MADE EASY

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Detailed Solutions

ESE-2025 Mains Test Series

E & T Engineering Test No : 7

Section A

Q.1 (a) Solution:

From the figure,

$$V_4 = 40 \text{ V} \quad \text{...(i)}$$

Nodes 2 and 3 form a supernode. We have,

$$V_3 = 5i_x + V_2 = 5\left[\left(\frac{V_2 - V_1}{5}\right)\right] + V_2 = 2V_2 - V_1 \quad \text{...(ii)}$$

Applying KCL at Node 1,

$$6 + \frac{V_1}{10} + \frac{V_1 - V_2}{5} + \frac{V_1 - V_4}{6} = 0$$

$$6 + \frac{V_1}{10} + \frac{V_1 - V_2}{5} + \frac{V_1 - 40}{6} = 0$$

$$\frac{7}{15}V_1 - \frac{1}{5}V_2 = \frac{2}{3} \quad \text{...(iii)}$$

Applying KCL for the supernode,

$$\frac{V_2 - V_1}{5} + \frac{V_2}{20} + \frac{V_3}{15} + \frac{V_3 - V_4}{2} = 0$$

$$\frac{V_2 - V_1}{5} + \frac{V_2}{20} + \frac{(2V_2 - V_1)}{15} + \frac{(2V_2 - V_1) - 40}{2} = 0 \quad \text{...using equation (ii)}$$

$$-\frac{23}{30}V_1 + \frac{83}{60}V_2 = 20 \quad \text{...(iv)}$$

Solving Eqs. (iii) and (iv), we get

$$V_1 = 10 \text{ V}$$

$$V_2 = 20 \text{ V}$$

From equation (ii), $V_3 = 2V_2 - V_1 = 40 - 10 = 30 \text{ V}$

Q.1 (b) Solution:

For full subtractor, the truth table is,

x	y	z	**Difference (D)**	**Borrow (B)**
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

For difference $D(x, y, z) = \Sigma m(1, 2, 4, 7)$

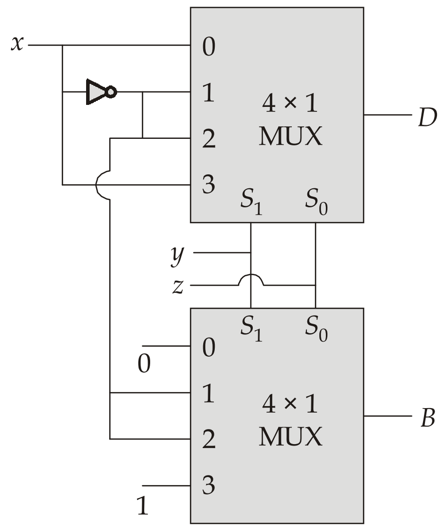
Borrow $B(x, y, z) = \Sigma m(1, 2, 3, 7)$

Consider y and z are connected to the select lines S_0 and S_1 of the Multiplexers to get the Difference D and Borrow B. The expressions for the input lines is obtained as below,

Sum	I_0	I_1	I_2	I_3
x'	0	(1)	(2)	3
x	(4)	5	6	(7)
	x	x'	x'	x

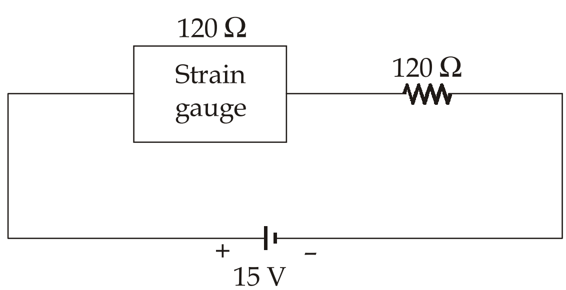
For Borrow B:

B	I_0	I_1	I_2	I_3
x'	0	(1)	(2)	(3)
x	4	5	6	(7)
	0	x'	x'	1



Q.1 (c) Solution:

Given,



When no stress is applied,

$$\text{Voltage across strain gauge,} \quad V_s = \frac{15 \times 120}{120 + 120} = 7.5 \text{ V}$$

When stress is applied, the resistance of the strain gauge changes. We have,

$$\text{Gauge factor } G = \frac{(\Delta R / R)}{(\Delta l / l)}$$

We know that,

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{\text{stress}}{\left(\frac{\Delta l}{l}\right)}$$

$$\therefore \quad \frac{\Delta l}{l} = \frac{\text{stress}}{\text{Young's modulus}} = \frac{150 \times 10^6}{200 \times 10^9} = 0.75 \times 10^{-3}$$

$$\therefore \quad \frac{\Delta R}{R} = G \times \frac{\Delta l}{l} = 2 \times 0.75 \times 10^{-3}$$

$\therefore \quad \Delta R = 1.5 \times 10^{-3} \times R = 1.5 \times 10^{-3} \times 120 = 0.18\ \Omega$

Voltage across strain gauge when stress is applied,

$$= 15 \times \frac{120.18}{120 + 120.18} = 7.50562\ \text{V}$$

$\therefore$ Difference in voltage $= 7.5 - 7.50562 = 0.00562$ V

$= 5.62$ mV

Q.1 (d) Solution:

Given, For BCC iron,

$$a_0 = 2.866 \times 10^{-8}\ \text{cm}$$

$$\text{atomic mass, } M = 55.847\ \text{g/mol}$$

$$\text{Density, } \rho = 7.87\ \text{g/cm}^3$$

A BCC unit cell has atoms located at each of its eight corners and one atom at the center of the cube. Each corner atom is shared by 8 unit cells, contributing 1/8 to the unit cell. The body-centered atom is entirely within the unit cell. Therefore, the total number of atoms per unit cell is given by

$$N = \frac{1}{8} \times 8 + 1 = 2\ \text{atoms/unit cell}$$

(i) Number of unit cells $= \dfrac{\text{mass}}{(\text{Volume of unit cell}) \times (\text{Density})} = \dfrac{M}{a_0^3 \times \rho}$

$$= \frac{0.59}{\left(7.87\ \text{g/cm}^3\right)\left(2.866 \times 10^{-8}\right)^3\ \text{cm}^3/\text{cell}}$$

$= 3.185 \times 10^{21}$ unit cells

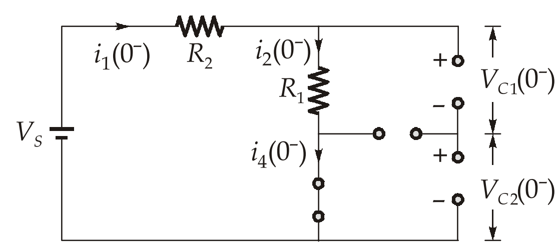
(ii) Number of Iron atoms = (No. of unit cells) × (No. of atoms per cell)

$= (3.185 \times 10^{21})\ (2)$

$= 6.37 \times 10^{21}$ atoms

Q.1 (e) Solution:

Consider the circuit at $t = 0^-$. At steady state, the inductor acts as short circuit and the capacitor acts as open-circuit.



Thus, $$i_1(0^-) = i_2(0^-) = i_4(0^-) = \frac{V_s}{R_1 + R_2} = i_L(0^-)$$

$$\text{Voltage drop in } R_1 = \frac{V_s \cdot R_1}{R_1 + R_2} = V_{R1}$$

As inductor is short, so V_{R1} will be distributed between C_1 and C_2. We get,

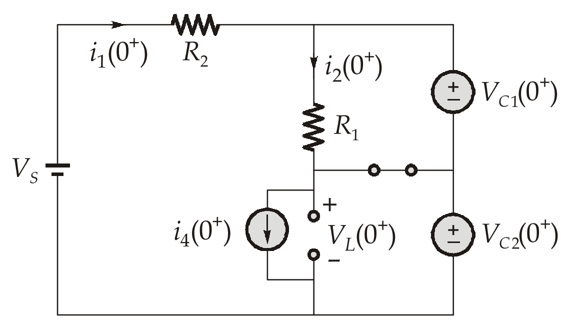
$$V_{C1}(0^-) = \frac{V_{R1}C_2}{C_1 + C_2}$$

and $$V_{C2}(0^-) = \frac{V_{R1}C_1}{C_1 + C_2}$$

Consider the circuit when switch is closed,

i.e., circuit at $t > 0$,

At $t = 0^+$;



The capacitor does not allow sudden change in voltage. Thus,

$$V_{C1}(0^+) = V_{C1}(0^-) = \frac{V_{R1}C_2}{C_1 + C_2}$$

$$V_{C2}(0^+) = V_{C2}(0^-) = \frac{V_{R1}C_1}{C_1 + C_2}$$

Similarly, the inductor does not allow sudden change in current. Therefore,

$$i_4(0^-) = i_4(0^+) = \frac{V_s}{R_1 + R_2}$$

$$V_{R1}(0^+) = V_{C1}(0^+) = \frac{C_2}{C_1 + C_2} \cdot \frac{V_s \cdot R_1}{R_1 + R_2}$$

$$i_2(0^+) = \frac{V_{R1}(0^+)}{R_1} = \frac{V_s C_2}{(R_1 + R_2)(C_1 + C_2)}$$

$$i_1(0^+) = \frac{V_s - [V_{C1}(0^+) + V_{C2}(0^+)]}{R_2}$$

$$= \frac{V_S - \left(\frac{V_{R1}C_2}{C_1 + C_2} + \frac{V_{R1}C_1}{C_1 + C_2}\right)}{R_2}$$

$$= \frac{V_S - \frac{V_S R_1}{R_1 + R_2}}{R_2} = \frac{V_S R_1 + V_S R_2 - V_S R_1}{(R_1 + R_2)R_2}$$

$$i_1(0^+) = \frac{V_S \cdot R_2}{(R_1 + R_2)R_2} = \frac{V_S}{R_1 + R_2}$$

Q.2 (a) Solution:

(i) Let the state of emergency shutdown switch be denoted by 'A' and, state of sensor 1, sensor 2 and sensor 3 denoted by B, C and D respectively.

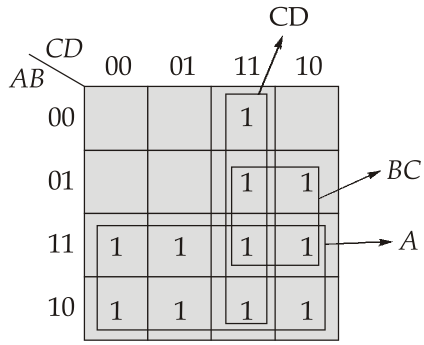
If the switch is pressed or sensor is activated, it is treated as logic 1.

The state of assembly line is denoted by F, with the shutdown of line considered as $F = 1$.

The truth table for the circuit is as shown below,

A (Emergency Switch)	B (Sensor 1)	C (Sensor 2)	D (Sensor 3)	F (Shutdown)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

By using 4-variable K-map,

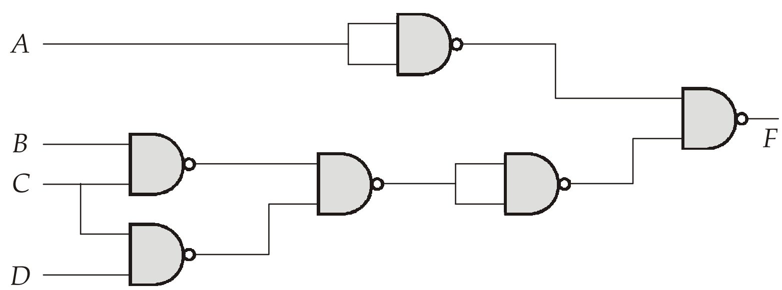


$\therefore$ The output logic expression, $F = A + BC + CD$

$$\bar{\bar{F}} = \overline{\overline{A + BC + CD}}$$

$$= \overline{\bar{A} \cdot \overline{BC} \cdot \overline{CD}}$$

Two input NAND realization for the logic expression F is,



(ii) Given, Hexadecimal number,

$$7B8_H = 011110111000_2$$

Also given, 5 bits represent integral part and 7 bits represent fractional part. Thus, the binary equivalent of the number is given by

$$N = 01111.0111000_2$$

Thus, the decimal equivalent can be obtained as below,

$$N = (2^4 \times 0 + 2^3 \times 1 + 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 1) + \left(\begin{array}{l} \frac{1}{2} \times 0 + \frac{1}{2^2} \times 1 + \frac{1}{2^3} \times 1 + \frac{1}{2^4} \times 1 \\ + \frac{1}{2^5} \times 0 + \frac{1}{2^6} \times 0 + \frac{1}{2^7} \times 0 \end{array} \right)$$

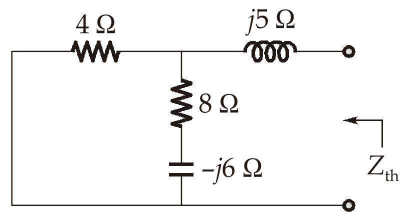
$$= (8 + 4 + 2 + 1) + (0 + 0.25 + 0.125 + 0.0625 + 0 + 0 + 0)$$

$$= 15.4375_{10}$$

Q.2 (b) Solution:

(i) For maximum power transfer in an AC circuit, $Z_L = Z_{th}^*$, where Z_{th} is the Thevenin equivalent impedance across the load Z_L. First we obtain thevenin equivalent circuit.

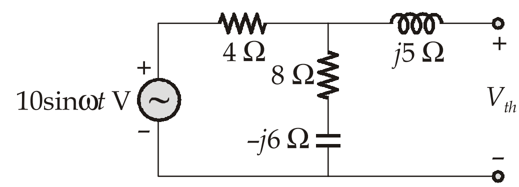
For Z_{th} :



$$Z_{\text{th}} = j5 + 4 \,||\, (8 - j6)$$

$$= j5 + \frac{4 \times (8 - j6)}{12 - j6} = 2.933 + j4.4667$$

For V_{th} : Using voltage division rule,



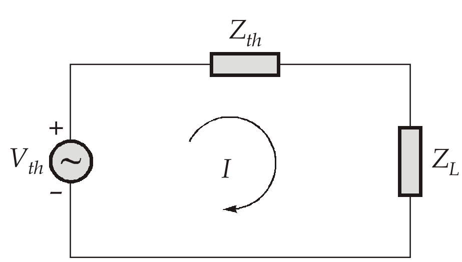
$$V_{\text{th}} = \frac{8 - j6}{4 + 8 - j6}(10\angle 0°)$$

$$= 7.4535\angle{-10.3°}$$

The load impedance draws the maximum power from the circuit when,

$$Z_L = Z_{th}^* = (2.933 - j4.467)\ \Omega = R_{\text{th}} - jX_{\text{th}}$$

The Thevenin's equivalent circuit is as shown below:



The current through the load is,

$$I = \frac{V_{th}}{Z_{th} + Z_L} = \frac{V_{th}}{R_{th} + jX_{th} + R_L + jX_L} = \frac{V_{th}}{2R_L}$$

$$P = \frac{1}{2}|I_m|^2 R_L = \frac{1}{2}\frac{|V_{th}|^2 R_L}{(2R_L)^2} = \frac{|V_{th}|^2}{8R_L}$$

The maximum power drawn from the circuit is,

$$P_{\max} = \frac{|V_{th}|^2}{8R_L} = \frac{(7.4535)^2}{8(2.933)} = 2.368 \text{ W}$$

(ii) Let the voltage and current at the terminal of the AC circuit be

$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_i)$$
$$P(t) = v(t)\, i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$
$$P(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

The average power is given by,

$$P_{\text{avg}} = \frac{1}{T}\int_0^T P(t)\,dt$$
$$P_{\text{avg}} = \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\,dt + \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

The second integrand is the average of a sinusoidal over its period, and is equal to zero. Thus, the second term vanishes and the average power becomes,

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$
$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

For purely resistive circuit or resistive load R,

$$P = \frac{1}{2} V_m I_m$$

Q.2 (c) Solution:

(i)
- Let I is the number of carriers which are injected per unit time at $x = 0$, then the number of carriers which have not yet recombined at a distance x can be given by,

$$I e^{-\frac{x}{L_p}}$$

- Considering a small slice of thickness dx at a distance x from the injecting surface, the number of carriers which have recombined in the slice can be given by,

$$I e^{-\frac{x}{L_p}} - I e^{-\left(\frac{x + dx}{L_p}\right)} = I e^{-\frac{x}{L_p}}\left[1 - e^{-\frac{dx}{L_p}}\right] \approx \frac{I dx}{L_p} e^{-\frac{x}{L_p}}$$

[$\because$ $e^{-x} \approx 1 - x$ for $|x| << 1$]

- Dividing the above expression by I, we get the probability of a carrier recombining between x and $x + dx$ as,

$$\frac{dx}{L_p} e^{-\frac{x}{L_p}}$$

- In the above expression, we can define x as the length travelled by a carrier before recombination and it can be modelled as a random variable with the probability density function,

$$f_X(x) = \frac{\frac{dx}{L_p} e^{-\frac{x}{L_p}}}{dx} = \frac{1}{L_p} e^{-\frac{x}{L_p}} \; ; x > 0$$

- The statistical average or the expectation of X will be equal to the average distance travelled by the carriers before they recombine and it can be calculated as,

$$E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\, dx = \int_0^{\infty} \frac{x}{L_p} e^{-\frac{x}{L_p}} dx = L_p \int_0^{\infty} u e^{-u} du = L_p \left[-ue^{-u} - e^{-u} \right]_0^{\infty} = L_p$$

- Hence, it is proved that the average distance travelled by the carriers before they recombine is L_p.

(ii) Since the collector is heavily doped compared to the base in a BJT, the depletion region width of the Collector-Base junction will primarily be determined by the width of depletion region of CB junction extended into the base region. Thus, the width of depletion region of CB junction extended into the base region, for uniform base doping, depends on V_{CB} as,

$$W_{\text{dep}} \propto \sqrt{V_{CB} + V_{bi}} \approx \sqrt{V_{CB}}$$

Given: $W_{\text{dep}} = 0.3\ \mu\text{m}$, when $V_{CB} = 5$ V.

Total base width = Neutral base width + Depletion layer width = 1.2 + 0.3 = 1.5 μm

When the neutral base region is completely depleted, $W_{\text{dep}} = 1.5\ \mu\text{m}$

So,
$$\frac{W_2}{W_1} = \sqrt{\frac{V_{CB2}}{V_{CB1}}}$$

$$V_{CB2} = \left(\frac{W_2}{W_1}\right)^2 \times V_{CB1} = \left(\frac{1.5}{0.3}\right)^2 \times 5\text{ V} = 125\text{ V}$$

Q.3 (a) Solution:

Let I_0, I_1, I_2 and I_3 represent the input data lines of MUX. Based on the select lines S_1S_0, one of the input data lines is propagated to the output.

From the given facts:

(i) If (S = 0 or S_1S_0 = 00) then $F = \bar{A}$ i.e., $I_0 = \bar{A}$ (use an inverter)

(ii) If (S = 1 or S_1S_0 = 01) then $F = A + B$

$\Rightarrow$ $I_1 = A + B$

Use full adder with $C_{in} = 0$

(iii) If (S = 2 or S_1S_0 = 10) then $F = A - B$,

$$\Rightarrow \quad F = A - B$$

$$F = A + (\text{2's complement of } B)$$

$$= A + (\text{1's complement of } B + 1)$$

$$= A + \bar{B} + 1$$

$$= A + (B \oplus 1) + 1$$

$$\therefore \quad F = A + (B \oplus 1) + 1$$

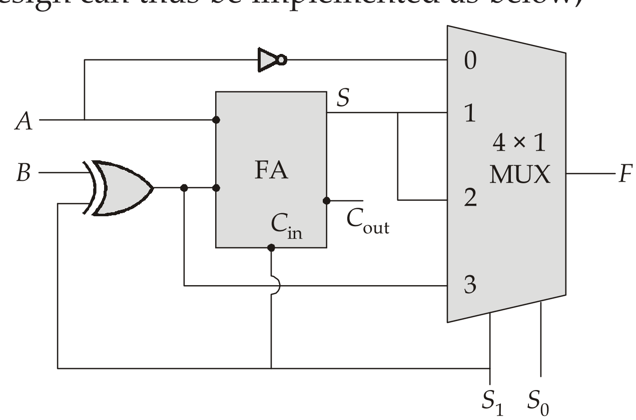
Use full adder with $C_{in} = 1$ and an XOR gate.

(iv) If (S = 3 or S_1S_0 = 11) then $F = \bar{B}$

$\Rightarrow$ $I_3 = \bar{B} = B \oplus 1$

Use XOR Gate with one input as B and other input as S_1. It is also used as an input to the full adder i.e. B when $S_1 = 0$ and $\bar{B}$ when $S_1 = 1$.

$\therefore$ The logic design can thus be implemented as below,



Q.3 (b) Solution

Given that there is one (Cs^+ – Br^-) pair per unit cell and a = 0.430 nm. So, the number of ion pairs per unit volume (N) can be given by,

$$N = \frac{1}{a^3} = \frac{1}{(0.430\times10^{-9})^3}\text{ m}^{-3} = 1.258\times10^{28}\text{ m}^{-3}$$

(i) From Clausius-Mossotti equation, at very low frequencies,

$$\frac{\varepsilon_{r(\text{low})}-1}{\varepsilon_{r(\text{low})}+2} = \frac{1}{3\varepsilon_0}[N\alpha_i + N\alpha_e] = \frac{1}{3\varepsilon_0}[N\alpha_i + N\alpha_{eB} + N\alpha_{eC}]$$

Given that, $\alpha_i = 5.8 \times 10^{-40}$ F-m², $\alpha_{eB} = 4.5 \times 10^{-40}$ F-m² and $\alpha_{eC} = 3.35 \times 10^{-40}$ F-m²

$$\frac{\varepsilon_{r(\text{low})}-1}{\varepsilon_{r(\text{low})}+2} = \frac{1.258\times10^{28}}{3\times8.854\times10^{-12}}\left[(5.8+4.5+3.35)\times10^{-40}\right] = 0.6465$$

$$(\varepsilon_{r(\text{low})} - 1) = (\varepsilon_{r(\text{low})} + 2)\,(0.6465)$$

$$(1 - 0.6465)\,\varepsilon_{r(\text{low})} = (2 \times 0.6465) + 1$$

$$\varepsilon_{r(\text{low})} = \frac{(2\times0.6465)+1}{(1-0.6465)} \approx 6.5$$

(ii) At optical frequencies, only electronic polarizability is dominant.

So,
$$\frac{\varepsilon_{r(\text{opt})}-1}{\varepsilon_{r(\text{opt})}+2} = \frac{N\alpha_e}{3\varepsilon_0} = \frac{N(\alpha_{eB}+\alpha_{eC})}{3\varepsilon_0}$$

$$= \frac{(1.258\times10^{28})\,(4.5+3.35)\times10^{-40}}{3\times8.854\times10^{-12}} = 0.372$$

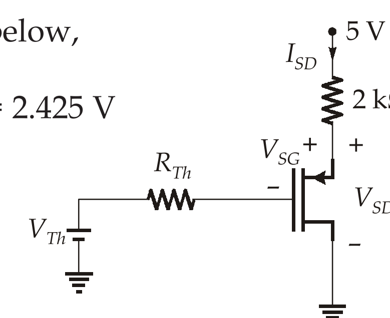
$$(\varepsilon_{r(\text{opt})} - 1) = (0.372)\,(\varepsilon_{r(\text{opt})} + 2)$$

$$(1 - 0.372)\,\varepsilon_{r(\text{opt})} = (2 \times 0.372) + 1$$

$$\varepsilon_{r(\text{opt})} = \frac{(2\times0.372)+1}{(1-0.372)} \approx 2.8$$

Q.3 (c) Solution:

(i) The Thevenin equivalent circuit can be obtained as below,



$$V_{Th} = \frac{5\times59.4+(-5)\times20.6}{59.4+20.6} = 2.425\text{ V}$$

$$V_{SG} = V_S - V_G$$

$$= 5 - 2\,I_{SD} - 2.425$$

$$= 2.575 - 2I_{SD}$$

$$\Rightarrow \quad I_{SD} = \frac{2.575 - V_{SG}}{2} \quad \text{...(i)}$$

Assume MOSFET is in saturation,

$$I_{SD} = \frac{\mu_p C_{ox}}{2} \cdot \frac{W}{L}(V_{SG} - |V_{TP}|)^2$$

$$\frac{2.575 - V_{SG}}{2} = 0.5\,(V_{SG} - 1.5)^2 \quad \Rightarrow \quad V_{SG}^2 - 2V_{SG} - 0.325 = 0$$

After solving, we get V_{SG} = 2.15 V, – 0.153 V

But, $V_{SG} > |V_{TP}|$ (For MOS transistor to be ON)

Therefore, V_{SG} = 2.15 V

(ii) $$I_{SD} = \frac{2.575 - 2.15}{2} = 0.2125\text{ mA} \quad \text{(From eqn. (i))}$$

(iii) $V_{SD} = 5 - 2\,I_{SD} = 4.575$ V

$V_{SD} > V_{SG} - |V_{TP}| \Rightarrow$ MOSFET is in saturation.

Q.4 (a) Solution:

(i) 1. Given $4\frac{1}{2}$ digit voltmeter is working on 100 V range. The 1/2 digit is used to represent either 0 or 1 and the full digit can represent any digit from 0 to 9. 100 V is represented on the voltmeter as shown below,

i.e.,

1	0	0	0.	0

In 100 V range, the minimum voltage that can be measured by $4\frac{1}{2}$ digit voltmeter is $\frac{100}{10^4}$ i.e. 10 mV.

∴ Resolution = 10 mV for $4\frac{1}{2}$ digit voltmeter.

For $3\frac{1}{2}$ digit voltmeter, range is 10 V with 10 V represesented on the voltmeter as shown below,

i.e.,

1	0	0.	0

In 10 V range, the minimum voltage that can be measured by $3\frac{1}{2}$ digit voltmeter is $\frac{10}{10^3} = 10\text{ mV}$.

∴ Resolution = 10 mV for $3\frac{1}{2}$ digit voltmeter.

2. In $4\frac{1}{2}$ digit voltmeter, 0.4312 V is displayed on 100 V range as,

0	0	0	4.	3

In $3\frac{1}{2}$ digit voltmeter, 0.4312 V is displayed on 10 V range as,

0	0	4.	3

(ii) **1.** When X = 0.0 cm, i.e. the displacement is zero, the entire space between the plates is filled by dielectric. Thus, capacitance of capacitive transducer is,

$$C = \frac{\in A}{d} = \frac{\in_0 \in_r A}{d}$$

where, Area, $A = 0.05 \times 0.05 \text{ m}^2$

$d = 0.01$ m

$$\therefore \quad C = \frac{4 \times 8.854 \times 10^{-12} \times (0.05 \times 0.05)}{0.01}$$

$$C = 8.854 \times 10^{-12} \text{ F}$$

2. When X = 2.0 cm, the region between the plates in the range x = 0 to x = 2 cm is filled with air and the remaining region with dielectric. Thus,

Total capacitance,

$$C_T = C_{air} + C_{dielect.}$$

$$= \frac{\in_0 A_1}{d} + \frac{\in_0 \in_r A_2}{d}$$

where Area, $AA_1 = 0.02 \times 0.05 \text{ m}^2$

$A_2 = 0.03 \times 0.05 \text{ m}^2$

$$C_T = \frac{(8.854 \times 10^{-12}) \times (0.02 \times 0.05)}{0.01} + \frac{(4)(8.854 \times 10^{-12})(0.03 \times 0.05)}{0.01}$$

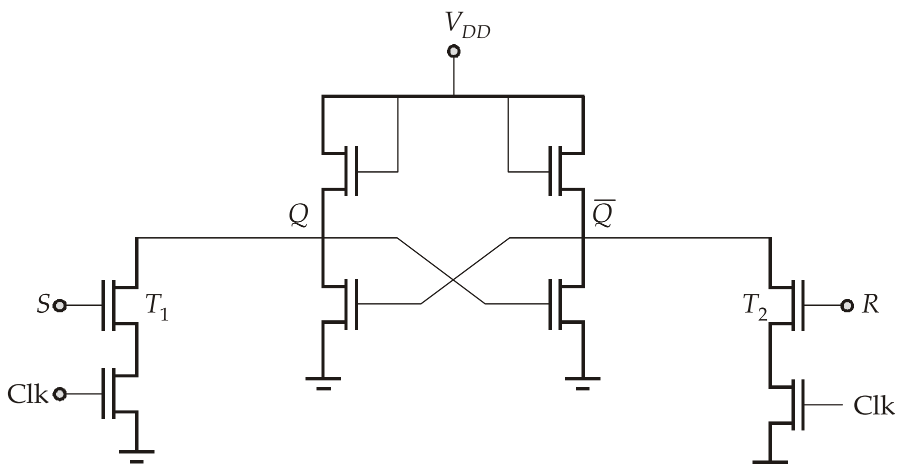
$$= 8.854 \times 10^{-13} + 5.312 \times 10^{-12}$$

$$\therefore \quad C_T = 6.197 \times 10^{-12} \text{ F}$$

Q.4 (b) Solution:

(i) RS flip-flop using NMOS

Operation of the circuit:



- When $S = 0$ and $R = 0$, transistor T_1 and T_2 are OFF and output Q remains in the same state.
- When $S = 1$ and $R = 0$, transistor T_1 is ON and thus, Q becomes one which makes $\bar{Q} = 1$ when clock = '1'.
- When $S = 0$ and $R = 1$, transistor T_2 is ON and thus, Q becomes zero when clock = '1'.
- When $S = 1$ and $R = 1$, both transistor T_1 and T_2 are ON, thus both Q and $\bar{Q}$ try to become 0 which leads to an invalid date.

The truth-table for the circuit is as below:

R	S	$Q(n+1)$
0	0	Q_n
0	1	1
1	0	0
1	1	×

(ii) The turns ratio k, for the transformer can be calculated as:

$$k = \frac{V_2}{V_1} = \frac{200}{2000} = 0.1$$

Referring the low voltage winding resistance to the high voltage side, we have

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{k^2}$$

$$R_{1e} = 0.6 + \frac{0.006}{(0.1)^2} = 1.2\Omega$$

The full-load current, $I_{1(\text{F.L.})} = \frac{kVA}{V_1} = \frac{50\times10^3}{2000} = 25\text{A}$

Hence, the full load copper losses is given as

$$P_{cu\ (\text{F.L.})} = I^2_{1(F.L.)} \times R_{1e}$$

$$= 625 \times 1.2 = 750\text{W}$$

When the transformer is operating at half load, the efficiency is given as

$$\%\eta = \frac{xVA\cos\phi}{xVA\cos\phi + \text{Iron loss} + x^2 P_{cu(F.L.)}} \times 100$$

For half load, $x = \frac{1}{2}$.

Therefore,

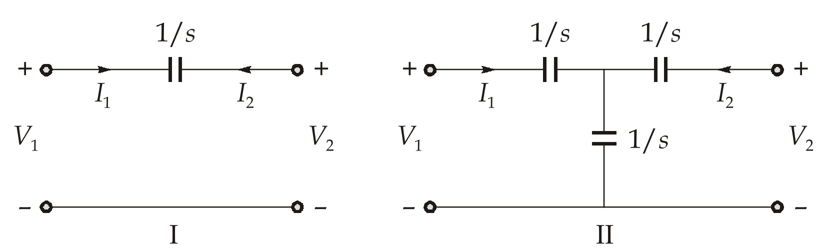
$$\%\eta = \frac{0.5\times50\times10^3\times0.8}{0.5\times50\times10^3\times0.8+400+(0.5)^2\times750}\times100$$

$$= \frac{20000}{20000+400+187.5}\times100$$

$$= 97.15\%$$

Q.4 (c) Solution:

(i) The given circuit can be considered as two networks connected in parallel as shown below,



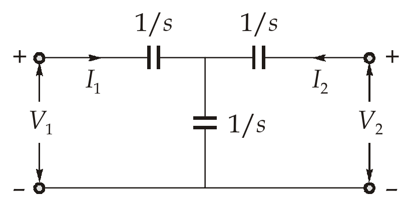
y-parameter of (I) network,

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = s = y_{22}$$

and

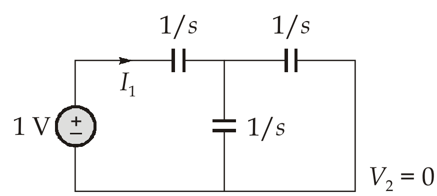
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -s = y_{21}$$

y-parameter of (II) network,



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

Considering $V_2 = 0$, the circuit can be drawn as below.

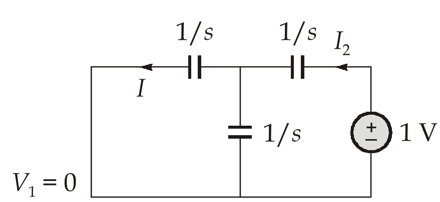


$$I_1 = \frac{1}{\frac{1}{s} + \frac{\frac{1}{s} \times \frac{1}{s}}{\frac{1}{s} + \frac{1}{s}}} = \frac{2s}{3}$$

$$y_{11} = \frac{2s}{3} = y_{22} \qquad (\because \text{Network is symmetrical})$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

Considering $V_1 = 0$, the circuit can be drawn as below.



$$I_2 = \frac{2s}{3}$$

Using current division rule,

$$I = \frac{\frac{2s}{3} \times \frac{1}{s}}{\frac{1}{s} + \frac{1}{s}} = \frac{s}{3}$$

We have, $\qquad I_1 = -I$

$\therefore$ $$y_{12} = \frac{-s}{3} = y_{21}$$ ($\because$ Network is reciprocal)

From above, we obtain

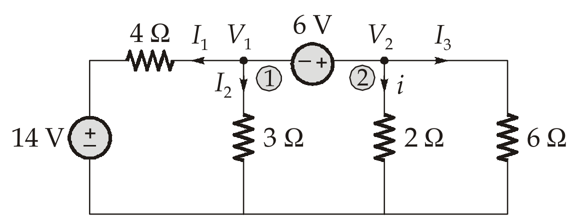
$$[Y_{(II)}] = \begin{bmatrix} \frac{2s}{3} & \frac{-s}{3} \\ \frac{-s}{3} & \frac{2s}{3} \end{bmatrix}$$

$$[Y_{(I)}] = \begin{bmatrix} s & -s \\ -s & s \end{bmatrix}$$

For parallel connection, y-parameters can be added. Thus,

$$[y] = [Y_{(I)}] + [Y_{(II)}] = \begin{bmatrix} \frac{5s}{3} & \frac{-4s}{3} \\ \frac{-4s}{3} & \frac{5s}{3} \end{bmatrix}$$

(ii)



Applying KCL at node-1 and 2 (super node),

$$I_1 + i + I_2 + I_3 = 0$$

$$\frac{V_1 - 14}{4} + \frac{V_1}{3} + \frac{V_2}{2} + \frac{V_2}{6} = 0$$

Substituting $$V_2 = V_1 + 6,$$

$$\frac{V_1 - 14}{4} + \frac{V_1}{3} + \frac{V_1 + 6}{2} + \frac{V_1 + 6}{6} = 0$$

$$\frac{3V_1 - 42 + 4V_1 + 6V_1 + 36 + 2V_1 + 12}{12} = 0$$

$$15\ V_1 + 6 = 0$$

$$V_1 = \frac{-6}{15} = -0.4\ \text{V}$$

$$V_2 = 6 - 0.4 = 5.6\ \text{V}$$

Thus, $$i = \frac{V_2}{2} = \frac{5.6}{2} = 2.8\ \text{A}$$

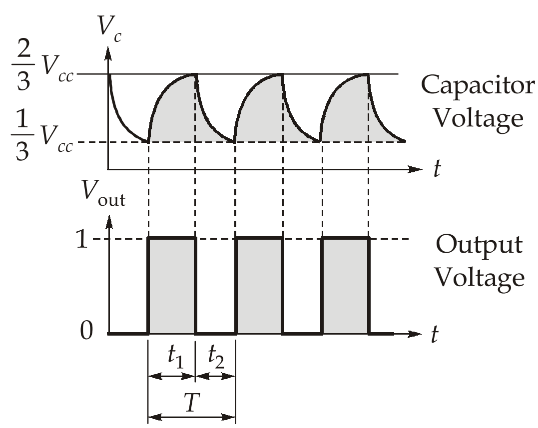
Section B

Q.5 (a) Solution:

Given, Astable multivibrator,

$$R_A = 2.2\ \text{k}\Omega$$
$$R_B = 3.9\ \text{k}\Omega$$
$$C = 0.1\ \mu\text{F}$$

During the positive pulse width, the capacitor C charges through R_A and R_B upto $2/3\ V_{cc}$ which cause the output to switch from HIGH to LOW. During the negative pulse width, the capacitor C discharges through R_B upto $1/3\ V_{cc}$ which triggers the lower comparator of 555 IC switching the output from LOW to HIGH. Thus, the capacitor charges and discharges continuously between $2/3\ V_{cc}$ and $1/3\ V_{cc}$ as shown below:



During the charging phase, voltage across the capacitor

$$V_c = V_{cc} + \left(\frac{V_{cc}}{3} - V_{cc}\right)e^{-t(R_A+R_B)C}$$

$At = t_1$, $V_c = 2V_{cc}/3$. Thus,

$$\frac{2V_{cc}}{3} = V_{cc} - \frac{2V_{cc}}{3}e^{-t_1(R_A+R_B)C}$$

$$t_1 = (R_A + R_B)C\ \ln 2 = 0.693(R_A + R_B)C$$

During the discharging phase, voltage across the capacitor

$$V_c = \frac{2V_{cc}}{3}e^{-tR_BC}$$

$At = t_2$, $V_c = V_{cc}/3$. Thus,

$$t_2 = R_BC\ \ln 2 = 0.693R_BC$$

(i) Positive pulse width,

$$t_{PH} = t_1 = 0.693\,(R_A + R_B)C$$
$$= 0.693(2.2k + 3.9k) \times 0.1 \times 10^{-6}$$
$$\therefore \quad t_{PH} = 0.423 \text{ msec}$$

(ii) Negative pulse width,

$$t_{PL} = t_2 = 0.693\,R_B C$$
$$= 0.693 \times 3.9k \times 0.1 \times 10^{-6}$$
$$\therefore \quad t_{PL} = 0.27 \text{ msec}$$

(iii) Total time period, $T = t_{PH} + t_{PL}$

$$\therefore \quad T = 0.423 \text{ ms} + 0.27 \text{ msec}$$
$$T = 0.693 \text{ msec}$$

$\therefore$ Free running frequency,

$$f_0 = \frac{1}{T} = 1.43 \text{ kHz}$$

(iv) Duty cycle, $D = \frac{t_{PH}}{T} \times 100\%$

$$= \frac{0.423 \times 10^{-3} \text{ sec}}{0.693 \times 10^{-3} \text{ sec}} \times 100\% = 0.6104 \times 100\%$$

$\therefore$ Duty cycle, $D = 61.04\%$

Q.5 (b) Solution:

Given, resistance temperature relationship

$$R = \alpha R_0^{\beta/T} \qquad \text{....(i)}$$

(i) At $T = 0°C$ or 273 K, $[\because T \text{ (in } K) = T\text{(in°C)} + 273]$

$$R = R_0 = 3.9 \text{ k}\Omega$$

$\therefore$ From equation (i),

$$3.9 \times 10^3 = \alpha(3.9 \text{ k})^{\beta/273} \qquad \text{...(ii)}$$

At $T = 50°C = 50 + 273 = 323$ K

Resistance, $R = 794\ \Omega$

$$\therefore \quad 794 = \alpha(3.9 \text{ k})^{\beta/323} \qquad \text{...(iii)}$$

Dividing equation (iii) by equation (ii), we get

$$\frac{794}{3.9\text{ k}} = \frac{\alpha(3.9\text{ k})^{\beta/323}}{\alpha(3.9\text{ k})^{\beta/273}} = (3.9\text{ k})^{\frac{\beta}{323}-\frac{\beta}{273}}$$

$$\ln\left(\frac{794}{3.9\text{ k}}\right) = \beta\left[\frac{1}{323}-\frac{1}{273}\right]\ln(3.9\text{ k})$$

$$-1.59 = \beta\left[\frac{1}{323}-\frac{1}{273}\right]\ln[3900]$$

$$\therefore \quad \beta = 339.5\text{ K}^{-1}$$

Substituting value of 'β' in equation (ii),

$$3900 = \alpha(3900)^{339.5/273}$$

$$\therefore \quad \alpha = 0.133$$

(ii) When $T = 40°\text{C} = 40 + 273 = 313\text{ K}$

$$\therefore \quad R = 0.133(3.9\text{ k})^{339.5/313} = 1.045\text{ k}\Omega$$

When, $T = 100°\text{C} = 100 + 273 = 373\text{ K}$

$$\therefore \quad R = 0.133(3.9\text{ k})^{339.5/373} = 246.8\ \Omega$$

Hence, the thermistor resistance decreases from 1.045 kΩ to 246.8 Ω as the temperature varies from 40°C to 100°C.

Q.5 (c) Solution:

(i) Truth table for the MUX is given as,

C	B	A	Y
0	0	0	D
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	$\bar{D}$
1	0	1	0
1	1	0	0
1	1	1	0

Using above, the truth table for the output Y showing all 16 possible combinations of input variables can be written as below,

D	C	B	A	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

(ii) In SOP form, we can write

$$Y = \bar{D}\bar{C}B\bar{A} + \bar{D}C\bar{B}\bar{A} + D\bar{C}\bar{B}\bar{A} + D\bar{C}B\bar{A}$$

$$= (\bar{D} + D)\bar{C}B\bar{A} + \bar{D}C\bar{B}\bar{A} + D\bar{C}\bar{B}\bar{A}$$

$$= \bar{C}B\bar{A} + \bar{D}C\bar{B}\bar{A} + D\bar{C}\bar{B}\bar{A}$$

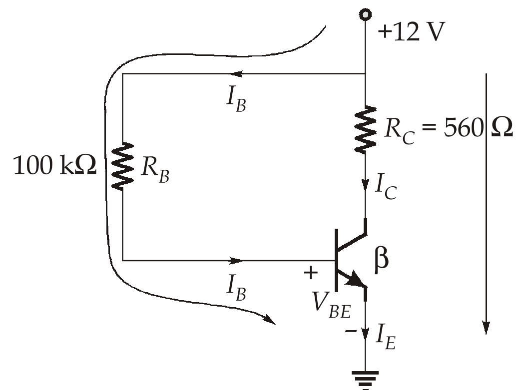
$$= \bar{C}\bar{A}(B + D\bar{B}) + \bar{D}C\bar{B}\bar{A}$$

$$= \bar{C}\bar{A}(B + D) + \bar{D}C\bar{B}\bar{A}$$

$$Y = \bar{C}B\bar{A} + D\bar{C}\bar{A} + \bar{D}C\bar{B}\bar{A}$$

Q.5 (d) Solution:

Given circuit,



At 25°C:

By applying KVL in input loop,

$$V_{CC} - I_B R_B - V_{BE} = 0$$

Base current; $$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 - 0.7}{100\ \text{k}\Omega}$$

$\therefore$ $$I_B = 0.113\ \text{mA}$$

$\therefore$ Collector current, $$I_C = \beta I_B = 100 \times 0.113\ \text{mA} = 11.3\ \text{mA}$$

Applying KVL in outerloop,

$$V_{CC} - I_C R_C - V_{CE} = 0$$

$\therefore$ $$V_{CE} = V_{CC} - I_C R_C$$

$$= 12 - 11.3 \times 10^{-3} \times 560$$

$$V_{CE} = 5.672\ \text{V}$$

At 75° C,

Base current from input loop,

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 - 0.7}{100\ \text{k}\Omega} = 0.113\ \text{mA}$$

$\therefore$ $$I_C = \beta I_B = 150 \times 0.113\ \text{mA} = 16.95\ \text{mA}$$

from outer loop,

$$V_{CE} = V_{CC} - I_C R_C$$

$$= 12 - 16.95 \times 10^{-3} \times 560$$

$$V_{CE} = 2.508\ \text{V}$$

percentage change in I_C,

$$= \frac{I_C|_{75°C} - I_C|_{25°C}}{I_C|_{25°C}} \times 100$$

$$= \frac{16.95 \text{ mA} - 11.3 \text{ mA}}{11.3 \text{ mA}} \times 100 = 50\%$$

$$\text{percentage change in } V_{CE} = \frac{V_{CE}|_{75°C} - V_{CE}|_{25°C}}{V_{CE}|_{25°C}} \times 100$$

$$= \frac{2.508 - 5.672}{5.672} \times 100 = -55.78\%$$

i.e., with increase in temperature from 25°C to 75°C, the percentage change in Q-point values are I_C = 50% (increases), V_{CE} = –55.78% (decreases).

Q.5 (e) Solution:

Output equations of full adder,

$$\text{Sum, } S = X \oplus Y \oplus Q$$

$$\text{Carry, } C = XY + XQ + YQ$$

Input to flip-flop is,

$$D = C$$

$$= XY + XQ + YQ$$

Characteristic equation of D-flip-flop,

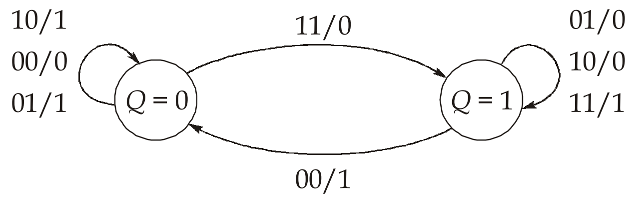
$$Q(t+1) = D = XY + XQ + YQ$$

$$\text{State equation, } Q(t+1) = C$$

State table:

Present state	Inputs		Next state	Output
Q	**X**	**Y**	**Q^+**	**S**
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

State Diagram:



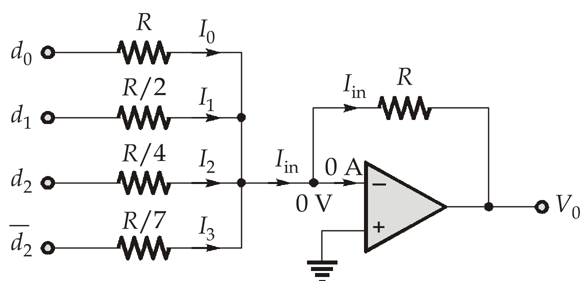
Q.6 (a) Solution

(i) • Let us denote the input binary values corresponding to d_i as b_i.

So,
$$d_i = \begin{cases} +\frac{1}{2}\text{ V}; & \text{when } b_i = 0 \\ -\frac{1}{2}\text{ V}; & \text{when } b_i = 1 \end{cases}$$

$$d_i = \left(\frac{1}{2}\bar{b}_i - \frac{1}{2}b_i\right)\text{ V}$$

• Redrawing the given circuit,



Output voltage,

$$V_0 = -I_{in}R$$

where
$$I_{in} = I_0 + I_1 + I_2 + I_3$$

$$= \frac{d_0}{R} + \frac{d_1}{\frac{R}{2}} + \frac{d_2}{\frac{R}{4}} + \frac{\bar{d}_2}{\frac{R}{7}}$$

$$V_0 = -\left[d_0 + 2d_1 + 4d_2 + 7\bar{d}_2\right]$$

$$= -\left[7\left(\frac{1}{2}b_2 - \frac{1}{2}\bar{b}_2\right) + 4\left(\frac{1}{2}\bar{b}_2 - \frac{1}{2}b_2\right) + 2\left(\frac{1}{2}\bar{b}_1 - \frac{1}{2}b_1\right) + \left(\frac{1}{2}\bar{b}_0 - \frac{1}{2}b_0\right)\right]\text{ V}$$

$$= \frac{-1}{2}\left[-\left(3\bar{b}_2 - 2\bar{b}_1 - \bar{b}_0\right) + \left(3b_2 - 2b_1 - b_0\right)\right]\text{ V}$$

- The output voltages for all the possible input binary combinations can be obtained as shown in the table below.

Digital input			V_0
b_2	b_1	b_0	
0	0	0	0 V
0	0	1	1 V
0	1	0	2 V
0	1	1	3 V
1	0	0	–3 V
1	0	1	–2 V
1	1	0	–1 V
1	1	1	0 V

- From the relation between input binary combinations and corresponding output voltages, it can be concluded that the given circuit can be used for converting digital signal in ones complement format to analog output.

(ii) **Resolution** (step size) of DAC is defined as the smallest change that can occur in the analog output as a result of a change in the digital binary input. The resolution is always equal to the weight of the LSB and is also known as the step size, since it is the amount that V_{out} will change as the digital input value is changed from one step to the next.

$$\text{Percentage resolution} = \frac{\text{Step size}}{\text{Full scale output}} \times 100$$

It can also be calculated as,

$$\%\ \text{Resolution} = \frac{1}{\text{Total no. of steps}} \times 100$$

In general, for an N-bit DAC, the total number of steps = $2^N - 1$

So, $$\%\ \text{Resolution} = \frac{1}{2^N - 1} \times 100$$

Settling time specifies the speed of a DAC, which is the time required for the DAC output to go from zero to full scale as the binary input changed from all 0's to all 1's.

Actually, the settling time is measured as the time for the DAC output to settle within ±1/2 step size of its final value.

Temperature sensitivity of a DAC determines the stability of D/A converter. For a fixed digital input, the analog input varies with temperature, normally from ±50 ppm/°C to ±1.5ppm/°C. This is produced due to temperature sensitivity of the reference voltages, the resistors used in the converters, the op-amp and its off-set voltages.

Q.6 (b) Solution:

Given, no. of turns in moving coil,

$$N = 500$$

$$\text{diameter, } D = 30 \text{ mm}$$

$$\text{power factor} = \cos\phi = 0.866, I = 0.05 \text{ A}$$

$$\text{flux density, } B = 15 \times 10^{-3} \text{ Wb/m}^2$$

The flux linking with moving coil

$$= \text{Area} \times \text{Component of flux perpendicular to area}$$

$$= \frac{\pi D^2}{4} \times B \times \cos\theta$$

The flux linkages with moving coil

$$= N \times \frac{\pi D^2}{4} \times B \times \cos\theta$$

where, D is mean diameter of pressure coil.

$$\text{Mutual inductance, } M = \frac{\text{flux linkages of moving coil}}{\text{(current of fixed coil)}}$$

$$= \frac{\pi}{4I} ND^2 B\cos\theta = M_{max}\cos\theta$$

where, M_{max} = maximum mutual inductance

$$= \left(\frac{\pi}{4I}\right) ND^2 B$$

Rate of change of flux linkages,

$$\frac{dM}{d\theta} = M_{max}\sin\theta$$

If I_p and I are the currents in the moving coil and the fixed coil respectively, then

$$\text{Deflecting torque, } T_d = I_p I\cos\phi\frac{dM}{d\theta} = \frac{V}{R_p} I\cos\phi\frac{dM}{d\theta}$$

$$= \frac{VI\cos\phi}{R_p} \times M_{max} \cdot \sin\theta$$

$$= (I_p I\cos\phi) M_{max}\sin\theta$$

$$= \frac{\pi}{4} \times ND^2 B I_P \cos\phi\sin\theta$$

(i) For θ = 60°, sin θ = 0.866

$$T_d = \frac{\pi}{4}\times 500\times(30\times10^{-3})^2\times15\times10^{-3}\times0.05\times0.866\times0.866$$

$$T_d = 198.8 \times 10^{-6} \text{ Nm}$$

(ii) For θ = 90°, sin θ = 1

$$T_d = \left(\frac{\pi}{4}\right)\times500\times(3\times10^{-2})^2\times15\times10^{-3}\times0.05\times0.866\times1$$

$$= 229.5 \times 10^{-6} \text{ Nm}$$

Q.6 (c) Solution:

(i) Electron injection efficiency (γ) = $\dfrac{\text{Electron current crossing the depletion region}}{\text{Total current}}$

$$\Rightarrow \qquad \gamma = \frac{I_n}{I_n + I_p} = \frac{J_n \cdot A}{(J_n + J_p)\cdot A} = \frac{J_n}{J_n + J_p},$$

where J_n = Electron current density, J_p = Hole current density

$$\Rightarrow \qquad \gamma = \frac{1}{1+\dfrac{J_p}{J_n}} \qquad \text{...(i)}$$

In a p-n junction, the electron and hole current density are given as:

$$J_p = \frac{eD_p p_{n0}}{L_p}[e^{eV/kT}-1] = \frac{en_i^2}{N_D}\cdot\sqrt{\frac{D_p}{\tau_{p_0}}}[e^{eV/kT}-1] \quad \left[\because L_p = \sqrt{D_p \tau_{p0}}\right]$$

Similarly,

$$J_n = \frac{eD_n n_{p0}}{L_n}[e^{eV/kT}-1] = \frac{en_i^2}{N_A}\cdot\sqrt{\frac{D_n}{\tau_{n_0}}}[e^{eV/kT}-1] \; [\because L_n = \sqrt{D_n \tau_{n0}}]$$

Substituting the values of J_p and J_n in equation (i), we get

$$\gamma = \frac{1}{1+\dfrac{N_A}{N_D}\sqrt{\dfrac{D_p}{D_n}}\cdot\sqrt{\dfrac{\tau_{n_0}}{\tau_{p_0}}}}$$

Since, $D_p = \mu_p V_T$ and $D_n = \mu_n V_T$. Hence,

$$\gamma = \frac{1}{1+\dfrac{N_A}{N_D}\sqrt{\dfrac{\mu_p}{\mu_n}}\cdot\sqrt{\dfrac{\tau_{n_0}}{\tau_{p_0}}}}$$

Using $\tau_{p0} = 0.1\,\tau_{n0}$ and $\mu_n = 2.4\,\mu_p$, we get

$$\gamma = \frac{1}{1 + 2.04\left(\frac{N_A}{N_D}\right)}$$

(ii) The critical magnetic field at a temperature T for the superconductor is given by

$$H_C(T) = H_C(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

It is given that $H_C\,(4\text{K}) = 0.02\text{T}$

$$H_C\,(3\text{K}) = 0.03\text{T}$$

Hence, $$\frac{0.02}{0.03} = \frac{1 - \left(\frac{4}{T_C}\right)^2}{1 - \left(\frac{3}{T_C}\right)^2}$$

$\Rightarrow$ $$\frac{0.02}{0.03} = \frac{T_C^2 - 16}{T_C^2 - 9}$$

$\Rightarrow$ $$0.01\,T_C^2 = 0.3$$

$\Rightarrow$ $$T_C = 5.48\text{K}$$

To calculate $H_C(0)$, $$0.02 = H_C(0)\left[1 - \left(\frac{4}{T_C}\right)^2\right]$$

$\Rightarrow$ $$0.02 = H_C(0) \times 0.467$$

$\Rightarrow$ $$H_C(0) = 0.043\text{T}$$

Critical magnetic field at 2K can be calculated as

$$H_C(2\text{K}) = 0.043\left[1 - \left(\frac{2}{5.48}\right)^2\right]$$

$$H_C(2\text{K}) = 0.037\text{T}$$

Q.7 (a) Solution:

(i) An even parity checker outputs a '0' if the number of '1' s in the input is even, and a '1' if the number of '1's is odd. Thus, the output '*P*' indicates an parity error. Let *A*, *B*, *C* and *D* be the inputs and *P* be the output of even parity checker.

A	B	C	D	P
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

By using 4-variable K-map,

AB \ CD	00	01	11	10
00		1		1
01	1		1	
11		1		1
10	1		1	

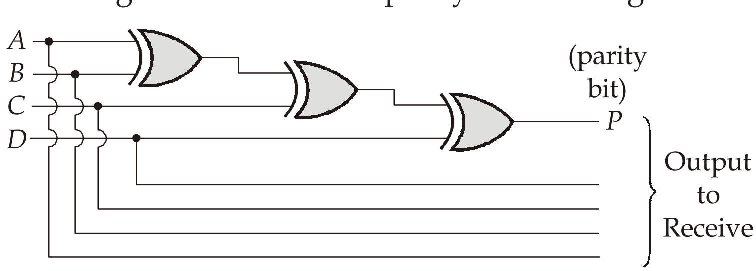
$$P = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + AB\bar{C}D + ABC\bar{D}$$

$$P = \bar{A}\bar{B}(C \oplus D) + \bar{A}B(\overline{C \oplus D}) + A\bar{B}(\overline{C \oplus D}) + AB(C \oplus D)$$

$$P = (\overline{A \oplus B})(C \oplus D) + (A \oplus B)(\overline{C \oplus D})$$

$$\therefore \qquad P = A \oplus B \oplus C \oplus D$$

The logic circuit diagram of 4-bit even parity checker is given as below,



(ii) **1.** Given, $(70)_8 + (122)_6 = (211)_x$

Converting the equation into decimal number system,

$$(8^1 \times 7 + 8^0 \times 0) + (6^2 \times 1 + 6^1 \times 2 + 6^0 \times 2) = (2 \times x^2 + 1 \times x^1 + 1 \times x^0)$$

$$56 + 50 = 2x^2 + x + 1$$

$$106 = 2x^2 + x + 1$$

$$x(2x + 1) = 105 = 7 \times 15$$

$$\therefore \quad x = 7$$

2. Given, $(135)_{12} = (x)_8 + (78)_9$

Converting the equation into decimal number system,

$$12^2 \times 1 + 12^1 \times 3 + 12^0 \times 5 = x + 7 \times 9^1 + 8 \times 9^0$$

$$144 + 36 + 5 = x + 63 + 8$$

$$185 - 71 = x$$

$$\therefore \quad x = 114$$

Q.7 (b) Solution:

At no load, terminal voltage is $V_t = 2500 \text{ V} = E_f$

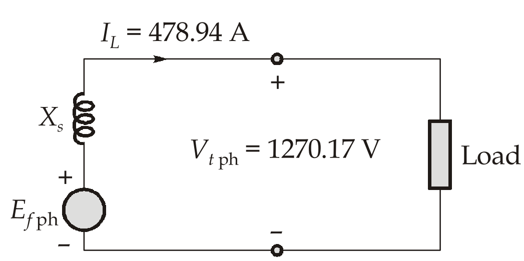
No load phase voltage, $E_{f\,ph} = \dfrac{2500}{\sqrt{3}} = 1443.37 \text{ V}$

At full load, $V_t = 2200 \text{ V} \Rightarrow V_{tph} = \dfrac{2200}{\sqrt{3}} = 1270.17 \text{ V}$

As we know that, $P_{3\text{-}\phi} = \sqrt{3}\, V_L\, I_L \cos\theta$

$$1460 \times 10^3 = \sqrt{3} \times 2200 \times I_L \times 0.8$$

$$I_L = 478.94 \text{ A}$$



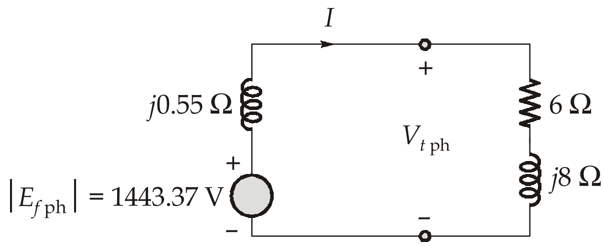
$$|E_{f\,ph}| = \sqrt{(V_{tph}\cos\theta + I_a R_a)^2 + (V_{tph}\sin\theta + I_a X_s)^2}$$

Neglecting the armature resistance, we get

$$1443.37 = \sqrt{(1270.17 \times 0.8)^2 + [(1270.17 \times 0.6) + 478.94 X_s]^2}$$

$$X_s = 0.55\ \Omega/\text{ph}$$

For a load $Z_L = 6 + j8\ \Omega$, the circuit can be drawn as below,



$$I = \frac{1443.37}{6 + j8.55} = 138.18\angle -54.9^\circ \text{ A}$$

$$V_{t\,ph} = (138.18\ \angle{-54.9^\circ}) \times (6 + j8)$$

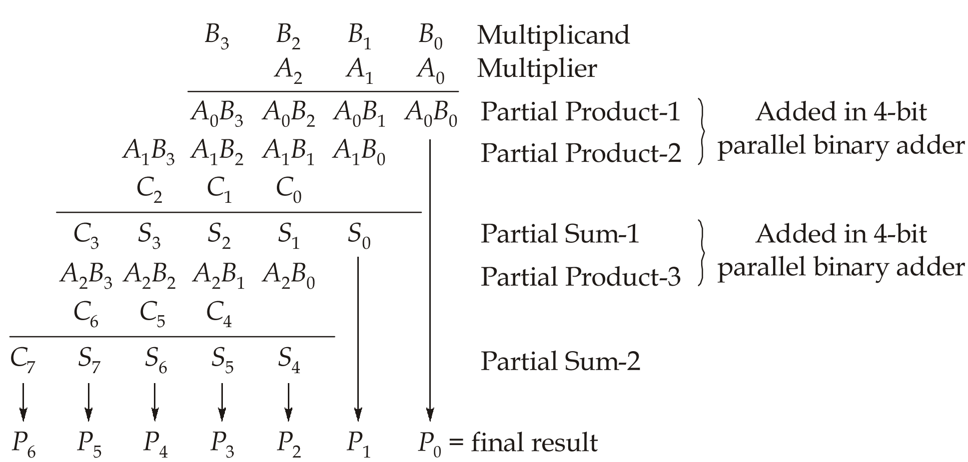
$$\left|V_{t\,ph}\right| = 1381.8 \text{ V}$$

For a star-connected load,

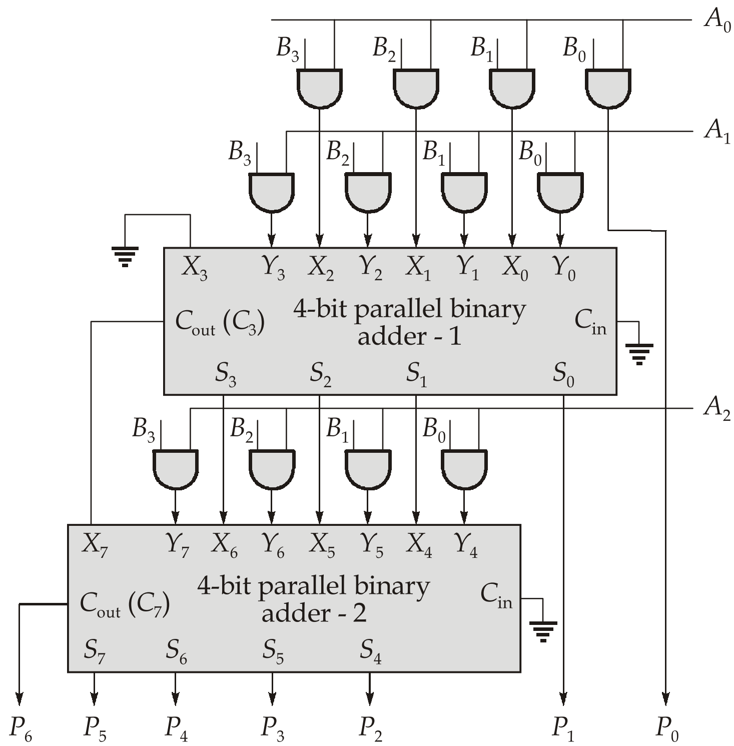
$$\text{terminal voltage, } V_t = \sqrt{3} \times 1381.8 = 2393.34 \text{ V}$$

Q.7 (c) Solution:

The operation of the parallel multiplier in symbolic form is given below,



It requires two 4-bit parallel binary adders and 12 numbers of 2-input AND gates. Here, each group of 4 AND gates is used to obtain partial products while 4-bit parallel adders are used to add the partial products. The circuit can be drawn as below,



Q.8 (a) Solution:

(i) Given data: Output = 10 kW ; Number of poles = 6 ; f = 50 Hz ; N = 960 rpm

η_{FL} = 90% ; p.f. = 0.88 lagging.

Full load line current drawn by 3 phase, delta connected induction motor can be obtained as below,

$$\eta_{FL} = \frac{\text{Output power in watts}}{\sqrt{3}\, V_L\, I_L\, \cos\phi}$$

$$I_L = \frac{10\times10^3}{\sqrt{3}\times400\times0.88\times0.90} = 18.22 \text{ A}$$

$$I_{ph} = \frac{18.22}{\sqrt{3}} = 10.52 \text{ A}$$

For direct on line start, the starting current drawn by the motor per phase is given by,

$$I_{fl,\,ph} = \frac{85}{\sqrt{3}} = 49.07 \text{ A}$$

$$\text{Synchronous speed} = N_s = \frac{120\times f}{P} = \frac{120\times50}{6} = 1000 \text{ rpm}$$

$$\text{Slip at full load, } s_{fl} = \frac{N_s - N_r}{N_s} = \frac{1000-960}{1000} = 0.04$$

The torque of induction motor is given by

$$T \propto \frac{I_2^2 R_2}{s}$$

If I_{st} is the starting current, then starting torque (T_{st}) is, (Slip at starting, s_{st} = 1)

$$T_{st} \propto I_{st}^2$$

If I_{fl} is the full-load current and s_{fl} is the full-load slip, then full load torque,

$$T_{fl} \propto \frac{I_{fl}^2}{s_{fl}}$$

Thus, for Y-Δ starter,

$$\frac{T_{st}}{T_{fl}} = \frac{1}{3}\left(\frac{I_{s,\,ph}}{I_{fl,ph}}\right)^2 s_{fl} = \frac{1}{3}\left(\frac{49.07}{10.52}\right)^2 \times 0.04 = 0.29$$

(ii) Synchronous speed = $N_s = \frac{120 \times f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$

Slip at maximum torque, $s_{max} = \frac{N_s - N_r}{N_s} = \frac{1000 - 900}{1000} = 0.1$

We know that, $s_{max} = \frac{R_2}{X_2} \quad \Rightarrow \quad 0.1 = \frac{0.25}{X_2}$

$\therefore$ $X_2 = 2.5\ \Omega$

Torque at any slip, $T = \frac{3}{\omega_s} \cdot \frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}$

At 5% slip, $T = \frac{3E_1^2}{\omega_s} \cdot \frac{0.05 \times 0.25}{(0.25)^2 + (0.05 \times 2.5)^2}$

$$T = \frac{3E_1^2}{\omega_s} \times 0.16 \quad \text{...(i)}$$

Given, $T_{max} = 200 \text{ N-m} = \frac{3}{\omega_s}\frac{E_1^2}{2X_2} = \frac{3E_1^2}{\omega_s} \times \frac{1}{5}$

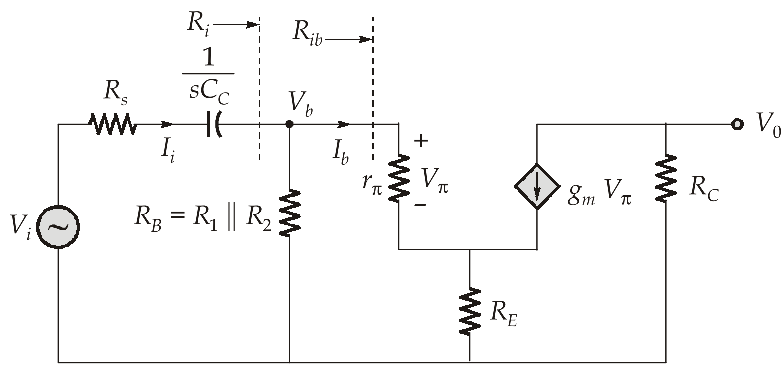
$$\frac{3E_1^2}{\omega_s} = 1000 \quad \text{...(ii)}$$

Substituting equation (ii) in equation (i), we get,

$$T = 1000 \times 0.16 = 160 \text{ N-m.}$$

Q.8 (b) Solution:

The small signal equivalent circuit of the given amplifier can be drawn as



Let the resistance $R_B = R_1 \| R_2$

The input current, $$I_i = \frac{V_i}{R_s + \frac{1}{sC_c} + R_i}, \text{ where } R_i = R_B \| R_{ib}$$

Now, resistance $$R_{ib} = \frac{V_b}{I_b}$$

We have, $$V_b = r_\pi I_b + (\beta + 1)R_E I_b$$

$$R_{ib} = \frac{V_b}{I_b} = r_\pi + (\beta + 1)R_E$$

and $$R_i = R_B \| R_{ib}$$

Using current division rule,

$$\text{Current, } I_b = \frac{R_B}{R_B + R_{ib}} \cdot I_i$$

We have, $$V_\pi = I_b r_\pi$$

and $$V_0 = -g_m R_C V_\pi$$

$$= -g_m R_C I_b \cdot r_\pi$$

$$= -g_m R_C r_\pi \cdot \left(\frac{R_B}{R_B + R_{ib}}\right) I_i$$

$$= -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}}\right) \cdot \frac{V_i}{R_s + \frac{1}{sC_C} + R_i}$$

$$A_V(s) = \frac{V_0(s)}{V_i(s)} = -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}}\right)\left[\frac{sC_C}{1+s(R_S+R_i)C_C}\right]$$

$$A_V(s) = \left(\frac{-g_m r_\pi R_C}{R_s + R_i}\right)\left(\frac{R_B}{R_B + R_{ib}}\right)\cdot\left(\frac{s\tau_s}{1+s\tau_s}\right)$$

where $\tau_s = (R_s + R_i)C_C$

Now,

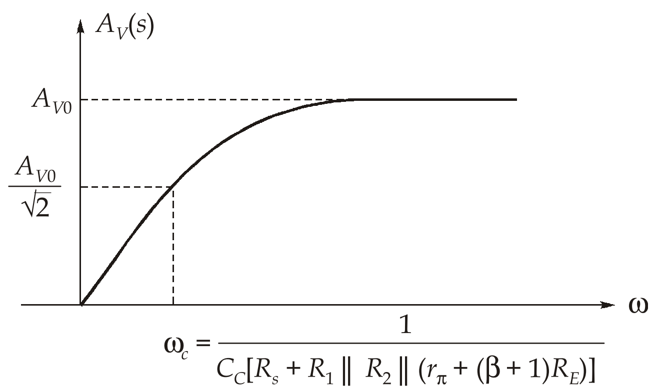
$$A_V(s) = \frac{-g_m r_\pi R_C}{R_s + R_1 \| R_2 \| [r_\pi + (\beta+1)R_E]}\left[\frac{R_1 \| R_2}{R_1 \| R_2 + [r_\pi + (\beta+1)R_E]}\right]\left[\frac{s(R_s + R_1 \| R_2 \| [r_\pi + (\beta+1)R_E])}{1 + s(R_s + R_1 \| R_2 \| [r_\pi + (\beta+1)R_E])}\right]$$

We can write, $A_v(s) = A_{v0}\left(\frac{s}{s+\omega_c}\right)$

$$A_{V0} = \frac{g_m r_\pi R_C}{R_s + R_1 \| R_2 \| [r_\pi + (\beta+1)R_E]}\left[\frac{R_1 \| R_2}{R_1 \| R_2 \| [r_\pi + (\beta+1)R_E]}\right]$$

and $\omega_c = \frac{1}{C_c\,[R_s + R_1 \| R_2 \| (r_\pi + (1+\beta)R_E)]}$

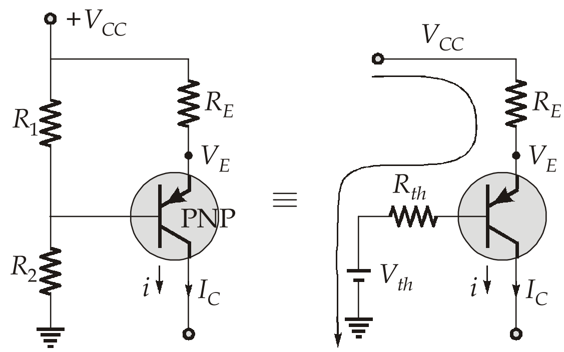
The transfer function $A_V(s)$ represents a high pass filter with cut-off frequency ω_c.



Q.8 (c) Solution:

In the above circuit, when trigger is applied, the transistor is turned ON and the capacitor starts charging. When the voltage across the capacitor reaches $2V_{cc}/3$, the upper comparator of 555 IC causes the output to switch to a low state and thus, the transistor is turned OFF.

The Thevenin equivalent circuit for the biasing circuit of linear ramp generator is drawn as below,



where,
$$V_{th} = \frac{V_{CC} \times R_2}{R_1 + R_2}$$

$$R_{th} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

By using KVL in base-emitter loop of PNP transistor,

$$V_{CC} - I_E R_E - V_{BE} - R_{th} I_B - V_{th} = 0$$

$$V_{CC} - (I_C + I_B) R_E - V_{BE} - R_{th} I_B - V_{th} = 0$$

Since 'β' is not given hence treated as very large. Thus, base current can be assumed to be zero.

$$\therefore \quad V_{CC} - I_C R_E - V_{BE} - V_{th} = 0$$

$$\because \quad I_C = i$$

$$V_{CC} - iR_E - V_{BE} - V_{th} = 0$$

$$\therefore \quad i = \frac{V_{CC} - V_{th} - V_{BE}}{R_E}$$

$$= \frac{V_{CC} - \frac{R_2 V_{CC}}{R_1 + R_2} - V_{BE}}{R_E}$$

$$i = \frac{V_{CC}[R_1 + R_2] - R_2 V_{CC} - V_{BE}(R_1 + R_2)}{(R_1 + R_2) R_E}$$

$$i = \frac{V_{CC} R_1 - V_{BE}(R_1 + R_2)}{R_E (R_1 + R_2)}$$

Voltage across capacitor, $V_C = \dfrac{1}{C}\displaystyle\int_0^t i\,dt$

$$= \frac{1}{C}\int_0^t \left[\frac{R_1 V_{CC} - V_{BE}(R_1 + R_2)}{R_E(R_1 + R_2)}\right] dt$$

$$V_C = \frac{1}{C}\left\{\frac{R_1 V_{CC} - V_{BE}(R_1 + R_2)}{R_E(R_1 + R_2)}\right\} t$$

From the given ramp waveform,

$$V_C = \frac{2}{3} V_{CC} \text{ at } t = T$$

$$\therefore \quad \frac{2}{3} V_{CC} = \frac{1}{C}\left\{\frac{R_1 V_{CC} - V_{BE}(R_1 + R_2)}{R_E(R_1 + R_2)}\right\} T$$

$$\therefore \quad T = \frac{\frac{2}{3} V_{CC} \times C R_E(R_1 + R_2)}{R_1 V_{CC} - V_{BE}(R_1 + R_2)}$$

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