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Detailed Solutions

# ESE-2025 Mains Test Series

# Mechanical Engineering Test No : 8

## Full Syllabus Test (Paper-II)

**Section : A**

**1. (a) Solution:**

Given : $d_i$ = 650 mm; $p_i$ = 35 MPa; $\sigma_c$ = 180 MPa; $\mu$ = 0.25

For thick cylinder,

Radial stress of pressure is $p_i$ [Compressive]

$$\text{Hoop stress, } \sigma_H = \frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} \times p_i = \frac{k^2+1}{k^2-1} \times p_i \qquad \left[\text{Where } k = \frac{d_o}{d_i}\right]$$

$$\text{Longitudinal stress, } \sigma_l = \frac{p_i d_i^2}{d_o^2 - d_i^2} = \frac{p_i}{k^2-1} \qquad \text{[Tensile]}$$

**(i)** Maximum principal stress theory : Failure occurs when the hoop stress exceeds the allowable tensile stress for the material. Thus, for safe design

$$\frac{k^2+1}{k^2-1} \times p_i \le \sigma_c$$

$$\frac{k^2+1}{k^2-1} \times 35 = 180$$

$$35k^2 + 35 = 180k^2 - 180$$

$$k = 1.218$$

$$\therefore \quad d_o = k \times d_i = 1.218 \times 650 = 791.7 \text{ mm}$$

$$\text{Wall thickness, } t = \frac{d_o - d_i}{2} = \frac{791.7 - 650}{2} = 70.85 \text{ mm}$$

**(ii)** Maximum shear stress theory:

For safe design, $\frac{1}{2}(\sigma_H + p_i) \le \frac{\sigma_c}{2}$

$$\frac{1}{2}\left(\frac{k^2+1}{k^2-1} \times p_i + p_i\right) \le \frac{\sigma_c}{2}$$

$$\frac{2k^2}{k^2-1}p_i \le \sigma_c$$

$$\frac{2k^2}{k^2-1} \times 35 = 180$$

$$70k^2 = 180k^2 - 180$$

$$k = 1.279$$

$$d_o = k \times d_i$$

$$= 1.279 \times 650$$

$$= 831.35 \text{ mm}$$

$$\text{Wall thickness, } t = \frac{d_o - d_i}{2} = \frac{831.35 - 650}{2} = 90.675 \text{ mm}$$

**1. (b) Solution:**

Density of NaC*l* = 2.18 g/cm$^3$

Number of effective atoms per unit cell in FCC structure = 4

Molecular weight NaC*l* = Atoms weight of Na + Atomic weight of C*l*

= 23 + 35.5 = 58.5 gm

As 58.5 gm of NaC*l* contains $6.023 \times 10^{23}$ molecules of NaC*l*

∴ Weight of $6.023 \times 10^{23}$ molecules = 58.5 gm

Therefore weight of 4 molecules = $\frac{58.5 \times 4}{6.023 \times 10^{23}}$ gm

This is the weight of unit cell whose volume is $a^3$.

We know that, $\rho = \frac{M}{V}$

$$\frac{58.5 \times 4}{6.023 \times 10^{23} \times a^3} = 2.18 \text{ gm/cm}^3$$

$$\therefore \quad a^3 = \frac{58.5 \times 4}{6.023 \times 10^{23} \times 2.18}$$

$$a = 5.63 \times 10^{-8} \text{ cm} \qquad (\because \ 1\text{Å} = 10^{-8} \text{ cm})$$

or $$a = 5.63\text{Å}$$

The lattice constant '$a$' is the unit cell dimension. In crystals of NaCl,

$$a = 2\sqrt{2}r \text{, distance between atoms} = 2r = \frac{a}{\sqrt{2}}$$

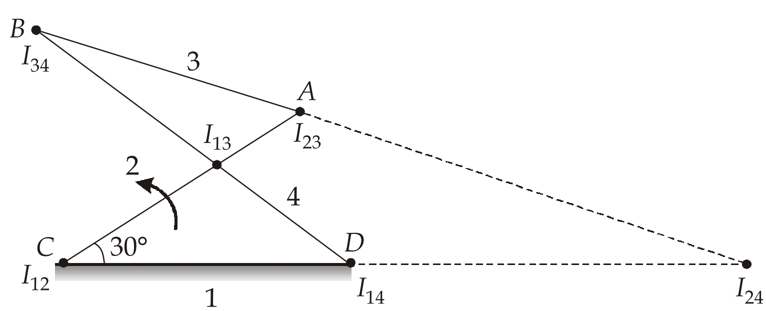
$\therefore$ Distance between two adjacent atoms $= \dfrac{5.63}{\sqrt{2}} = 3.98\text{Å}$

**1. (c) Solution:**

Given : $CD$ = 65 mm; $CA$ = 60 mm; $DB$ = 80 mm; $AB$ = 55 mm; $N_{AC}$ = 100 rpm

$$\text{Number of } I\text{-centre} = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 2 \times 3 = 6$$

All the instantaneous centres of given mechanism are shown in figure.




***I*-centre**

$$\text{Angular velocity of link AC, } \omega_2 = \frac{2\pi N}{60} = \frac{2\pi \times 100}{600} = 10.472 \text{ rad/s}$$

We know that, $$\omega_2 \times I_{12}I_{23} = \omega_3 \times I_{13}I_{23}$$

$$\omega_3 = \omega_2 \times \frac{I_{12}I_{23}}{I_{13}I_{23}}$$

$$\omega_3 = \frac{10.472 \times 60}{22} = 28.56 \text{ rad/s}$$

$\therefore$ $$\omega_{AB} = \omega_3 = 28.56 \text{ rad/s}$$

Also, $$\omega_2 \times I_{12}I_{24} = \omega_4 \times I_{14}I_{24}$$

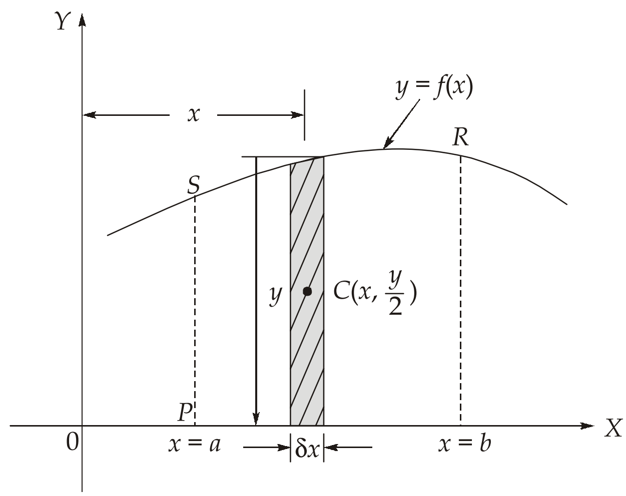
$$\omega_4 = \omega_2 \times \frac{I_{12}I_{24}}{I_{14}I_{24}}$$

$$\omega_4 = \frac{10.472 \times 209}{144} = 15.199 \text{ rad/s}$$

$$\therefore \qquad \omega_{BD} = \omega_4 = 15.199 \text{ rad/s}$$

**1. (d) Solution:**

The curve $y = x(5 - x)$ cuts the $x$-axis at 0 and 5 as shown in figure below. Let the coordinates of the centroid be $(\bar{x}, \bar{y})$.



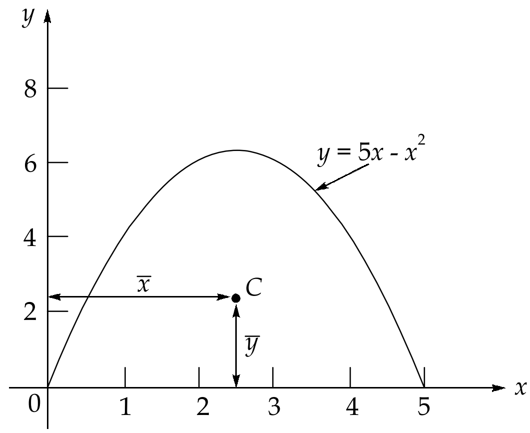

Let $\bar{x}$ and $\bar{y}$ be the distance of the centroid of area $A$ about $OY$ and $OX$ respectively then;

$$(\bar{x})(A) = \text{Total first moment of area A about axis}$$

$$OY = \int_a^b xy\, dx$$

From which, $\bar{x} = \dfrac{\int_a^b xy\, dx}{\int_a^b y\, dy}$ and $(\bar{y})(A) = $ Total moment of area A about axis $OX = \dfrac{1}{2}\int_a^b y^2 dx$

from which, $\bar{y} = \dfrac{\frac{1}{2}\int_a^b y^2 dx}{\int_a^b y dx}$




$$\bar{x} = \frac{\int_0^5 xy\,dx}{\int_0^5 y\,dx} = \frac{\int_0^5 x(5x - x^2)dx}{\int_0^5 (5x - x^2)dx} = \frac{\int_0^5 (5x^2 - x^3)dx}{\int_0^5 (5x - x^2)dx} = \frac{\left|\frac{5x^3}{3} - \frac{x^4}{4}\right|_0^5}{\left|\frac{5x^2}{2} - \frac{x^3}{3}\right|_0^5}$$

$$= \frac{\frac{625}{3} - \frac{625}{4}}{\frac{125}{2} - \frac{125}{3}} = \frac{625}{12} \times \frac{6}{125} = \frac{5}{2} = 2.5$$

$$\bar{y} = \frac{\frac{1}{2}\int_0^5 y^2 dx}{\int_0^5 y\,dx} = \frac{\frac{1}{2}\int_0^5 (5x - x^2)^2 dx}{\int_0^5 (5x - x^2)dx}$$

$$= \frac{\frac{1}{2}\int_0^5 (25x^2 - 10x^3 + x^4)dx}{\left(\frac{125}{6}\right)} \qquad \text{[From the calculation of } \bar{x}\text{]}$$

$$= \frac{\frac{1}{2}\left[\frac{25x^3}{3} - \frac{10x^4}{4} + \frac{x^5}{5}\right]_0^5}{\frac{125}{6}}$$

$$= \frac{\frac{1}{2}\left[\frac{25 \times 125}{3} - \frac{10 \times 625}{4} + 625\right]}{\frac{125}{6}} = \frac{6 \times 625}{2 \times 125}\left[\frac{5}{3} - \frac{10}{4} + 1\right]$$

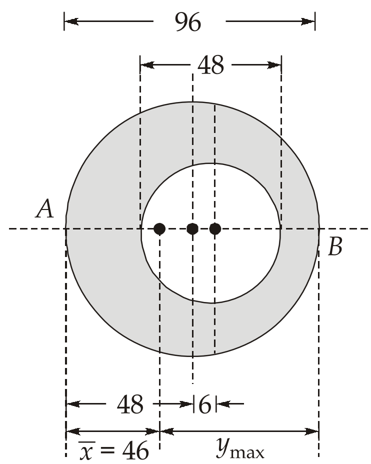
$$= (3\times5)\left[\frac{5\times4-10\times3}{12}+1\right] = 15\left[1-\frac{10}{12}\right] = 15\times\frac{2}{12} = \frac{15}{6}$$

$$= \frac{5}{2} = 2.5$$

Hence the centroid of the area lies at (2.5, 2.5) Ans.

**1. (e) Solution:**

Given : $D$ = 96 mm; $d$ = 48 mm




*(All dimensions are in mm)*

Area of hollow section, $A = \frac{\pi}{4}(96)^2 - \frac{\pi}{4}(48)^2 = 5428.67 \text{ mm}^2$

CG of hollow section from end $A$,

$$\bar{x} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

$$= \frac{\left(\frac{\pi}{4}\times96^2\right)\times48 - \left(\frac{\pi}{4}\times48^2\right)\times54}{\left(\frac{\pi}{4}\times96^2\right)-\left(\frac{\pi}{4}\times48^2\right)} = 46 \text{ mm}$$

Moment of inertia of section, $I = I_1 - I_2$

$$= \left(I_{G_1} + A_1h_1^2\right)-\left(I_{G_2} + A_2h_2^2\right)$$

$$= \left(\frac{\pi}{64}\times96^4 + \frac{\pi}{4}\times96^2\times2^2\right)-\left(\frac{\pi}{64}\times48^4 + \frac{\pi}{4}\times48^2\times8^2\right)$$

$$= \frac{\pi}{64}\left(96^4 - 48^4\right)+\frac{\pi}{4}\left(96^2 + 2^2 - 48^2\times8^2\right)$$

$$= 3908643.916 + (-86858.7537) = 3821785.16 \text{ mm}^4$$

Section modulus of hollow section,

$$Z = \frac{I}{y_{max}} = \frac{3821785.16}{50} \qquad (\because \quad y_{max} = 2 + 48 = 50 \text{ mm})$$

For not tensile, stress, $e \leq \frac{Z}{A}$

$\Rightarrow$ $e \leq \frac{76435.70}{5428.67}$

$\Rightarrow$ $e \leq 14.08$ mm

For no tensile stress, place the load within the distance of 14.08 mm either side of the CG of the hollow section.

**2. (a) Solution:**

Given : $m$ = 1 tonne, $F_0$ = 2500 N; $\omega$ = 1440 rpm; $\delta$ = 2.5 mm; $\xi$ = 0.22

**(i)** Natural frequency, $\omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{2.5 \times 10^{-3}}} = 62.642$ rad/s

Excitation frequency, $\omega = \frac{2 \times \pi \times N}{60} = \frac{2 \times \pi \times 1440}{60} = 150.8$ rad/s

Frequency ratio, $r = \frac{150.8}{62.642} = 2.407$

$\therefore$ Transmissibility, $T = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}}$

$$T = \frac{\sqrt{1 + (2 \times 0.22 \times 2.407)^2}}{\sqrt{(1 - (2.407)^2)^2 + (2 \times 0.22 \times 2.407)^2}}$$

$$T = 0.2967$$

Force transmitted to foundation,

$$F_T = TF_0 = 0.2967 \times 2500$$

$$F_T = 741.75 \text{ N}$$

**(ii)** Amplitude of vibration,

$$A = \frac{F_0 / k}{\sqrt{\left(1 - r^2\right)^2 + (2\xi r)^2}}$$

$$\text{Stiffness, } k = \frac{mg}{\delta} = \frac{1000 \times 9.81}{2.5 \times 10^{-3}} = 3.924 \times 10^6 \text{ N/m}$$

$$A = \frac{2500}{3.924 \times 10^6 \sqrt{\left[1-(2.407)^2\right]^2 + (2 \times 0.22 \times 2.407)^2}}$$

$$A = 1.298 \times 10^{-4} \text{ m}$$

$$A = 0.1298 \text{ mm}$$

**(iii)**

Let $\alpha$ be the angle made by the transmitted force (vector sum of spring force and damping force) with spring force.

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{c\omega}{k}\right) = \tan^{-1}(2\xi r)$$

$$\alpha = \tan^{-1}(2 \times 0.22 \times 2.407)$$

$$\alpha = 46.643°$$

**Q.2 (b) Solution:**

As per given information

Power, $P = 10$ kW

Speed, $N = 1440$ rpm

Speed reduction, $G = 4$

Pinion and gear made of same material ($\sigma_{ut} = 600$ N/mm$^2$)

Factor of safety = 1.5

The minimum number of teeth for 20° pressure angle is 18.

Pinion, $Z_P = 18$

Gear, $Z_G = G.Z_P = 4 \times 18 = 72$

$$\text{Torque, } T = \frac{60 \times 10^6 \times P}{2\pi N_P} = \frac{60 \times 10^6 \times 10}{2\pi \times 1440} = 66314.56 \text{ N.mm}$$

The Lewis form factor

$$Y = \pi y$$

$$Y = \pi\left(0.154 - \frac{0.912}{Z}\right) \text{ for 20° full depth involute system}$$

$$\text{For Pinion, } Y = \pi\left(0.154 - \frac{0.912}{18}\right)$$

$$Y = 0.324$$

When both materials are same then design is done on the basis of pinion.

$$\text{Service factor, } C_S = \frac{\text{Starting torque}}{\text{Rated torque}} = 1.5$$

$$C_V = \frac{3}{3+v}$$

$$v = \frac{\pi d_P N}{60 \times 1000} = \frac{\pi m Z_P N}{60 \times 1000} = \frac{\pi \times m \times 18 \times 1440}{60 \times 1000}$$

$$v = 1.357\, m \text{ m/s}$$

To avoid failure of pinion teeth,

$$S_b > P_{\text{effective}}$$

Introducing a factor of safety,

$$S_b = P_{\text{eff}} \times \text{FOS}$$

$$P_{\text{effective}} = \frac{P_t \times C_s}{C_v}$$

$$P_{\text{effective}} = \frac{T \times 2 \times C_s}{m z_P \times C_v}$$

$$S_b = m.b \times \sigma_b y$$

$$S_b > P_{\text{effective}}$$

$$m \times b \times \frac{\sigma_{ut}}{3} \times y \geq \frac{T \times 2 \times C_s \times \text{FOS}}{m.z_p \times C_v}$$

$$m \times 10m \times \frac{\sigma_{ut}}{3} \times y \geq \frac{T \times 2 \times C_s \times 1.5}{m.z_p C_v}$$

$$m \times 10 \times m \times \frac{600}{3} \times 0.324 \geq \frac{66314.55962 \times 2 \times 1.5 \times 1.5}{m \times 18 \times \left(\frac{3}{3+1.357m}\right)}$$

$$m^3 \times \left(\frac{3}{3+1.357m}\right) \times 10 \times 18 \times 200 \times 0.324 \geq 66314.55962 \times 2 \times 1.5 \times 1.5$$

$$34992\, m^3 \geq (895246.555 + 404949.86 \text{ m})$$

On solving $m \geq 4.202$

Using standard module of 5 mm

Width of tooth, $b = 10\, m = 10 \times 5 = 50$ mm

Diameter of pinion, $d_p = m z_p = 5 \times 18 = 90$ mm

Diameter of gear, $d_g = m z_g = 5 \times 72 = 360$ mm

Velocity of pinion, $V = m \times 1.357 = 6.785$ m/s

$$P_{\text{effective}} = \frac{C_s}{C_v} \times \frac{T \times 2}{m z_P} = \frac{1.5 \times 66314.55962 \times 2}{\left(\frac{3}{3+1.357 \times 5}\right) \times 5 \times 18}$$

$$= 7209.87 \text{ N}$$

Surface hardness of gears,

$$Q = \frac{2Z_G}{Z_G + Z_P} = \frac{2\times 72}{72+18} = 1.6$$

$$K = 0.16\left(\frac{BHN}{100}\right)^2 = 0.16\left(\frac{BHN}{100}\right)^2$$

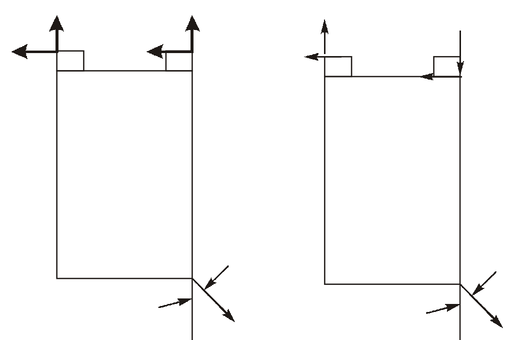
$$S_w = bQd_pK \geq P_{\text{effective}} \times \text{FOS}$$

$$10\times 5\times 1.6\times 90\times 0.16\left(\frac{BHN}{100}\right)^2 \geq 7209.87 \times 1.5$$

$$BHN \geq 306.396$$

**2. (c) Solution:**

There will two possible situations of loads acting on the panel as shown in the FBDs below:



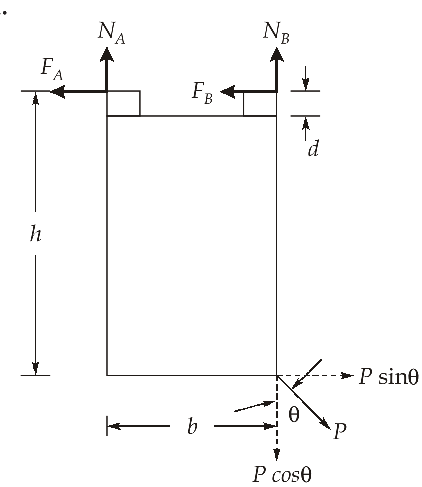
FBD -case (i)
Small angle

FBD -case (ii)
Large angle

**Case(i):**

$\mu_s = 0.4$, $\mu_k = 0.3$, $b = 400$ mm, $h = 625$ mm, $d = 25$ mm.

From equilibrium conditions of the panel we get,




$\Sigma M_A = 0$

$$bN_B = bP\cos\theta - hP\sin\theta$$

$\therefore$ $N_B \geq 0$

If $\tan\theta \leq \frac{b}{h} \Rightarrow \theta \leq 32.61°$

$\Sigma F_y = 0$

$$N_A + N_B = P\cos\theta$$

Panel will start moving to the right if:

$$P \sin\theta = F_A + F_B = \mu_s N_A + \mu_s N_B = \mu_s P \cos\theta$$

$\Rightarrow$ $\tan\theta = \mu_s \Rightarrow \theta = 21.8°$

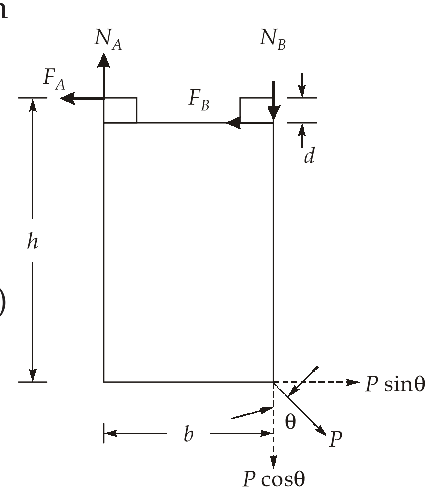
If $\theta > 21.8°$ the panel will move to the right under a net force of $P(\sin\theta - \mu_k \cos\theta)$

Therefore for case (i) the panel will move to the right if

$$21.8° \leq \theta \leq 32.61°$$

**Case (ii):**

$\mu_s = 0.4$, $b = 400$ mm, $\mu_k = 0.3$, $h = 625$ mm, $d = 25$ mm




From equilibrium conditions of the panel we get

$\Sigma M_A = 0$

$$bN_B + dF_B = hP \sin\theta - bP \cos\theta$$

As $F_B = \mu_s N_B$

$\therefore$ $$N_B = \frac{hP\sin\theta - bP\cos\theta}{b + \mu_s d} \quad \text{... (i)}$$

$\therefore$ $N_B \geq 0$

If $\tan\theta > \frac{b}{h} \Rightarrow \theta \geq 32.61°$

$\Sigma F_y = 0$

$$N_A - N_B = P \cos\theta$$

Panel will start moving to the right if:

$$P \sin\theta = F_A + F_B = \mu_s N_A + \mu_s N_B = \mu_s (P \cos\theta + 2N_B)$$

Replacing $N_B$ from (i) in the above we get

$$\tan\theta = \frac{2\mu_s b - \mu_s(b + \mu_s d)}{2\mu_s h - (b + \mu_s d)} = \frac{320 - 0.4 \times 410}{500 - 410} \Rightarrow \theta = 60.02°$$

$$32.61° \leq \theta \leq 60.02°$$

$$21.8° \leq \theta \leq 32.61° \qquad \text{From case (i)}$$

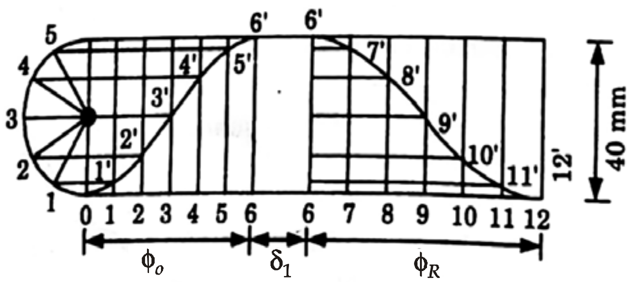
Hence, the range is between,

$$21.8° \leq \theta \leq 60.02°$$

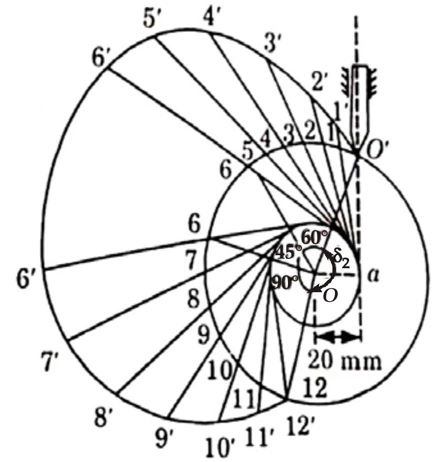
**3. (a) Solution:**

Given : $h = 40$ mm; $\phi_o = 60°$; $N = 300$ rpm; $\delta_1 = 45°$; $r_c = 50$ mm; $\phi_R = 90°$; $x = 20$ mm

The displacement diagram is drawn by following the usual procedure is shown in figure (a).




(a) Displacement diagram




(b) Cam profile

Construct the cam profile shown in figure (b) as follows :

1. Draw a circle with radius $r_c$ (= 50 mm)
2. Draw another circle concentric with the previous circle with radius $x$ (= 20 mm). If the cam is assumed stationary, the follower will be tangential to this circle in all the positions. Let the initial position be *O*-*O*′.
3. Join *O*-*O*′. Divide the circle of radius $r_c$ into four parts as usual with angles $\phi_a$, $\delta_1$, $\phi_d$ and $\delta_2$ starting from *O*-*O*′.
4. Divide the angles $\phi_o$ and $\phi_R$ into same number of parts as is done in the displacement diagram and obtain the points 1, 2, 3, etc., on the circumference of circle with radius $r_c$.
5. Draw tangents to the circle with radius $x$ from the points 1, 2, 3, etc.
6. On the extension of the tangent lines, mark the distances from the displacement diagram.
7. Draw a smooth curve through *O*′, 1′, 2′, etc. This is the required cam profile.

During outward stroke, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 300}{60} = 10\,\pi \text{ rad/s}$

We know that velocity, $v_{max} = \dfrac{h}{2}\dfrac{\pi\omega}{\phi_0}$

$$v_{max} = \frac{40}{2} \times \left[\frac{\pi \times 10\pi}{60 \times \frac{\pi}{180}}\right] = 1884.96 \text{ mm/s} = 1.88 \text{ m/s}$$

$$a_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\phi_o}\right)^2 = \frac{40}{2} \times \left[\frac{\pi \times 10\pi}{60 \times \frac{\pi}{180}}\right]^2$$

$$= 177652.88 \text{ mm/s}^2 = 177.652 \text{ m/s}^2$$

Similarly, during return stroke,

$$v_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\phi_R}\right) = \frac{40}{2} \times \left[\frac{\pi \times 10\pi}{90 \times \frac{\pi}{180}}\right]$$

$$= 1256.64 \text{ mm/s} = 1.25 \text{ m/s}$$

$$a_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\phi_R}\right)^2 = \frac{40}{2}\left[\frac{\pi \times 10\pi}{90 \times \frac{\pi}{180}}\right]^2$$

$$= 78956.84 \text{ mm/s}^2 = 78.956 \text{ m/s}^2$$

**3. (b)**

The beam is supported on the points $B$ and $D$, there will be support reactions say $R_B$ and $R_D$. Consider the three portions of the beam is $AB$, $BC$ and $CD$. Consider a section $X$-$X$ at a distance of $x$ from the end $A$. Taking upward force on the left side of the section to be positive.

**Positive AB:**

$F_x$ = –40 kN (Constant in the portion AB).

This shows that there is a vertical load of 40 kN acting on the point A.

**Positive BC:**

$F_x$ = +80 kN (Constant in the portion BC).

At the point B, SF has changed from –40 kN to +80 kN showing thereby that

$$-40 + R_B = 80 \text{ kN}$$

$$R_B = 120 \text{ kN}$$

Reaction at the support B,

$$R_B = 120 \text{ kN}$$

**Portion CD:,** At the point $C$,

$$SF = 0$$

Which shows that a vertical load of 80 kN is acting on this point.

From C to D, the SF is not constant but has a straight line relation which shows gradual decrease in SF, showing uniformly distributed load over the portion *CD*.

Let, the rate of loading = $w$

Then, *SF*, $F_C - F_D = -w \times 4$ (4 m is the length of the portion CD) where

$0 - 40\text{ kN} = -4w,$ $F_C$ = Shear force at *C* and $F_D$ = Shear force at *D*

$w = 10$ kN/metre run

At the point D, there is a SF,

$$F_D = -40\text{ kN}$$

(Note that vertically upward force on the right side of the section is taken as a negative SF).

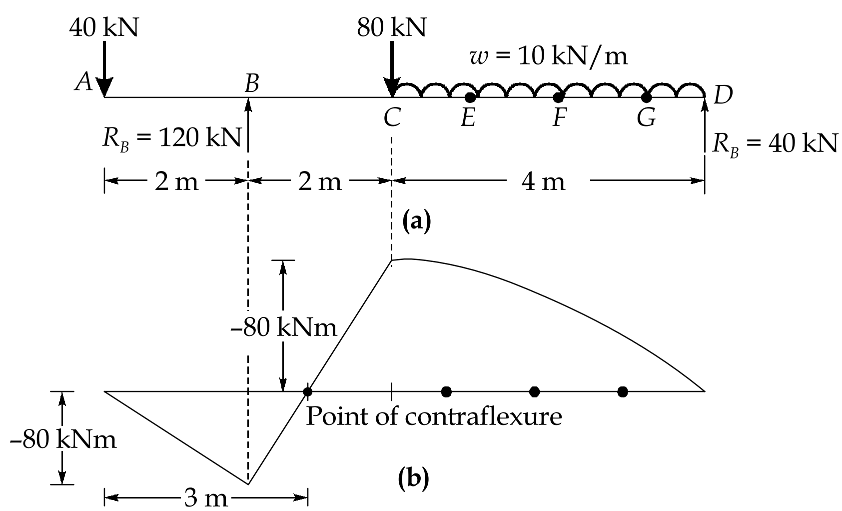
So, reaction at D, $R_D = 40$ kN

The load diagram on the beam is shown in figure(a) below.

Total vertical load on the beam = 40 + 80 + 4 × 10 = 160 kN

Reactions, $R_B + R_D = 120 + 40 = 160$ kN

which shows that load diagram is correct.




**BM diagram**

**BM diagram:** Taking clockwise moments on the left side of the section to be positive.

**Portion AB:** BM at any section,

$$M_x = -40x$$

$$M_A = 0, \text{ (at } x = 0\text{ m)}$$

$$M_B = -80\text{ kNm (at } x = 2\text{ m)}$$

**Portion BC:**

$$M_x = -40x + 120\,(x - 2)$$
$$M_B = -80 \text{ kNm, (at } x = 2 \text{ m)}$$
$$M_C = +80 \text{ kNm (at } x = 4 \text{ m)}$$

**Portion CD:**

$$M_x = -40x + 120\,(x - 2) - 80(x - 4) - \frac{w(x-4)^2}{2}$$

where, $w = 10$ kN/m

$$M_x = 80 - 5(x - 4)^2$$
$$M_C = 80 \text{ kNm (at } x = 4 \text{ m)}$$
$$M_E = 75 \text{ kNm (at } x = 5 \text{ m)}$$
$$M_F = 60 \text{ kNm (at } x = 6 \text{ m)}$$
$$M_G = 35 \text{ kNm (at } x = 7 \text{ m)}$$
$$M_D = 0 \text{ kNm (at } x = 8 \text{ m)}$$

**Figure (b)** shows the BM diagram. Maximum bending moment ±80 kNm occurs at distances of 2 and 4 m from the end A. Point of contraflexure lies at a distance of 3 m from the end A.

3. **(c) Solution:**

Given : $D = 450$ mm; $R = 225$ mm; $d = 800$ mm; $r = 400$ mm; OB = 25 mm; OA = 100 mm;

$$\theta = 250° = 250 \times \frac{\pi}{180} = 4.364 \text{ rad}\ ;\ \mu = 0.25;\ P = 750 \text{ N};\ l = OC = 3000 \text{ mm}$$

Since OA is greater than OB, therefore the operating force ($P$ = 750 N) will act downwards.

**Case I :** When the drum rotates in clockwise direction, the end of band attached to $A$ will be slack with tension $T_2$ and the end of the band attached to $B$ will be tight with tension $T_1$, as shown in figure below.

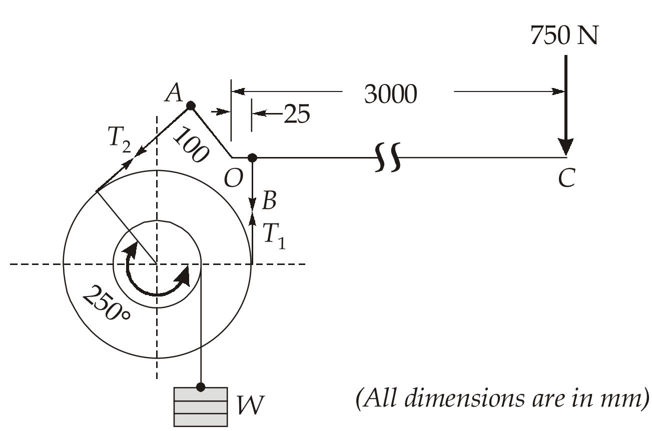



**Fig :** When drum rotates clockwise

We know that, $\frac{T_1}{T_2} = e^{\mu\theta} = e^{(0.25 \times 4.364)}$

$$\frac{T_1}{T_2} = 2.98 \text{ or } T_1 = 2.98T_2$$

Now, taking moments about the fulcrum *O*,

$$750 \times 3000 + T_1 \times 25 = T_2 \times 100$$

$$T_2 \times 100 - 2.98T_2 \times 25 = 2250 \times 10^3 \qquad (\because \quad T_1 = 2.98T_2)$$

$$25.5T_2 = 2250 \times 10^3$$

$$T_2 = 88.24 \times 10^3 \text{ N}$$

and $T_1 = 2.98T_2 = 2.98 \times 88.24 \times 10^3 = 262.94 \times 10^3$ N

We know that braking torque,

$$T_B = (T_1 - T_2)r$$

$$= (262.94 - 88.24) \times 10^3 \times 400$$

$$= 69.88 \times 10^6 \text{ N-mm} \qquad \text{...(i)}$$

And the torque due to load *W* newtons,

$$T_W = WR = W \times 225 = 225 \text{ W N-mm} \qquad \text{...(ii)}$$

Since the braking torque must be equal to the torque due to load *W* therefore from equation (i) and equation (ii)

$$W = 69.88 \times \frac{10^6}{225} = 310.58 \times 10^3 \text{N} = 310.58 \text{ kN}$$

**Case II :** When the drum rotates in anticlockwise direction, the end of the band attached to *A* will be tight with tension $T_1$ and end of the band attached to *B* will be slack with tension $T_2$, as shown in figure below. The ratio of tensions $T_1$ and $T_2$ will be same as calculated above, i.e.

$$\frac{T_1}{T_2} = 2.98 \text{ or } T_1 = 2.98T_2$$

Now, taking moments about the fulcrum *O*,

$$750 \times 3000 + T_2 \times 25 = T_1 \times 100$$

$$2.98T_2 \times 100 - T_2 \times 25 = 2250 \times 10^3 \qquad (\because \quad T_1 = 2.98T_2)$$

$$273T_2 = 2250 \times 10^3$$

$$T_2 = 8242 \text{ N}$$

and $T_1 = 2.98T_2 = 2.98 \times 8242 = 24561$ N

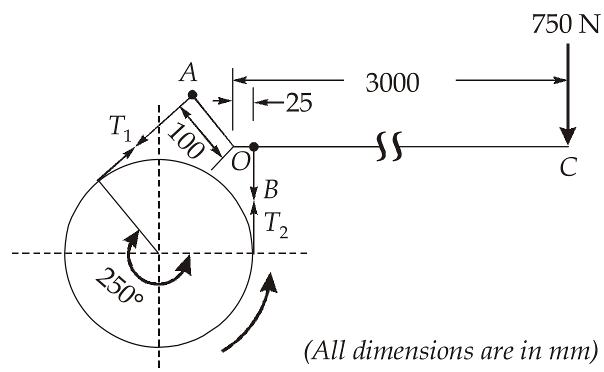



**Fig :** When drum rotates anticlockwise

Braking torque, $T_B = (T_1 - T_2)r$

$= (24561 - 8242) \times 400 = 6.53 \times 10^6$ N-mm ...(iii)

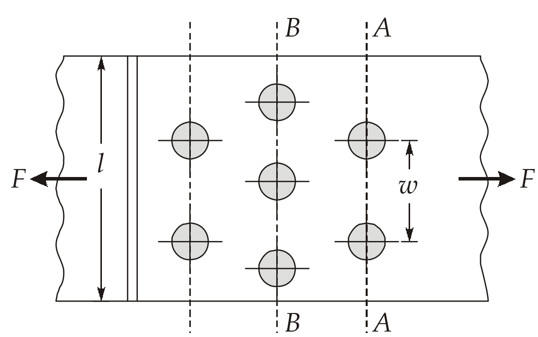
from equation (ii) and (iii)

$$W = 6.53 \times \frac{10^6}{225} = 29 \times 10^3 \text{N} = 29 \text{ kN}$$

From above, we see that the maximum load (*W* that can be supported by the brake is 309 kN, when the drum rotates in clockwise direction).

**4. (a) Solution:**

Given : $l \times t$ = 180 mm × 12 mm; $d$ = 18 mm; $w$ = 50 mm; $\sigma_{yt}$ = 200 MPa; t = 150 MPa; $\sigma_p$ = 300 MPa; $F$ = 200 kN




Load carrying capacity according to shear strength of rivets.

$$F_1 = n\frac{\pi}{4}d^2\tau = 7 \times \frac{\pi}{4} \times (18)^2 \times (150)$$

$$= 267.19 \text{ kN}$$

Load carrying according to crushing strength,

$$F_2 = n \times d \times t(\sigma_p)$$
$$= 7 \times 18 \times 12 \times 300$$
$$F_2 = 453.6 \text{ kN}$$

Load carrying capacity according to tensile strength, there are two cases.

**Case I :** If the plate fails in tension at cross-section A-A

$$(F_{\text{tension}})_{\text{A-A}} = (l - 2d)t(\sigma_{yt})$$
$$= (180 - 2 \times 18) \times 12 \times 200$$
$$= 345.6 \text{ kN}$$

**Case II :** If the plate fails in tension at cross-section B-B and rivets on cross-section A-A must either fail in shear or in crushing.

$$(F_{\text{tension}})_{\text{B-B}} = (l - 3d)t(\sigma_{yt})$$
$$= (180 - 3 \times 18) \times 12 \times 200$$
$$= 302.4 \text{ kN}$$

Force causes the shear failure of the rivet at cross-section A-A

$$(F_{\text{shear}})_{\text{A-A}} = 2 \times \frac{\pi}{4} d^2 \times \tau$$

$$= 2 \times \frac{\pi}{4}(8)^2 \times 150 = 76.34 \text{ kN}$$

Force causes the crushing failure of the rivet at cross-section A-A,

$$(F_{\text{crushing}})_{\text{A-A}} = 2 \times \text{d} \times t \times \sigma_p$$
$$= 2 \times 18 \times 12 \times 300$$
$$= 129.6 \text{ kN}$$

The force required for tensile failure at B-B and simultaneous shear (A-A) is

$$F_3 = (F_{\text{tension}})_{\text{B-B}} + (F_{\text{shear}})_{\text{A-A}}$$
$$= 302.4 + 76.34$$
$$= 378.74 \text{ kN}$$

The force required for tensile failure at B-B and simultaneous crushing of rivet at A-A is

$$F_4 = (F_{\text{tension}})_{\text{B-B}} + (F_{\text{crushing}})_{\text{A-A}}$$
$$= 302.4 + 129.6$$
$$= 432 \text{ kN}$$

Therefore maximum allowable load for joint is

minimum ($F_1$, $F_2$, $F_3$, $F_4$) = 267.19 kN

and applied load is 200 kN. It means strength of joint is sufficient. No need to improve design.

**4. (b) Solution:**

**(i) Effort of a Governor:**

The effort of the governor is the mean force acting on the sleeve to raise or lower it for a given change of speed. At constant speed, the governor is in equilibrium and the resultant force acting on the sleeve is zero.

However, when the speed of the governor increases or decreases, a force is exerted on the sleeve which tends to move it. When the sleeve occupies a new steady position, the resultant force acting on it again becomes zero.

If the force acting at the sleeve changes gradually from zero (when the governor is in the equilibrium position) to a value $E$ for an increased speed of the governor, the mean force or the effort is $\frac{E}{2}$.

For a Porter governor, the height is given by

$$h = \frac{g}{\omega^2} + \frac{M(1+k)}{2m\omega^2} = \frac{2mg + Mg(1+k)}{2m\omega^2} \quad \text{...(i)}$$

Let ω be increased by $c$ times ω where $c$ is a factor and $E$ be the force applied on the sleeve to prevent it from moving. Thus, the force on the sleeve is increased to ($Mg + E$).

Then,
$$h = \frac{2mg + (Mg+E)(1+k)}{2m(1+c)^2\omega^2} \quad \text{...(ii)}$$

Dividing equation (ii) by equation (i), we have

$$\frac{2mg + (Mg+E)(1+k)}{2mg + Mg(1+k)} = \frac{(1+c)^2}{1}$$

Subtracting 1 from both sides and solving,

$$\frac{[2mg + (Mg+E)(1+k) - [2mg + Mg(1+k)]]}{2mg + Mg(1+k)} = \frac{1 + c^2 + 2c - 1}{1}$$

$$\frac{E(1+k)}{2mg + Mg(1+k)} = 2c \qquad [\because \; c^2 \text{ being a small quantity is neglected}]$$

$$E = \frac{2c}{(1+k)}[2mg + Mg(1+k)]$$

$$\text{Effort, } \frac{E}{2} = \frac{cg}{1+k}[2m + M(1+k)]$$

**(ii) Power of a Governor :**

The power of a governor is the work done at the sleeve for a given percentage change of speed, i.e., it is the product of the effort and the displacement of the sleeve.

For a Porter governor, having all equal arms which intersect on the axis or pivoted at points equidistant from the spindle axis,

$$\text{Power} = \frac{E}{2}(2 \times \text{Height of governor})$$

The height of the governor changes from $h$ to $h_1$, when the speed changes from ω to $(1 + c)\omega$,

$$\therefore \quad h = \frac{2m + Mg(1+k)}{2m\omega^2} \text{ and } h_1 = \frac{2m + Mg(1+k)}{2m(1+c)^2\omega^2}$$

$$\text{or} \quad \frac{h_1}{h} = \frac{1}{(1+c)^2}$$

$$\text{Displacement of sleeve} = 2(h - h_1)$$

$$= 2h\left(1 - \frac{h_1}{h}\right) = 2h\left(1 - \frac{1}{(1+c)^2}\right)$$

$$= 2h\left(1 - \frac{1}{1+2c}\right) \qquad [\because \text{ Neglecting } c^2]$$

$$= 2h\left(\frac{2c}{1+2c}\right)$$

$$\text{Power} = (m+M)cg \times 2h\left(\frac{2c}{1+2c}\right)$$

$$= (m+M)gh\left(\frac{4c^2}{1+2c}\right)$$

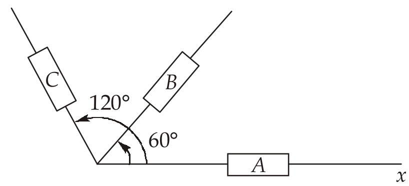
In case $k \neq 1$, displacement of sleeve,

$$= (1+k)(h - h_1) = (1+k)h\left(\frac{2c}{1+2c}\right)$$

$$\therefore \quad \text{Power} = \frac{cg}{1+k}[2m + M(1+k)](1+k)h\left(\frac{2c}{1+2c}\right)$$

$$= \left[m + \frac{M}{2}(1+k)\right]gh\left(\frac{4c^2}{1+2c}\right)$$

**4. (c) Solution:**




$\in_A = 60\ \mu\in$ at $\theta_A = 0°$

$\in_B = 135\ \mu\in$ at $\theta_B = 60°$

$\in_C = 264\ \mu\in$ at $\theta_C = 120°$

$\in_A = 60 = \in_x \cos^2 0° + \in_y \sin^2 0° + \gamma_{xy} \sin 0° \cos 0°$

$60 = \in_x + 0 + 0$

or $\quad \in_x = 60 = \in_A$ ...(i)

$\in_B = 135 = \in_x \cos^2 60° + \in_y \sin^2 60° + \gamma_{xy} \sin 60° \cos 60°$

$135 = 0.25\in_x + 0.75\in_y + 0.433\ \gamma_{xy}$ ....(ii)

$\in_C = 264 = \in_x \cos^2 120° + \in_y \sin^2 120° + \gamma_{xy} \sin 120° \cos 120°$

$264 = 0.25\in_x + 0.75\in_y - 0.433\ \gamma_{xy}$ ...(iii)

We have three equations and three unknowns.

By solving equations (i), (ii) and (iii), we get

$\in_x = 60\ \mu\in,\ \in_y = 246\ \mu\in$ and $\gamma_{xy} = -149\ \mu\in$

Principal strains and principal angles are given as

$$\in_{1,2} = \frac{\in_x + \in_y}{2} \pm \sqrt{\left(\frac{\in_x - \in_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\in_x - \in_y}$$

$$\in_{1,2} = \frac{60+246}{2} \pm \sqrt{\left(\frac{60-246}{2}\right)^2 + \left(\frac{-149}{2}\right)^2}$$

$$= 153 \pm \sqrt{8649 + 5550.25}$$

$$= 153 \pm 119.16$$

$\in_1 = 272.16 \simeq 272\ \mu\in$ Answer

$\in_2 = 33.84 \simeq 34\ \mu\in$ Answer

$$\tan 2\theta_P = \frac{-149}{60-246} = 0.8$$

$$\theta_{P1} = -70.6° \text{ or } 109.4°$$

and $\theta_{p2} = 19.4°$ Answer

Principal stresses are

$$\sigma_1 = \frac{E}{1-\nu^2}(\epsilon_1 + \nu\epsilon_2)$$

$$= \frac{210\times10^9}{1-(0.3)^2}(272+0.3\times34)\times10^{-6}$$

$= 65.123 \times 10^6$ N/m$^2 \simeq 65$ MPa Answer

$$\sigma_2 = \frac{E}{1-\nu^2}(\epsilon_2 + \nu\epsilon_1)$$

$$= \frac{210\times10^9}{1-0.09}(34+0.3\times272)\times10^{-6}$$

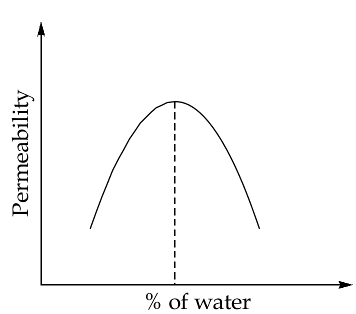
$= 26.67 \times 10^6 \simeq 27$ MPa Answer

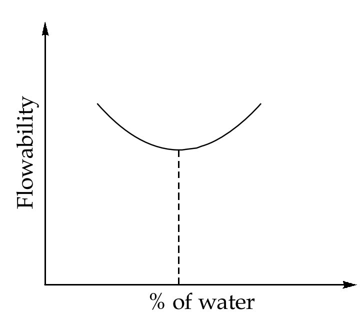
## Section : B

**Q.5 (a) Solution:**

Desirable properties of moulding sand :

1. **Strength :** This property is required for withstanding the force during casting process applied by the liquid metal on the mould surface.
2. **Refractoriness :** It is the ability of moulding sand to withstand high temperature of liquid metal without fusion.
3. **Permeability :** It is the ability of moulding sand to allow gases to escape. The permeability of uniform grain sand is more than the non-uniform grain sand.
4. **Flowability :** It is the ability of moulding sand to flow to all corners of mould box due to ramming.



5. **Mould Hardness :** It is represented by mould hardness number, which has an average value of 60-80. Hardness is required to avoid mould erosion. It is inverse of permeability.
6. **Collapsibility :** It is the property of moulding sand due to which surface will not offer any resistance to solid contraction of the casting.

**Additives used are :**

1. Saw Dust and Woodflour : These are used for collapsibility and permeability.
2. Linseed Oil and Molasses etc. : These are used for mould hardness.
3. Coal Dust and Graphite : These are used to get good surface finish.
4. Starch and Dextrin : This is used to improve resistance to deformation.

**5. (b) Solution:**

$$\text{Stock to be removed} = \frac{60-50}{2} = 5 \text{ mm}$$

$$\text{Finish allowance} = 0.8 \text{ mm}$$

**Roughing pass:**

$$\text{Roughing stock available} = 5 - 0.8 = 4.2 \text{ mm}$$

Since maximum depth of cut to be taken as 2 mm, there will be 3 roughing passes.

$$\text{Cutting speed, } v = 36 \text{ m/min}$$

$$\text{Average diameter} = \frac{60+50}{2} = 55 \text{ mm}$$

$$\text{Spindle speed, } N = \frac{36\times1000}{\pi\times55} = 208.35 \text{ rpm}$$

This rpm lies between spindle speed range of 180 rpm and 290 rpm.

$$\text{Nearest RPM} = \text{Minimum of } [(208.35-180)(290-208.35)]$$
$$= \text{Minimum of } (28.35, 81.65)$$

Therefore, nearest rpm available from the list is 180 rpm.

$$\text{Machining time for one pass} = \frac{160+2}{0.36\times180} = 2.5 \text{ minutes}$$

$$\text{Machining time for 3 passes} = 2.5 \times 3 = 7.5 \text{ minutes}$$

**Finishing pass:**

$$\text{Cutting speed, } v = 60 \text{ m/min}$$

$$\text{Spindle speed, } N = \frac{1000\times60}{\pi\times40} = 477.465 \text{ rpm}$$

The nearest RPM available from the list is 430 rpm.

$$\text{Machining time for one pass} = \frac{160+2}{0.2 \times 430} = 1.884 \text{ minutes}$$

$$\text{Total machining time} = \text{Roughing time} + \text{Finishing time}$$
$$= 7.5 + 1.884 = 9.384 \text{ minutes}$$

**5. (c) Solution:**

**Austempering or Isothermal Quenching :**

1. It is very similar to martempering. Steel is austenitized and then quenched in a salt bath maintained at a constant temperature in the range of 260°C to 400°C.
2. The article is held at this temperature for long enough to allow isothermal transformation to be completed.
3. After the complete transformation of austenite to bainite, steel is cooled to room temperature in air. It is also called isothermal quenching.
4. The temperature of quenching lies below the nose of the TTT curve and above the $M_S$ temperature.
5. Heat treatment cycle for austempering is shown in figure below.

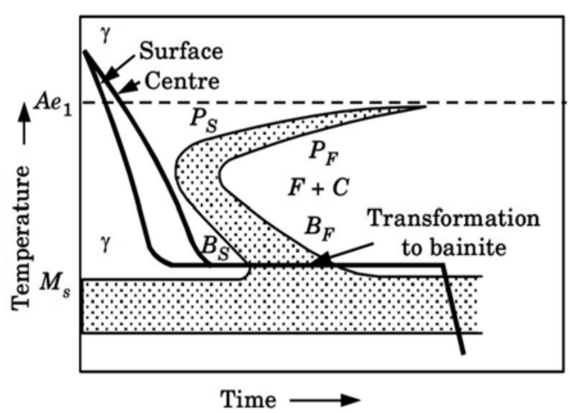



**Fig.** Heat treatment cycle for austempering

**Martempering or Stepped Quenching:**

1. This is a hardening method that produces martensite. This method is also known as hardening by interrupted quenching.
2. First the steel is heated to the hardening temperature then quenched in a medium (salt bath) having a temperature slightly above the point where martensite starts to form (usually from 150°C to 300°C).

3. It is held until it reaches the temperature of the medium and then cooled further to room temperature in air or oil.
4. The holding time in quenching medium or bath should be sufficient to enable a uniform temperature to be reached throughout the cross section but not long enough to cause austenite decomposition.
5. Austenite is transformed into martensite during the subsequent period of cooling to room temperature.
6. This treatment provides a structure of martensite and retained austenite in the hardened steel.

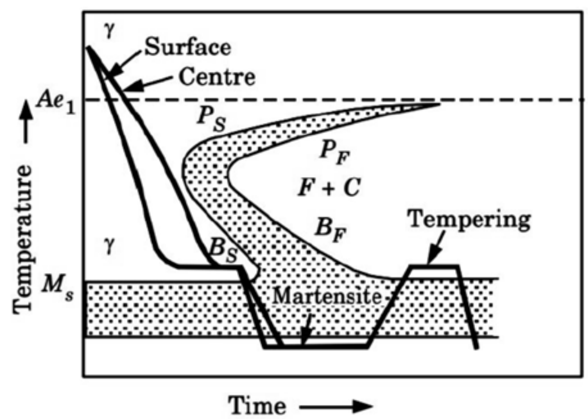



**Fig.** Heat treatment cycle for martempering

**5. (d) Solution:**

Rotation of 90° about $z$-axis:

$$\text{Rot}(z, 90°) = \begin{bmatrix} \cos 90° & -\sin 90° & 0 & 0 \\ \sin 90° & \cos 90° & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation of 90° about $y$-axis:

$$\text{Rot}(y, 90°) = \begin{bmatrix} \cos 90° & 0 & \sin 90° & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90° & 0 & \cos 90° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation of [4, –3, 7]

$$\text{Trans}(4,-3,7) = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{\text{noa}} = \begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

The matrix $x$ equation representing the transformation is

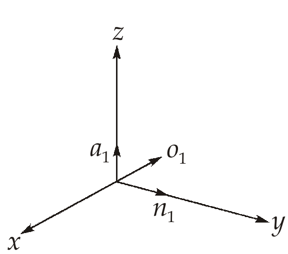
$$P_{xyz} = \text{Trans}(4,-3,7)\,\text{Rot}(y, 90°)\,\text{Rot}(z, 90°)\,P_{noa}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

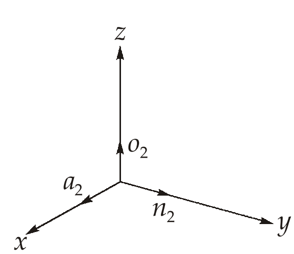
$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -3 \\ 7 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+4 \\ 7-3 \\ 3+7 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 10 \\ 1 \end{bmatrix}$$

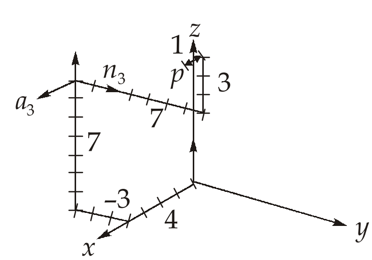
The first transformation of 90° about the $z$-axis rotates the $F_{\text{noa}}$ frame as shown in figure followed by the second rotation about the $y$-axis, followed by the translation relative to the reference frame $F_{xyz}$. The point $p$ in the frame can then be found relative to the $F_{\text{noa}}$ as shown. The final coordinates of the point can be traced on the $x$-, $y$-, $z$-axes to be 4 + 1 = 5, -3 + 7 = 4, and 7 + 3 = 10.




After the first transformation




After the second transformation




After the third transformation

**5. (e) Solution:**

Oil flow rate from pump, $Q = 0.002\ \text{m}^3/\text{s}$

Diameter of the cylinder, $D = 50\ \text{mm} = 0.05\ \text{m}$

Diameter of the rod, $d = 20\ \text{mm} = 0.02\ \text{m}$

Load during the extension and retraction, $F = 6000\ \text{N}$

Piston velocity during extension stroke, $V_E = \dfrac{Q}{A_P}$

$$V_E = \frac{0.002}{\frac{\pi}{4}\left(0.05^2 - 0.02^2\right)} = 1.2\ \text{m/s}$$

Cylinder pressure during extension stroke,

$$P_E = \frac{F}{A_P} = \frac{6000}{\frac{\pi}{4} \times 0.05^2} = 30.6\ \text{bar}$$

Cylinder pressure during retraction stroke,

$$P_R = \frac{F}{A_P - A_R}$$

$$P_R = \frac{6000}{\frac{\pi}{4}\left(0.05^2 - 0.02^2\right)} = 36.4\ \text{bar}$$

Cylinder power during extension stroke $= \dfrac{P_E Q}{1000} = \dfrac{30.6 \times 10^5 \times 0.002}{1000} = 6.12\ \text{kW}$

Cylinder power during retraction stroke $= \dfrac{P_E Q}{1000} = \dfrac{36.4 \times 10^5 \times 0.002}{1000} = 7.28\ \text{kW}$

**6. (a) Solution:**

The forecasts using each of the above values are shown calculated in table below.

| Week $t$ | Sales (in '000) $D_t$ | $\alpha = 0.2$ | | $\alpha = 0.5$ | | $\alpha = 0.8$ | |
|---|---|---|---|---|---|---|---|
| | | $F_t$ | $\lvert D_t - F_t \rvert$ | $F_t$ | $\lvert D_t - F_t \rvert$ | $F_t$ | $\lvert D_t - F_t \rvert$ |
| 1 | 113 | 108.0 | 5.0 | 108.0 | 5.0 | 108.0 | 5.0 |
| 2 | 101 | 109.0 | 8.0 | 110.5 | 9.5 | 112.0 | 11.0 |
| 3 | 98 | 107.4 | 9.4 | 105.8 | 7.8 | 103.2 | 5.2 |
| 4 | 107 | 105.5 | 1.5 | 101.9 | 5.1 | 99.0 | 8.0 |
| 5 | 120 | 105.8 | 14.2 | 104.4 | 15.6 | 105.4 | 14.6 |
| 6 | 132 | 108.7 | 23.3 | 112.2 | 19.8 | 117.1 | 14.9 |
| 7 | 110 | 113.3 | 3.3 | 122.1 | 12.1 | 129.0 | 19.0 |
| 8 | 117 | 112.7 | 4.3 | 116.1 | 0.9 | 113.8 | 3.2 |
| 9 | 112 | 113.5 | 1.5 | 116.5 | 4.5 | 116.4 | 4.4 |
| 10 | 125 | 113.2 | 11.8 | 114.3 | 10.7 | 112.9 | 12.1 |
| 11 | – | 115.6 | – | 119.6 | – | 122.6 | – |
| | | Total | 82.3 | | 91.0 | | 97.4 |

For $t = 1$, let $F_1 = 108$. Now, with $D_1 = 113$ and $\alpha = 0.2$, the forecast

For $t = 2$,

$$F_2 = F_1 + 0.2(D_1 - F_1)$$

i.e.,

$$F_2 = 108 + 0.2(113 - 108)$$

$$= 108 + 0.2 \times 5 = 109$$

For each case, the absolute difference between actual and forecasted values are obtained and averaged to get MAD.

From the above table,

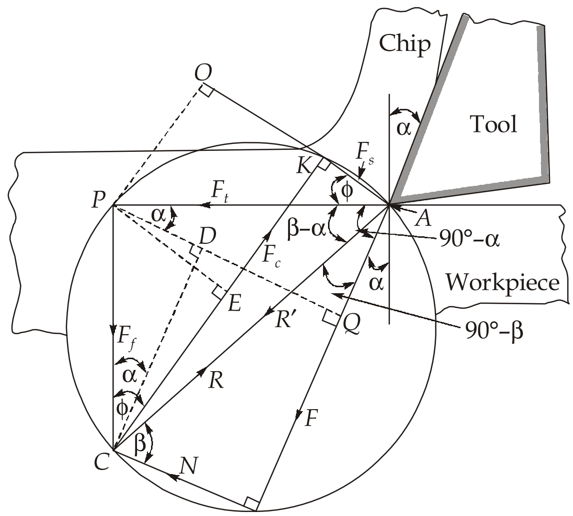
i. if $\alpha = 0.2$ then MAD = 82.3 /10= 8.23,

ii. if $\alpha = 0.5$ then MAD = 91/10 = 9.1,

iii. if $\alpha = 0.8$ then MAD = 97.4/10 = 9.74.

5. Since, MAD is found to be minimum for $\alpha = 0.2$ therefore, this smoothing constant is most preferable of these.

**6. (b)**




Chip thickness ratio: $r = \dfrac{t}{t_c} = \dfrac{\text{feed/rev(i.e. 0.25)}}{0.32} = 0.78$

From the given tool signature, α = 10°

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} = 0.888$$

$$\phi = \tan^{-1} 0.888 = 41.6°$$

$$\text{Shear force } (F_s) = F_t \cos\phi - F_f \sin\phi = 180 \cos 41.6° - 100 \sin 41.6°$$

$$= 68.2 \text{ kg}$$

Normal force acting on shear plane ($F_c$):

$$F_c = F_f \cos\phi + F_t \sin\phi = 100 \cos 41.6° + 180 \sin 41.6°$$

$$= 194.29 \text{ kg}$$

$$\text{Friction force } (F) = F_f \cos\alpha + F_t \sin\alpha = 100 \cos 10° + 180 \sin 10°$$

$$= 129.74 \text{ kg}$$

$$\text{Coefficient of friction } (\mu) = \frac{F_f \cos\alpha + F_t \sin\alpha}{F_t \cos\alpha - F_f \sin\alpha} = \frac{100\cos 10° + 180\sin 10°}{180\cos 10° - 100\sin 10°}$$

$$= 0.811$$

$$\text{Angle of friction } (\beta) = \tan^{-1}\mu = \tan^{-1} 0.811 = 39°$$

**Chip flow velocity ($V_c$):** $V_c = V_r$ where $V$ = Cutting velocity = 280 m/min

$$V_c = 280 \times 0.78 = 218.4 \text{ m/min}$$

**6. (c)**

Figure shows a 3-link elbow manipulator wherein frames are already assigned.

D-H table

| Joint | $\theta$ | $d$ | $a$ | $\alpha$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | $d_1$ | 0 | 90° |
| 2 | $\theta_2$ | 0 | $a_2$ | 0 |
| 3 | $\theta_3$ | 0 | $a_3$ | 0 |

Ans. (i)

$$A_1^0 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^1 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Overall matrix,

$$T_3^0 = A_1^0 A_2^1 A_3^2$$

$$= \begin{bmatrix} C_1C_2C_3 - C_1S_2S_3 & -C_1C_2S_3 - C_1S_2S_3 & S_1 & C_1C_2a_3 + C_1a_2 \\ S_1C_2C_3 - S_1S_2S_3 & -S_1C_2C_3 - S_1S_2C_3 & -C_1 & S_1C_2a_3 + S_1a_2 \\ S_2C_3 + C_2S_3 & -S_2S_3 + C_2C_3 & 0 & S_2a_3 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For $(\theta_1, \theta_2, \theta_3) = (90°, 0, 0)$

$$C_1 = \cos 90° = 0,\ S_1 = \sin 90° = 1,$$

$$C_2 = \cos 0° = 1,\ S_2 = \sin 0° = 0$$

$$C_3 = \cos 0° = 1,\ S_3 = \sin 0° = 0$$

$$C_1C_2C_3 = 0,\ C_1S_2S_3 = 0$$

$$C_1C_2S_3 = 0,\ C_1S_2S_3 = 0$$

$$S_1 = 1,\ S_1C_2C_3 = 1$$

$$S_1S_2S_3 = 0,\ S_1S_2C_3 = 0$$

$$S_2C_3 = 0, C2S3 = 0$$
$$S_2S_3 = 0, C_2C_3 = 1$$
$$C_1C_2 = 0, S_1C_2 = 1$$

Substituting these values in overall matrix, we get

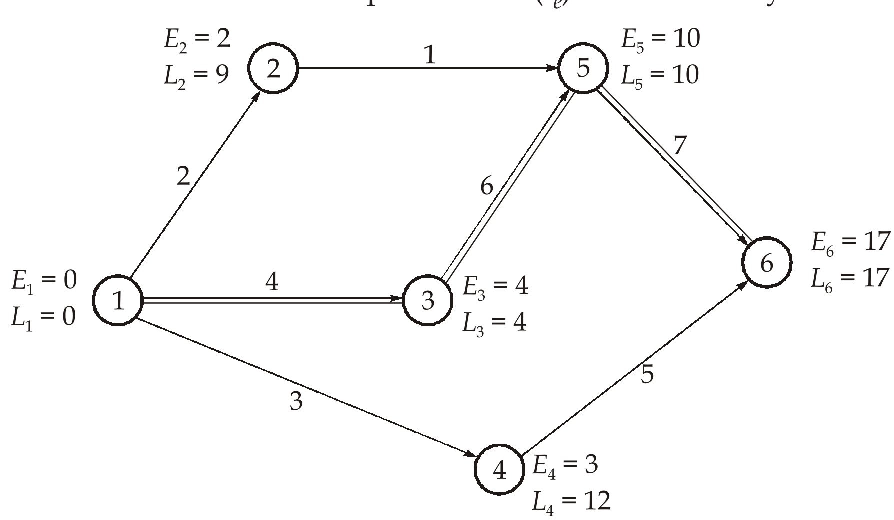
$$T_3^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & a_3 + a_2 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ Ans. (ii)

**7. (a) Solution**

The network diagram of activities in the project is shown in figure. The earliest and latest expected time for each event is calculated by considering the expected time of each activity, as shown in table.

| Activity | $t_o$ | $t_m$ | $t_p$ | $t_e = \frac{1}{6}(t_o + 4t_m + t_p)$ | $\sigma^2 = \left[\frac{1}{6}(t_p - t_o)\right]^2$ |
|---|---|---|---|---|---|
| 1 – 2 | 1 | 1 | 7 | 2 | 1 |
| 1 – 3 | 1 | 4 | 7 | 4 | 1 |
| 1 – 4 | 2 | 2 | 8 | 3 | 1 |
| 2 – 5 | 1 | 1 | 1 | 1 | 0 |
| 3 – 5 | 2 | 5 | 14 | 6 | 4 |
| 4 – 6 | 2 | 5 | 8 | 5 | 1 |
| 5 – 6 | 3 | 6 | 15 | 7 | 4 |

The E-values and L-values based on expected time ($t_e$) of each activity are shown in figure.



**(i)** Critical path is: 1-3-5-6.

**(ii)** The expected duration and variance for each activity is shown in table. The expected project length is the sum of the duration of each critical activity:

Expected project length = 1 - 3 - 5 - 6 = 4 + 6 + 7 = 17 weeks

**(iii)** Variance of the project length is the sum of the variances of each critical activity:

Variance of project length = 1 - 3 - 5 - 6 = 1 + 4 + 4 = 9 weeks

Therefore, standard deviation, $\sigma = \sqrt{9} = 3$

1. Probability that the project will be completed at least 4 weeks earlier (i.e. 13 weeks) than the expected project duration of 17 weeks is given by

$$\text{Prob.}\left[Z \le \frac{T_s - T_e}{\sigma} = \frac{13-17}{3}\right] = \text{Prob. } \{Z \le -1.33\} = 1 - P(Z < 1.33)$$

$$= 1 - 0.9082 = 0.0918$$

Thus the probability of completing the project in less than 13 weeks is 9.18 percent.

2. Probability that the project will be completed in 4 weeks later (i.e. 21 weeks) than expected project duration of 17 weeks is given by

$$P\left[Z \ge \frac{21-17}{3}\right] = P\{Z \ge -1.33\} = P(Z < 1.33) = 0.9082$$

Thus the probability that the project will be completed no more than 4 weeks later than the expected time is 90.82%.

Now, if project due data, $T_s$ = 19 weeks,

Then, probability that the project completed within 19 weeks.

$$P\left[Z < \frac{T_s - T_e}{\sigma}\right] = P\left[Z < \frac{19-17}{3}\right] = P[Z < 0.67] = 0.7486$$

Probability of completion of project = 74.86%

So, the probability of not meeting due date = (1 – 0.7486) × 100

= 24.14%

**7. (b) Solution:**

Given : machining cost, $c_m$ = ₹360 per hour = ₹6 per min

Total cost, $c_t$ = ₹60

Total change time, $t_c$ = 1 min

Idle time (loading and unloading) for component change, $t_o$ = 3 min

Setup time, $t_i$ = 2 hour

Number of parts, $p$ = 1200

Taylor's tool life equation, $VT^{0.2} = 600$

$n = 0.2;\ c = 600$

Optimum tool life for minimum cost,

$$T_0 = \left(t_c + \frac{c_t}{c_m}\right)\left(\frac{1-n}{n}\right) = \left(1+\frac{60}{6}\right)\left(\frac{1-0.2}{0.2}\right) = 44 \text{ min}$$

$$\text{Optimum cutting speed, } v_0 = \frac{c}{T_o^n} = \frac{600}{(44)^{0.2}} = 281.488 \text{ m/min}$$

$$\text{Machining time, } T_m = \frac{\pi DL}{100 f v_0} = \frac{\pi \times 100 \times 200}{1000 \times 0.15 \times 281.488} = 1.488 \text{ min}$$

Total time per piece,

$$T_{\text{total}} = \text{Idle time }(t_0) + \frac{\text{Initial setup time for a batch }(t_i)}{\text{Number of parts produced per batch } P}$$

$$+ \text{Machining time }(T_m) + \frac{\text{Total change }(T_i) \times \text{Machining time }(T_m)}{\text{Optimum tool life }(T_o)}$$

$$= 3 + \frac{2\times 60}{1200} + 1.488 + 1 \times \frac{1.488}{44}$$

$$T_{\text{total}} = 4.62 \text{ min/piece}$$

Total production time for a batch (1200 component)

$$T_{\text{total}} = 5544 \text{ min}$$

$$\text{Total cost per piece} = c_m\left(t_o + \frac{t_e}{p} + t_m\right) + \left(c_t + T_c \times c_m\right) \times \frac{T_m}{T_o}$$

$$= 6\left(3 + \frac{2\times 60}{1200} + 1.488\right) + (60 + 1\times 6) \times \frac{1.488}{44} = 29.76$$

$$= 27.528 + 2.232$$

= ₹29.76 per piece

Hence, the total production cost for batch of 1200 component = ₹35712.

**7. (c)**

**Corrosion :** It is defined as the deterioration or destruction of material due to its interaction with environment or in other words it is reverse of metallurgy. In metallurgy, metals are extracted from their ores and refined and then converted into slabs, tubes etc. The material then starts transforming into oxides or hydrides again due to interaction with the environment, therefore, in the material world nothing is permanent.

**There are generally 8 forms of corrosion:**

(a) Uniform attack
(b) Galvanic corrosion
(c) Crevice corrosion
(d) Pitting corrosion
(e) Intergranular corrosion
(f) Selective leaching
(g) Erosion corrosion
(h) Stress corrosion

**Following are methods to control corrosion**

(a) Material selection
(b) Alteration of environment
(c) Design
(d) Cathodic protection
(e) Anodic protection
(f) Coatings

**Selective leaching :** It is nothing but loss of one chemical element from an alloy. For example, dezincification, where the element Zn is depleted from yellow brass (30% Zn and 70% Cu) consequently, it will be turned into red.

It is because Zn is more active as compared to Cu, therefore Zn will be oxidized initially, there is reduction of $H_2O$ into $H_2$ and hydroxide ions without oxygen but later on the rate increases with environmental oxygen. Similarity in some alloy systems, A*l*, Cobalt, iron, Cr and other elements are removed and they are called as dealuminumification, decobaltification, graphitization etc.

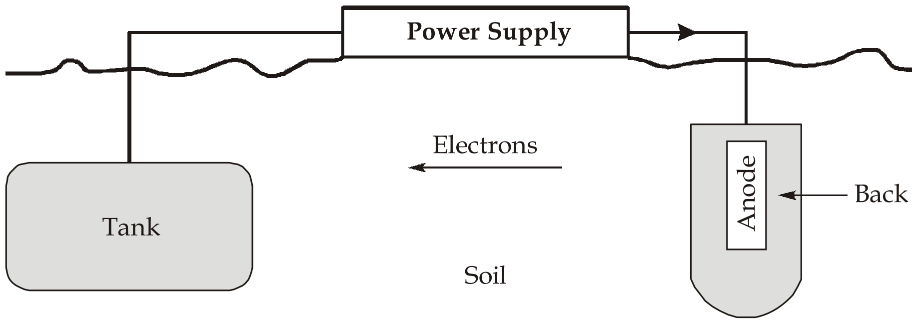
**Cathodic Protection :** In essence, the electrochemical reactions involved in corrosion are metal dissolution and hydrogen evolution as shown below:

$$M \longrightarrow M^{n+} + ne \quad ...(vii)$$

$$2H^+ + 2e \longrightarrow H_2 \quad ...(viii)$$

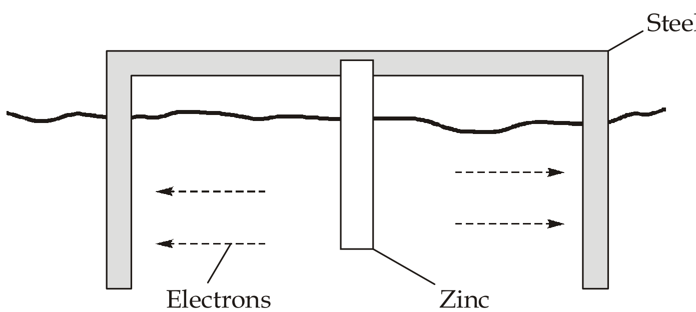
Therefore, if the electrons are released from metal, into electrolyte, the metal corrodes rigorously. Conversely, as long as electrons are supplied to the metal it is protected. Hence, this principle, *supplying electrons to metal through electrolyte is called cathodic protection.* It can be accomplished in two ways (1) by external power supply (2) by galvanic effect.

The accomplishment of cathodic protection of a tank in soil by external DC supply with an inert anode is depicted in the figure given below. Negative terminal of the supply is connected to the buried tank whereas that of positive is connected to an anode which is surrounded by backfill consisting of coke breeze, gypsum, or bentonite to improve electric contact between soil and anode. Then, the electrons travel through soil (electrolyte) to tank therefore tank is protected.




**Cathodic protection by an external power supply**

The cathodic protection by galvanic effect of steel and zinc is illustrated in the figure below. The galvanic effect involves the increase of the rate of corrosion of anode, zinc, and protection of steel component.




**Cathodic protection by galvanic effect**

8. **(a) Solution:**

Development of microstructure in isomorphous alloys for non-equilibrium cooling.

1. Let us begin cooling from a temperature of about 1300°C this is indicated by point $a'$ in the liquid region.
2. This liquid has a composition of 35 wt% Ni-65 wt% Cu noted as L(35 Ni) as shown in figure below, and no changes occur while cooling through the liquid phase region (moving down vertically from point $a'$).
3. At point $b'$ (approximately 1260°C) α-phase particles begin to form, which, from the tie line constructed have a composition of 46 wt% Ni-54 wt% Cu [α(46 Ni)].
4. Upon further cooling to point $c'$ (about 1240°C ), the liquid composition has shifted to 29 wt% Ni-71 wt% Cu; furthermore, at this temperature the composition of the a phase that solidified is 40 wt% Ni-60 wt% Cu [α(40 Ni)].
5. However, since diffusion in the solid a phase is relatively slow, the a phase that formed at point b' has not changed composition appreciably that is, it is still about 46 wt% Ni-and the composition of the α grains has continuously changed with

radial position, from 46 wt% Ni at grain centers to 40 wt% Ni at the outer grain perimeters.

6. Thus, at point *c*′, the average composition of the solid α grains that have formed would be some volume weighted average composition, lying between 46 and 40 wt% Ni. Take this average composition to be 42 wt% Ni-58 wt% Cu [α(42 Ni)].
7. Furthermore, we would also find that, on the basis of lever-rule computations, a greater proportion of liquid is present for these non equilibrium conditions than for equilibrium cooling.
8. The implication of this non-equilibrium solidification phenomenon is that the solidius line on the phase diagram has been shifted to higher Ni contents-to the average compositions of the a phase, (e.g., 42 wt% Ni at 1240°C) and is represented by the dashed line in figure below.

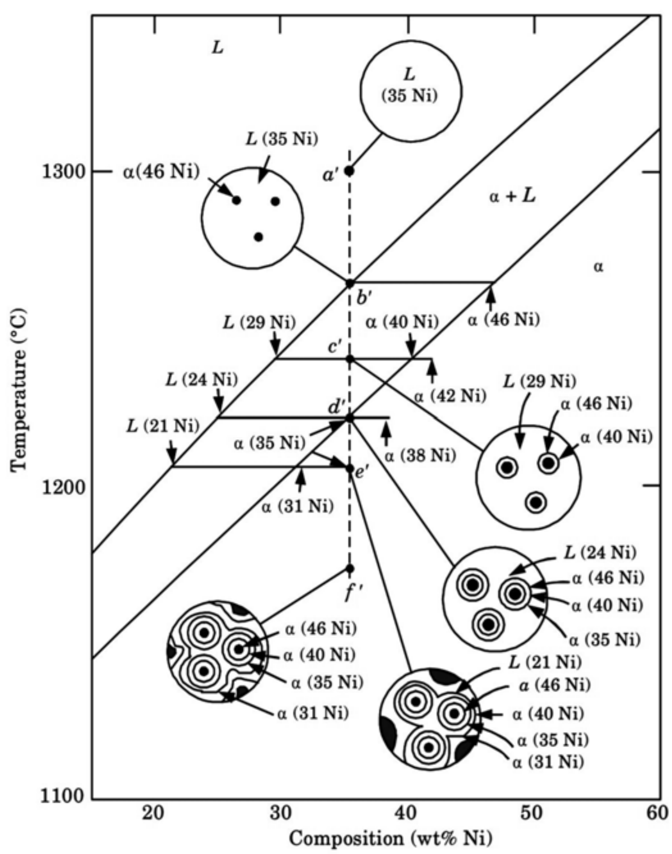



**Fig.** Development of micro structure during non equilibrium solidification of 35% Ni-65% Cu alloy

9. There is no comparable alteration of the liquidus line in as much as it is assumed that equilibrium is maintained in the liquid phase during cooling because of sufficiently rapid diffusion rates.

10. At point $d'$ (~1220°C) and for equilibrium cooling rates, solidification should be completed. However, for this non equilibrium situation, there is still an appreciable proportion of liquid remaining, and the a phase that is forming has a composition of 35 wt% Ni [α(35 Ni)]; also the average α-phase composition at this point is 38 wt% Ni [α(38 Ni)].
11. Non equilibrium solidification finally reaches completion at point $e'$ (~1205 °C), composition of the last a phase to solidify at this point is about 31 wt% Ni, the average composition of the a phase at complete solidification is 35 wt% Ni.
12. The inset at point $f'$ shows the microstructure of the totally solid material.
13. The degree of displacement of the non-equilibrium solidus curve from the equilibrium one will depend on rate of cooling.
14. The slower the cooling rate, the smaller this displacement, that is, the difference between the equilibrium solidus and average solid composition is lower.

**8. (b) Solution:**

From the data of the problem, we have

$$D = 2000 \text{ units/year}, \; r = 0.25, \; C = \text{Rs } 10 \text{ per item}$$

$$C_0 = \text{Rs } 50/\text{order}; \qquad C_h = C \times r = 10 \times 0.25 = \text{Rs } 2.50$$

When no discount is offered, the optimal order quantity is given by:

$$Q^* = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2 \times 2,000 \times 50}{2.5}} = 283 \text{ units (approx.)}$$

Also, the number of orders per year is given by:

$$N = \frac{D}{D^*} = \frac{2,000}{283} = 7 \text{ orders}$$

The total inventory cost for $Q^* = 283$ becomes

$$TC = DC + \frac{D}{Q^*}C_0 + \frac{Q^*}{2}C_h$$

$$= 2,000 \times 10 + \frac{2,000}{283} \times 50 + \frac{283}{2} \times 2.5 = \text{Rs. } 20,707.10$$

When quantity discounts are offered, the following information is available:

| Quantity | Price per unit (Rs) |
|---|---|
| $0 < Q_1 < 399$ | 10 |
| $400 \le Q_2 < 699$ | 9(10% discount) |
| $700 \le Q_3$ | 8(20% discount) |

The optimal order quantity $Q_3^*$ based on price $C_3$ = Rs 8 is given by:

$$Q_3 = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2 \times 2,000 \times 50}{8 \times 0.25}} = 316 \text{ units (approx.)}$$

The value $Q_3^* = 316$ lies in the first range, $0 < Q_1 < 399$. Thus computing and then comparing $TC(Q_1^*)$.

$TC(b_1 = 400)$ and $TC$ $(b_2 = 700)$ with each other.

$$TC(Q_1^*) = DC_1 + \frac{D}{Q_1^*}C_0 + \frac{Q_1^*}{2}(C_1 \times r)$$

$$= 2,000 \times 10 + \frac{2,000}{283} \times 50 + \frac{283}{2} \times 2.50 = \text{Rs. } 20,707.10$$

$$TC(b_1) = DC_2 + \frac{D}{b_1}C_0 + \frac{b_1}{2}(C_2 \times r)$$

$$= 2,000 \times 9 + \frac{2,000}{400} \times 50 + \frac{400}{2} \times 9 \times 0.25 = \text{Rs. } 18,700.00$$

$$TC(b_2) = DC_3 + \frac{D}{b_2}C_0 + \frac{b_2}{2}(C_3 \times r)$$

$$= 2,000 \times 8 + \frac{2,000}{700} \times 50 + \frac{700}{2} \times 8 \times 0.25 = \text{Rs. } 16,842.86$$

Since $TC(b_2 = 700)$ is the lowest cost, therefore the optimal order quantity is $Q^* = b_2 = 700$ units.

Hence, the shopkeeper should accept the offer of 20 per cent discount only because in this case his net saving per year would be Rs (20,707.10 – 16,842.86) = Rs 3,864.14.

**8. (c)**

1. **Vibration monitoring :** The noise and vibration are the most important parameters to monitor a machine, particularly in the moving parts such as shafts, rotors, cutting tools, gears etc. The vibration level is recorded by attaching a transducer like velocity probe, accelerometer, or proximity probe to the machine. Special equipment is also available for using the output from the sensor to indicate the nature of vibration problem and even its precise cause. In some cases, it may become necessary to use the principles of sonics and acoustics.
2. **Wear debris monitoring :** This works on the principle that the working surfaces of a machine are washed by the lubricating oil, and any damage to them should be detectable from particles of wear debris in the oil. If the debris consists of relatively large ferrous lumps such as those generated by the fatigue of rolling element bearings and gears or the pitting of cams and taproots, these can be picked up by removable magnetic plugs inserted in the oil return lines. For small debris particle, spectrographic analysis or microscopic examination of oil samples after magnetic separation are commonly used techniques. Another popular technique is SOAP analysis for debris analysis.
3. **Thermography** is a technique that can be used to monitor the condition of plant machinery, structures and system. It uses instrumentation designed to monitor the emission of infrared energy, that is temperature to determine their operating condition. By detecting thermal anomalies or areas which are hotter or colder than they should be, an experienced surveyor can locate and define incipient problems within the plant. The intensity of infrared radiation from an object is function of its surface temperature. Inclusion of thermography into a condition monitoring programme will enable the user to monitor the thermal efficiency of critical processes system relying on heat transfer or retention that will improve reliability of the plant.
4. **Visual inspection :** Regular visual inspection of the machinery and systems in a plant is a necessary part of any condition monitoring programme. In many cases, visual inspection will detect potential problems that will be missed using other condition monitoring techniques. Routine visual inspection of all critical part system will augment the other gytechniques and ensure potential that problems are detected before serious damage can occur, since the incremental cost of visual observations are small, this technique should be incorporated in all condition monitoring programme.

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