Detailed Solutions

ESE-2025 Mains Test Series

E & T Engineering Test No : 8

Section A

Q.1 (a) Solution:

Given, Data rate, R_b = 1 kbps

Single-sided noise power spectral density,

$$\eta = 10^{-10} \text{ W/Hz}$$

(i) Total bits received in a day = $24 \times 3600 \times R_b$

$$= 8.64 \times 10^7 \text{ bits}$$

Thus, average probability of bit error (P_{er})

$$= \frac{\text{Average rate of errors in a day}}{\text{Total bits received in a day}}$$

$$= \frac{100}{8.64\times10^7} = 1.16\times10^{-6}$$

(ii) Received average signal power = $E_b \times R_b$

$$10^{-6} = E_b \times 10^3$$

$\therefore$ Energy per bit, $E_b = \dfrac{10^{-6}}{10^3} = 10^{-9}$ J

For BPSK, Probability of error,

$$P_{er} = Q\left(\sqrt{\frac{2E_b}{\eta_0}}\right) = Q\left(\sqrt{\frac{2\times10^{-9}}{10^{-10}}}\right)$$

$$= Q(\sqrt{20}) = Q(4.5)$$

$$\therefore \quad P_{er} = 3.9 \times 10^{-6}$$

The probability of error is more than that obtained in part (i). Thus, the signal power is not adequate to maintain the error rate.

Q.1 (b) Solution:

Let, $h(t) = h_1(t) + h_2(t) + h_3(t)$

$$\text{Output } y(t) = h_1(t) * x(t) + h_2(t) * x(t) + h_3(t) * x(t)$$

For a sinusoidal input $A\cos(\omega_0 t)$ to an LTI system with impulse response $H(j\omega)$, the output is given by $A\,|H(j\omega_0)|\cos(\omega_0 t + \angle H(j\omega_0))$. Here, $x(t)$ is a sinusoidal input. For $x(t) = 7\cos(2t)$,

$$y(t) = 7|H_1(j2)|\cos(2t + \angle H_1(j2)) + 7|H_2(j2)|\cos(2t + \angle H_2(j2)) + 7|H_3(j2)|\cos(2t + \angle H_3(j2))$$

$\therefore$ We have to calculate the magnitude and phase of impulse response $h(t)$.

$$H_1(j\omega) = \frac{5}{1 + j\omega}, \quad H_2(j\omega) = \frac{-16}{2 + j\omega}, \quad H_3(j\omega) = \frac{13}{3 + j\omega}$$

For $\omega = 2$,

$$H_1(j2) = \frac{5}{\sqrt{5}}\angle -\tan^{-1}(2) = \sqrt{5}\angle -63.43^\circ$$

$$H_2(j2) = \frac{16}{\sqrt{8}}\angle 180^\circ - \tan^{-1}(1) = 2\sqrt{8}\angle 135^\circ$$

$$H_3(j2) = \frac{13}{\sqrt{13}}\angle -\tan^{-1}\left(\frac{2}{3}\right) = \sqrt{13}\angle -33.7^\circ$$

We can write, $y(t) = y_1(t) + y_2(t) + y_3(t)$

where
$$y_1(t) = 7\sqrt{5}\cos(2t - 63.43^\circ)$$
$$y_2(t) = 39.6\cos(2t + 135^\circ)$$
$$y_3(t) = 25.24\cos(2t - 33.7^\circ)$$

Thus, $y(t) = 15.65\cos(2t - 63.43^\circ) + 39.6\cos(2t + 135^\circ) + 25.24\cos(2t - 33.7^\circ)$

Q.1 (c) Solution:

Given, $V_g = 10$ V, $R_g = 50\ \Omega$, $X_g = 0\ \Omega$, $R_r = 48\ \Omega$, $R_L = 2\ \Omega$, $X_A = 50\ \Omega$

The input impedance of a transmission line seen from generator, is given by

$$Z_{in} = Z_0 \frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$$

Here, $$Z_0 = 100\ \Omega;\ l = \frac{\lambda}{4}$$

$\therefore$ $$Z_{\text{in}} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{R_r + R_L + jX_A} = \frac{Z_0^2[(R_r + R_L) - jX_A]}{(R_r + R_L)^2 + X_A^2}$$

$$= R_{\text{in}} + jX_{\text{in}} \quad \text{...(i)}$$

The current I_g is,

$$I_g = \frac{V_g}{Z_g + Z_{in}} = \frac{V_g}{R_g + R_{in} + jX_{in}}$$

$$|I_g| = \frac{|V_g|}{\sqrt{(R_g + R_{in})^2 + X_{in}^2}}$$

where, from equation (i), $$R_{\text{in}} = \frac{Z_0^2(R_r + R_L)}{(R_r + R_L)^2 + X_A^2}$$

$$= \frac{100^2(48+2)}{(48+2)^2 + 50^2} = \frac{100^2 \times 50}{2 \times 50^2}$$

$\therefore$ $$R_{\text{in}} = 100\ \Omega$$

and $$X_{\text{in}} = -\frac{Z_0^2 X_A}{(R_r + R_L)^2 + X_A^2} = \frac{-100^2 \times 50}{(48+2)^2 + 50^2} = -100\ \Omega$$

Power radiated, $$P_r = \frac{1}{2}|I_g|^2 R_r$$

$$= \frac{1}{2} \times \frac{(V_g)^2 R_r}{(R_g + R_{\text{in}})^2 + X_{\text{in}}^2}$$

$$= \frac{1}{2} \times \frac{10^2 \times 48}{(50+100)^2 + 100^2} = \frac{24}{15^2 + 10^2}$$

$$P_r = 0.0738\ \text{W} \quad \text{(or)} \quad 73.8\ \text{mW}$$

Q.1 (d) Solution:

(i) The system response is

$$c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$$

Taking the Laplace transform

$$C(s) = \frac{1}{s} + \frac{0.2}{s+60} - \frac{1.2}{s+10}$$

$$= \frac{(s+10)(s+60)+0.2(s+10)s-1.2(s+60)s}{s(s+10)(s+60)}$$

$$= \frac{600}{s(s^2+70s+600)}$$

For a unit-step input, $r(t) = u(t)$, therefore, $R(s) = \frac{1}{s}$.

Hence, the transfer function is

$$\frac{C(s)}{R(s)} = \frac{\frac{600}{s(s^2+70s+600)}}{\frac{1}{s}} = \frac{600}{s^2+70s+600}$$

(ii) Calculation of ω_n and ξ:

Comparing the obtained transfer function with the standard form of the transfer function of a second-order system, we get

$$\frac{C(s)}{R(s)} = \frac{600}{s^2+70s+600} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\omega_n^2 = 600$$

or $$\omega_n = \sqrt{600} = 24.49 \text{ rad/s}$$

and $$2\xi\omega_n = 70$$

or $$\xi = \frac{70}{2\omega_n} = \frac{70}{2\times 24.49} = 1.43$$

Hence, the undamped natural frequency of the system is ω_n = 24.49 rad/s, and the damping ratio of the system is ξ = 1.43.

Q.1 (e) Solution:

Given, angle modulated signal,

$$\phi_M(t) = 10\cos(13000\pi t + 0.3\pi);\ |t| \le 1$$

$$\text{Carrier frequency, } \omega_c = 12000\,\pi \text{ rad/sec}$$

(i) If given angle modulated signal is a PM signal with k_p = 1000 rad/V, we would have

$$\phi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$$

$$= 10\cos[12000\pi t + 1000\, m(t)]$$

Clearly on comparing,

$$12000\pi t + 1000m(t) = 13000\pi t + 0.3\pi$$

$$1000m(t) = 1000\pi t + 0.3\pi$$

$$\therefore \quad m(t) = \pi t + 3 \times 10^{-4}\pi; \ |t| \le 1$$

(ii) If the given angle modulated signal is an FM signal with $k_f = 1000$ Hz/V, we can write

$$\phi_{FM}(t) = A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(t)dt\right]$$

$$= 10\cos\left[12000\pi t + 1000\int_{-\infty}^{t} m(t)dt\right]$$

Clearly on comparing,

$$12000\pi t + 1000\int_{-\infty}^{t} m(t)dt = 13000\pi t + 0.3\pi$$

$$1000\int_{-\infty}^{t} m(t)dt = 1000\pi t + 0.3\pi$$

$$\int_{-\infty}^{t} m(t)dt = \pi t + 3 \times 10^{-4}\pi$$

By differentiating on both sides, we get

$$m(t) = \pi; \qquad |t| \le 1$$

Q.2 (a) Solution:

- Each regenerative repeater will have a receiver system with a detector. So, including $(n - 1)$ repeaters and one terminal receiver, there are "n" detectors in cascade.
- The probability of "i" out of "n" detectors to produce an error can be given by the binomial distribution as,

$$P_i = \binom{n}{i} p^i (1-p)^{n-i}$$

- As the channel is used for binary transmission, an error will be produced at the output of final detector, when the odd number of detectors produces an error. Hence, the required probability can be given as,

$$P_n = P_{odd} = \sum_{i=\text{odd}} \binom{n}{i} p^i (1-p)^{n-i}$$

- Let P_{even} be the probability that an even number of detectors produces an error.

Then,
$$P_{\text{even}} = \sum_{i=\text{even}} \binom{n}{i} p^i (1-p)^{n-i}$$

$$P_{\text{even}} + P_{\text{odd}} = \sum_{i=0}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

Using the binomial expansion theorem $\left[(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}\right]$,

$$P_{\text{even}} + P_{\text{odd}} = [p + (1-p)]^n = 1 \quad \text{...(i)}$$

$$P_{\text{even}} - P_{\text{odd}} = \sum_{i=\text{even}} \binom{n}{i} p^i (1-p)^{n-i} - \sum_{i=\text{odd}} \binom{n}{i} p^i (1-p)^{n-i}$$

$$= \sum_{i=\text{even}} \binom{n}{i} (-p)^i (1-p)^{n-i} + \sum_{i=\text{odd}} \binom{n}{i} (-p)^i (1-p)^{n-i}$$

$$= \sum_{i=0}^{n} \binom{n}{i} (-p)^i (1-p)^{n-i}$$

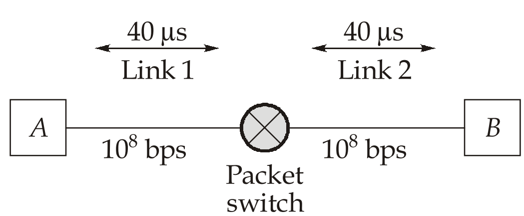
$$P_{\text{even}} - P_{\text{odd}} = [(-p) + (1-p)]^n = (1-2p)^n \quad \text{...(ii)}$$

By subtracting equation (ii) from equation (i), we get,

$$2P_{\text{odd}} = [1 - (1-2p)^n]$$

$$P_n = P_{\text{odd}} = \frac{1}{2}\left[1 - (1-2p)^n\right]$$

Q.2 (b) Solution:



Extra delay at switch = 60 µs for each packet.

$$\text{Data} = 100000 \text{ bits}$$

$$\text{No. of packets} = \frac{100000}{25000} = 4 \text{ packets}$$

$$\text{Transmission delay for one packet} = \frac{1}{2 \times 10^8} \times 25000$$

$$= 125 \,\mu\text{sec}$$

At t = 125 µsec, last bit of packet 1 is placed on link 1 by A and transmission of packet begins.

It is given that each link has a propagation delay of 40 μs. Thus, at t = 165 μsec, last bit of packet 1 reaches switch.

At t = 225 μs, first bit of packet 1 is placed on link 2 by switch.

At t = 250 μsec, last bit of packet 2 is placed on link 1 by A.

At t = 290 μs, last bit of packet 2 reaches switch.

Note: Packet 2 need to wait upto 350 μsec before switch transfer it.

Last bit of packet 1 will be placed on link 2 by switch at 225 + 125 = 350 μsec.

At t = 350 + 40 = 390 μsec, last bit of packet 1 reaches B.

At t = 350 μs, first bit of packet 2 is placed on link 2.

At t = 475 μs, last bit of packet 2 is placed on link 2 which reaches B at t = 515 μs.

Similarly,

At t = 375 μs, last bit of packet 3 is placed on link 1 by A.

At t = 415 μs, last bit of packet 3 reaches switch.

At t = 475 μs, first bit of packet 3 is placed on link 2.

At t = 600 μs, last bit of packet 3 reaches B.

At t = 600 + 40 = 640 μs, last bit of packet 3 reaches B.

At t = 500 μs, last bit of packet 4 is placed on link 1 by A.

At t = 540 μs, last bit of packet 4 reaches switch.

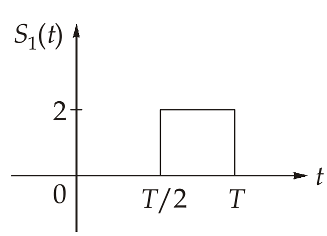
At t = 600 μs, last bit of packet 4 is placed on link 2.

At t = 725 μs, last bit of packet 4 reaches B.

At t = 725 + 40 = 765 μs, last bit of packet 4 reaches B.

∴ Total 765 μs i.e., 0.765 msec is the time lapsed between the transmission of the first bit of data and the reception of the last bit of the data.

Q.2 (c) Solution:



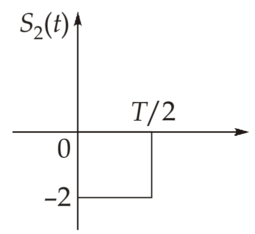
$$S_{11} = \left[\int_0^T S_1^2(t)\cdot dt\right]^{1/2} = \left[\int_{T/2}^{T} 2^2\cdot dt\right]^{1/2}$$

$$= \left[4\left(\frac{T}{2}\right)\right]^{1/2} = (2T)^{1/2}$$

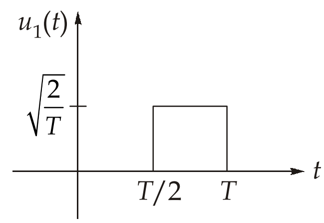
$$S_{11} = \sqrt{2T}$$

$$u_1(t) = \frac{S_1(t)}{S_{11}} = \frac{2}{\sqrt{2T}}; \qquad \frac{T}{2} \le t \le T$$

$$u_1(t) = \sqrt{\frac{2}{T}}; \qquad \frac{T}{2} \le t \le T$$



$$S_{21} = \int_0^T S_2(t)u_1(t)\cdot dt$$



∴ $$S_{21} = 0$$

$$S_{22} = \left[\int_0^T [S_2(t) - S_{21}u_1(t)]^2 \cdot dt\right]^{1/2}$$

$$S_{22} = \left[\int_0^{T/2} 4\cdot dt\right]^{1/2}$$

$$S_{22} = \sqrt{2T}$$

$$u_2(t) = \frac{S_2(t) - S_{21}u_1(t)}{S_{22}} = \frac{-2}{\sqrt{2T}}; \qquad 0 \le t \le \frac{T}{2}$$

$$u_2(t) = -\sqrt{\frac{2}{T}} \qquad 0 \le t \le \frac{T}{2}$$

Here, $u_1(t)$ and $u_2(t)$ represents the basis functions. Thus, $S_1(t)$ and $S_2(t)$ can be represented in terms of basis functions as,

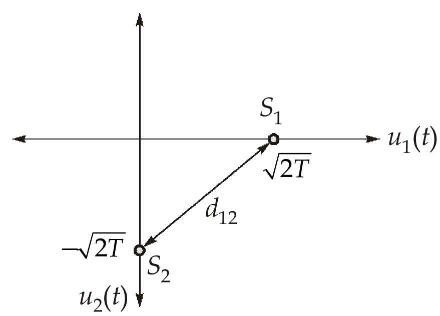
$$S_1(t) = S_{11}u_1(t) = \sqrt{2T}u_1(t)$$

$$S_1(t) = \sqrt{2T}\sqrt{\frac{2}{T}} = 2; \qquad \frac{T}{2} \le t \le T$$

$$S_2(t) = S_{21}u_1(t) + S_{22}u_2(t) = \sqrt{2T}u_2(t)$$

i.e.,
$$S_2(t) = \sqrt{2T}\left(-\sqrt{\frac{2}{T}}\right) = -2\,; \qquad 0 \le t \le \frac{T}{2}$$

Constellation diagram



$$d_{min} = d_{12} = 2\sqrt{T}$$

Q.3 (a) Solution:

Given, y-component of TE mode is,

$$E_y = \sin\left(\frac{2\pi}{a}x\right)\cos\left(\frac{3\pi}{b}y\right)\sin(10\pi\times10^{10}t-\beta z)\text{ V/m}$$

(i) The field components for the TE mode in a rectangular waveguide is given by

$$E_x = E_{x0}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin(\omega t-\beta z)$$

$$E_y = E_{y0}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin(\omega t-\beta z)$$

$$E_z = 0$$

On comparing with standard TE mode equations,

$$\frac{m\pi}{a} = \frac{2\pi}{a} \Rightarrow m = 2$$

$$\frac{3\pi}{b} = \frac{n\pi}{b} \Rightarrow n = 3$$

$\therefore$ The mode of operation is TE_{23}.

(ii) For the mode to propagate,

$$\gamma = j\beta$$

where $$\beta = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \frac{\omega}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

where f_c is the cut-off frequency of TE_{23} mode

From the given expression of electric field, we get

$$\omega = 10\pi \times 10^{10} \text{ rad/sec}$$

$$\therefore \quad f = \frac{10\pi\times 10^{10}}{2\pi} = 50 \text{ GHz}$$

Cut off frequency, f_c for TE_{23} mode is

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$= \frac{3\times 10^{10}}{2}\sqrt{\left(\frac{2}{2.286}\right)^2+\left(\frac{3}{1.016}\right)^2}$$

$$f_c = 46.19 \text{ GHz}$$

$$\therefore \quad \beta = \frac{10\pi\times 10^{10}}{3\times 10^8}\sqrt{1-\left(\frac{46.19}{50}\right)^2}$$

$$\therefore \quad \beta = 400.9 \text{ rad/m}$$

Thus, propagation constant, $\gamma = 0 + j400.9$ rad/m

(iii) For rectangular waveguide, the field expression for H_x is

$$H_x = \frac{-\beta}{\omega\mu}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{m\pi}{b}y\right)\sin(\omega t-\beta z) \text{ A/m}$$

$$= \frac{-400.9}{10\pi\times 10^{10}\times 4\pi\times 10^{-7}}\sin\left(\frac{2\pi}{a}x\right)\cos\left(\frac{3\pi}{b}y\right)\sin(10\pi\times 10^{10}t-\beta z)$$

$$H_x = -1.016\sin\left(\frac{2\pi}{a}x\right)\cos\left(\frac{3\pi}{b}y\right)\sin(10\pi\times 10^{10}t-\beta z) \text{mA/m}$$

Q.3 (b) Solution:

The 8237 operates in two cycles, viz. idle or passive cycle and active cycle. Each cycle contains a fixed number of states. The 8237 can assume six states, when it is in active cycle. During idle cycle, it is in state SI (Idle State).

The 8237 is in the idle cycle if there is no pending request or the 8237 is waiting for a request from one of the DMA channels. Once a channel requests a DMA service, the 8237 sends the HOLD request to the CPU using the HRQ pin. If the CPU acknowledges the hold request on HLDA, the 8237 enters an active cycle. In the active cycle, the actual data transfer takes place in one of the following transfer modes, as is programmed.

Single Transfer Mode : In this mode, the device transfers only one byte per request. The word count is decremented and the address is decremented or incremented (depending on programming) after each such transfer. The Terminal Count (TC) state is reached, when the count becomes zero. For each transfer, the DREQ must be active until the DACK is activated, in order to get recognized. After TC, the bus will be relinquished for the CPU. For a new DREQ to 8237, it will again activate the HRQ signal to the CPU and the HLDA signal from the CPU will push the 8237 again into the single transfer mode. This mode is also called as "cycle stealing".

Block Transfer Mode : In this mode, the 8237 is activated by DREQ to continue the transfer until a TC is reached, i.e. a block of data is transferred. The transfer cycle may be terminated due to $\overline{\text{EOP}}$ (either internal or external) which forces Terminal Count (TC). The DREQ needs to be activated only till the DACK signal is activated by the DMA controller. Auto-initialization may be programmed in this mode.

Demand Transfer Mode : In this mode, the device continues transfers until a TC is reached or an external $\overline{\text{EOP}}$ is detected or the DREQ signal goes inactive. Thus, a transfer may exhaust the capacity of data transfer of an I/O device. After the I/O device is able to catch up, the service may be re-established activating the DREQ signal again. Only the EOP generated by TC or external EOP can cause the auto-initialization, and only if it is programmed for.

Cascade Mode : In this mode, more than one 8237 can be connected together to provide more than four DMA channels. The HRQ and HLDA signals from additional 8237s are connected with DREQ and DACK pins of a channel of the host 8237 respectively. The priorities of the DMA requests may be preserved at each level. The first device is only used for prioritizing the additional devices (slave 8237s), and it does not generate any address or control signal of its own. The host 8237 responds to DREQ generated by slaves and generates the DACK and the HRQ signals to co-ordinate all the slaves. All other outputs of the host 8237 are disabled.

Memory to Memory Transfer Mode : To perform the transfer of a block of data from one set of memory address to another one, this transfer mode is used. Programming the corresponding mode bit in the command word, sets the channel 0 and 1 to operate as source and destination channels, respectively. The transfer is initialized by setting the $DREQ_0$ using software commands. The 8237 sends HRQ (Hold Request) signal to the CPU as usual and when the HLDA signal is activated by the CPU, the device starts operating in block transfer mode to read the data from memory. The channel 0 current address register acts as a source pointer. The byte read from the memory is stored in an internal temporary register of 8237. The channel 1 current address register acts as a destination pointer to write the data from the temporary register to the destination memory location. The pointers are automatically incremented or decremented, depending upon the programming. The channel 1 word count register is used as a counter and is decremented after each transfer. When it reaches zero, a TC is generated, causing $\overline{\text{EOP}}$ to terminate the service.

The 8237 also responds to external $\overline{\text{EOP}}$ signals to terminate the service. This feature may be used to scan a block of data for a byte. When a match is found the process may be terminated using the external $\overline{\text{EOP}}$.

Under all these transfer modes, the 8237 carries out three basic transfers namely, write transfer, read transfer and verify transfer. In write transfer, the 8237 reads from an I/O device and writes to memory under the control of $\overline{\text{IOR}}$ and $\overline{\text{MEMW}}$ signals. In read transfer, the 8237 reads from memory and writes to an I/O device by activating the $\overline{\text{MEMR}}$ and $\overline{\text{IOW}}$ signals. In verify transfer, the 8237 works in the same way as the read or write transfer but does not generate any control signal.

Q.3 (c) Solution:

(i) The normalized magnitude response of the n^{th} order low-pass butterworth filter transfer function is given as,

$$\left|\frac{H(j\omega)}{H_0}\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}} \qquad \text{...(i)}$$

An attenuation of 40 dB corresponds to

$$20\log\left|\frac{H(j\omega)}{H_0}\right| = -40$$

$$\frac{H(j\omega)}{H_0} = 10^{-2}$$

$$\therefore \qquad \frac{H(j\omega)}{H_0} = 0.01$$

Substituting in equation (i), we get,

$$(0.01)^2 = \frac{1}{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}$$

Given, $$\frac{\omega}{\omega_0} = 2$$

$$(0.01)^2 = \frac{1}{1+(2)^{2n}}$$

$$\Rightarrow \qquad 1 + 2^{2n} = 10^4$$

$$2^{2n} = 10^4 - 1$$

Taking logarithm on both sides,

$$\log(2^{2n}) = \log(10^4 - 1)$$

$$2n = \frac{\log(10^4 - 1)}{\log 2}$$

$$\therefore \qquad n = 6.64$$

Since the order of the filter must be an integer, hence $n = 7$.

(ii) The alternate pattern of 0/1 bits to generate a square wave can be provided by loading the register with AA H(10101010) and rotating the pattern after a delay of 560/2 = 280 μs. Bit D_0 of the output port is used to provide the output logic 0 and 1. Therefore, all other bits can be masked by ANDing the accumulator with 01H.

Label	Mnemonics	T-states	Comments
	MVI C, AA H		; Move AA H to register C. LSB of Register C is used to store the number to be displayed at output port.
ROTATE:	MOV A, C	4	; Moves the number from register C to A;
	RLC	4	; Rotates the contents of A to left without carry flag.
	MOV C, A	4	; Moves the shifted contents of A to C.
	ANI 01 H	7	; Masks D_1 - D_7 bits

	OUT PORT 0	10	;	Displays contents of A at output device connected at PORT 0.
	MVI B, 36 H	7	:	Load B with 36 H, Register B is used as 8-bit counter for loop delay. It stores the count number.
DELAY:	DCR B	4	;	Decrements the content of B counter by one.
	JNZ DELAY	10/7	;	Jumps to DELAY until B = 00H
	JMP ROTATE	10	;	Jumps to rotate to switch the output.

Time Delay Calculations:

Time delay provided by loop DELAY = 4 + 10 = 14 T-states; when condition is TRUE.

= 4 + 7 = 11 T-states ; when condition is false.

Time delay outside DELAY loop from: ROTATE : MOV A, C to JMP ROTATE = 46 T-states

$$\text{Time Delay} = (\text{Count-1}) \times (14T) + 11T + 46T$$
$$= (\text{Count-1}) \times (14T) + 57T$$

Given, T = 350 ns and time period of WAVE = 560 μs

Time after which the output switches = 280 μs. Hence, we require a time delay of 280 μs.

$$\Rightarrow \quad 280 \times 10^{-6} = (\text{Count-1}) \times 14 \times 350 \times 10^{-9} + 57 \times 350 \times 10^{-9}$$

$$\text{Count} = \frac{280 \times 10^{-6} - 57 \times 350 \times 10^{-9}}{14 \times 350 \times 10^{-9}} + 1$$

$$\text{Count} = 54.07 \cong 54 = (36)_{16}$$

Q.4 (a) Solution:

The closed-loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{\frac{4}{s^2+s}}{1+\frac{4}{s^2+s}} = \frac{4}{s^2+s+4}$$

Therefore, the characteristic equation is

$$s^2 + s + 4 = 0,$$

Comparing it with the standard second order characteristic equation, we get

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = 4$$

$$\therefore \quad \omega_n = 2 \text{ rad/s}$$

and $$2\xi\omega_n = 1$$

$$\therefore \quad \xi = \frac{1}{2\omega_n} = \frac{1}{2\times 2} = 0.25$$

The damping ratio with derivative control is

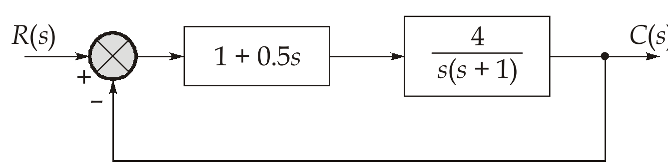
$$\xi' = \xi + \frac{\omega_n T_d}{2}$$

Since the damping ratio with derivative control is to be 0.75 and $\omega_n = 2$ rad/sec, we have

$$0.75 = 0.25 + \frac{2\times T_d}{2}$$

$$\therefore \quad T_d = 0.5$$

Therefore, the block diagram with derivative control is as shown in figure below:



Without derivative control, the rise time

$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}$$

$$= \frac{\pi - \tan^{-1}\dfrac{\sqrt{1-0.25^2}}{0.25}}{2\times\sqrt{1-0.25^2}} = \frac{1.823}{1.936} = 0.942s$$

The peak time $$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$= \frac{\pi}{2\sqrt{1-0.25^2}} = 1.62s$$

The peak overshoot is given by

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$= e^{-\pi\times 0.25/\sqrt{1-0.25^2}} = 0.4443$$

The overall transfer function with derivative control is

$$\frac{C(s)}{R(s)} = \frac{\frac{4(1+0.5s)}{s(s+1)}}{1+\frac{4(1+0.5s)}{s(s+1)}} = \frac{4(1+0.5s)}{s^2+3s+4}$$

$$= \frac{4+2s}{s^2+3s+4}$$

For a unit-step input, $r(t) = 1$

$$\therefore \quad R(s) = \frac{1}{s}$$

$$C(s) = \frac{4+2s}{s(s^2+3s+4)}$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+3s+4}$$

$$= \frac{1}{s} - \frac{(s+1)}{s^2+3s+4} = \frac{1}{s} - \frac{s+1}{(s+1.5)^2+1.75}$$

$$= \frac{1}{s} - \frac{(s+1)}{(s+1.5)^2+1.32^2} = \frac{1}{s} - \frac{(s+1.5)-0.5}{(s+1.5)^2+1.32^2}$$

$$= \frac{1}{s} - \frac{(s+1.5)}{(s+1.5)^2+1.32^2} + \frac{0.5}{1.32}\frac{1.32}{(s+1.5)^2+1.32^2}$$

Taking inverse Laplace Transform, we get

$$c(t) = 1 - e^{-1.5t}\cos 1.32t + 0.378e^{-1.5t}\sin 1.32t$$

$$= 1 - e^{-1.5t}(\cos 1.32t - 0.378\sin 1.32t)$$

At peak time (t_r), $c(t) = 1$ i.e, $c(t_r) = 1$

$$\therefore \quad 1 = 1 - e^{-1.5t_r}(\cos 1.32t_r - 0.378\sin 1.32t_r)$$

$$\therefore \quad \cos 1.32t_r = 0.378\sin 1.32t_r$$

$$\tan 1.32t_r = 1/0.378 = 2.645$$

$$\therefore \qquad 1.32t_r = \tan^{-1} 2.645 = 69.28° = 1.209 \text{ rad}$$

$$\therefore \qquad t_r = 1.209/1.32 = 0.916s$$

For peak time, differentiating the output equation with respect to t and equating to zero,

$$\frac{d}{dt}c(t)\Big|_{t=t_p} = \frac{d}{dt}\left[1-e^{-1.5t}(\cos 1.32t - 0.378\sin 1.32t)\right]\Big|_{t=t_p} = 0$$

i.e.,

$$-e^{-1.5t}[-1.32\times\sin 1.32t - 0.378\times 1.32\cos 1.32t] - e^{-1.5t}(-1.5)[\cos 1.32t - 0.378\sin 1.32t]\big|_{t=t_p} = 0$$

i.e., $-1.32 \sin 1.32t_p - 0.498 \cos 1.32t_p + 0.567 \sin 1.32t_p - 1.5 \cos 1.32t_p = 0$

i.e., $-0.753 \sin 1.32t_p - 1.998 \cos 1.32t_p = 0$

$$\tan 1.32t_p = -1.998/0.753 = -2.653$$

$$\therefore \qquad t_p = \frac{\tan^{-1}(-2.63)}{1.32} = \frac{\pi - \tan^{-1} 2.63}{1.32} = \frac{\pi - 1.207}{1.32} = 1.46s$$

Peak output occurs at $t = t_p$. Thus,

$$c(t_p) = 1 - e^{-1.5t_p}(\cos 1.32t_p - 0.378\sin 1.32t_p)$$

$$= 1 - e^{-1.5 \times 1.46}(\cos 1.32 \times 1.46 - 0.378 \sin 1.32 \times 1.46)$$

$$= 1 - e^{-2.19}(\cos 1.927 - 0.378 \sin 1.927)$$

$$= 1 - 0.1119 (\cos 110.4° - 0.378 \sin 110.4°)$$

$$= 1 - 0.1119(-0.348 - 0.354)$$

$$= 1.08$$

Therefore, the peak overshoot

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)}$$

$$= 1.08 - 1 = 0.08$$

$$\%M_p = 8\%$$

Q.4 (b) Solution:

(i) In a preemptive priority scheduling algorithm, a higher priority process is executed first. If a new process arrives with a higher priority than the one currently running, the currently executing process is pre-empted (suspended) and process with higher priority is executed.

Pid	Priority	AT (msec)	BT (msec)	CT	TAT	WT
P_1	3	0	10	16	16	6
P_2	1	1	1	2	1	0
P_3	3	2	2	18	16	14
P_4	4	3	1	19	16	15
P_5	2	4	5	9	5	0

Turn around time (TAT) = CT – AT

Waiting time (WT) = TAT – BT

Gantt Chart:

P_1	P_2	P_1	P_5	P_1	P_3	P_4

0 1 2 4 9 16 18 19

∴ Average turn around time (TAT),

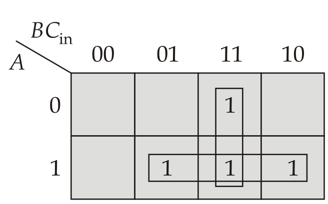
$$= \frac{16+1+16+16+5}{5} = 10.8 \text{ msec}$$

$$\text{Average waiting time (WT)} = \frac{6+0+14+15+0}{5} = 7 \text{ msec}$$

(ii) Full Adder Truth Table:

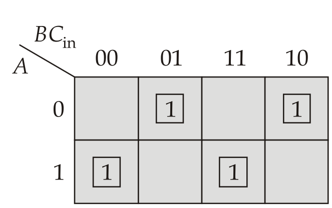
A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

K-map for C_{out}



$C_{out} = AB + BC_{in} + C_{in} \cdot A$

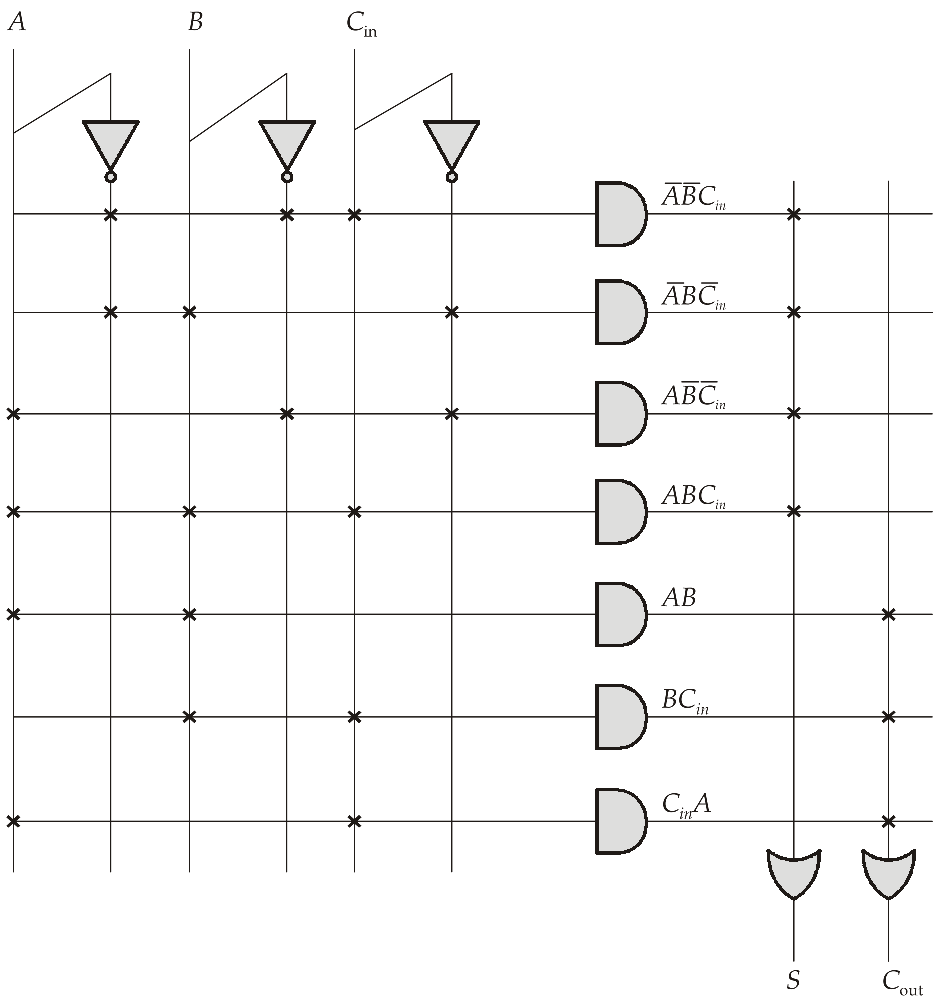
K-Map for Sum, *S*:



$$S = A \oplus B \oplus C_{in} = \bar{A}\bar{B}C_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + ABC_{in}$$

Implementation of full adder using (3 × 8 × 2) PLA

Programmable Logic Array (PLA) has a programmable AND gate array and programmable OR gate array. A (3 × 8 × 2) PLA has 8 programmable AND gates which produces the product terms and 2 programmable OR gates to generate the output. Thus, the full adder can be realized as below:



Q.4 (c) Solution:

(i) Given: $h = 400$ km ; $n = 0.9$; $f_{\text{muf}} = 10$ MHz

We know that, the refractive index of the ionosphere layer is given as

$$n = \sqrt{1 - \frac{81N_{\text{max}}}{f^2}}$$

where N_{max} is the maximum electron density in the ionosphere layer

For $f = 10$ MHz, $$0.9 = \sqrt{1 - \frac{81N_{\text{max}}}{(10 \times 10^6)^2}}$$

$$N_{max} = \frac{\left[1-(0.9)^2\right]\times(10\times10^6)^2}{81} = 2.345\times10^{11}\ /\text{m}^3$$

Therefore, $$f_c = 9\sqrt{N_{max}} = 9\sqrt{2.345\times10^{11}} = 4.358\text{ MHz}$$

When earth's curvature is taken into account, considering R = 6371 km and h as the height of ionospheric layer from the earth, we have

$$\frac{f_{muf}}{f_c} = \sqrt{\frac{D^2}{4\left(h+\frac{D^2}{8R}\right)^2}+1}$$

$$\frac{D^2}{4\left(h+\frac{D^2}{8R}\right)^2} = \left(\frac{f_{muf}}{f_c}\right)^2-1$$

$$\text{Skip Distance, } D = 2\left\{h+\frac{D^2}{8R}\right\}\sqrt{\left(\frac{f_{\text{MUF}}}{f_c}\right)^2-1}$$

$$D = 2\left\{400+\frac{D^2}{8\times6371}\right\}\sqrt{\left(\frac{10\times10^6}{4.358\times10^6}\right)^2-1}$$

$$D = 2\left\{400+\frac{D^2}{8\times6371}\right\}\times2.065$$

$$D = 1652.16 + 0.8103\times10^{-4}\,D^2$$

$$D^2 - 12341.108D + 20.389\times10^6 = 0$$

After solving the equation,

$$D = 10376.114,\ 1964.993\text{ km}$$

Since skip distance is the shortest distance from the transmitter and represents the ground range, hence

$$D \cong 1965\text{ km}$$

(ii) **Given data:** Propagation losses = 200 dB ; Margin and other losses = 3 dB

G/T = 11 dB ; EIRP = 45 dBW ; $[C/N]$ = ?

We have, $$[C/N] = [\text{EIRP}]+\left[\frac{G}{T}\right]-[\text{Losses}]-[K]-[B_N] \quad ...(i)$$

$$= [\text{EIRP}] + \left[\frac{G}{T}\right] - [\text{Losses}] - [K] - [B_N]$$

$$= 45 + 11 - 200 - 3 - 10\log(1.38 \times 10^{-23}) - 10\log(36 \times 10^6)$$

$$= 6.038 \text{ dB}$$

$$\therefore \quad \left[\frac{C}{N}\right](\text{dB}) = 6.038 \text{ dB}$$

Section B

Q.5 (a) Solution:

The dimensions of rectangular waveguides are given as,

$$a = 2b$$

Therefore at this dimension, the single mode propagation is possible between $f_c|_{TE_{10}}$ to $f_c|_{TE_{20}}$ where TE_{10} is the dominant mode with the lowest cut-off frequency and the next dominant mode is TE_{20}/TE_{01}.

For waveguide 1 (air filled),

$$\text{cutoff frequency, } f_c|_{TE_{10}} = \frac{c}{2a}$$

$$f_c|_{TE_{20}} = \frac{c}{a}$$

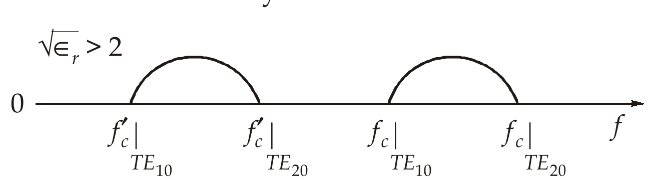
For waveguide 2 (dielectric filled ($\in_r$)),

$$\text{cutoff frequency } f'_c|_{TE_{10}} = \frac{c}{2a\sqrt{\in_r}}$$

$$f'_c|_{TE_{20}} = \frac{c}{a\sqrt{\in_r}}$$

If $\sqrt{\in_r} > 2$, $\quad f'_c|_{TE_{10}} < f'_c|_{TE_{20}}$

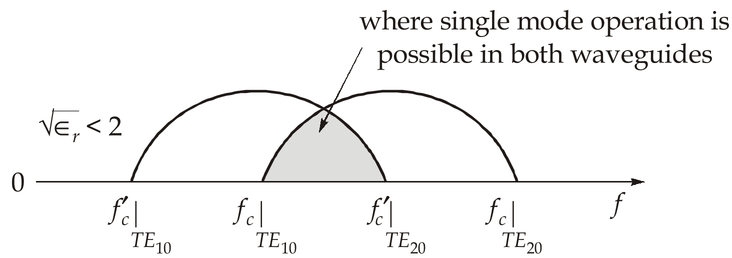
but $f_c|_{TE_{10}}$ and $f_c|_{TE_{20}}$ is more than $f'_c|_{TE_{20}}$. Thus, single mode operation is not possible in both the waveguides simultaneously.



If $\sqrt{\epsilon_r} < 2$, $f_c|_{TE_{10}} < f'_c|_{TE_{20}}$

but $$f'_c|_{TE_{20}} > f_c|_{TE_{10}}$$

also $$f'_c|_{TE_{20}} < f_c|_{TE_{20}}$$



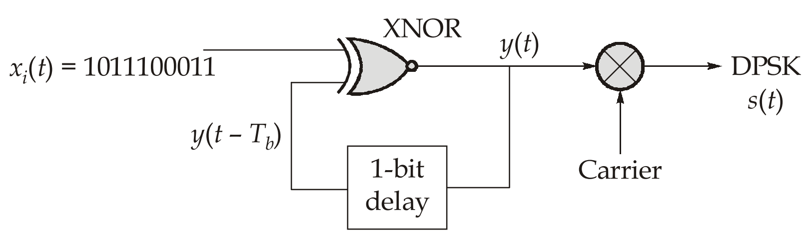
∴ The range of dielectric constant required for single mode operation in both the rectangular waveguides is $\sqrt{\epsilon_r} < 2$ i.e., $\epsilon_r < 4$.

Q.5 (b) Solution:

Given input bit stream for DPSK is,

Let, $$x_i(t) = 1011100011$$

The below figure shows the scheme to generate DPSK signal,



where $y(t - T_b)$ is the encoded sequence delayed by 1-bit period.

Let the transmitted signal be taken as

$$s(t) = y(t) \times A\cos(2\pi f_0 t)$$
$$= \pm A\cos(2\pi f_0 t)$$

i.e., $$s(t) = \begin{cases} A\cos(2\pi f_0 t) \text{ when } y(t) = 1 \\ -A\cos(2\pi f_0 t) \text{ when } y(t) = 0 \end{cases}$$

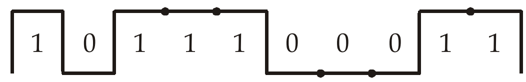
The above equations can also be written as,

$$s(t) = \begin{cases} A\cos(2\pi f_0 t) \text{ when } y(t) = 1 \\ A\cos(2\pi f_0 t + \pi) \text{ when } y(t) = 0 \end{cases}$$

$\therefore$ The transmitted phase sequence is

$$\phi(t) = \begin{cases} 0 \text{ for } y(t) = 1 \\ \pi \text{ for } y(t) = 0 \end{cases}$$

Input sequence $x_i(t)$

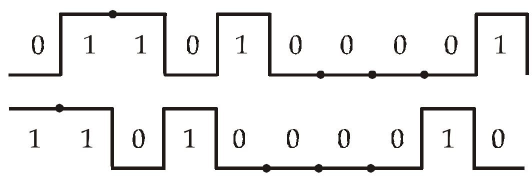


The delayed output sequence, $y(t - T_b)$ is assumed '0' initially. The encoded sequence $y(t)$ is given as

$$y(t) = x_i(t) \oplus y(t - T_b)$$

$y(t - T_b)$

$y(t)$



Transmitted phase sequence 0 0 π 0 π π π π 0 π

Q.5 (c) Solution:

The given signal flow graph shown in figure has four forward paths and five loops. All the loops are touching all the forward paths. There are five pairs of two non-touching loops and one combination of three non-touching loops.

The forward paths and the gains associated with them are as follows:

Forward path R-x_1-x_2-x_3-x_4-x_5-x_6-C $\quad M_1 = (1)(G_1)(G_2)(G_3)(G_4)(G_5)(1)$

$= G_1G_2G_3G_4G_5$

Forward path R-x_1-x_3-x_4-x_5-x_6-C $\quad M_2 = (1)(G_6)(G_3)(G_4)(G_5)(1) = G_6G_3G_4G_5$

Forward path R-x_1-x_2-x_3-x_4-x_6-C $\quad M_3 = (1)(G_1)(G_2)(G_3)(G_7)(1) = G_1G_2G_3G_7$

Forward path R-x_1-x_3-x_4-x_6-C $\quad M_4 = (1)(G_6)(G_3)(G_7)(1) = G_6G_3G_7$

The loops and the gains associated with them are as follows:

Loop x_1-x_2-x_1 $\quad L_1 = (G_1)(H_1) = G_1H_1$

Loop x_3-x_4-x_3 $\quad L_2 = (G_3)(H_2) = G_3H_2$

Loop x_5-x_6-x_5 $\quad L_3 = (G_5)(H_3) = G_5H_3$

Loop x_4-x_5-x_6-x_4 $\quad L_4 = (G_4)(G_5)(H_4) = G_4G_5H_4$

Loop x_4-x_6-x_4 $\quad L_5 = (G_7)(H_4) = G_7H_4$

The pairs of two non-touching loops and the product of gains associated with them are as follows:

Loop x_1-x_2-x_1 and x_3-x_4-x_3 $\quad L_{12} = (G_1H_1)(G_3H_2) = G_1H_1G_3H_2$

Loop x_1-x_2-x_1 and x_5-x_6-x_5 $\quad L_{13} = (G_1H_1)(G_5H_3) = G_1H_1G_5H_3$

Loop x_3-x_4-x_3 and x_5-x_6-x_5 $\quad L_{23} = (G_3H_2)(G_5H_3) = G_3G_5H_2H_3$

Loop x_1-x_2-x_1 and x_4-x_5-x_6-x_4 $\quad L_{14} = (G_1H_1)(G_4G_5H_4) = G_1H_1G_4G_5H_4$

Loop x_1-x_2-x_1 and x_4-x_6-x_4 $\quad L_{15} = (G_1H_1)(G_7H_4) = G_1G_7H_1H_4$

Combinations of three non-touching loops and the product of gains associated with them are as follows:

Loop L_1, L_2 and L_3 $\quad L_{123} = (G_1H_1)(G_3H_2)(G_5H_3)$

$$= G_1H_1G_3H_2G_5H_3$$

Since all the loops are touching all the forward paths, $\Delta_1 = 1$, $\Delta_2 = 1$, $\Delta_3 = 1$, and $\Delta_4 = 1$. The determinant of the signal flow graph is given by

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_{12} + L_{13} + L_{14} + L_{15} + L_{23}) - (L_{123})$$

Therefore, using Mason's Gain Formula, the closed-loop transfer function $\frac{C}{R}$ is

$$\frac{C}{R} = \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3 + M_4\Delta_4}{\Delta}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_6G_3G_4G_5 + G_1G_2G_3G_7 + G_6G_3G_7}{\Delta}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_6G_3G_4G_5 + G_1G_2G_3G_7 + G_6G_3G_7}{1-(G_1H_1 + G_3H_2 + G_5H_3 + G_4G_5H_4 + G_7H_4) + (G_1H_1G_3H_2 + G_1H_1G_5H_3 + G_3G_5H_2H_3 + G_1H_1G_4G_5H_4 + G_1G_7H_1H_4) - G_1H_1G_3H_2G_5H_3}$$

Q.5 (d) Solution:

Given,

Output at satellite antenna, $V = 5\ \mu\text{V}$ (rms)

frequency, $f = 1\ \text{GHz}$

Distance between ground station and satellite, $d = 41935$ km

Gain of transmitting antenna,

$$G_t = 25\ \text{dBi}$$

Gain of receiving antenna,

$$G_r = 15\ \text{dBi}$$

$$R = 50\ \Omega$$

$\therefore$ Power received at satellite antenna, $P_R = \frac{V^2}{R}$

$$= \frac{(5\times10^{-6})^2}{50}$$

$$P_R = 5 \times 10^{-13}$$

In dBm,
$$P_{R(\text{dBm})} = 10\log\left[\frac{P_R}{1\text{ mW}}\right]$$

$$= 10\log\left[\frac{5\times10^{-13}}{10^{-3}}\right]$$

$$P_{R(\text{dBm})} = -93 \text{ dBm}$$

From the Friis transmission equation (in dBm)

$$10\log\left(\frac{P_R}{P_t}\right) = -32.44 - 20\log(f_{\text{in MHz}}) - 20\log(d_{\text{in km}})$$

$$= -32.44 - 20\log(1000) - 20\log(41935)$$

$$= -185 \text{ dB}$$

$\therefore$
$$P_{R(\text{dBm})} = P_{t(\text{dBm})} + G_{t(\text{dBi})} - 185 + G_{r(\text{dBi})}$$

$$-93 \text{ dBm} = P_t + 25 - 185 + 15$$

$\therefore$
$$P_{t(\text{dBm})} = 52 \text{ dBm}$$

$$P_t = 10^{5.2} \text{ mW} = 158.5 \text{ Watts}$$

Q.5 (e) Solution:

Given, radiation intensity of antenna,

$$U(\theta,\phi) = \begin{cases} 1 & ; \quad 0° < \theta < 20° \\ 0.342\,\text{cosec}(\theta) & ; \quad 20° \le \theta < 60° \\ 0 & ; \quad 60° \le \theta \le 180° \end{cases}$$

also,
$$0° \le \phi \le 360°$$

Radiated power,
$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

$$= 2\pi\left[\int_0^{20°}\sin\theta\, d\theta + \int_{20°}^{60°} 0.342\,\text{cosec}(\theta)\times\sin\theta\, d\theta\right]$$

$$= 2\pi\left\{[-\cos\theta]_0^{\pi/9} + 0.342\,[\theta]_{\pi/9}^{\pi/3}\right\}$$

$$= 2\pi\left\{\left[-\cos\left(\frac{\pi}{9}\right)+1\right]+0.342\left(\frac{\pi}{3}-\frac{\pi}{9}\right)\right\}$$

$$= 2\pi\left\{(-0.93969+1)+0.342\pi\left(\frac{2}{9}\right)\right\}$$

$$= 2\pi\{0.06031 + 0.23876\} = 1.87912$$

$\therefore$ Directivity, $D = \dfrac{4\pi U_{max}}{P_{rad}} = \dfrac{4\pi\times(1)}{1.87912} = 6.68737$

In decibels, $D = 10\log_{10}6.68737 = 8.25255$ dB

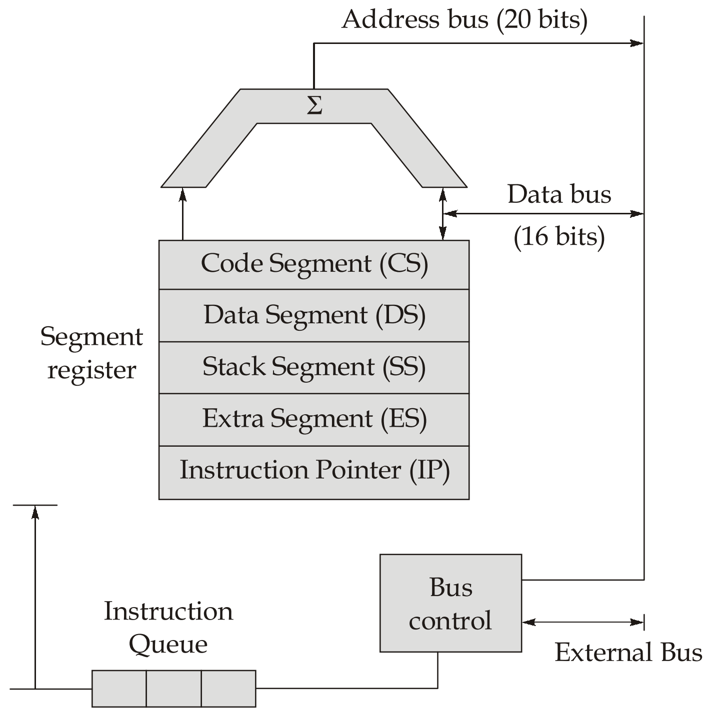
Q.6 (a) Solution:

(i)

0400 H	MOV AX, 2000 H	:	Moves base address 2000H in AX register
0403 H	MOV DS, AX	:	Loads data segment register to 2000 H
0405 H	MOV SI, 500 H	:	Set the value of SI to 500
0408 H	MOV CL, [SI]	:	Load data from offset SI to register CL. It works as a counter.
040A H	MOV CH, 00 H	:	Set the value of register CH to 00
040C H	INC SI	:	Increase the value of SI by 1
040D H	MOV AL, [SI]	:	Load value from offset SI to register AL (Load the data from memory)
040F H	DEC CL	:	Decrease value of register CL by 1
0411 H	INC SI	:	Increase the value of SI by 1
0412 H	CMP AL, [SI]	:	Compare value of register AL and SI (AL - [SI])
0414 H	JNC 0413 H	:	Jump to address 0413 H if carry not generated
0417 H	MOV AL, [SI]	:	Transfer data from offset SI to register AL if [SI] > AL
0419 H	INC [SI]	:	Increase value of SI by 1 i.e. update the pointer to next memory location.
041A H	LOOP 040D H	:	Decrease value of register CX by 1 and jump to address 040D H if value of register CX is not zero
041D H	MOV [600 H], AL	:	Store the value of register AL to offset 600 H
0420 H	HLT	:	STOP

(ii) Bus Interface Unit (BIU):

8086 Microprocessor is divided into two functional units, i.e., EU (Execution Unit) and BIU (Bus Interface Unit). The BIU takes care of all data and addresses transfers on the buses for the Execution Unit (EU) like sending addresses, fetching instructions from the memory, reading data from the ports and the memory as well as writing data to the ports and the memory. Execution unit has no direct connection with system buses, so this is possible with the help of Bus Interface Unit (BIU). EU and BIU are connected with the internal bus.



BIU has the following functional parts:

- **Instruction Queue:** BIU contains instruction queue which is used to store upto 6 bytes of next instructions. When EU executes instructions and is ready for its next instruction, then it simply reads the instruction from instruction queue so that the execution speed is increased.
- **Segment Registers:** BIU has 4 segment registers i.e., CS, DS, SS and ES. It holds the address of instructions and data in memory , which are used by the processor to access memory locations. The segment registers holds the base address of the memory segment.

1. **Code Segment (CS):** It is used for addressing a memory location in the code segment of the memory, where the executable program is stored.
2. **Data segment (DS):** The data segment consists of data used by the program and is accessed by the data segment register and an offset address or the content of other register that holds the offset address.
3. **Stack Segment (SS):** It handles stack memory used to store data and addresses during execution.
4. **Extra Segment (ES):** It is additional data segment which is used by the string instructions to hold the extra destination data.

- **Instruction Pointer:** It is a 16-bit register used to hold the address of the next instruction to be executed.
- The bus control logic of the BIU generates all the bus control signals such as read and write signals for memory and I/O.
- The BIU has a Physical Address Generation Circuit. It generates the 20-bit physical address using Segment and Offset addresses using the formula:

$$\text{Physical Address} = \text{Segment Address} \times 10\text{H} + \text{Offset Address}$$

Q.6 (b) Solution:

(i) This pipeline shows combined operation:

$$A_i \times B_i + C_i$$

The operations performed can be grouped into 3 segments each performing a sub-operation as shown:

S-1 $R_1 \leftarrow A_i\,;\, R_2 \leftarrow B_i$

S-2 $R_3 \leftarrow R_1 \times R_2\,;\, R_4 \leftarrow C_i$

S-3 $R_5 \leftarrow R_3 + R_4$

$$\therefore \qquad K = 3$$

Segment 1 takes = 40 nsec

Segment 2 takes = 45 + 5 = 50 nsec

Segment 3 takes = 15 nsec

$\therefore$ Minimum clock cycle time = Max (Seg1, Seg2, Seg3)

$$= \text{Max}[40, 50, 15]$$

$$t_p = 50 \text{ nsec}$$

(ii) Without pipeline, the operations are performed sequentially. The total time will be the sum of propagation times. Thus,

$$t_n = 40 + 45 + 5 + 15$$

$$t_n = 105 \text{ nsec}$$

$\therefore$ If there is no pipeline, then it will take 105 nsec.

(iii) Speedup for n tasks

$$S = \frac{n \times t_n}{[k+(n-1)]t_p}$$

$\therefore$ For $n = 10$,

$$S = \frac{10 \times 105}{(3+9)\times 50}$$

$$S = 1.75$$

(iv) Maximum speedup,

$$S_{max} = \frac{t_n}{t_p} = \frac{105}{50}$$

$$S_{max} = 2.1$$

Q.6 (c) Solution:

For the given open-loop transfer function $G(s)H(s)$:

The open-loop poles are at $s = 0$, $s = -6$, $s = \frac{-4 \pm \sqrt{16-52}}{2} = -2 \pm j3$. Therefore, $n = 4$.

There are no open-loop zeros. Therefore, $m = 0$.

Hence, the number of branches of root locus is $n = 4$ and the number of asymptotes is $n - m = 4 - 0 = 4$.

The complete root locus is drawn as shown in figure, as per the rules given as follows:

1. The root locus will be symmetrical about the real axis because the pole-zero location is symmetrical with respect to the real axis.
2. The four branches of the root locus start at the open-loop poles $s = 0$, $s = -6$, $s = -2 + j3$ and $s = -2 - j3$, where $K = 0$ and terminate at the open-loop zeros at infinity, where $K = \infty$.
3. The four branches of the root locus go to the zeros at infinity along asymptotes making angles of $\theta_q = \frac{(2q+1)\pi}{n-m}$, $q = 0, 1, 2, 3$ with the real axis, i.e.,

$$\theta_0 = \frac{\pi}{4}, \theta_1 = \frac{3\pi}{4}, \theta_2 = \frac{5\pi}{4}, \theta_3 = \frac{7\pi}{4}$$

4. The point of intersection of the asymptotes on the real axis (centroid) is given by

$$\sigma = \frac{\text{sum of real parts of poles – sum of real parts of zeros}}{\text{number of poles – number of zeros}}$$

$$= \frac{(0-6-2-2)-(0)}{4-0} = -2.5$$

5. The root locus exists on the real axis where there is an odd number of open-loop poles and zeros to the right i.e. from $s = 0$ to $s = -6$.

6. The break points are given by the solution of the equation $\frac{dK}{ds} = 0.$

We have, $$1 + G(s)H(s) = \frac{K}{s(s+6)(s^2+4s+13)} + 1 = 0$$

$$\therefore \quad K = s^4 + 10s^3 + 37s^2 + 78s$$

$$\frac{dK}{ds} = 4s^3 + 30s^2 + 74s + 78 = 0$$

i.e., $s^3 + 7.5s^2 + 18.5s + 19.5 = 0$

i.e., $(s + 4.2)(s + 1.65 + j1.39)(s + 1.65 - j1.39) = 0$

Therefore, the break points are $s = -4.2$, $s = -1.65 + j1.39$ and $s = -1.65 - j1.39$

Out of these three points, the actual break point is $s = -4.2$, because this point lies on the root locus. The other two are not actual break points, because the root locus does not exist there. They can be ignored.

The break angles at $s = -4.1$ are

$$\pm\frac{\pi}{r} = \pm\frac{180°}{2} = \pm 90°$$

7. The angle of departure of the root locus branch from the pole at $s = -2 + j3$ is given by

$$\theta_d = \pi + \phi$$

where $$\phi = -(\theta_1 + \theta_2 + \theta_3) = -\left(180° - \tan^{-1}\frac{3}{2} + 90° + \tan^{-1}\frac{3}{4}\right)$$

$$= -(123° + 90° + 37°) = -250°$$

$$\therefore \quad \theta_d = 180° - 250° = -70°$$

Therefore, the angle of departure of the root locus branch from the pole at $s = -2 - j3$ is $\theta_d = +70°$.

8. The point of intersection of the root locus with the imaginary axis and the critical value of K are obtained using the Routh Hurwitz criterion. The characteristic equation is

$$1 + G(s)H(s) = 0$$

i.e., $$1+\frac{K}{s(s+6)(s^2+4s+13)} = 0$$

i.e., $s^4 + 10s^3 + 37s^2 + 78s + K = 0$

The Routh table is as follows:

s^4	1	37	K
s^3	10	78	
s^2	$\frac{370-78}{10} = 29.2$	K	
s^1	$\frac{29.2\times78-10K}{29.2}$		
s^0	K		

For stability, all the elements in the first column of the Routh array must be positive.

Therefore, $K > 0$

and $29.2 \times 78 - 10K > 0$

i.e., $$K < \frac{29.2\times78}{10} = 227.76$$

So, the range of values of K for stability is $0 < K < 227.76$.

The marginal value of K for stability is $K_m = 227.76$. For $K > 227.76$, the system has two closed-loop poles in the right half of the s-plane and is thus unstable.

For that value of K, the frequency of sustained oscillations is given by the solution of the auxiliary equation,

$$29.2s^2 + K = 0$$

i.e., $$29.2s^2 + K_m = 0$$

i.e., $$29.2s^2 + 227.76 = 0$$

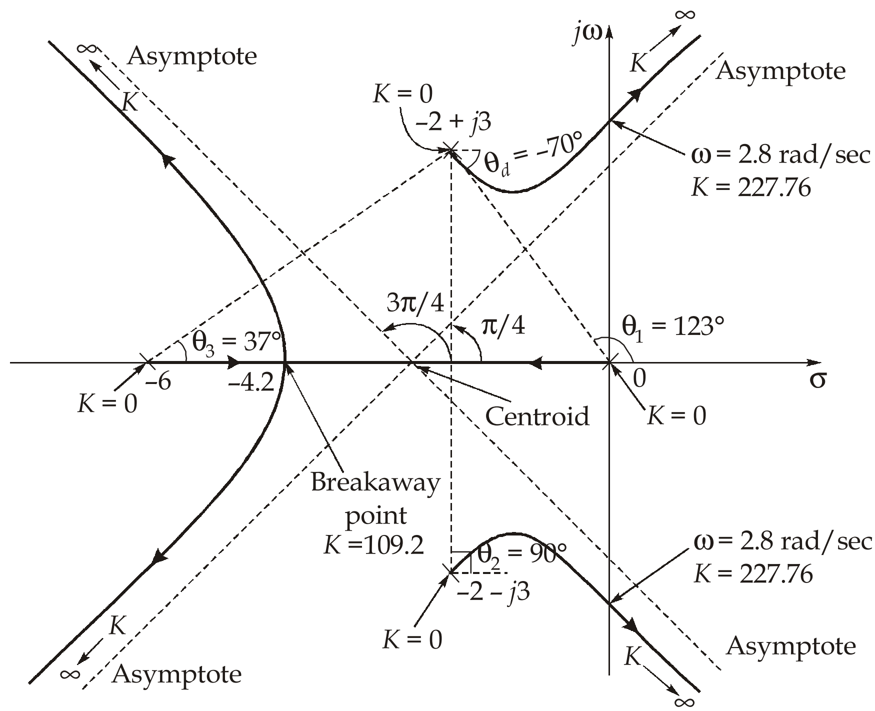
$\therefore$ $$s^2 = -\frac{227.76}{29.2} = -7.8$$

or $$s = \pm j2.8$$

$\therefore$ $$\omega = 2.8 \text{ rad/s}$$

Hence, the frequency of sustained oscillations is $\omega = 2.8$ rad/s.

The complete root locus is drawn as shown in figure.



Q.7 (a) Solution:

(i) Given,

$$m(t) = e^{-t^2/100}$$

$$f_c = 10^4 \text{ Hz}$$

$$k_f = 500\pi \text{ rad/sec/V and } k_p = 1.2\pi \text{ rad/V}$$

For FM:

Frequency deviation, $\Delta f = k_f A_m$

where, $A_m = m(t)|_{max} = 1$

$$\therefore \quad \Delta f = \left(\frac{500\pi}{2\pi} \text{Hz/V}\right) \times 1 \text{ V} = 250 \text{ Hz}$$

For PM:

$$\theta(t) = k_p m(t)$$

$$\text{Frequency deviation } \Delta f = \left|\frac{1}{2\pi}\frac{d}{dt}\theta(t)\right|_{max} = \frac{k_p\left|\frac{d}{dt}m(t)\right|_{max}}{2\pi}$$

We have,

$$\frac{d}{dt}m(t) = \frac{d}{dt}\left[e^{-t^2/100}\right] = \frac{-t}{50}e^{-t^2/100}$$

For the peak value of $\frac{d}{dt}m(t)$, let $\dot{m}_p$, we need to set $\frac{d^2}{dt^2}m(t) = 0$

$$\Rightarrow \quad \frac{d^2}{dt^2}\left[\frac{-t}{50}e^{-t^2/100}\right] = 0$$

$$\Rightarrow \quad \frac{-1}{50}e^{-t^2/100} + \frac{2t^2}{5000}e^{-t^2/100} = 0$$

$$\Rightarrow \quad \frac{1}{50} = \frac{2t^2}{5000}$$

$$\therefore \quad t^2 = 50 \quad \Rightarrow \quad t = \sqrt{50}\ \text{sec}$$

Thus,

$$\dot{m}_p = \left|\frac{d}{dt}m\left(\sqrt{50}\right)\right| = \left|\frac{-\sqrt{50}}{50}e^{-50/100}\right|$$

$$\therefore \quad \dot{m}_p = 0.0858$$

and frequency deviation,

$$\Delta f = \frac{1.2\pi \times 0.0858}{2\pi}$$

$$\therefore \quad \Delta f = 0.0515\ \text{Hz}$$

(ii) Given,

$$\text{SNR} = 30\ \text{dB}$$

$$n = 10\ \text{bits/sample}$$

$$\text{Desired SNR} = 42\ \text{dB}$$

Signal to noise ratio of PCM is given as,

$$\text{SNR} = (1.8 + 6n)\ \text{dB}$$

Thus, with increase in n by 1-bit, the value of SNR is increases by 6 dB.

∴ For increase in value of SNR by 12 dB, it is necessary to increase n by 2.

∴ The new value of n is 12-bit.

Bandwidth (BW) of PCM system,

$$BW = \frac{R_b}{2} = \frac{1}{2}nf_s$$

∴ BW with n = 10 is $\text{BW}_{10} = \frac{1}{2}\times 10 \times f_s = 5f_s$

and BW with n = 12 is $\text{BW}_{12} = \frac{1}{2}\times 12 \times f_s = 6f_s$

$\therefore$ The change in BW = $\Delta BW = 6f_s - 5f_s = f_s$

Thus, fractional change in BW = $\dfrac{\Delta BW}{BW_{10}} \times 100\%$

$$= \frac{f_s}{5f_s} \times 100\% = 20\%$$

Q.7 (b) Solution:

Comparing the given angle modulated signal with

$$\phi(t) = A_c \cos(\omega_c t + m(t))$$

We get, $m(t) = 5 \sin 3000t + 10 \sin 2000\pi t$

The signal bandwidth is the highest frequency in $m(t)$ (or its derivative).

In this case, $B = \dfrac{2000\pi}{2\pi} = 1000 \text{ Hz}$

(i) The carrier amplitude is $A_c = 10$ and the power is $P = \dfrac{10^2}{2} = 50 \text{ W}$.

(ii) Instantaneous frequency,

$$\omega_i = \frac{d[\theta(t)]}{dt} = \frac{d}{dt}[\omega_c t + 5\sin 3000t + 10\sin 2000\pi t]$$

$$\omega_i = \omega_c + 15000 \cos(3000t) + 20{,}000\pi \cos(2000\pi t)$$

The phase deviation is $15000 \cos(3000t) + 20000\pi \cos(2000\pi t)$. The two sinusoids will add in phase at some point, and the maximum value of this expression is $15000 + 20000\pi$.

$\therefore$ frequency deviation, $\Delta f = \dfrac{\Delta\omega}{2\pi}$

$$= \frac{20000\pi + 15000}{2\pi} = 10000 + \frac{7500}{\pi}$$

$$\Delta f = 12387.32 \text{ Hz}$$

(iii) $$\beta = \frac{\Delta f}{B} = \frac{12387.32}{1000} = 12.387$$

(iv) $$B_{\phi(t)} = 2(\Delta f + B)$$

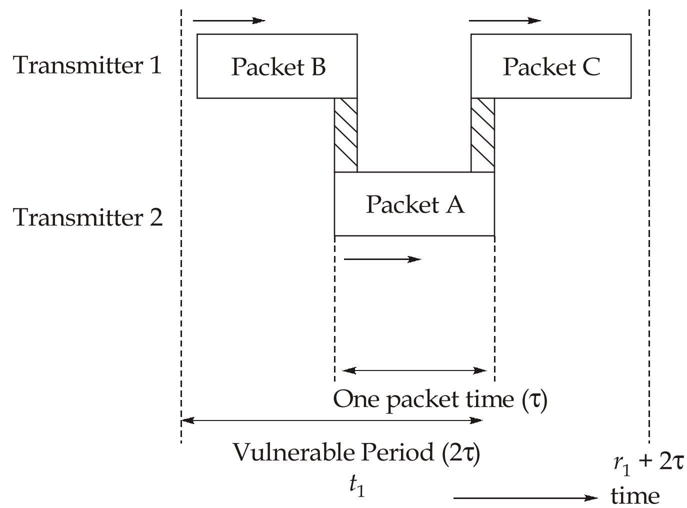
$$= 2(12387.32 + 1000)$$

$$= 26774.65 \text{ Hz}$$

Q.7 (c) Solution:

(i) **Pure ALOHA:** The pure ALOHA protocol is a random access protocol used for data transfer. In ALOHA protocol, a user accesses a channel as soon as a message is ready to be transmitted. After a transmission, the user waits for an acknowledgement on either the same channel or a separate feedback channel. In case of collisions (i.e., when a NACK is received), the terminal waits for a random period of time and retransmits the message. As the number of users increase, a greater delay occurs because the probability of collision increases.

For ALOHA protocol, the vulnerable period i.e. the time interval during which the packets are susceptible to collisions with transmissions from other users is double the packet duration as shown in the figure below and the probability of no collision during the vulnerable period is e^{-2R}, where R is the average number of packets generated by the system in one packet time.



The throughput of the ALOHA protocol is

$$T = Re^{-2R}$$

Slotted ALOHA: In slotted ALOHA, time is divided into equal time slots of length greater than packet duration τ. The subscribers each have synchronized clocks and transmit a message only at the beginning of a new time slot, thus resulting in a discrete distribution of packets. This prevents partial collisions, where one packet collides with a portion of another. As the number of users increase, a greater delay will occur due to complete collisions resulting in the repeated transmissions of those packets originally lost. The number of slots which a transmitter waits prior to retransmitting also determines the delay characteristics of the traffic.

The vulnerable period for slotted ALOHA is only one packet duration, since partial collisions are prevented through synchronization. The probability that no other packets will be generated during the vulnerable period is e^{-R}.

The throughput for the case of slotted ALOHA is thus given by

$$T = Re^{-R}$$

(ii) ALOHA: Consider the packet arrival has a Poisson distribution with an average number of R arrivals/X seconds. The probability of a successful transmission is the probability that they are no additional packet transmission in the vulnerable period ($2X$).

For a poison distribution, the number of packets transmitted in time interval 't' is given by

$$P_k = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

where λ is the average rate of packet transmission (packets per unit of time).

Hence, Throughput (T) = $R.P$(0 transmissions in $2X$ seconds)

$$= R.[(2R)^0/0!].e^{-2R} = Re^{-2R}.$$

The rate of arrival which maximizes the throughput for ALOHA is found by taking the derivative of $T = Re^{-2R}$ and equating it to zero.

$$\therefore \quad \frac{d}{dR}(T) = \frac{d}{dR}(Re^{-2R})$$

$$\frac{dT}{dR} = Re^{-2R}(-2) + e^{-2R} = 0$$

$$R = \frac{1}{2}$$

$$\therefore \quad R_{max} = \frac{1}{2}$$

Maximum throughput achieved by using the ALOHA protocol is found by substituting R_{max} in $T = Re^{-2R}$, and this value can be calculated as

$$T = Re^{-2R}\Big|_{R=1/2}$$

$$T = \frac{1}{2}e^{-1}$$

$$T = 0.1839$$

Thus, the best traffic utilization one can hope for using ALOHA is 0.184 Erlangs.

Slotted ALOHA: In slotted ALOHA, the vulnerable period is X. Hence,

Throughput (T) = $R.P$(no arrival in X seconds) = $R.((R)^0/0!)e^{-R} = Re^{-R}$

The maximum throughput for slotted ALOHA is found by taking the derivative of $T = Re^{-R}$ and equating it to zero, thus

$$\frac{dT}{dR} = \frac{d}{dR}(\text{Re}^{-R})$$

$$\frac{dT}{dR} = Re^{-R}(-1) + e^{-R} = 0$$

$$R = 1$$

$$\therefore \quad R_{max} = 1$$

Maximum throughput is found by substituting R_{max} in $T = Re^{-R}$ and this value can be calculated as

$$T = 1 \cdot e^{-1}$$

$$= e^{-1}$$

$$T = 0.3679$$

Notice that slotted ALOHA provides a maximum channel utilization of 0.368 Erlangs, double that of ALOHA.

Q.8 (a) Solution:

(i) The positional error constant,

$$K_p = \underset{s\to 0}{\text{Lt}}\, G(s)$$

$$= \underset{s\to 0}{\text{Lt}} \frac{10}{s(0.1s+1)} = \infty$$

Therefore, the steady-state error for a unit-step input is

$$e_{ss}(t) = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

The velocity error constant,

$$K_v = \underset{s\to 0}{\text{Lt}}\, sG(s)$$

$$= \underset{s\to 0}{\text{Lt}}\, s \times \frac{10}{s(0.1s+1)} = 10$$

Therefore, the steady-state error for a unit-ramp input is

$$e_{ss}(t) = \frac{1}{K_v} = \frac{1}{10} = 0.1$$

The acceleration error constant

$$K_a = \underset{s\to 0}{\text{Lt}}\, s^2 G(s)$$

$$= \underset{s\to 0}{\text{Lt}}\, s^2 \times \frac{10}{s(0.1s+1)} = 0$$

Therefore, the steady-state error for a unit-parabolic input $(t^2/2)\,u(t)$ is

$$e_{ss}(t) = \frac{1}{K_a} = \frac{1}{0} = \infty$$

(ii) For a linear system, we can apply the principle of superposition. Therefore, the steady-state error for an input

$$r(t) = \left(a_0 + a_1 t + a_2 \frac{t^2}{2}\right) u(t)$$

$= a_0 \times$ (steady-state error for a unit-step input) +
$a_1 \times$ (steady-state error for a unit-ramp input) +
$a_2 \times$ (steady-state error for a unit-parabolic input)

$$= a_0 \times 0 + a_1 \times (0.1) + a_2 \times (\infty) = 0 + 0.1a_1 + \infty = \infty$$

For the given unity feedback system,

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = \frac{1}{1+\dfrac{10}{s(0.1s+1)}} = \frac{0.1s^2+s}{0.1s^2+s+10}$$

For expanding it into an infinite series in ascending powers of s, write the numerator and denominator polynomials in ascending powers of s, i.e.,

$$\frac{E(s)}{R(s)} = \frac{s+0.1s^2}{10+s+0.1s^2} = \frac{s}{10} - \frac{s^3}{1000} + \frac{s^4}{10000} \ldots\ldots$$

$$\begin{array}{r|l} & \dfrac{s}{10} - \dfrac{s^3}{1000} + \dfrac{s^4}{10000} \ldots \\ \hline 10 + s + 0.1s^2 & s + 0.1s^2 \\ & \underline{s + 0.1s^2 + 0.01s^3} \\ & -0.01s^3 \\ & \underline{-0.01s^3 - 0.001s^4 - 0.0001s^5} \\ & 0.001s^4 - 0.0001s^5 \end{array}$$

Equating it with

$$\frac{E(s)}{R(s)} = C_0 + C_1 s + \frac{C_2}{2!}s^2 + \frac{C_3}{3!}s^3 + \ldots$$

The dynamic error coefficients are as follows:

$$C_0 = 0,\ C_1 = \frac{1}{10} = 0.1,\ C_2 = 0,\ C_3 = \frac{1}{1000}$$

Therefore, the dynamic error for input $r(t)$ is

$$e(t) = C_0 r(t) + C_1 \frac{dr(t)}{dt} + \frac{C_2}{2!}\frac{d^2 r(t)}{dt^2} + \frac{C_3}{3!}\frac{d^3 r(t)}{dt^3} + \dots$$

$$e(t) = 0 + \frac{1}{10}\frac{d}{dt}\left(a_0 + a_1 t + a_2 \frac{t^2}{2}\right) + 0 + \frac{\left(\frac{1}{1000}\right)}{3!}\frac{d^3}{dt^3}\left[a_0 + a_1 t + a_2\frac{t^2}{2}\right]$$

$e(t) = 0.1[a_1 + a_2 t]$

Therefore, the steady-state error

$$e_{ss}(t) = \underset{t\to\infty}{\text{Lt}}\ e(t) = \underset{t\to\infty}{\text{Lt}}\ 0.1(a_1 + a_2 t) = \infty$$

Q.8 (b) Solution:

(i) The channel matrix :

$$P\left[\frac{Y}{X}\right] = \begin{bmatrix} 0.2 & 0.4 & 0.4 \\ 0.4 & 0.2 & 0.4 \\ 0.4 & 0.4 & 0.2 \end{bmatrix}$$

We know for symmetric channel ;

$$\text{Channel capacity 'C'} = \log_2 M - \left[\text{Entropy of any row of } P\left[\frac{Y}{X}\right]\right]$$

$$M = \text{Number of output symbols}$$

$$\therefore \text{ Entropy of any row of } P\left[\frac{Y}{X}\right] = -\sum_{i=1}^{3} P_i \log_2 \frac{1}{P_i}$$

$$= 0.2\log_2\frac{1}{0.2} + 0.4\times 2\times\log_2\frac{1}{0.4} = 1.52 \text{ bits/symbol}$$

$$\therefore \quad \text{Channel capacity 'C'} = \log_2 3 - 1.52$$

$$= 0.065 \text{ bits/symbol}$$

(ii) We know, $1\ Np = 8.686$ dB

$$\alpha = \frac{8}{8.686} = 0.921 \text{ Np/m}$$

$$\gamma = \alpha + j\beta = 0.921 + j\ (\text{meter})^{-1}$$

$$\gamma\cdot l = 2[0.921 + j] = 1.84 + j2$$

Now, for lossy transmission line, the input impedance is given as ;

$$Z_{in} = Z_o\left[\frac{Z_L + Z_o \tanh\gamma l}{Z_o + Z_L \tanh\gamma l}\right]$$

$\because$ $\tan h\gamma l = \tanh[1.84 + j2] = 1.033 - j0.03929$

$\therefore$ $$Z_{in} = (60+j40)\left[\frac{(20+j50)+(60+j40)(1.033-j0.03929)}{(60+j40)+(20+j50)(1.033-j0.03929)}\right]$$
$$= 60.25 + j38.79\ \Omega$$

Remember: $$\tanh(x \pm iy) = \frac{\sin h2x}{\cos h2x + \cos 2y} \pm i\frac{\sin 2y}{\cos h2x + \cos 2y}$$

Q.8 (c) Solution:

(i) Using partial fraction expansion, $H(s)$ can be written as

$$H(s) = \frac{1}{(s+0.5)(s^2+0.5s+2)} = \frac{A_1}{s+0.5} + \frac{A_2 s + A_3}{s^2+0.5s+2}$$

Therefore, $A_1(s^2 + 0.5s + 2) + (A_2 s + A_3)(s + 0.5) = 1$

Comparing the coefficients of s^2, s and the constants on either side of the above expression, we get

$$A_1 + A_2 = 0$$
$$0.5A_1 + 0.5A_2 + A_3 = 0$$
$$2A_1 + 0.5A_3 = 1$$

Solving the above simultaneous equations, we get $A_1 = 0.5$, $A_2 = -0.5$ and $A_3 = 0$.
The system response can be written as,

$$H(s) = \frac{0.5}{s+0.5} - \frac{0.5s}{s^2+0.5s+2}$$
$$= \frac{0.5}{s+0.5} - 0.5\left(\frac{s}{(s+0.25)^2+(1.3919)^2}\right)$$
$$= \frac{0.5}{s+0.5} - 0.5\left(\frac{s+0.25}{(s+0.25)^2+(1.3919)^2} - \frac{0.25}{(s+0.25)^2+(1.3919)^2}\right)$$

Using impulse invariant transformation,

$$\frac{s+a}{(s+a)^2+b^2} \xrightarrow{\text{(is transformed to)}} \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$\frac{b}{(s+a)^2+b^2} \xrightarrow{\text{(is transformed to)}} \frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$\frac{A_i}{s+p_i} \xrightarrow{\text{(is transformed to)}} \frac{A_i}{1-e^{-p_iT}z^{-1}}$$

$$H(s) = \frac{0.5}{s+0.5} - 0.5\left(\frac{s+0.25}{(s+0.25)^2+(1.3919)^2}\right) + 0.0898\left(\frac{1.3919}{(s+0.25)^2+(1.3919)^2}\right)$$

We get, $H(z) = \dfrac{0.5}{1-e^{-0.5T}z^{-1}} - 0.5\left[\dfrac{1-e^{-0.25T}(\cos 1.3919T)z^{-1}}{1-2e^{-0.25T}(\cos 1.3919T)z^{-1}+e^{-0.5T}z^{-2}}\right]$

$$+0.0898\left[\frac{e^{-0.25T}(\sin 1.3919T)z^{-1}}{1-2e^{-0.25T}(\cos 1.3919T)z^{-1}+e^{-0.5T}z^{-2}}\right]$$

Assume $T = 1$ s,

$$H(z) = \frac{0.5}{1-0.6065z^{-1}} - 0.5\left(\frac{1-0.1385z^{-1}}{1-0.277z^{-1}+0.6065z^{-2}}\right)$$

$$+0.0898\left[\frac{0.7663z^{-1}}{1-0.277z^{-1}+0.6065z^{-2}}\right]$$

(ii) From the fact 2,

$$F^{-1}[(1+j\omega)X(\omega)] = Ae^{-2t}u(t)$$

By taking Fourier transform,

$$(1+j\omega)X(\omega) = \frac{A}{2+j\omega}$$

$$X(\omega) = \frac{A}{(1+j\omega)(2+j\omega)}$$

$$= A\left[\frac{A'}{1+j\omega}+\frac{B'}{2+j\omega}\right]$$

where $$A' = \left.\frac{1}{2+j\omega}\right|_{j\omega=-1} = 1$$

$$B' = \left.\frac{1}{1+j\omega}\right|_{j\omega=-2} = -1$$

$\therefore$ $$X(\omega) = A\left(\frac{1}{1+j\omega}-\frac{1}{2+j\omega}\right)$$

By taking inverse Fourier transform of the above $X(\omega)$, we get

$$x(t) = Ae^{-t}u(t) - Ae^{-2t}u(t)$$

Using Parseval's theorem,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

$$\therefore \quad \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \times 2\pi = 1$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} \left|Ae^{-t}u(t) - Ae^{-2t}u(t)\right|^2 dt = 1$$

$$\int_{-\infty}^{\infty} \left[A^2e^{-2t} - 2A^2e^{-3t} + A^2e^{-4t}\right]u(t)dt = 1$$

$$\int_{0}^{\infty} \left[A^2e^{-2t} - 2A^2e^{-3t} + A^2e^{-4t}\right]dt = 1$$

$$\frac{A^2}{-2}\left[e^{-2t}\right]_0^{\infty} - \frac{2A^2}{(-3)}\left[e^{-3t}\right]_0^{\infty} + \frac{A^2}{(-4)}\left[e^{-4t}\right]_0^{\infty} = 1$$

$$\frac{A^2}{2} - \frac{2}{3}A^2 + \frac{A^2}{4} = 1$$

$$6A^2 - 8A^2 + 3A^2 = 12$$

$$A^2 = 12$$

$$\therefore \quad A = \pm\sqrt{12}$$

Since from the fact 1, $x(t)$ is non-negative

$$\therefore \quad A = \sqrt{12} = 2\sqrt{3}$$

Thus, Closed form of $x(t) = 2\sqrt{3}\left[e^{-t} - e^{-2t}\right]u(t)$

❍❍❍❍