Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Electrical Engineering Test No : 8

Section-A

Q.1 (a) Solution:

Quantity of water available for utilization per second

$$= \frac{\text{Catchment area in km}^2 \times 10^6 \times \text{average rainfall in m} \times \text{yield factor}}{365 \times 24 \times 60 \times 60}$$

$$= \frac{500 \times 10^6 \times 1.30 \times 0.80}{8760 \times 3600} = 16.489 \text{ m}^3$$

Available head, $H = 30$ m

Overall efficiency of power plant

η = Penstock efficiency × Turbine efficiency × Generator efficiency

$\eta = 0.97 \times 0.87 \times 0.92$

$\eta = 0.7764$

Average power, $P = WQH \times 9.81 \times \eta \times 10^{-6}$ MW

$= 1000 \times 16.489 \times 30 \times 0.7764 \times 10^{-6} \times 9.81$

$P = 3.767$ MW

So, capacity of power station

$$P_{capacity} = \frac{P}{\text{Load factor}} = \frac{3.767}{0.60}$$

$P_{capacity} = 6.27$ MW

Q.1 (b) Solution:

(i) Using equation,

$$P_R = \frac{|V_s||V_r|}{|B|}\cos(\beta - \delta) - \frac{|A||V_r|^2}{|B|}\cos(\beta - \alpha)$$

$$50 \times 0.8 = \frac{132 \times 132}{110}\cos(\beta - \delta) - \frac{0.98 \times 132^2}{110}\cos(75° - 3°)$$

$$40 = 158.4 \cos(\beta - \delta) - 47.97$$

(or) $$(\beta - \delta) = 56.26°$$

Using equation:

$$Q_R = \frac{|V_S||V_R|}{|B|}\sin(\beta - \delta) - \frac{|A||V_R|^2}{|B|}\sin(\beta - \alpha)$$

$$Q_R = \frac{132 \times 132}{110}\sin(56.26°) - \frac{(0.98)(132)^2}{110}\sin(75° - 3°)$$

$$Q_R = -15.91 \text{ MVAR}$$

Thus for the given operating conditions, a leading MVAR of 15.91 must be drawn from the line along with real power of 40 MW. Since the load requires 50 × 0.6 i.e., 30 MVAR lagging, the static capacitors must deliver (30 + 15.91) i.e. (45.91) MVAR lagging.

The capacity of shunt compensation equipment is therefore 45.91 MVAR.

(ii) Using equation,

$$P_R = \frac{|V_S||V_R|}{|B|}\cos(\beta - \delta) - \frac{|A||V_R^2|}{|B|}\cos(\beta - \alpha)$$

$$0 = \frac{132 \times 132}{110}\cos(\beta - \delta) - \frac{0.98(132)^2}{110}\cos(75° - 3°)$$

$$0 = 158.4 \cos(\beta - \delta) - 47.97$$

$$(\beta - \delta) = 72.37°$$

$$Q_R = \frac{132 \times 132}{110}\sin(72.37°) - \frac{0.98(132^2)}{110}\sin 72°$$

$$= 3.33 \text{ MVAR}$$

Thus under no load condition, the line delivers 3.33 MVAR at the receiving end. This reactive power must be absorbed by shunt reactor at the receiving end.

Q.1 (c) Solution:

Given, $R_a = 0.5\ \Omega,\ V_s = 230$ V,

$f = 50$ Hz, $K\phi = 3$ V-sec

Load torque when operating in upward direction = 69 N-m

$$\alpha_1 = 15°$$

Load torque when operating in downward direction = 180 N-m

Electromagnetic torque T_{em} during upward motion,

$$T_{em1} = (K\phi)\, I_{a1}$$

$$\Rightarrow \quad 69 = 3 \times I_{a1}$$

$$\Rightarrow \quad I_{a1} = 23 \text{ A}$$

Also we know that, $V_{t1} = E_b + I_{a1}R_a$

$$\Rightarrow \quad \frac{2V_m}{\pi}\cos\alpha_1 = E_{b1} + (23 \times 0.5)$$

$$\Rightarrow \quad \frac{2 \times \sqrt{2} \times 230 \times \cos 15°}{\pi} = E_{b1} + (23 \times 0.5)$$

$$\Rightarrow \quad E_{b1} = 188.5169 \text{ V}$$

Electromagnetic torque during downward motion,

$$T_{em2} = (K\phi)\, I_{a2}$$

$$\Rightarrow \quad 180 = (K\phi)\, I_{a2}$$

$$\Rightarrow \quad I_{a2} = \frac{180}{3} = 60 \text{ A}$$

Now as direction of speed is reversed, E_b will become negative

$$V_{t2} = -E_{b2} + I_{a2}\, R_a$$

Since $E_b \propto N$ as speed is same in both the cases we take the magnitude of E_b as same in both cases.

$$\Rightarrow \quad \frac{2V_m}{\pi}\cos\alpha_2 = -188.5169 + (60 \times 0.5)$$

$$\frac{2 \times \sqrt{2} \times 230 \times \cos\alpha_2}{\pi} = -158.5169$$

On solving, $\alpha_2 = 140°$

During holding position speed is zero so back emf is zero

$$\therefore \quad V_3 = I_{a1}R_a$$

$$\Rightarrow \qquad \frac{2\sqrt{2}\times 230}{\pi}\times\cos\alpha_3 = 23 \times 0.5$$

On solving triggering angle, $\alpha_3 = 86.82°$

Q.1 (d) Solution:

By the definition of z-transform,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Given, $x(n) = \sin(2\omega_0 n)\, u[n]$

$$X(z) = \sum_{n=0}^{\infty}\sin(2\omega_0 n)z^{-n} = \sum_{n=0}^{\infty}\left(\frac{e^{j2\omega_0 n}-e^{-j2\omega_0 n}}{j2}\right)z^{-n}$$

$$= \frac{1}{j2}\sum_{n=0}^{\infty} e^{j2\omega_0 n}\cdot z^{-n} - \frac{1}{2j}\sum_{n=0}^{\infty} e^{-j2\omega_0 n}z^{-n}$$

$$= \frac{1}{j2}\sum_{n=0}^{\infty}\left(e^{j2\omega_0}\cdot z^{-1}\right)^n - \frac{1}{j2}\sum_{n=0}^{\infty}\left(e^{-j2\omega_0}\cdot z^{-1}\right)^n$$

$$= \frac{1}{j2}\left[\frac{1}{(1-e^{j2\omega_0}\cdot z^{-1}} - \frac{1}{(1-e^{-j2\omega_0}\cdot z^{-1}}\right]$$

$$= \frac{z}{j2}\left[\frac{1}{z-e^{j2\omega_0}} - \frac{1}{z-e^{-j2\omega_0}}\right] = \frac{z}{j2}\left[\frac{z-e^{-j2\omega_0}-z+e^{j2\omega_0}}{(z-e^{j2\omega_0})(z-e^{-j2\omega_0})}\right]$$

$$= \frac{z}{j2}\left[\frac{e^{j2\omega_0}-e^{-j2\omega_0}}{(z-e^{j2\omega_0})(z-e^{-j2\omega_0})}\right] = z\left[\frac{\dfrac{e^{j2\omega_0}-e^{-j2\omega_0}}{j2}}{\left(z-e^{j2\omega_0}\right)\left(z-e^{-j2\omega_0}\right)}\right]$$

$$= \frac{z\sin(2\omega_0)}{z^2 - ze^{-j2\omega_0} - ze^{j2\omega_0}+1} = \frac{z\sin(2\omega_0)}{z^2-2z\left(\dfrac{e^{j2\omega_0}+e^{-j2\omega_0}}{2}\right)+1}$$

$$X(z) = \frac{z\sin(2\omega_0)}{z^2-2\cos(2\omega_0)z+1} \qquad \text{for } |z| > 1.$$

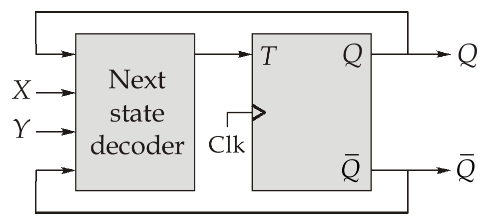
Q.1 (e) Solution:

Truth table:

X	Y	Q_{n+1}
0	0	Q_n
0	1	$\bar{Q}_n$
1	0	0
1	1	1

Realization of XY flip-flop using T-flip flop. Here the desired flip-flop is XY flip flop and the chosen one is T-flip flop.

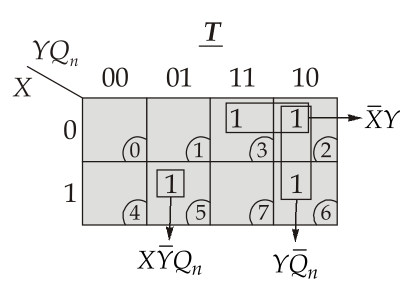
The block diagram of this realization is shown below,



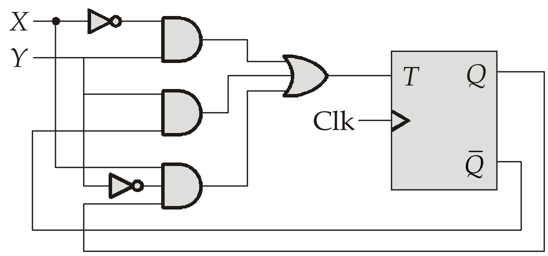
State table:

X	Y	Q_n	Q_{n+1}	T
0	0	0	0	0
0	0	1	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	0

K-map simplification:



$$\therefore \quad T = \bar{X}Y + Y\bar{Q}_n + X\bar{Y}Q_n$$



Q.2 (a) Solution:

(i) Subscripts 1 and 2 will be used to refer to 50 Hz and 60 Hz quantities respectively.

Since the voltage and current at both the frequencies of 50 Hz and 60 Hz are the same, the output also remains the same.

Since ohmic losses depends on current and here current remains unchanged.

So ohmic loss at 50 Hz

$$P_{0h1} = \text{ohmic loss at 60 Hz,}$$

$$P_{0h2} = 1.6\%$$

Hysterisis losses :

$$P_h = K_h \cdot f \cdot B_m^x \qquad \left\{\because B_m \propto \frac{V}{f}\right\}$$

So,

$$\frac{P_{h2}}{P_{h1}} = \frac{f_2}{f_1} \cdot \left(\frac{B_{m2}}{B_{m1}}\right)^x$$

$$\frac{P_{h2}}{P_{h1}} = \frac{f_2}{f_1} \cdot \left(\frac{V_2}{V_1}\right)^x \cdot \left(\frac{f_1}{f_2}\right)^x \qquad \{\because V_1 = V_2\}$$

$$\frac{P_{h2}}{P_{h1}} = \frac{60}{50}\left(\frac{50}{60}\right)^{1.6}$$

$$P_{h2} = 0.9 \times \frac{6}{5}\left(\frac{5}{6}\right)^{1.6} = 0.806\%$$

Eddy current losses:

$$P_e = K_e \cdot f^2 \cdot B_m^2$$

$$\frac{P_{e2}}{P_{e1}} = \frac{f_2^2}{f_1^2} \cdot \frac{B_{m2}^2}{B_{m1}^2} = \left(\frac{f_2}{f_1}\right)^2 \cdot \left(\frac{f_1}{f_2}\right)^2$$

$$= \left(\frac{60}{50}\right)^2 \cdot \left(\frac{50}{60}\right)^2 = 1$$

$$P_{e2} = P_{e1} = 0.6\%$$

Thus the ohmic, hysterisis and eddy current losses at 60 Hz are 1.6%, 0.806% and 0.6% respectively.

(ii) The core loss depends on voltage and frequency only. Therefore P_{C1} = 1.5% (= 0.9 + 0.6) and P_{C2} = 1.406% (= 0.806 + 0.6) can't be changed for given values of voltage and frequency. For the total losses remains the same, the ohmic loss alone can be varied.

$\therefore$ Total losses at 50 Hz = Total losses at 60 Hz

$$1.6 + 0.9 + 0.6 = 0.806 + 0.6 + \text{New ohmic losses}$$

Permissible value of ohmic losses at 60 Hz

$$= 3.1 - 1.406$$

$$= 1.694\%$$

Since, ohmic $\propto$ (current)^2

$$(\text{New permissible current})^2 = \frac{\text{New ohmic losses}}{\text{Original ohmic losses}} \cdot (\text{original current})^2$$

$$\text{New permissible currrent} = \sqrt{\frac{1.694}{1.60}} \cdot (\text{Original current})$$

$$\text{New permissible current} = 1.028\ (\text{original current})$$

For the same voltage,

$$\text{Output at 60 Hz} = (1.028)\ (\text{Output at 50 Hz})$$

Q.2 (b) Solution:

l = 80 km, f = 50 Hz

r = 3.75 mΩ/km, L_1 = 15.92 μH/km

$\therefore$ Resistance of the line, $R = rl = 3.75 \times 10^{-3} \times 80$

$= 0.3\ \Omega$ per phase

Reactive of the line, $X = 2\pi f l L_1$

$= 2\pi \times 50 \times 80 \times 15.92 \times 10^{-6}$

$= 0.4\ \Omega$ per phase

Impedance of the line, $Z = R + jX = (0.3 + j0.4)\Omega$ per phase

$= 0.5\angle 53.13°\ \Omega$ per phase

Total receiving end apparent power,

$$S_R = 3 \times 375 \text{ kVA} = 1125 \text{ kVA}$$

and $$\text{p.f.} = \cos\phi_R = 0.8 \text{ lag}$$

$\Rightarrow$ $$\phi_R = 36.87°$$

Receiving end active power,

$$P_R = S_R \cos\phi_R = 1125 \times 0.8$$
$$= 900 \text{ kW}$$

Let the per phase receiving end voltage and current be $\bar{V}_R$ and $\bar{I}_R$ respectively and receiving end line voltage be V_{R2}.

So, $$I_R = \frac{S_R}{\sqrt{3}\times V_{RL}} = \frac{1125\times10^3}{\sqrt{3}\times\left(\sqrt{3}\times V_R\right)} = \frac{1125\times10^3}{3\times V_R}$$

$$I_R = \frac{375}{V_R}\times10^3 \quad \text{...(i)}$$

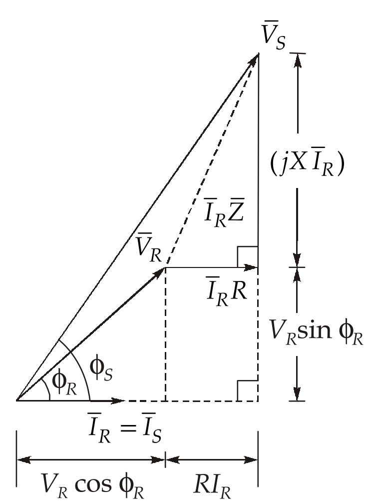
Sending end line voltage,

$$V_{SL} = 3300 \text{ V}$$

Sending end phase voltage,

$$V_S = \frac{V_{SL}}{\sqrt{3}} = \frac{3300}{\sqrt{3}} = 1905.26 \text{ volts}$$

Assuming reference phasor to be $\bar{V}_R$



$$V_S^2 = (V_R \cos\phi_R + RI_R)^2 + (V_R \sin\phi_R + XI_R)^2$$

$$\Rightarrow \quad (1905.26)^2 = \left(V_R \times 0.8 + 0.3 \times \frac{375}{V_R} \times 10^3\right)^2 + \left(V_R \times 0.6 + \frac{0.4 \times 375 \times 10^3}{V_R}\right)^2$$

$$\Rightarrow \quad 3630015.668 = V_R^2 + \frac{35156.25}{V_R^2} \times 10^6 + 360 \times 10^3$$

$\Rightarrow$ $V_R{}^4 - 3270015.668\ V_R{}^2 + 35156.25 \times 10^6 = 0$

(i) $V_R = 1805.33, 103.86$

or $\bar{V}_R = 1805.33\angle 0°$ volts

So receiving end line voltage,

$$V_{RL} = \sqrt{3} \times V_R$$

$$= \sqrt{3} \times 1805.33 = 3126.92 \text{ volts}$$

Now receiving end current,

$$I_R = \frac{375}{V_R} \times 10^3 = \frac{375 \times 10^3}{1805.33} = 207.718 \text{ A}$$

i.e., $\bar{I}_S = \bar{I}_R = I_R\angle -\phi_R = 207.718\angle -36.87°$ A

Now, $\vec{V}_S = \bar{V}_R + \bar{I}_R Z$

$$= 1805.33\angle 0° + (207.718\angle -36.87°)\ (0.3 + j0.4)$$

$$\vec{V}_S = 1905.26\angle 0.87° \text{ volts}$$

i.e. $\phi_S = \angle \bar{V}_S - \angle \bar{I}_S = 0.87° - (-36.87°) = 37.74°$

(ii) Sending end power, $P_S = \sqrt{3}\, V_{SL} I_S \cos\phi_s$

$$= \sqrt{3} \times 3300 \times 1905.26 \times \cos 37.74°$$

$$= 8611.79 \text{ kW}$$

and sending end p.f., $\cos\phi_s = \cos 37.74°$

$$= 0.7908 \text{ lagging}$$

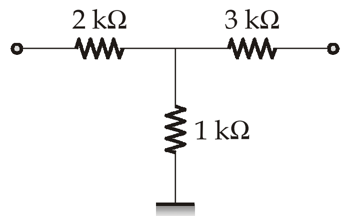
(iii) Voltage regulation of the line,

$$\%V_{reg} = \frac{V_{SL} - V_{RL}}{V_{RL}} \times 100$$

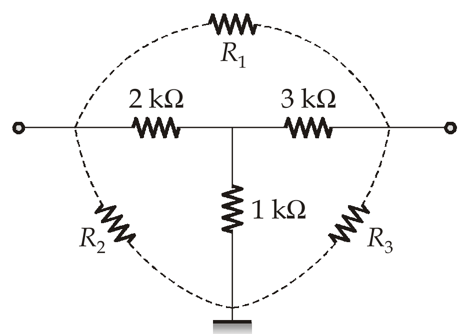
$$= \frac{3300 - 3126.92}{3126.92} \times 100 = 5.535\%$$

Q.2 (c) Solution:

The two port network can be drawn as

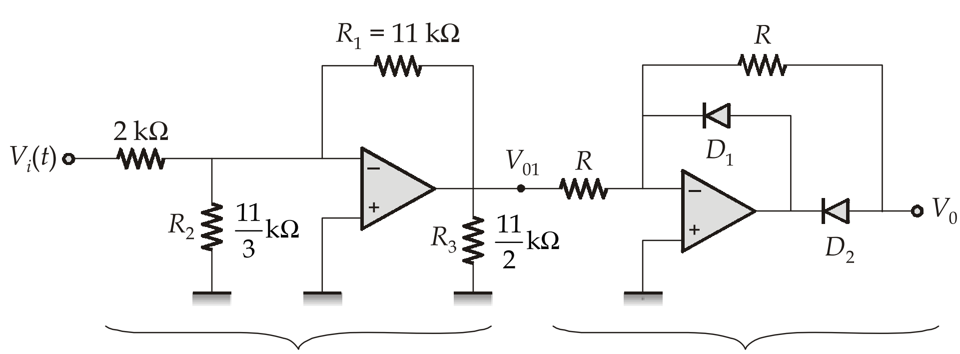


on performing Y-Δ transformation on above network.



where,
$$R_1 = 2+3+\frac{2\times 3}{1} = 11 \text{ k}\Omega$$
$$R_2 = 1+2+\frac{1\times 2}{3} = \frac{11}{3}\text{k}\Omega$$
$$R_3 = 3+1+\frac{3\times 1}{2} = \frac{11}{2}\text{k}\Omega$$

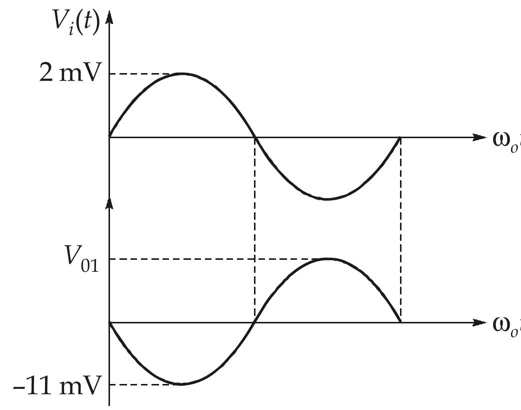
Now, the above circuit can be drawn as,



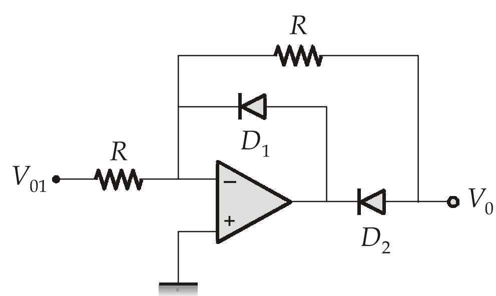
For Section-1 :

$$V_{o1} = -\frac{11}{2}V_i(t) = -\frac{11}{2}\times 2\sin(100\pi t) \text{ mV}$$

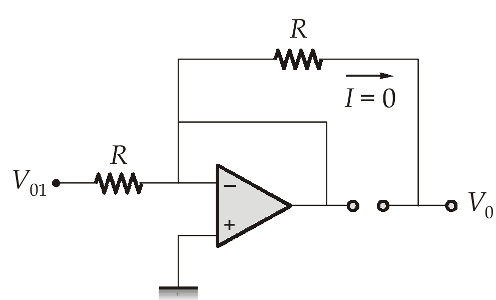
$$V_{o1} = -11 \sin(100\pi t) \text{ mV}$$



For Section-2 :

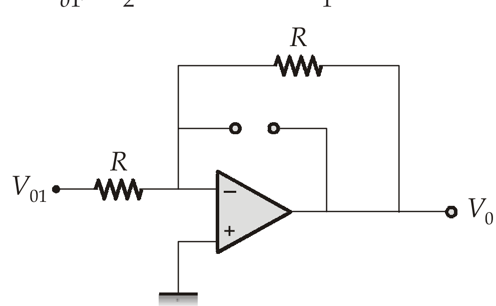


During –ve half cycle of V_{o1}, D_1 is ON and D_2 is OFF.

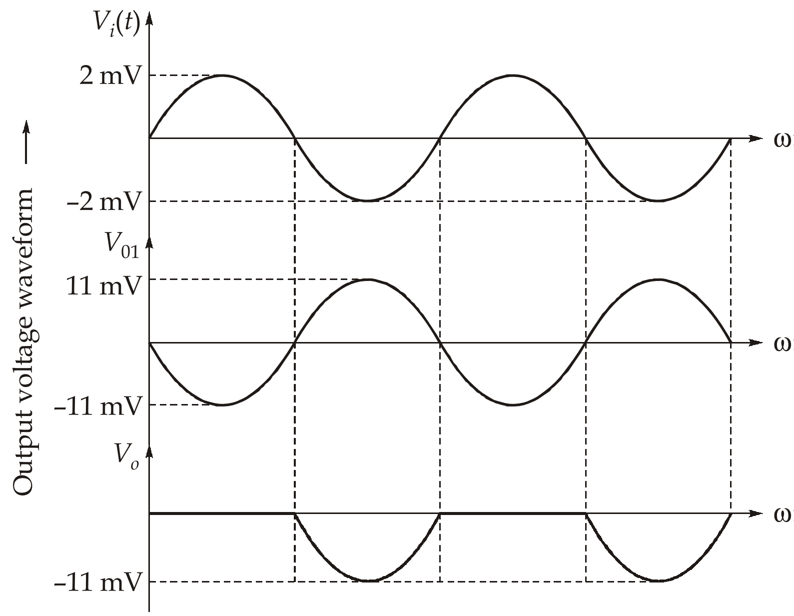


Therefore, $V_o = 0$ in the –ve half cycle of V_{o1}.

During the +ve half cycle of V_{o1}, D_2 is ON and D_1 is OFF.



$$V_o = -\frac{R}{R} \times V_{o1} = -V_{o1}$$



Average value of half cycle sinusoidal waveform is given as

$$V_{o(\text{avg})} = \frac{V_m}{\pi} = \frac{-11}{\pi} \text{ mV}$$

$$= -3.50 \text{ mV}$$

Q.3 (a) Solution:

Given, $x[n] = \text{IDFT}\,[X(k)],\ n\ k = 0, \ldots, N-1$

$$X(k) = \text{DFT}\,\{1, -j2, j, -j4\}$$

$\therefore$ $x[n] = \{1, -j2, j, -j4\}$

(i) Given, IDFT$\{X^*(k)\}$

By the definition of IDFT,

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(k) W_N^{-nk} = \frac{1}{N}\sum_{K=0}^{N-1} X^*(k) W_N^{-nk}$$

$$= \left[\frac{1}{N}\sum_{K=0}^{N-1} X(k) W_N^{-nk}\right]^*$$

$\therefore$ $\text{IDFT}\{X^*(k)\} = x^*(-n)_N$

$\therefore$ $x(-n) = \{1, -j4, j, -j2\}$

$\therefore$ $x^*(-n) = \{1, j4, -j, j2\}$

(ii)

$$\text{IDFT}\{X(-k)_N\} = \frac{1}{N}\sum_{K=0}^{N-1} X(-k)_N W_N^{-nk}$$

$$= \frac{1}{N}\sum_{K=0}^{N-1} X(k)_N W_N^{nk}$$

$$= x(-n)_N$$

$$\therefore \quad x(n) = \{1, -j4, j, -j2\}$$

(iii)

$$\text{IDFT}\{\text{Re}[X(k)]\} = \frac{1}{2}IDFT\{X(k)\} + \frac{1}{2}IDFT\{X^*(k)\}$$

$$= \frac{1}{2}x(n) + \frac{1}{2}x^*(-n)_N$$

$$= \frac{1}{2}\left[x(n) + x^*(-n)_N\right]$$

$$= \frac{1}{2}[(1,-j2,+j,-j4)+(1,+j4,-j,+j2)]$$

$$= \frac{1}{2}[2,+j2,0,-j2]$$

$$= [1, j, 0, -j]$$

(iv)

$$\text{IDFT}\{\text{Im}[X(k)]\} = \frac{1}{j2}IDFT[X(k)] + \frac{1}{j2}IDFT\left[x^*(k)\right]$$

$$= \frac{1}{j2}[x(n)] - \frac{1}{j2}\left[x^*(-n)_N\right]$$

$$= \frac{1}{j2}[(1,-2j, j, -j4)] - (1, j4, -j, j2)]$$

$$= \frac{1}{j2}(0,-j6, j2, -j6]$$

$$= [0, -3, 1, -3]$$

Q.3 (b) Solution:

(i) Open-circuit test has been conducted on l.v. side because 250 V during this test is equal to the rated voltage on l.v. side,

Short-circuit test has been performed on h.v. side, since 60 V is a fraction (2 to 12%) of the rated voltage on h.v. side.

Full load (or rated) current on h.v. side

$$= \frac{10000}{2500} = 4 \text{ A}$$

Ohmic loss of 45 W is due to short-circuit current of 3 A

∴ ohmic loss at full load current of 4 A

$$P_{SC} = 45\left(\frac{4}{3}\right)^2 = 80 \text{ W}$$

$$\text{Fixed core loss, } P_C = 50 \text{ W}$$

At 0.8 power factor and at $\frac{1}{4}$ full load, the core loss

$$P_C = 50 \text{ W},$$

$$\text{Ohmic loss} = \left(\frac{1}{4}\right)^2 P_{sc} = \left(\frac{1}{4}\right)^2 \times 80 = 5 \text{ W}$$

and

$$\text{output} = \frac{1}{4} \times 10000 \times 0.8 = 2000 \text{ W}$$

∴ η at $\frac{1}{4}$ full load

$$= 1 - \frac{50+5}{2000+50+5} = 0.9732 = 97.32\%$$

(ii) Let the maximum efficiency occur at x times the rated kVA

∴ Ohmic loss at maximum efficiency,

$$\eta = x^2(80) \text{ watts}$$

But maximum η occurs when ohmic loss

$$= \text{core loss}$$

i.e.

$$x^2(80) = 50$$

$$x = \sqrt{\frac{50}{80}} = 0.79$$

∴ kVA output at maximum,

$$\eta = x(10) = (0.79) \times 10 = 7.9 \text{ kVA}$$

Note that the kVA output at maximum efficiency does not depend on the load power factor

For maximum efficiency,

$$\text{ohmic loss} = \text{core loss} = 50 \text{ W}$$

$\therefore$ Total losses at maximum,

$$P_L = 50 + 50 = 100 \text{ W}$$

The maximum efficiency is

$$\eta_{max} = 1 - \frac{100}{7900 \times 0.8 + 100} = 0.98443 \text{ or } 98.443\%$$

(iii) From short-circuit test,

$$R_{eH} = \frac{45}{9} = 5\,\Omega$$

$$Z_{eH} = \frac{60}{3} = 20\,\Omega$$

$$\therefore \quad X_{eH} = \sqrt{(20)^2 - (5)^2} = 19.35\ \Omega$$

$$\therefore \quad R_{pu} = \frac{I_H \cdot r_{eH}}{V_H} = \frac{4 \times 5}{2500} = 0.008 \text{ p.u.}$$

$$X_{pu} = \frac{I_H \cdot x_{eH}}{V_H} = \frac{4 \times 19.35}{2500} = 0.031$$

$\therefore$ Voltage requisition at 0.8 power factor lagging

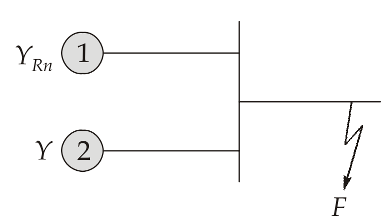
$$= R_{pu} \cos\theta_2 + X_{pu} \sin\theta_2$$
$$= 0.008 \times 0.8 + 0.031 \times 0.6$$
$$= 0.0250 \text{ p.u. or } 2.5\%$$

Now $$\frac{E_2 - V_2}{E_2} = 0.025$$

$$V_2 = 0.975\, E_2$$

$$V_2 = 0.975 \times 250 = 243.75 \text{ V}$$

Q.3 (c) Solution:



Let the base MVA = 25 MVA

Base kV = 11 kV

Feeder reactance;

$$\text{Positive sequence } (X_1) = \frac{j0.4 \times 25}{121} = j0.0826 \text{ p.u.}$$

$$\text{Negative sequence } (X_2) = j0.0826 \text{ p.u.}$$

$$\text{Zero sequence } (X_o) = 2 \times X_1 = 0.165 \text{ p.u.}$$

$$\text{Grounding resistance} = \frac{1 \times 25}{121} = 0.2066 \text{ p.u.}$$

So, $3R_n = 0.62$ p.u.

Now, L – G fault at feeder end,

$$X_0, \text{ equation} = \frac{j0.08}{2} + j0.165 = j0.205 \text{ p.u.}$$

$$X_1, \text{ equation} = \frac{j0.2}{2} + j0.826 = j0.1826 \text{ p.u.}$$

$$X_2, \text{ equation} = \frac{j0.15}{2} + j0.0826 = j0.1576 \text{ p.u.}$$

(i)

$$I_f = I_a = \frac{3}{X_{1eq} + X_{2,eq} + X_{0eq} + 3R_n}$$

$$= \frac{3}{0.62 + j[0.1826 + 0.1576 + 0.205]}$$

$$= 3.63\angle{-41.32°} \text{ p.u.}$$

Now,

$$I_{a1} = I_{a2} = I_{a0} = \frac{3.63\angle -41.32°}{3} = 1.21\angle{-41.32°} \text{ p.u.}$$

(ii)

$$V_{a1} = E_{a1} - I_{a1} X_{1,\,eq}$$

$$= 1 - (j0.1826)\,(1.21\angle{-41.32°})$$

$$= 0.87\angle{-11°} \text{ p.u.}$$

$$V_{a2} = -I_{a2} X_{2,\,eq}$$

$$= -(1.21\angle{-41.32°})\,(0.1576\angle 90°)$$

$$= 0.19\angle{-131.32°} \text{ p.u.}$$

$$V_{a0} = -I_{a0} \times 3\,Z_{0,\,eq}$$

$$= -(1.21\angle{-41.32°})\,(0.62 + j0.0205)$$

$$= 0.79\angle 156.92° \text{ p.u.}$$

As we know,

Phase voltage of phase 'b' after fault in phase 'a'.

$$V_b = V_{ab} + \lambda^2 V_{a1} + \lambda V_{a2}$$

$$V_b = 0.79\angle 156.92° + (1\angle 240°)(0.87\angle -11°) + (1\angle 120°)(0.19\angle -131.32°)$$

$$= 1.1757\angle -160° \text{ p.u.}$$

$$V_c = V_{a0} + \lambda V_{a1} + \lambda^2 V_{a2}$$

$$= 0.79\angle 156.92° + (1\angle 120°)(0.87\angle -11°) + (1\angle 240°)(0.19\angle 131.32°)$$

$$= 1.69\angle 129.20° \text{ p.u.}$$

So phase voltage of 'b'

$$V_{b\,\text{ph}} = 1.1757\angle -160° \times \frac{11}{\sqrt{3}} \text{kV} = 7.46\angle -160° \text{ kV}$$

$$V_{c\,\text{ph}} = 1.69\angle 129.20° \times \frac{11}{\sqrt{3}} \text{kV} = 10.75\angle 129.20 \text{ kV}$$

(iii) Voltage of star point w.r.t. ground

$$3I_{a0}R_n = 3 \times 1.21 \times 0.2066 = 0.75 \text{ p.u.}$$

So the voltage of star point w.r.t. ground is

$$= 0.75 \times 11 \text{ kV} = 8.25 \text{ kV}$$

Q.4 (a) Solution:

(i) **Power factor improvement:** Forced commutation techniques for ac-dc converters can improve input power factor. Some methods are:

1. **Extinction angle control:** The switching actions performed by gate-turn-off thyristors (GTOs). The characteristics of GTOs allows turning on and turning off by applying short positive and negative pulses on its gate respectively.

   Performance of semi and full converters with this control method is similar to that with phase angle control except the power factor is leading.

2. **Symmetric angle control:** Allows one quadrant operation. The fundamental component of input current is in phase with the input voltage and the displacement factor is unity. Therefore, the power factor is improved.

   This control can be applied for the same half controlled forced commutated bridge converter with two switches.

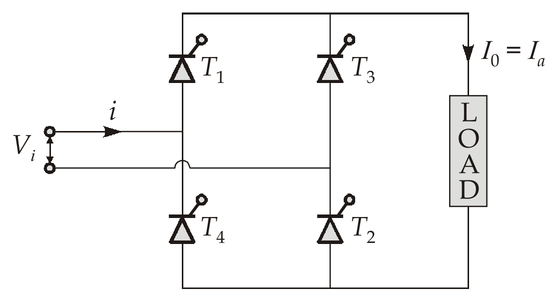
3. **Pulse width modulation control:** The converter switches are turned-on and off several times during a half-cycle. Thus, the harmonics can be eliminated or reduced and the output voltage is controlled by varying width of pulses.

The lower order harmonics can be eliminated by selecting the number of pulses per half cycle. However, increasing the pulses can increase the magnitude of higher order harmonics, which can be easily filtered out.

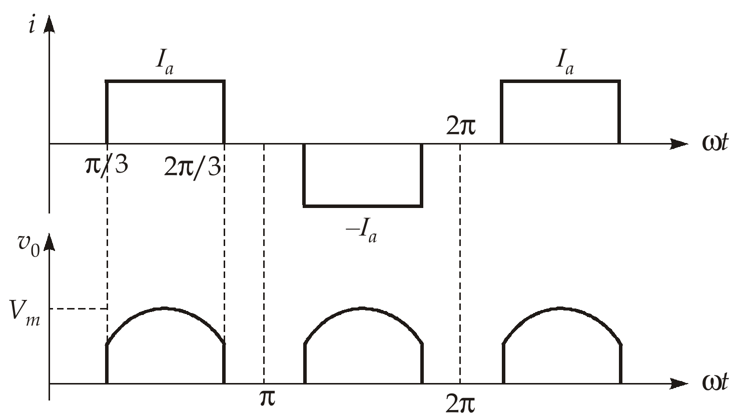
4. **Sinusoidal pulse-width modulation:** In this pulse widths are not uniform (like in PWM), the pulse widths are generated by comparing a triangular voltage with a sinusoidal voltage. The power factor is further improved.

The width of pulses (and the output voltage) are varied by changing the amplitude of sinusoidal voltage or the modulation index. In sinusoidal PWM control, the displacement factor is unity and power factor is improved.

(ii)



(Single-phase full converter)



$$i(t) = I_{dc} + \sum_{n=1,2,3}^{2\pi} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$I_{dc} = \frac{1}{2\pi}\left\{\int_{\pi/3}^{2\pi/3} I_a \, d(\omega t) - \int_{4\pi/3}^{5\pi/3} I_a \, d(\omega t)\right\} = 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} i(t)\cos(n\omega t)\, d(\omega t) = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} i(t)\sin(n\omega t)\, d(\omega t)$$

$$= \frac{4I_a}{n\pi}\sin\frac{n\pi}{6}, \quad n = 1, 3, 5.... \text{ and } b_n = 0 \text{ for } n = \text{even}$$

$$i(t) = 4I_a \sum_{n=1,3,5}^{\infty} \frac{\sin\left(\frac{n\pi}{6}\right)}{n\pi}\sin(n\omega t)$$

Rms input current, $I_{s,r} = I_a\sqrt{\frac{\pi/3}{\pi}} = \frac{I_a}{\sqrt{3}}$ $(\because I_a = I_0)$

$$I_{s1} = \left(\frac{4I_a}{\sqrt{2}}\right)\times\frac{1}{\pi}\sin\left(\frac{\pi}{6}\right) = 0.4502\, I_a$$

Hence, $\text{HF} = \left[\left(\frac{I_{sr}}{I_{s1}}\right)^2 - 1\right]^{1/2}$

Also, harmonic factor,HF $= \left[\left(\frac{I_{s,r}}{I_{s1}}\right)^2 - 1\right]^{1/2} = \left[\frac{\pi\times\frac{\pi}{3}}{4\left(1-\cos\frac{\pi}{3}\right)} - 1\right]^{1/2} = 0.803$

Q.4 (b) (i) Solution:

$$\frac{C(s)}{E(s)} = \frac{1}{0.5\times 0.2s(s+2)(s+5)} = \frac{10}{s(s+2)(s+5)}$$

$$= \frac{10}{s^3+7s^2+10s}$$

$$\frac{C(s)}{E(s)} = \frac{G(s)}{1+G(s)\times 1} = \frac{10}{s^3+7s^2+10s}$$

$\because$ $G(s) = \frac{10}{s^3+7s^2+10s}$

The characteristic equation of the system is given by

$$s^3 + 7s^2 + 10s + 10 = 0$$

To a fair approximation one of three roots of the above characteristic equation can be determined as $s = -5.5$, thus the quadratic term of the above equation can be determined as

$$\frac{s^3+7s^2+10s+10}{s+5.5} = s^2 + 1.5s + 1.8$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{10}{(s+5.5)(s^2+1.5s+1.8)}$$

The characteristic equation of the system can be rewritten as

$$(s + 5.5)(s^2 + 1.5s + 1.8) = 0$$

For oscillatory roots

$$\omega_n^2 = 1.8$$

or $$\omega_n = \sqrt{1.8} = 1.34 \text{ rad/sec}$$

or $$2\xi\omega_n = 1.5$$

$$\therefore \quad \xi = \frac{1.5}{2\xi\omega_n} = \frac{1.5}{2\times 1.34} = 0.56$$

The time constant of quadratic term is

$$\frac{1}{\xi\omega_n} = \frac{1}{0.56\times 1.34} = 1.33 \text{ sec}$$

The time constant due to first order term is $\frac{1}{5.5} = 0.18$ sec its effect is neglected as compared to quadratic term time constant is 1.33 sec

$$\%M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}\times 100 = e^{-\frac{0.56\pi}{\sqrt{1-0.56^2}}}\times 100 = e^{-\frac{1.76}{0.83}}\times 100$$

$$= e^{-2.12}\times 100 = 12.03\%$$

Q.4 (b) (ii) Solution:

For a unity feedback control system

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\therefore \quad \frac{Ks+\beta}{s^2+\alpha s+\beta} = \frac{G(s)}{1+G(s)}$$

or $$(K_s + \beta)[1 + G(s)] = G(s)(s^2 + \alpha s + \beta)$$

or $$G(s) = \frac{Ks+\beta}{s^2+s(\alpha-K)}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$

$\therefore \quad \dfrac{E(s)}{R(s)} = \dfrac{1}{1+\dfrac{Ks+\beta}{s^2+s(\alpha-K)}}$

$\therefore \quad E(s) = R(s)\cdot\dfrac{s(s+\alpha-K)}{s^2+s\alpha+\beta}$

$\because \quad R(s) = \dfrac{1}{s^2}$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\cdot\frac{1}{s^2}\times\frac{s(s+\alpha-K)}{s^2+\alpha s+\beta}$$

or $\quad e_{ss} = \dfrac{\alpha-K}{\beta}$

Q.4 (c) Solution:

Given that the field current required for rated voltage,

$$I_{F(OC)} = 2.4\text{ Amp}$$

The field current required for rated short circuit,

$$I_{F(SC)} = 0.8\text{ A}$$

So, $\quad \text{SCR} = \dfrac{I_{F(OC)}}{I_{F(SC)}} = \dfrac{2.4}{0.8} = 3\text{ pu}$

So, $\quad X_d = \dfrac{1}{SCR} = \dfrac{1}{3}\text{ pu}$

The base impedance, $\quad Z_B = \dfrac{V_{ph}}{I_{ph}} = \dfrac{220}{\sqrt{3}\times 27} = 4.704\ \Omega$

So, $\quad X_d = \dfrac{1}{3}(4.704) = 1.568\ \Omega$

Given : $\quad \dfrac{X_d}{X_q} = 1.5$

$$X_q = \frac{1.568}{1.5} = 1.045\ \Omega$$

(i) The power equation, $\quad P = \dfrac{V_tE_f}{X_d}\sin\delta + \dfrac{V_t^2}{2}\left[\dfrac{1}{X_d}-\dfrac{1}{X_q}\right]\sin 2\delta$

$$P = \frac{220\times 250}{1.568}\sin\delta + \frac{220^2}{2}\left[\frac{1}{1.045}-\frac{1}{1.568}\right]\sin 2\delta$$

$$P = 35076.53 \sin\delta + 7724.22 \sin 2\delta$$

$$\frac{dP}{d\delta} = 35076.53 \cos\delta + 15448.44 \cos 2\delta = 0$$

Solving above equation, $\delta = 1.22$ rad

$$\delta = 1.22 \times \frac{180}{\pi} = 70.71°$$

So, $P_{max} = 35076.53 \sin(70.71) + 7724.22 \sin(2 \times 70.71)$

$P_{max} = 37.926$ kW (maximum power)

Load angle at maximum power,

$$\delta = 70.71°$$

(ii) When there is sudden loss of excitation

$$E_f = 0$$

So, power $P = \frac{V_t^2}{2}\left[\frac{1}{X_d} - \frac{1}{X_d}\right]\sin 2\delta$

For maximum power, $\delta = 45°$

$$P_{(max)} = \frac{V_t^2}{2}\left[\frac{1}{X_d} - \frac{1}{X_q}\right] = \frac{220^2}{2}\left[\frac{1}{1.045} - \frac{1}{1.568}\right] = 7.72 \text{ kW}$$

Section-B

Q.5 (a) Solution:

$$f_{clock} = 3 \text{ MHz}$$

$\Rightarrow$ $T = \frac{1}{3}\mu s$

Instruction	Execution time
MVI	7T
DCX	6T
JNZ	7T

$\Rightarrow$ Time to execute MVI B and MVI C = $(7 + 7)\ T = 14\ T$

If the program loops 'n' times, then time required

$= \text{time for 1 loop} \times n$

$= (6 + 7) \times T \times n = 13\ Tn$

Thus, $10 \times 10^{-3} = 14\ T + 13\ Tn$

$\Rightarrow$ $n = 2306.62 \approx 2307 = (2307)_{10} = (0903)_H$

Q.5 (b) Solution:

(i) String = $\{h, e, l, l, o\}$

Symbol (x_i)	n	Probability $p(x_i)$
h	1	$\frac{1}{5}$
e	1	$\frac{1}{5}$
l	2	$\frac{2}{5}$
o	1	$\frac{1}{5}$

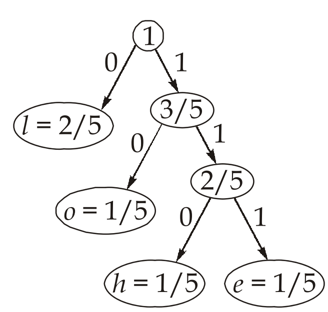
(ii) Entropy, $H(X)$;

$$H(X) = -\sum_{i=1}^{N} p(x_i)\log_2 P(x_i)$$

$$= -\frac{1}{5}\log_2\left(\frac{1}{5}\right) - \frac{1}{5}\log_2\left(\frac{1}{5}\right) - \frac{2}{5}\log_2\left(\frac{2}{5}\right) - \frac{1}{5}\log_2\left(\frac{1}{5}\right)$$

$$H(X) = 1.9219 \text{ bits/symbol}$$

(iii) Huffman Tree :



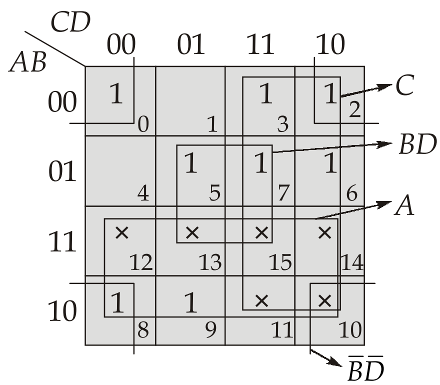
(iv) Huffman Code Book :

Symbol	Encoded Bits
h	110
e	111
l	00
o	10

Bit stream = 110 111 00 10

Q.5 (c) Solution:

For the given function, *K*-map is shown below,

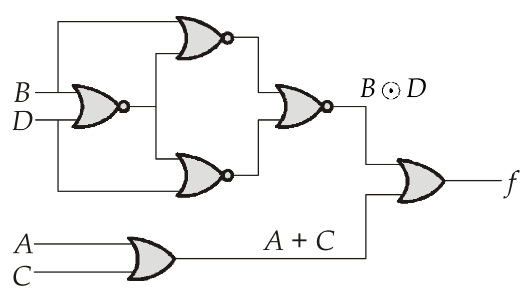


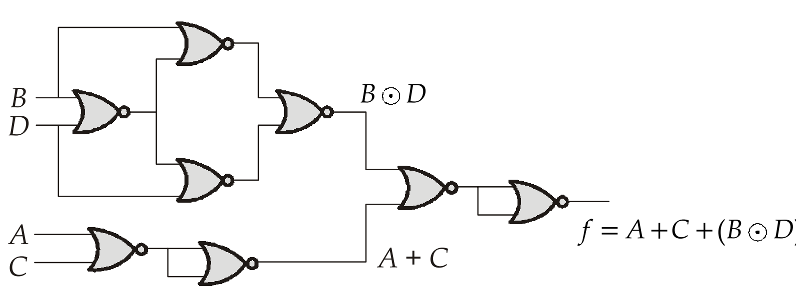
In the 4-variable *K*-map shown above, there are two octet and two quads. The simplified expression is given by,

$$f = A + C + BD + \bar{B}\bar{D}$$

$$f = A + C + (B \odot D)$$

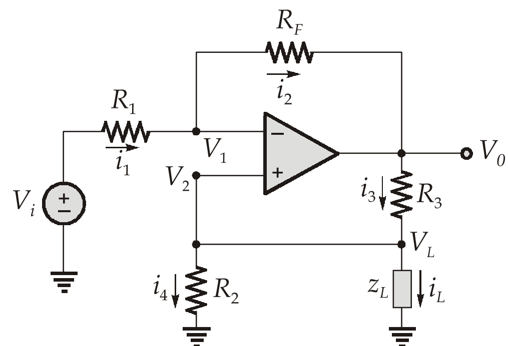
The logic circuit for the minimized function by using only NOR gates is





Q.5 (d) Solution:

The circuit can be redrawn by showing the currents,



From virtual short concept, $V_1 = V_2$ and also, we know that

$$V_1 = V_2 = V_L = i_L z_L$$

and $i_1 = i_2$

$$\frac{V_i - i_L z_L}{R_1} = \frac{i_L z_L - V_0}{R_F} \quad \text{...(i)}$$

Taking the sum of currents in the non-inverting terminal,

$$i_3 = i_4 + i_L$$

$$\frac{V_0 - i_L z_L}{R_3} = i_L + \frac{i_L z_L}{R_2} \quad \text{...(ii)}$$

From equation (i) and (ii) solving for $(V_0 - i_L z_L)$

$$\frac{R_F}{R_1 R_3}(i_L z_L - V_i) = i_L + \frac{i_L z_L}{R_2}$$

Combining terms in i_L, we get,

$$i_L\left(\frac{R_F z_L}{R_1 R_3} - \frac{z_L}{R_2} - 1\right) = \frac{V_i R_F}{R_1 R_3} \quad \text{...(iii)}$$

In order to make i_L independent of z_L, we can design the circuit such that the coefficient of z_L is zero.

i.e.
$$\frac{R_F}{R_1 R_3} = \frac{1}{R_2}$$

$\Rightarrow$
$$R_F = \frac{R_1 R_3}{R_2}$$

then equation (iii) reduces to

$$i_L = \frac{-V_i (R_F)}{R_1 R_3} = \frac{-V_i}{R_2}$$

Which means that load current is proportional to input voltage and is independent of the load impedance z_L.

Q.5 (e) Solution:

The characteristic equation is

$$1 + G(s) = 1+\frac{K}{s(s+3)(s^2+s+1)} = 0$$

or $$s(s+3)(s^2+s+1) = 0$$

or $$s^4 + 4s^3 + 4s^2 + 3s + K = 0$$

The Routh's array is

$$\begin{array}{c|ccc} s^4 & 1 & 4 & K \\ s^3 & 4 & 3 & 0 \\ s^2 & \frac{13}{4} & K & \\ s^1 & \frac{\left(\frac{39}{4}-4K\right)}{\frac{13}{4}} & & \\ s^0 & K & & \end{array}$$

The condition for system stability is

$$K \geq 0$$

and $$\left(\frac{39}{4}-4K\right) > 0$$

Therefore for stability,

K should lie in the range $\frac{39}{4} > K > 0$

When, $$K = \frac{39}{16}$$

There will be a zero at the first entry in the fourth row (s^1 row). This will indicate presence of a symmetrical roots, which as shown below, will be pure imaginary,

$\therefore$ $K = \frac{39}{16}$ will cause sustained oscillations

The subsidiary equation of third row for $K = \frac{39}{16}$, is

$$\frac{13}{4}s^2 + \frac{39}{16} = 0$$

$\therefore \quad s = \pm j0.866$

Therefore, the frequency of sustained oscillation is 0.866 rad/sec

Q.6 (a) Solution:

(i) The battery terminal voltage (V_0)

$$V_0 = \frac{3V_{mL}}{\pi}\cos\alpha = E + I_0 R$$

So, $\quad V_0 = 190 + 22 \times 0.6 = 203.6 \text{ V}$

So, $$V_0 = \frac{3V_{mL}}{\pi}\cos\alpha = 203.6$$

$$\frac{3\times 240\times\sqrt{2}}{\pi}\cos\alpha = 203.6$$

$$\cos\alpha = \frac{203.6\times\pi}{3\times 240\times\sqrt{2}} = 0.62817$$

Firing angle, $\quad \alpha = 51.08°$

For constant load current, the supply current can be given as

$$I_s = \left(\frac{2}{3}\right)^{1/2} I_o$$

$$I_s = \sqrt{\frac{2}{3}}\times 22 = 17.96 \text{ Amp}$$

So, power delivered to the load

$$P_o = EI_0 + I_{0r}^2 R$$
$$= 190 \times 22 + (22)^2 \times 0.6$$
$$= 4470.4 \text{ Watt}$$

So, $\quad$ Input power = Output power

$$\sqrt{3}V_s I_s \cos\phi = P_0$$

Input supply power factor,

$$\cos\phi = \frac{4470.4}{\sqrt{3}\times 240\times 17.96} = 0.599 \text{ lagging}$$

(ii) When power flow from dc to ac source,

Output voltage, $\quad V_0 = E - I_o R$

$$V_0 = 190 - 22 \times 0.6$$

$$V_0 = 176.8 \text{ V}$$

When power flows from dc source to ac source, the 3-ϕ full converter works as 3-phase line commutated inverter.

$$\frac{3V_{mL}}{\pi}\cos\alpha = -176.8$$

$$\frac{3\times 240\times\sqrt{2}}{\pi}\cos\alpha = -176.8$$

$$\cos\alpha = -0.5454$$

$$\alpha = \cos^{-1}(-0.5454)$$

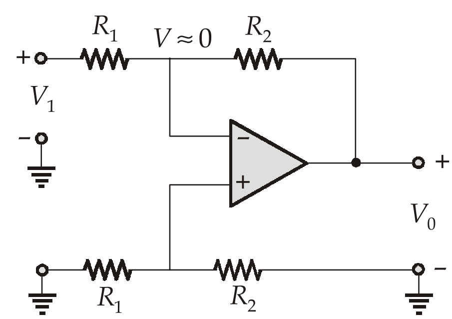
Firing angle, $\alpha = 123.06°$

Q.6 (b) Solution:

(i) Output voltage, $V_0 = V_{01} + V_{02}$

where V_{0_1} is the output voltage only due to V_1.

and V_{0_2} is the output voltage only due to V_2.

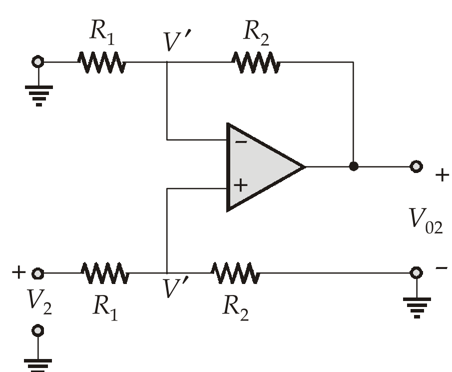


Let, $V_2 = 0$

$$\because \quad \frac{V_1}{R_1} = \frac{-V_{0_1}}{R_2}$$

$$\Rightarrow \quad V_{0_1} = -\frac{R_2}{R_1}\cdot V_1 \qquad \text{...(i)}$$

Let, $V_1 = 0$



$$\therefore \quad V' = \frac{R_2}{R_1+R_2}\cdot V_2$$

$$\Rightarrow \quad \frac{V_{0_2}-V'}{R_2} = \frac{V'}{R_1}$$

$$\Rightarrow \quad \frac{V_{0_2}}{R_2} = V'\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{V_{0_2}}{R_2} = V'\left(\frac{R_1+R_2}{R_1R_2}\right)$$

$$\Rightarrow \quad \frac{V_{0_2}}{R_2} = \frac{R_2}{(R_1+R_2)}\cdot\frac{(R_1+R_2)}{R_1R_2}\cdot V_2$$

$$\Rightarrow \quad \frac{V_{0_2}}{R_2} = \frac{V_2}{R_1}$$

$$\Rightarrow \quad V_{0_2} = \frac{R_2}{R_1}\cdot V_2 \qquad \text{...(ii)}$$

By using V_{0_2} superposition principle we get,

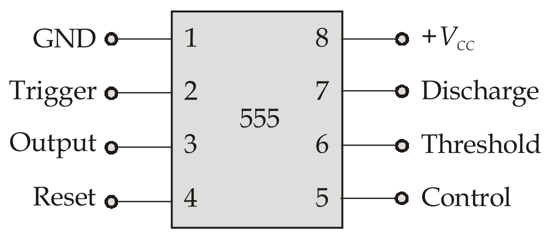
$$V_0 = V_{01} + V_{02}$$

$$\Rightarrow \quad V_0 = \frac{-R_2}{R_1}V_1 + \frac{R_2}{R_1}V_2$$

$$\Rightarrow \quad V_0 = \frac{R_2}{R_1}(V_2 - V_1)$$

(ii) Pin diagram of the 555 Timer:

The below figure shows the schematic of 555 integrated timer. The device may be connected to carry out either astable or monostable operation.



Pin Description:

Pin-1: Ground Terminal: This pin represents ground. All voltages are measured with respect to this terminal.

Pin-2: Trigger: This pin is an inverting input to a comparator that is responsible for transition of flip-flop from set to reset. The external trigger pulse is applied to this terminal. The output voltage of the timer depends on the amplitude of the trigger pulse.

Pin-3: Output: A load may be connected between this pin and the ground pin.

Pin-4: Reset: To reset or disable the timer a negative pulse is applied to this pin due to which it is referred as reset terminal. When this pin is not to be used for reset purpose, it should be connected to $+V_{cc}$ to avoid any possibility of false triggering.

Pin-5: Control Voltage: When an external voltage is applied to control voltage, both trigger and threshold voltages vary, so that output voltage pulse width changes.

Pin-6: Threshold Voltage: This is the non inverting input terminal of comparator-1. When the voltage at this terminal exceeds the control voltage $2/3\ V_{CC}$, the output of comparator-1 goes high, as a result of the timer output goes low.

Pin-7: Discharge: This pin is connected to the collector terminal of the discharge transistor Q_1. During operation, when the timer output is low, Q_1 is saturated and it short circuits the capacitor connected across it externally. When the timer output is high, Q_1 is cut-off.

Pin-8: +V_{CC} Supply: A supply voltage of +5 V to +18 V is applied to this terminal with respect to ground (pin-1).

⇒ **For astable multivibrator using 555 timer**

1. Output frequency, $f = \dfrac{1}{0.693(R_A + 2R_B)C}$

$$= \frac{1}{0.693(0.1\times10^{-6})(10+4.6)\times10^3} = 988.36 \text{ Hz}$$

2. Duty cycle, $D = \dfrac{1 + R_B / R_A}{(1 + 2R_B / R_A)} \times 100\%$

$$= \frac{(1+2.3/10)}{\left(1+\frac{2\times2.3}{10}\right)}\times100\% = 84.24\%$$

3. Current sourcing, $P_{avg} = \dfrac{1}{(T_1+T_2)}\int_0^{T_1}\left(\dfrac{V_{CC}}{R_L}\right)^2 R_L dt = \dfrac{T_1}{T_1+T_2}\times\left(\dfrac{V_{CC}}{R_L}\right)^2 R_L$

$$= \text{Duty cycle}\times\left(\frac{12^2}{1\times10^3}\right) = 0.8424\times\left(\frac{12^2}{1000}\right) = 121 \text{ mW}$$

Q.6 (c) (i) Solution:

From the given response curve

Peak time, $t_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = 1 \text{ sec}$

and settling time, $t_s = \dfrac{4}{\xi\omega_n} = 1.7 \text{ sec}$

Hence $\dfrac{t_p}{t_s} = \dfrac{1}{1.7} = \dfrac{\pi}{4}\cdot\dfrac{\xi}{\sqrt{1-\xi^2}}$

$$\frac{\xi}{\sqrt{1-\xi^2}} = \frac{4}{1.7\pi}$$

Solving for ξ, $\xi = 0.6$

So, $\omega_n = \dfrac{4}{\xi t_s} = \dfrac{4}{0.6\times1.7} = 3.92 \text{ rad/sec}$

The transfer function, $\dfrac{C(s)}{R(s)} = \dfrac{K}{s^2+2\xi\omega_n s+\omega_n^2} = \dfrac{K}{s^2+2\times0.6\times3.92s+(3.92)^2}$

$$\frac{C(s)}{R(s)} = \frac{K}{(s^2 + 4.70s + 15.37)}$$

For unit step function, $R(s) = \frac{1}{s}$

Hence, $C(s) = \frac{K}{s(s^2 + 4.70s + 15.37)}$

$$c(\infty) = \lim_{s\to 0} sC(s) = \frac{K}{15.37} = 0.8$$

$$K = 12.30$$

So transfer function, $\frac{C(s)}{R(s)} = \frac{12.30}{(s^2 + 4.7s + 15.37)}$

Q.6 (c) (ii) Solution:

The characteristic equation is

$$1 + G(s) = 0$$

$$s^4 + 6s^3 + 30s^2 + 60s + K = 0$$

Substituting $s = p - 1$, the equation can be modified as,

$(p - 1)^4 + 6(p - 1)^3 + 30(p - 1)^2 + 60(p - 1) + K = 0$

$p^4 - 4p^3 + 6p^2 - 4p + 1 + 6(p^3 - 3p^2 + 3p - 1) + 30(p^2 - 2p + 1) + 60(p - 1) + K = 0$

$\Rightarrow \quad p^4 + 2p^3 + 18p^2 + 14p + K - 35 = 0$

The Routh array is formed as shown below,

p^4	1	18	$K-35$
p^3	2	14	
p^2	11	$K-35$	
p^1	$\frac{2(112-K)}{11}$		
p^0	$K-35$		

For all roots of this equation to lie on the left half of the p-plane (or to the left of $s = -1$ in the s-plane), all the first-column elements should be of the same sign,

So, $112 - K > 0$ and $K - 35 > 0$

$K < 112$ and $K > 35$

So combining these two conditions, the required range of K is

$$35 < K < 112$$

Q.7 (a) Solution:

(i) The relationship between the dc supply V_s and dc machine back emf is given by,

$$I_o = \frac{E - V_o}{R} = \frac{E - V_s(1-\alpha)}{R}$$

$$10 = \frac{150 - 200(1-\alpha)}{1}$$

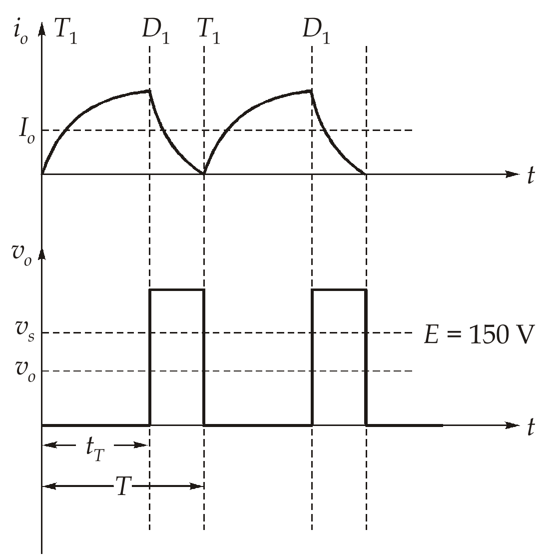
$$200(1-\alpha) = 150 - 10$$

$$1 - \alpha = \frac{140}{200} = 0.7$$

$$\alpha = 0.3 \text{ or } 30\% \text{ duty cycle}$$

Load time constant, $\tau = \frac{L}{R} = \frac{1}{1}\text{ ms} = 1 \text{ msec}$

Waveforms :



The expression for average dc machine output current is based on continuous armature inductance current. Therefore, the switching period must be shorter than the time t_x, given by below expression, for the current to reach zero, before the next switch on-period. That is $t_x = T$ and $\alpha = 0.30$.

$$t_x = t_T + \tau \ln\left[1 + E\left(1 - e^{-\frac{t_T}{T}}\right)\right]$$

$$1 = 0.3 + \frac{1\,\text{mS}}{T}\ln\left[1 + \frac{150}{50}\left(1 - e^{\frac{-0.3T}{1\,\text{mS}}}\right)\right]$$

$$e^{0.7T} = 4 - 3e^{-0.3T}$$

On solving for T,

$$T = 0.494 \text{ msec}$$

Therefore, switching frequency must be greater than $f_s = \frac{1}{T} = 2.024$ kHz, else machine output current discontinuous.

(ii) The operational boundary giving by equation,

$$\frac{E}{V_s} = \frac{1 - e^{-\frac{T+t_T}{\tau}}}{1 - e^{-\frac{T}{\tau}}}$$

$$\frac{150}{200} = \frac{1 - e^{\frac{(\alpha-1)\times 1\text{ msec}}{1\text{ msec}}}}{1 - e^{-\frac{1\text{ mS}}{1\text{ mS}}}}$$

On solving for α;

$$\alpha = 0.357$$

Therefore, on-state duty cycle must be at least 35.7%. For continuous machine output current at a switching frequency of 1 kHz,

$$I_o = \frac{E - V_o}{R} = \frac{150 - 200(1 - 0.357)}{1} = 21.4 \text{ A}$$

$$V_o = 150 - 21.4 \times 1 = 128.6 \text{ Volt}$$

(iii) At an increased switching frequency of 5 kHz, the duty cycle would be expected to be much lower that the 35.7% as at 1 kHz. The operational boundary between continuous and discontinuous armature current is given by equation

$$\frac{E}{V_s} = \frac{1 - e^{\frac{-T+t_T}{\tau}}}{1 - e^{-\frac{T}{\tau}}}$$

$$\frac{150}{200} = \frac{1 - e^{(-1+\alpha)\times\frac{0.2}{1}}}{1 - e^{\frac{-0.2}{1}}} \Rightarrow \alpha = 26.9\%$$

Machine average output current,

$$I_o = \frac{E - V_0}{R} = \frac{150 - 200 \times (1 - 0.269)}{1} = 3.8 \text{ A}$$

and average output current,

$$V_o = (1 - \alpha) V_s = 146.2 \text{ V}$$

Q.7 (b) Solution:

(i) Kinetic energy, K.E. = G.H

$$= 3.5 \times 100 = 350 \text{ MJ}$$

(ii) System base(s) = 500 MVA

Then, the acceleration power,

$$P_a = P_i - P_u = (0.18 - 0.16) \times 500 = 10 \text{ MW}$$

$$M = \frac{2H}{\omega_s} \times S_{\text{rated}} = \frac{2 \times 3.5}{360 \times 50} \times 100 = \frac{3.5}{90}$$

Now,

$$M \cdot \frac{d^2\delta}{dt^2} = P_a = 10$$

$$\frac{d^2\delta}{dt^2} = \frac{10 \times 90}{3.5} = 257.143 \text{ ele-degree/sec}^2 \quad \text{...(1)}$$

or 4.488 rad/sec^2

(iii) Acceleration period in seconds = $\frac{7.5}{50}$ = 0.15 sec

$$N_s = \frac{120 f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\frac{d^2\delta}{dt^2} = 4.488$$

$$\frac{d\delta}{dt} = 4.488t + K$$

$$K = \left.\frac{d\delta}{dt}\right|_{t=0} = 0$$

$$\delta = \frac{4.488t^2}{2} + \delta_o$$

$$\delta - \delta_o = \Delta\delta = \frac{4.488}{2} \times t^2$$

$$\Delta\delta = 0.05049 \text{ rad}$$

$$\Delta\delta = 2.89°$$

(iv) Rotor velocity is calculated as

$$\Delta\omega = \frac{d^2\delta}{dt^2} \times t$$

$$= 4.488 \times 0.15$$

$$= 0.6732 \text{ elec. rad/sec}$$

$$\Delta\omega = 0.6732 \times \frac{60}{2\pi} \text{ rpm} \times \frac{2}{P}$$

$$\Delta N = 6.429 \text{ rpm}$$

$$N' = N_s + \Delta N$$

$$N' = 1503.2145 \text{ rpm}$$

Q.7 (c) Solution:

Power loss in terms of MW generation is

$$P_L = \left[0.0125\left(\frac{P_1}{100}\right)^2 + 0.00625\left(\frac{P_2}{100}\right)^2\right] \times 100 \text{ MW}$$

$$P_L = 0.000125P_1^2 + 0.0000625P_2^2 \text{ MW}$$

For the numerical solution using the gradient method, the initial guess of $\lambda^{(1)} = 7.0$.

Now, we know that from coordination equation

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

Then, $$P_i^{(1)} = \frac{\lambda_i^{(1)} - \beta_i}{2\gamma_i}$$

Therefore, $$P_1^{(1)} = \frac{7 - 6.2}{2(0.004 + 7 \times 0.000125)} = 82.05128 \text{ MW}$$

and $$P_2^{(1)} = \frac{7 - 6}{2(0.003 + 7 \times 0.0000625)} = 145.4545 \text{ MW}$$

The real power loss is

$$P_L^{(1)} = 0.000125 \times (82.05228)^2 + 0.0000625 \times (145.4545)^2$$
$$= 2.16388 \text{ MW}$$

Since, $P_D = 412.35 \text{ MW}$

The error $\Delta P^{(1)}$ is

$$\Delta P^{(1)} = 412.35 + 2.16388 - (82.05628 + 145.4545)$$
$$\Delta P^{(1)} = 187.003$$

Now, $$\sum_{i=1}^{2}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(1)} = \frac{0.004 + 0.000125 \times 6.2}{2(0.004 + 7 \times 0.000125)^2} + \frac{0.003 + 0.0000625 \times 6}{2(0.003 + 7 \times 0.0000625)^2}$$
$$= 243.2701$$

Now, $$\Delta\lambda^{(1)} = \frac{187.003}{243.2701} = 0.7687$$

The new value of λ is

$$\lambda^{(2)} = 7 + 0.7687 = 7.7687$$

Now, continuing the process, for second iteration, we have

$$P_1^{(2)} = \frac{7.7687 - 6.2}{2(0.004 + 7.7687 \times 0.000125)} = 157.7823 \text{ MW}$$

$$P_2^{(2)} = \frac{7.7687 - 6}{2(0.003 + 7.7687 \times 0.0000625)} = 253.7194 \text{ MW}$$

The real power loss

$$P_L^{(2)} = 0.000125 \times (157.7823)^2 + 0.0000625 \times (253.7194)^2$$
$$= 7.13525 \text{ MW}$$

Since, $P_D = 412.35$ MW, the error $\Delta P^{(2)}$ is

Error, $$\Delta P^{(2)} = 412.35 + 7.13525 - (157.7823 + 253.7194) = 7.98355$$

$$\sum_{i=1}^{2}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(2)} = \frac{0.004 + 0.000125 \times 6.2}{2(0.004 + 7.7687 \times 0.000125)^2} + \frac{(0.003 + 0.0000625 \times 6.0)}{2(0.003 + 7.7687 \times 0.0000625)^2}$$
$$= 235.514$$

$\Rightarrow$ $$\Delta\lambda^{(2)} = \frac{7.98355}{235.514} = 0.0339$$

Therefore, the new value of λ is,

$$\lambda^{(3)} = 7.7687 + 0.0339 = 7.8026$$

For the third iteration, we have

$$P_1^{(3)} = \frac{7.8026 - 6.2}{2(0.004 + 7.8026 \times 0.000125)} = 161.0548 \text{ MW}$$

$$P_2^{(3)} = \frac{7.8026 - 6}{2(0.003 + 7.8026 \times 0.0000625)} = 258.4252 \text{ MW}$$

The real power loss is

$$P_L^{(3)} = 0.000125 \times (161.0548)^2 + 0.0000625 \times (258.4252)^2$$

$$= 7.4163 \text{ MW}$$

Since, $P_D = 412.35$ MW, the error $\Delta P^{(3)}$ is

Error, $$\Delta P^{(3)} = 412.35 + 7.4163 - (161.0548 + 258.4252)$$

$$\Delta P^{(3)} = 0.2863$$

And $$\sum_{i=1}^{2}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(3)} = \frac{0.004 + 0.000125 \times 6.2}{2(0.004 + 7.8026 \times 0.000125)^2} + \frac{0.003 + 0.0000625 \times 6}{2(0.003 + 7.8026 \times 0.0000625)^2} = 235.18$$

$$\Delta\lambda^{(3)} = \frac{0.2683}{235.18} = 0.0014$$

Therefore, the new value of λ after 3rd iteration is,

$$\lambda^{(4)} = 7.8026 + 0.0014 = 7.80374$$

Hence, after 3rd iteration

$$P_1 = 161.0548 \text{ MW}$$

$$P_2 = 258.4252 \text{ MW}$$

$$\lambda = 7.80374$$

Q.8 (a) (i) Solution:

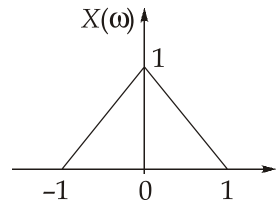
Given LTI system is the interval [–5, 5],

$$|H(\omega)| = 1$$

and $$\angle H(\omega) = -\omega$$

Hence the input signal selected within this internal [–5, 5]

Therefore input be $X(\omega)$ chosen as,



$$\therefore \qquad X(\omega) = \begin{cases} 1-|\omega|; & |\omega| \le 1 \\ 0; & \text{otherwise} \end{cases}$$

Output of LTI system, $Y(\omega) = H(\omega) \times G(\omega)$

or

$$Y(\omega) = H(\omega)\, X(\omega)$$

$$= |H(\omega)| \angle H(\omega) \cdot X(\omega)$$

$$Y(\omega) = 1 \cdot X(\omega) \cdot e^{-j\omega}$$

by taking inverse Fourier transform,

$$y(t) = x(t-1)$$

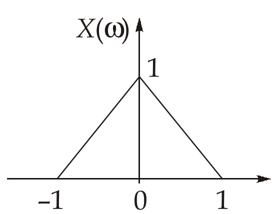
but,

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

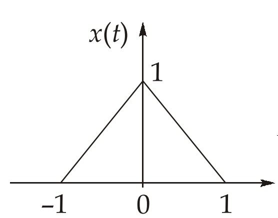
We can use duality property,

Since $X(\omega)$ is an triangular signal

i.e.

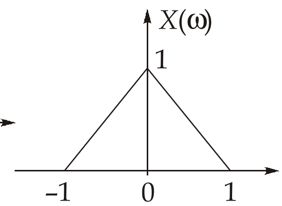


$$\Rightarrow \qquad X(\omega) = \frac{1}{2\pi} \text{sin}c^2\left(\frac{\omega}{2}\right)$$



$$\longleftrightarrow \frac{1}{2\pi} \text{sin}c^2\left(\frac{\omega}{2}\right)$$

$$x(t) = \frac{1}{2\pi}\sin c^2\left(\frac{t}{2}\right) \longleftrightarrow$$



$$x(t) = \frac{1}{2\pi}\sin c^2\left(\frac{\omega}{2}\right)$$

$$\therefore \quad y(t) = x(t-1)$$

$$= \frac{1}{2\pi}\sin c^2\left(\frac{t-1}{2}\right)$$

Q.8 (a) (ii) Solution:

Given, $$F(s) = \frac{1}{s^2(s+1)^2}$$

Let, $$F_1(s) = \frac{1}{s^2};$$

$$F_2(s) = \frac{1}{(s+1)^2}$$

$$f_1(t) = L^{-1}[F_1(s)] = t;$$

$$f_2(t) = L^{-1}[F_2(s)] = te^{-t}$$

by using continuous convolution method,

$$L^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(\tau)f_2(t-\tau)d\tau \text{ or } \int_0^t f_1(t-\tau)f_2(\tau)d\tau$$

$$L^{-1}\left(\frac{1}{s^2}\cdot\frac{1}{(s+1)^2}\right) = \int_0^t \tau e^{-\tau}(t-\tau)d\tau$$

$$= t\int_0^t \tau e^{-\tau}d\tau - \int_0^t \tau^2 e^{-\tau}d\tau$$

$$= t\left[-\tau e^{-\tau} - e^{-\tau}\right]_0^t - \left[-\tau^2 e^{-\tau} - 2\tau e^{-\tau} - 2e^{-\tau}\right]_0^t$$

$$= t\left[-te^{-t} - (e^{-t} - 1)\right] - \left[-t^2e^{-t} - 2te^{-t} - 2(e^{-t} - 1)\right]$$

$$= (te^{-t} + 2e^{-t} + t - 2)u(t)$$

Q.8 (b) Solution:

$$G(s)H(s) = \frac{K(1+2s)}{s(1+s)(1+s+s^2)}$$

$$M = \frac{K\sqrt{1+4\omega^2}}{\omega\sqrt{1+\omega^2}\sqrt{(1-\omega^2)^2+\omega^2}}$$

$$\phi = \tan^{-1}2\omega - 90° - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{1-\omega^2}$$

When $\omega = 0,\quad M\angle\phi = \infty\angle -90°,$

$\omega = 1,\quad M\angle\phi = \sqrt{\frac{5}{2}}\angle 161.5°,$

$\omega = 0.5,\quad M\angle\phi = 2.8\angle -105.5°,$

$\omega = \infty,\quad M\angle\phi = 0\angle -270°,$

$$G(j\omega)H(j\omega) = \frac{K(1+j2\omega)}{j\omega(1+j\omega)\{(1-\omega^2)+j\omega\}}$$

Separating into real and imaginary parts and equating the imaginary part to zero, i.e.

$$1 + 2\omega^2 - 2\omega^4 = 0$$

We get, by hit and trial method,

$$\omega = 1.17$$

$$M\big|_{\omega=1.17} = \frac{K\sqrt{1+(1.17\times 2)^2}}{1.17\sqrt{1+1.17^2}\sqrt{(1-1.17^2)^2+1.17^2}} = 1.153\text{ K}$$

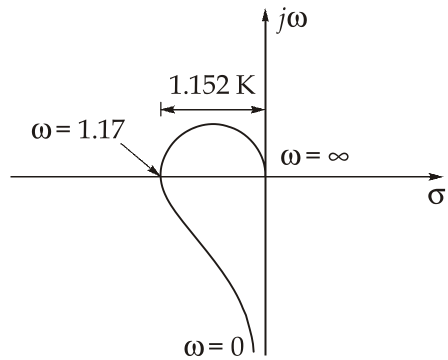
$$\phi\big|_{\omega=1.17} = \tan^{-1}(2\times 1.17) - 90° - \tan^{-1} - \tan^{-1}\frac{1.17}{1-1.17^2}$$

$$= -180°$$

$\therefore\quad G(j\omega)H(j\omega) = M\angle\phi = 1.153K\angle -180°$

The polar plot is shown in figure

If $K = 1$, then point $(-1 + j0)$ will be encircled and the system will be unstable,



If $K < \frac{1}{1.153}$

or $K < 0.87$, the system will be stable

Phase cross over frequency

$$\frac{K\sqrt{1+4\omega^2}}{\omega\sqrt{1+\omega^2}\sqrt{\omega^2+(1-\omega^2)^2}} = 1$$

or $$\frac{(1+4\omega^2)\times K}{\omega^2(1+\omega^2)\left\{\omega^2+(1-\omega^2)^2\right\}} = 1$$

$$\text{Gain margin} = 20\log\frac{1}{a} = 3$$

$$20\log\frac{1}{1.153K} = 3$$

or $$\frac{1}{1.153K} = 1.413$$

which gives $$K = \frac{1}{1.153\times 1.413} = 0.614$$

Putting the value of K in the equation

$$\frac{0.614(1+4\omega^2)}{\omega^2(1+\omega^2)\left\{\omega^2+(1-\omega^2)\right\}} = 1$$

By hit and trial method, $\omega = 0.97$ is the phase crossover frequency

$$\text{P.M.} = \tan^{-1}(2\times 0.97) - 90° - \tan^{-1}0.97 - \tan^{-1}\frac{0.97}{1-0.97^2} + 180°$$

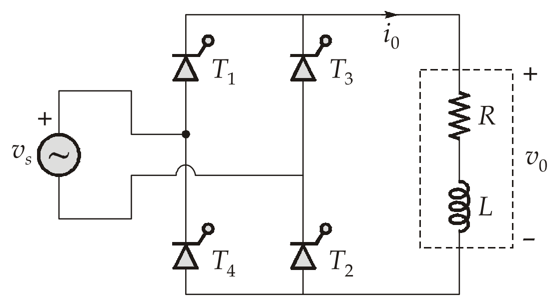
$$= 22.5°$$

Q.8 (c) Solution:

$$V = 230 \text{ V}, \quad f = 50 \text{ Hz}$$
$$R = 20\ \Omega;$$
$$X = 2\pi fL = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$$
$$\alpha = 60°$$

When thyristors are in ON-state

applying KVL from source to load,



$$v_s = L\frac{di_0}{dt} + Ri_0$$

$$v_m \sin\omega t = L\frac{di_0}{dt} + Ri_0$$

The general solution to this equation is

$$i_0(t) = \frac{v_m}{|z|}\sin(\omega t - 0) + Ae^{-t/\tau}$$

Where, $v_m = 230\sqrt{2}$

$$|z| = \sqrt{20^2 + 62.83^2} = 65.94\,\Omega$$

$$\theta = \tan\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{62.83}{20}\right) = 72.34°$$

and $$T = \frac{L}{R} = \frac{0.2}{20} = 0.01$$

∴ $$i_0(t) = \frac{230\sqrt{2}}{65.94}\sin(\omega t - 72.34°) + Ae^{-t/0.01}$$

$$= 4.933 \sin(\omega t - 72.34°) + Ae^{-100t}$$

At $\omega t = \alpha = 60°$,

$$i_0 = 0$$

$$\therefore \quad 0 = 4.933\sin(60° - 72.34°) + Ae^{-100\times\frac{60°}{\omega}\times\frac{\pi}{180°}}$$

$$0 = -1.054 + Ae^{-104.72/\omega} = -1.054 + Ae^{-104.72/100\pi}$$

$$1.054 = Ae^{-1/3}$$

$$A = 1.471$$

$$\therefore \quad i_0(t) = 4.933\sin(\omega t - 72.34°) + 1.471e^{-100t}$$

At $\omega t = \beta$, $i_0 = 0$

$$0 = 4.933\sin(\beta - 72.34°) + 1.471e^{\frac{-100\times\beta}{\omega}}$$

$$0 = 4.933\ \sin(\beta - 1.263) + 1.471e^{-0.318\beta}$$

[$\because$ 72.34° = 1.263 radian and β is in radian]

Solving this transcedental equation we get

$$\beta = 1.0476 \text{ radian, } 7.52 \text{ rad}$$

$$= 60°, 430°$$

(i) It is clear from the values of β that the load current is continous in nature.

(ii) Average output voltage, $V_0 = \frac{2v_m}{\pi}\cos\alpha = \frac{2\times\sqrt{2}\times 230}{\pi}\cos 60° = 103.536$ volts

Average/dc load current, $I_0 = \frac{V_0}{R} = \frac{103.536}{20} = 5.177$ A

(iii) The Fourier series of output voltage will be

$$v_0 = V_0 + \sum_{n=1}^{\infty}[a_n\cos n\omega t + b_n \sin n\omega t], \quad \text{where, } a_n = \frac{2}{T}\int_0^T v_0(\omega t)\cos n\omega t\, d\omega t$$

$$= \frac{2}{\pi}\int_{\alpha}^{(\pi+\alpha)} v_0 \sin\omega t\cos n\omega t\, d\omega t$$

$$= \frac{v_m}{\pi}\int_{\alpha}^{(\pi+\alpha)}[\sin(n+1)\omega t - \sin(n-1)\omega t]\cdot d\omega t$$

$$= \frac{v_m}{\pi}\left[\frac{-\cos(n+1)\omega t}{(n+1)} + \frac{\cos(n-1)\omega t}{(n-1)}\right]_{\alpha}^{(\pi+\alpha)}$$

$$= \frac{v_m}{\pi}\left[\left\{\frac{\cos(n+1)(\pi+\alpha) - \cos(n+1)\alpha}{(n+1)}\right\} + \frac{\cos(n-1)(\pi+\alpha) - \cos(n-1)\alpha}{(n-1)}\right]$$

For n = odd ;

$$\cos(n+1)(\pi+\alpha) = \cos(n+1)\alpha$$

$$\cos(n-1)(\pi+\alpha) = \cos(n-1)\alpha$$

For n = even ;

$$\cos(n+1)(\pi+\alpha) = -\cos(n+1)\alpha$$

$$\cos(n-1)(\pi+\alpha) = -\cos(n-1)\alpha$$

$$\therefore \qquad a_n = \frac{v_m}{\pi}\left[-\left\{\frac{-\cos(n+1)\alpha-\cos(n+1)\alpha}{(n+1)}\right\}+\frac{-\cos(n-1)\alpha-\cos(n-1)\alpha}{(n-1)}\right]$$

.... n is even

$$= \frac{2v_m}{\pi}\left[\frac{\cos(n+1)\alpha}{(n+1)}-\frac{\cos(n-1)\alpha}{(n-1)}\right]$$

Similarly,

$$b_n = \frac{2}{T}\int_0^T v_0(\omega t)\sin n\omega t\, d\omega t$$

$$= \frac{2}{\pi}\int_{\alpha}^{(\pi+\alpha)} v_m \sin\omega t \sin n\omega t\, d\omega t$$

$$= \frac{v_m}{\pi}\int_{\alpha}^{(\pi+\alpha)} [-\cos(n+1)\omega t+\cos(n-1)\omega t]\, d\omega t$$

$$= \frac{v_m}{\pi}\left[-\frac{\sin(n+1)\omega t}{(n+1)}+\frac{\sin(n-1)\omega t}{(n-1)}\right]_{\alpha}^{(\pi+\alpha)}$$

$$= \frac{v_m}{\pi}\left[-\left\{\frac{\sin(n+1)(\pi+\alpha)-\sin(n+1)\alpha}{(n+1)}\right\}+\frac{\sin(n-1)(\pi+\alpha)-\sin(n-1)\alpha}{(n-1)}\right]$$

For n = odd;

$$\sin(n+1)(\pi+\alpha) = \sin(n+1)\alpha$$

$$\sin(n-1)(\pi+\alpha) = \sin(n-1)\alpha$$

For n = even ;

$$\sin(n+1)(\pi+\alpha) = -\sin(n+1)\alpha$$

$$\sin(n-1)(\pi+\alpha) = -\sin(n-1)\alpha$$

$$\therefore \quad b_n = \frac{2v_m}{\pi}\left[\frac{\sin(n+1)\alpha}{(n+1)} - \frac{\sin(n-1)\alpha}{(n-1)}\right] \quad \text{... } n \text{ is even}$$

Since the dominant harmonic is 2nd harmonic

So,
$$a_2 = \frac{2v_m}{\pi}\left[\frac{\cos 3\alpha}{3} - \frac{\cos\alpha}{1}\right]$$

$$= \frac{2\sqrt{2}\times 230}{\pi}\left[\frac{\cos 3\times 60^\circ}{3} - \cos 60^\circ\right] = -172.561 \text{ V}$$

and
$$b_2 = \frac{2v_m}{\pi}\left[\frac{\sin 3\alpha}{3} - \frac{\sin\alpha}{1}\right]$$

$$= \frac{2\times\sqrt{2}\times 230}{\pi}\left[\frac{\sin 3\times 60^\circ}{3} - \sin 60^\circ\right] = -179.33 \text{ V}$$

Peak value of 2nd harmonic voltage

$$\hat{V}_2 = \sqrt{a_2^2 + b_2^2} = \sqrt{(-172.561)^2 + (-179.33)^2} = 248.871 \text{ V}$$

Rms 2nd harmonic voltage,

$$V_2 = \frac{\hat{V}_2}{\sqrt{2}} = \frac{248.871}{\sqrt{2}} = 175.978 \text{ V}$$

2nd harmonic impedance,

$$Z_2 = \sqrt{R^2 + [(2\omega)L]^2} = \sqrt{20^2 + [2\times 100\pi\times 0.2]^2} = 127.242\ \Omega$$

Rms 2nd harmonic current,

$$I_2 = \frac{V_2}{Z_2} = \frac{175.798}{127.242} = 1.382 \text{ A}$$

Rms output current,

$$I_{0r} = \sqrt{I_0^2 + I_2^2} = \sqrt{5.177^2 + 1.382^2} = 5.358 \text{ A}$$

$\therefore$ Power absorbed by the load is

$$P_0 = I_{0r}^2 R = 5.358^2 \times 20 = 574.163 \text{ W}$$

❍❍❍❍