frac{8 \times \frac{\pi}{4} \times 16^2 \times 100}{138544.236} = 1.161\%$$

This satisfies the IS 456 limits of 0.8 percent minimum and 6 percent maximum. The minimum bar diameter requirement of 12 mm and minimum number of bars for a circular column, which is 6, are also satisfied.

Core diameter for helical reinforcement is

$$D_c = 420 - 2 \times 40 = 340 \text{ mm}$$

Core area is $$A_c = \frac{\pi}{4} \times 340^2 = 90792.028 \text{ mm}^2$$

The diameter of helical reinforcement shall not be less than 6 mm and not less than one-fourth of the longitudinal bar diameter.

$$\frac{16}{4} = 4 \text{ mm} < 6 \text{ mm}$$

Hence minimum governs. Provide 8 mm diameter helix,

$$A_{sp} = \frac{\pi}{4} \times 8^2 = 50.265 \text{ mm}^2$$

The volumetric ratio requirement for helical reinforcement is

$$\frac{V_h}{V_c} \geq 0.36 \times \left(\frac{A_g}{A_c} - 1\right) \times \frac{f_{ck}}{f_y}$$

$$\Rightarrow \quad \frac{V_h}{V_c} \geq 0.36 \times \left(\frac{138544.236}{90792.028} - 1\right) \times \frac{25}{415}$$

$$\Rightarrow \quad \frac{\frac{\pi}{4} \times 8^2 \times \pi \times (340 - 8)}{\frac{\pi}{4} \times 340^2 \times s} \geq 0.0114$$

$$\Rightarrow \quad s \leq 50.653 \text{ mm}$$

Pitch checks as per IS 456

$$s \leq 75 \text{ mm}$$

$$s \leq \frac{D_k}{6} = \frac{340}{6} = 56.667 \text{ mm}$$

$$s \geq 25 \text{ mm}$$

$$s \leq 3 \times d_h = 24 \text{ mm}$$

Adopt helical reinforcement pitch s = 50 mm, which satisfies all requirements.

Final design

Circular column diameter = 420 mm

Longitudinal reinforcement = 8 bars of 16 mm diameter

Helical reinforcement = 8 mm diameter at 50 mm pitch

6. (c) Solution:

Given: $b = 500$ mm, $D = 700$ mm

$V_u = 150$ kN, $T_u = 15$ kNm and $M_u = 200$ kNm

Equivalent shear force, $V_e = V_u + \frac{1.6T_u}{b} = 150 + \frac{1.6 \times 15}{0.5} = 198$ kN

Effective cover = 35 mm

Equivalent nominal shear stress,

$$\tau_e = \frac{V_e}{bd} = \frac{198 \times 1000}{500 \times (700 - 35)} = 0.6 \text{ N/mm}^2$$

Maximum shear strength of M20 concrete

$$\tau_{max} = 0.625\sqrt{f_{ck}} = 0.625\sqrt{20}$$

$$= 2.8 \text{ N/mm}^2 > 0.6 \text{ N/mm}^2$$

Since tension steel is 0.25%

∴ Design shear strength of concrete,

$$\tau_c = 0.36 \text{ N/mm}^2 < \tau_e$$

Thus, $\tau_c < \tau_e < \tau_{c\,max}$

∴ Shear reinforcement is required which can be provided in the form of longitudinal and transverse steel.

Longitudinal reinforcement,

Equivalent B.M., $M_e = M_u + M_t$

$$= M_u + T_u\left(\frac{1 + \frac{D}{b}}{1.7}\right) = 200 + 15 \times \left(\frac{1 + \frac{700}{500}}{1.7}\right)$$

$$= 200 + 21.2 = 221.2 \text{ kNm}$$

Since, M_t (= 21.2 kNm) < M_u (= 200 kNm), there is no need of compression reinforcement due to twisting moment.

Maximum depth of neutral axis,

$$x_{u,\,lim} = 0.48d = 0.48 \times 665$$

$$= 319.2 \text{ mm} \simeq 319 \text{ mm}$$

Depth of neutral axis, $x_u = \frac{0.87 f_y A_t}{0.36 f_{ck} b} = 98.8 \text{ mm} < x_{u,\,lim}$

∴ Section is under-reinforced.

The area of tension steel can be computed as:

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6M_u}{f_{ck}bd^2}}\right]bd$$

$$\Rightarrow \quad A_{st} = \frac{0.5\times 20}{415}\left[1-\sqrt{1-\frac{4.6\times 221.2\times 10^6}{20\times 500\times 665^2}}\right]\times 500\times 665$$

$$\Rightarrow \quad \text{Ast} = 982 \text{ mm}^2$$

$$= 0.87 f_y \times A_t\left(665-\frac{415\times A_t}{20\times 500}\right)$$

$$A_t = 985 \text{ mm}^2$$

Minimum tension reinforcement,

$$A_{st,\,\min} = \frac{0.85bd}{f_y} = \frac{0.85\times 500\times 665}{415}$$

$$= 681 \text{ mm}^2 < A_t \qquad \text{(OK)}$$

Provide 5-16 mmϕ bars,

$$A_{t,\text{provided}} = 5\times\frac{\pi}{4}(16)^2\, 1005 \text{ mm}^2 > 982 \text{ mm}^2 \qquad \text{(OK)}$$

Transverse reinforcement

$$0.87 f_y A_{sv} = \left(\frac{T_u}{b_1 d_1}+\frac{V_u}{2.5d_1}\right)S_v$$

A clear cover of 25 mm is assumed all around the shear stirrups. Use 8 mm-2 legged vertical stirrups,

b_1 = centre to centre distance between corner bars in the direction of width

$$= 500-25-25-8-8-\frac{16}{2}-\frac{16}{2} = 418 \text{ mm}$$

Provide 2-12 mm diameter bars at top as holder bars for shear stirrups.

d_1 = centre to centre distance between corner bars in the direction of depth

$$= d-25-8-\frac{12}{2} = 665-25-8-6 = 626 \text{ mm}$$

$$A_{sv} = 2\times\frac{\pi}{4}\times 8^2 = 100.5\text{mm}^2$$

Spacing of shear reinforcement x is given by

$$0.87 \times 415 \times 100.5 = \left[\frac{15\times10^6}{418\times626}+\frac{150\times1000}{2.5\times626}\right]S_v$$

$\Rightarrow$ $S_v = 237$ mm

x_1 = Centre to centre distance between the shear stirrups in the direction of width

= 500 – 25 – 25 – 8 = 442 mm

y_1 = Centre to centre distance between the shear stirrups in the direction of depth

= 700 – 25 – 25 – 8 = 642 mm

Check $S_v < \frac{(x_1+y_1)}{4} = \frac{442+642}{4} = 271$ mm > 237 mm (OK)

Adopt a spacing of 200 mm c/c

Minimum shear reinforcement,

$$A_{sv\,\min} \geq \frac{(\tau_e-\tau_c)b.S_v}{0.87\,f_y}$$

% tension steel, $p = \frac{100A_t}{bd} = \frac{100\times5\times\frac{\pi}{4}\times16^2}{500\times665} = 0.3\%$

From table,

∴ Design shear strength of concrete,

$\tau_c = 0.384$ N/mm²

∴ $A_{sv\,\min} = \frac{(0.6-0.384)\times500\times200}{0.87\times415}$

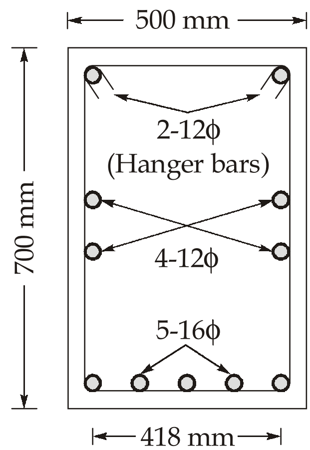
$= 59.825\text{ mm}^2 < 100.5\text{ mm}^2$ (OK)

Use 8 mm-2 legged vertical stirrups @ 200 mm c/c

Since the depth exceeds 450 mm, provide 0.1% steel as side-face reinforcement equally distributed on the two vertical faces.

Side force reinforcement $= \frac{0.1}{100}bD = \frac{0.1}{100}\times500\times700 = 350\text{ mm}^2$

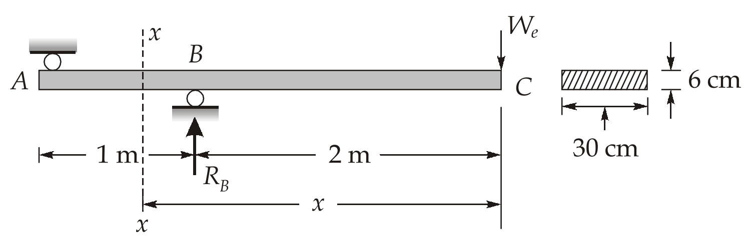
Hence provide 2-12 mm ϕ bar on each side of face.



Reinforcement details

7. (a) Solution:

Let W_e be equivalent gradually applied load which produced same deflection as produced by man.



Taking moment about A,

$$W_e \times 3 = R_B \times 1$$

$$R_B = 3W_e$$

Taking moment about x-x section

$$M_x = -W_e \times x + R_B(x-2)$$

$$EI\left(\frac{d^2y}{dx^2}\right) = -W_e x + 3W_e(x-2)$$

$$EI\left(\frac{dy}{dx}\right) = \frac{W_e \times x^2}{2} - \frac{3W_e(x-2)^2}{2} + C_1$$

$$EI(y) = \frac{W_e \times x^3}{6} - \frac{W_e(x-2)^3}{2} + C_1 x + C_2 \quad \text{...(I)}$$

At $x = 2, y = 0$

$$\Rightarrow \quad 0 = \frac{W_e \times 2^3}{6} + 2C_1 + C_2$$

$$\Rightarrow \qquad 2C_1 + C_2 = \frac{-4W_e}{3} \qquad \text{...(ii)}$$

At $x = 3$, $y = 0$

$$\Rightarrow \qquad 0 = \frac{W_e \times 3^3}{6} - \frac{W_e(1)^3}{2} + 3C_1 + C_2$$

$$\Rightarrow \qquad 3C_1 + C_2 = -4W_e \qquad \text{...(iii)}$$

On solving equation (ii) and (iii)

$$C_1 = \frac{-8}{3}W_e,\ W_2 = 4W_e$$

On putting the value of C_1 and C_2 in equation (i)

$$EIy = \frac{W_e x^3}{6} - \frac{W_e(x-2)^3}{2} - \frac{8}{3}W_e x + 4W_e$$

At $x = 0$

$$EIy_c = 4W_e$$

$$y_c = \frac{4W_e}{EI}$$

$$I = \frac{1}{12}\times 300 \times 60^3 = 540 \times 10^4 \text{ mm}^4$$

$$EI = 1 \times 10^4 \times 540 \times 10^4 \text{ N-mm}^2 = 54000 \text{ N-m}^2$$

$$\therefore \qquad y_c = \frac{4W_e}{54000}$$

Work done be man when he jumps

$$= 600(h + y_c)$$

$$= 600 \times \left(0.5 + \frac{4W_e}{54000}\right) \qquad \text{...(iv)}$$

Work done by equivalent applied load

$$= \frac{1}{2}W_e \times \delta = \frac{1}{2}W_e \times \frac{4W_e}{54000} = \frac{W_e^2}{27000} \qquad \text{...(v)}$$

From equation (iv) and (v)

$$600\left(0.5 + \frac{4W_e}{54000}\right) = \frac{W_e^2}{27000}$$

$$\Rightarrow \qquad 8100000 + 1200\,W_e = W_e^2$$

$$\Rightarrow \qquad W_e^2 - 1200W_e = 8100000$$

$$\Rightarrow \quad W_e^2 - 1200W_e - 8100000 = 0$$

$$W_e = 3508.6 \text{ N}, -2308.6$$

Take + ve sign $\quad W_e = 3508.6$ N

$$\text{Maximum bending moment} = 3508.6 \times 2 \text{ Nm}$$

$$= 7017.2 \text{ Nm} = 7017200 \text{ Nmm}$$

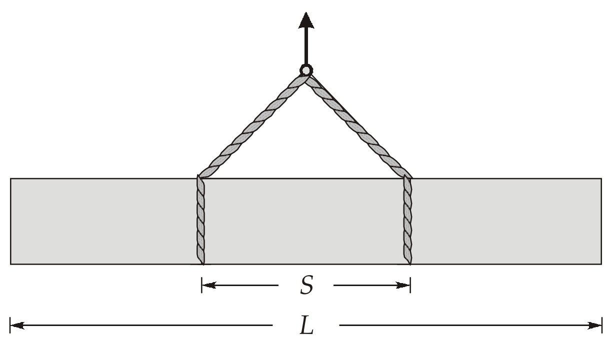
Maximum stress produced

$$f_{max} = \frac{M_{max}}{y} \times y_{max}$$

$$\Rightarrow \quad f_{max} = \frac{7017200}{540 \times 10^4} \times \left(\frac{60}{2}\right) = 38.97 \text{ N/mm}^2$$

7. (b) Solution:

Given data : $d_o = 180$ mm; $t = 6$ mm; $\gamma = 20$ kN/m^3; $L = 18$ m; $S = 5$ m



$$d_i = d_o - 2t$$

$$= 180 - 2 \times 6 = 168 \text{ mm}$$

$$a = \frac{L-S}{2} = \frac{18-5}{2} = 6.5 \text{ m}$$

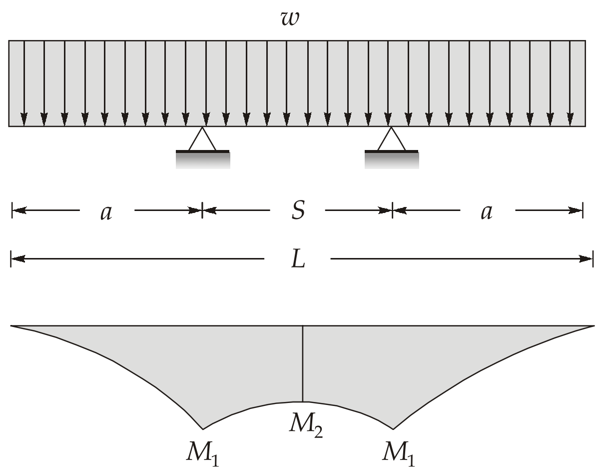
$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4} \times \left(180^2 - 168^2\right)$$

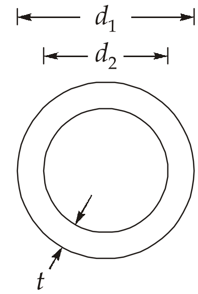
$$= 3279.8 \text{ mm}^2$$

$$w = \gamma \times A = \frac{20 \times 10^3 \times 3279.8}{10^6} = 65.596 \text{ N/m}$$

$$I = \frac{\pi}{64} \times \left(d_o^4 - d_i^4\right) = \frac{\pi}{64} \times \left(180^4 - 168^4\right)$$

$$I = 12.427 \times 10^6 \text{ mm}^4$$





$$(M_1)_{\text{at } s\,=\,10.544\text{ m}} = -\frac{wa^2}{2} = \frac{-65.596\times6.5^2}{2} = -1385.72 \text{ kNm}$$

$$M_2 =$$

$$-\frac{wL}{4}\left(\frac{L}{2}-5\right) = \frac{-65.596\times18}{4}\times\left[\frac{18}{2}-5\right] = -1180.728 \text{ Nm}$$

(i) Maximum bending stress

$$\sigma_{max} = \frac{M_{max}}{I}y = \frac{M_1}{I}\left(\frac{d_o}{2}\right)$$

$$\sigma_{max} = \frac{1385.72\times10^3}{12.427\times10^6}\times\left(\frac{180}{2}\right) = 10.0358 \text{ MPa}$$

(ii) For bending stress to be minimized, beam must have a point of contraflexture and $M_1 + M_2 = 0$

$$M_1(s) = -w\frac{\left(\frac{L-S}{2}\right)^2}{2}$$

$$M_2(s) = -\frac{wL}{4}\left(\frac{L}{2}-S\right)$$

$\because$ $$M_1 + M_2 = 0$$

$$-w\frac{\left(\frac{L-S}{2}\right)^2}{2} - \frac{wL}{4}\left(\frac{L}{2}-S\right) = 0$$

On solving, $$S = 0.58579L = 10.544 \text{ m}$$

$$S = 10.544 \text{ m}$$

So,
$$[M_1]_{\text{at } S = 10.544 \text{ m}} = -\frac{w}{2}\left(\frac{18-10.544}{2}\right)^2 = -455.826 \text{ Nm}$$

$$\sigma_{min} = \frac{[M_1]_{\text{at } 10.544 \text{ m}}}{I}\left(\frac{d_o}{2}\right)$$

$$\sigma_{min} = \frac{455.826 \times 10^3}{12.427 \times 10^6} \times \left(\frac{180}{2}\right)$$

$$= 3.301 \text{ MPa}$$ **Ans.**

(iii) Either $M_{1,\, max}$ ($S = 0$) or $M_{2,\, max}$ ($S = L$) will lead to maximum bending stress

1. Support at $\frac{L}{2}$ or $\frac{1}{2}$ beam is a cantilever with maximum moment $\frac{wL^2}{8}$

$$\sigma_{max_1} = \left(\frac{wL^2}{8}\right) \times \frac{(d_o/2)}{I} = \frac{65.596 \times 18^2 \times 10^3 \times 180}{8 \times 12.427 \times 10^6 \times 2}$$

$$= 19.24 \text{ MPa}$$

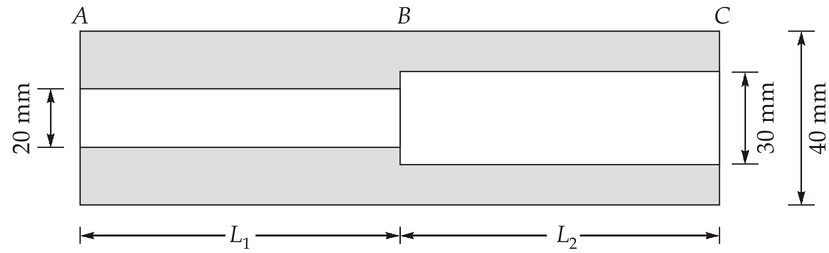
2. Or simply supported beam $S = L$ under uniform load, so max moment, $\frac{wL^2}{8}$

$$\sigma_{max} = \frac{wL^2}{8} \times \frac{d_o/2}{I} = 19.24 \text{ MPa}$$

7. (c) Solution:

Given,
Total length, L = 500 mm
External diameter, D = 40 mm

Let
L_1 = Length of AB
d_1 = Internal diameter of AB = 20 mm
L_2 = Length of shaft BC = $(500 - L_1)$
d_2 = Internal diameter of BC = 30 mm
τ = Maximum shear stress = 80 N/mm^2
N = Speed = 200 rpm
T_1 = Torque transmitted by shaft AB
T_2 = Torque transmitted by shaft BC



Using the general torque equation,

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \qquad ...(i)$$

$$\Rightarrow \qquad T = \frac{\tau J}{R} = \tau \times \frac{\pi}{16} \times \left[\frac{D_0^4 - D_i^4}{D_0}\right]$$

Hence, torque transmitted by hollow shaft AB is,

$$T_1 = \frac{\pi}{16} \times 80 \times \left[\frac{40^4 - 20^4}{40}\right] \text{ Nmm}$$

$$T_1 = 942477.8 \text{ Nmm} = 942.5 \text{ Nm}$$

Similarly torque transmitted by hollow shaft BC is

$$T_2 = \frac{\pi}{16} \times 80 \times \left[\frac{40^4 - 30^4}{40}\right]$$

$$T_2 = 687223.4 \text{ Nmm} = 687.2 \text{ Nm}$$

So, the safe torque transmitted by the whole shaft is the minimum of T_1 and T_2

Hence, safe torque transmitted = 687.2 Nm

Power transmitted, $P = \omega \times T = \frac{2\pi NT}{60} = \frac{2 \times \pi \times 200 \times 687.2}{60} = 14392.68 \text{ W}$

$\therefore$ $P = 14.39 \text{ kW}$

Now from equation (i)

$$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow \theta = \frac{TL}{GJ}$$

The safe torque T and shear modulus C are same for the given shaft section AB and BC,

Hence angle of twist in shaft $AB = \frac{TL_1}{GJ_1}$

Angle of twist in shaft $BC = \dfrac{TL_2}{GJ_2}$

Hence, $\dfrac{TL_1}{GJ_1} = \dfrac{TL_2}{GJ_2}$

$\Rightarrow \quad \dfrac{L_1}{J_1} = \dfrac{L_2}{J_2}$

$$\Rightarrow \quad \frac{L_1}{\frac{\pi}{32}\left[40^4 - 20^4\right]} = \frac{L_2}{\frac{\pi}{32}\left[40^4 - 30^4\right]}$$

$$\Rightarrow \quad \frac{L_1}{L_2} = \frac{40^4 - 20^4}{40^4 - 30^4} = 1.37$$

$\Rightarrow \quad L_1 = 1.37L_2 = 1.37(500 - L_1)$

$\Rightarrow \quad L_1 = 289.03$ mm

$\therefore \quad L_2 = 210.97$ mm

Total angle of twist of shaft, $\theta = 2 \times \dfrac{TL_1}{GJ_1}$

$$\Rightarrow \quad \theta = \frac{2 \times 687.2 \times (0.28903)}{80 \times 10^9 \times \frac{\pi}{32} \times \left[(0.04)^4 - (0.02)^4\right]}$$

$\Rightarrow \quad \theta = 0.021$ radian $= 1.203°$

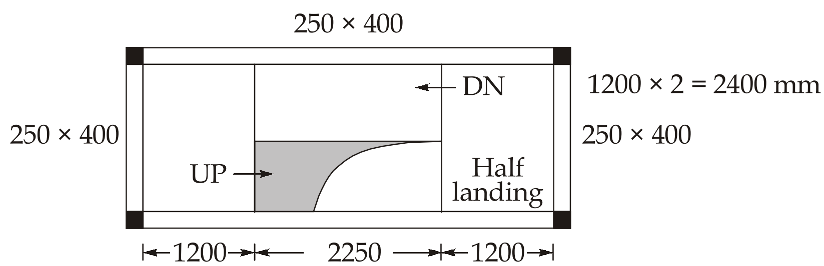
OR

Total angle of twist of shaft, $\theta = 2 \times \dfrac{TL_2}{GJ_2}$

$$\Rightarrow \quad \theta = \frac{2 \times 687.2 \times (0.21097)}{80 \times 10^9 \times \frac{\pi}{32} \times \left[(0.04)^4 - (0.03)^4\right]}$$

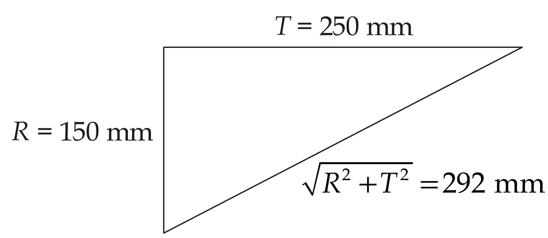
$\Rightarrow \quad \theta = 0.021$ radian $= 1.203°$

8. (a) Solution:



Given: Floor to floor height = 3 m, Live load = 2.5 kN/m^2,

Thickness of stair case slab = 150 mm, Rise (R) = 150 mm, Tread (T) = 250 mm



$$f_{ck} = 25 \text{ N/mm}^2 \text{ and } f_y = 500 \text{ N/mm}^2$$

$$\text{Width of flight} = \text{Landing width} = 1200 \text{ mm}$$

Assuming 20 mm clear cover (mild exposure) and 12ϕ main bars.

Effective depth of stair case slab = $150 - 20 - \frac{12}{2} = 124$ mm

Loads on projected plan area are as follows:

1. Self weight of slab, $25 \times 0.15 \times \frac{292}{250} = 4.38 \text{ kN/m}^2$
2. Self weight, $25 \times \frac{1}{2} \times 0.15 = 1.875 \text{ kN/m}^2$
3. Live load of steps = 2.5 kN/m^2

$$\text{Total load} = 8.755 \text{ kN/m}^2$$

$$\text{Factored load} = 8.755 \times 1.5 = 13.13 \text{ kN/m}^2$$

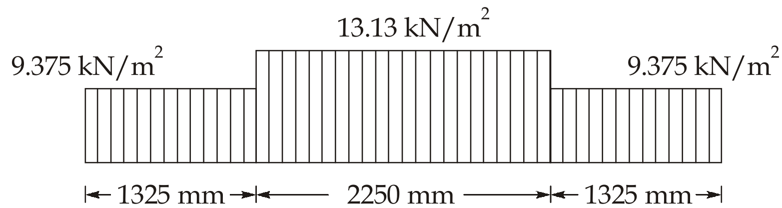
Loads on landing:

Assume landing slab thickness = 150 mm

1. Self weight of slab, 25 × 0.15 = 3.75 kN/m^2
2. Live loads = 2.5 kN/m^2

$$\text{Total load} = 6.25 \text{ kN/m}^2$$

$$\text{Factored live load} = 1.5 \times 6.25 = 9.375 \text{ kN/m}^2$$



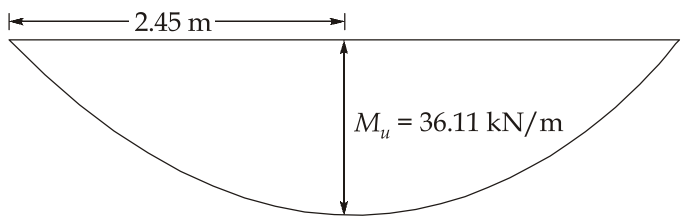
Design moment

$$\text{Reaction (R)} = \frac{2\times 9.375\times 1.325 + 13.13\times 2.25}{2} = 27.193 \text{ kN/m}$$

Maximum moment at mid span

$$M_u = (27.193\times 2.45) - (9.375\times 1.325)\left(2.45 - \frac{1.325}{2}\right) - \frac{13.13\times(2.45-1.325)^2}{2}$$

$$= 44.42 - 8.31 = 36.11 \text{ kNm/m}$$



BMD

Main reinforcement:

$$R = \frac{M_u}{bd^2} = \frac{36.11\times 10^6}{10^3\times(124)^2} = 2.35$$

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{25}{2\times 500}\left[1-\sqrt{1-\frac{4.598\times 2.35}{25}}\right] = 0.616\times 10^{-2}$$

$\Rightarrow$ $$A_{st} = 0.616\times 10^{-2}\times 1000\times 124 = 764 \text{ mm}^2\text{/m}$$

$$\text{Required spacing of } 12\phi \text{ bars} = \frac{\frac{\pi}{4}\times 12^2\times 10^3}{764} = 147.9 \text{ mm}$$

Thus provide 140 mm spacing c/c

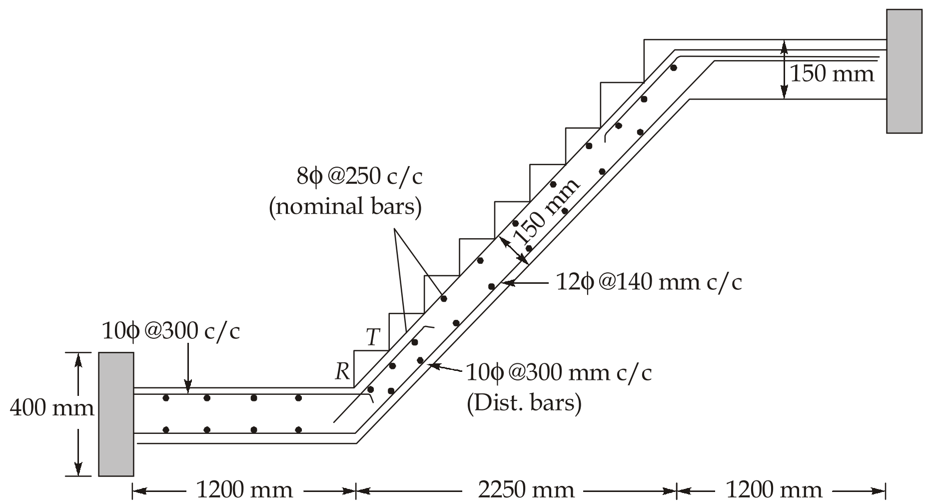
Distribution bars required $(A_{st})_{req} = 0.0012 \times bD$

$$= 0.0012\times 1000\times 150$$

$$= 180 \text{ mm}^2\text{/m}$$

$$\text{Spacing of } 10\phi \text{ bars} = \frac{\frac{\pi}{4}\times 10^2\times 10^3}{180} = 436 \text{ mm but } \not> 300 \text{ mm}$$

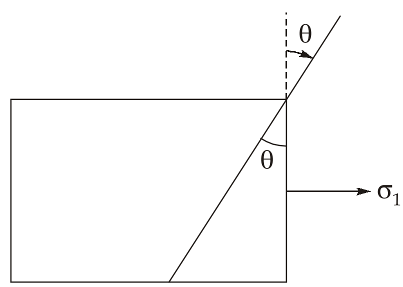
Provide 10ϕ @ 300 c/c as distributors.



Reinforcement details

8. (b) (i) Solution:

Given Data



Major principal stress: $\sigma_1 = 120\ \text{N/mm}^2$ (tensile)

Minor principal stress: $\sigma_2 = 60\ \text{N/mm}^2$ (tensile)

Angle of inclination: $\theta = 20°$ (clockwise)

Poisson's ratio: $\mu = 0.25$

Stresses on the Inclined Plane

Normal Stress (σ_n)

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$\Rightarrow \quad \sigma_n = \frac{120 + 60}{2} + \frac{120 - 60}{2}\cos 40°$$

$$\Rightarrow \quad \sigma_n = 90 + 30\cos 40°$$

$$\Rightarrow \quad \sigma_n = 90 + 30(0.7660)$$

$$\Rightarrow \quad \sigma_n \approx 112.98 \text{ N/mm}^2$$

Shear Stress (τ)

$$\tau = -\left(\frac{\sigma_1 - \sigma_2}{2}\right)\sin 2\theta$$

$$\Rightarrow \quad \tau = -\left(\frac{120-60}{2}\right)\sin(40°)$$

$$\Rightarrow \quad \tau = -19.28 \text{ N/mm}^2$$

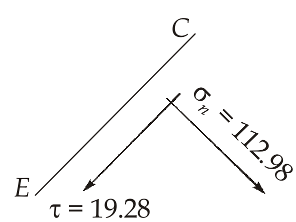
Resultant Stress (σ_R)

$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$$

$$\Rightarrow \quad \sigma_R = \sqrt{112.98^2 + 19.28^2}$$

$$\Rightarrow \quad \sigma_R \approx 114.62 \text{ N/mm}^2$$

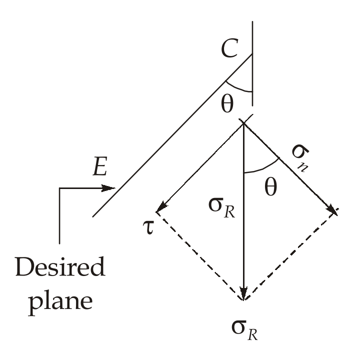
Direction sence of stress on plane *CE*



Obliquity (ϕ)

$$\tan\phi = \left|\frac{\tau}{\sigma_n}\right| = \left|\frac{-19.28}{112.98}\right|$$

$$\phi \approx 9.69°$$



Intensity of Stress Producing Maximum Strain

Maximum principal strain:

$$\in_1 = \frac{1}{E}(\sigma_1 - \mu\sigma_2)$$

$$\Rightarrow \quad \in_1 = \frac{1}{E}(120 - 0.25 \times 60)$$

$$\Rightarrow \quad \in_1 = \frac{1}{E}(120 - 15)$$

$$\Rightarrow \quad \in_1 = \frac{105}{E}$$

For an equivalent single stress σ:

$$\frac{\sigma}{E} = \frac{105}{E}$$

$$\Rightarrow \quad \sigma = 105 \text{ N/mm}^2$$

8. (b) (ii) Solution:

Given: Power transmitted, $P = 75 \text{ kW} = 75 \times 10^3 \, W$

Speed, $N = 150 \text{ rpm}$

Diameter, $d = 10 \text{ cm} = 100 \text{ mm}$

Length, $L = 4 \text{ m} = 4000 \text{ mm}$

Modulus of rigidity, $G = 8 \times 10^4 \text{ N/mm}^2$

1. Torque Transmitted

$$P = \frac{2\pi NT}{60}$$

$$\Rightarrow \quad 75 \times 10^3 = \frac{2\pi(150)T}{60}$$

$$\Rightarrow \quad T = \frac{75 \times 10^3 \times 60}{2\pi \times 150}$$

$$\Rightarrow \quad T = 4774.65 \text{ N-m} = 4.77465 \times 10^6 \text{ N-mm}$$

(i) Maximum Shear Stress

For a solid circular shaft,

$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$\Rightarrow \quad \tau_{max} = \frac{16(4.77465 \times 10^6)}{\pi(100)^3} = 24.32 \text{ N/mm}^2$$

(ii) Strain Energy Stored

Strain energy in shaft, $U = \dfrac{T^2L}{2GJ}$

$$\Rightarrow \quad U = \frac{(4.77465\times10^6)^2\times4000}{2\times(8\times10^4)\times\left(\dfrac{\pi(100)^4}{32}\right)}$$

$\Rightarrow$ $U = 5.81 \times 10^4$ N-mm

8. (c) Solution:

Given Data

External diameter, $D = 100$ mm

Internal diameter, $d = 50$ mm

Diameter ratio, $k = \dfrac{d}{D} = 0.5$

Twisting moment, $T = 13.2 \times 10^6$ N-mm

Bending moment, $M = 4.125 \times 10^6$ N-mm

Poisson's ratio, $\mu = 0.25$

Bending Stress, $\sigma_b = \dfrac{M}{\dfrac{\pi}{64}(D^4 - d^4)}\times\dfrac{D}{2}$

$$\Rightarrow \quad \sigma_b = \frac{32M}{\pi D^3(1-k^4)}$$

$$\Rightarrow \quad \sigma_b = \frac{32\times4.125\times10^6}{\pi\times100^3(1-0.5^4)}$$

$\sigma_b = 44.82$ N/mm^2

Shear Stress, $\tau = \dfrac{T}{\dfrac{\pi}{32}(D^4 - d^4)}\times\dfrac{D}{2}$

$$\Rightarrow \quad \tau = \frac{16T}{\pi D^3(1-k^4)}$$

$$\Rightarrow \quad \tau = \frac{16\times13.2\times10^6}{\pi\times100^3(1-0.5^4)}$$

$$\tau = 71.71 \text{ N/mm}^2$$

Principal Stresses

$$\sigma_{1,2} = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$\Rightarrow \quad \sigma_{1,2} = \frac{44.82}{2} \pm \sqrt{\left(\frac{44.82}{2}\right)^2 + (71.71)^2}$$

$$\Rightarrow \quad \sigma_{1,2} = 22.41 \pm \sqrt{(22.41)^2 + (71.71)^2}$$

$$\Rightarrow \quad \sigma_{1,2} = 22.41 \pm 75.13$$

$$\sigma_1 = 97.54 \text{ N/mm}^2 \text{ (tensile)}$$

$$\sigma_2 = -52.72 \text{ N/mm}^2 \text{ (compressive)}$$

Equivalent Direct Stress

(i) Maximum Elastic Strain Energy Theory

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2$$

$$\Rightarrow \quad \sigma^2 = (97.54)^2 + (-52.72)^2 - 2(0.25)(97.54)(-52.72)$$

$$\Rightarrow \quad \sigma^2 = 9514.05 + 2779.40 + 2571.15$$

$$\Rightarrow \quad \sigma^2 = 14864.60$$

$$\Rightarrow \quad \sigma = 121.92 \text{ N/mm}^2$$

(ii) Maximum Elastic Shear Strain Energy Theory

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2$$

$$\Rightarrow \quad \sigma^2 = (97.54)^2 + (-52.72)^2 - (97.54)(-52.72)$$

$$\Rightarrow \quad \sigma^2 = 9514.05 + 2779.40 + 5142.31$$

$$\Rightarrow \quad \sigma^2 = 17435.76$$

$$\Rightarrow \quad \sigma = 132.04 \text{ N/mm}^2$$