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Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025
Mains Test Series

Electrical Engineering
Test No : 7

Section-A

Q.1 (a) Solution:

Given, $$(D^2 - 4D + 3)y = \sin 3x \cos 2x$$

Auxiliarly equation is $D^3 - 4D + 3 = 0$

$$D^2 - 3D - D + 3 = 0$$
$$D(D - 3) - 1(D - 3) = 0$$
$$(D - 1)(D - 3) = 0$$
$$D = 1, 3 \text{ are roots}$$
$$\text{C.F.} = C_1 e^x + C_2 e^{3x}$$

$$\text{P.I.} = \frac{1}{(D^2 - 4D + 3)} \sin 3x \cos 2x = \frac{1}{D^2 - 4D + 3} \frac{1}{2}[\sin(5x) + \sin(x)]$$

$$= \frac{1}{2}\left\{\frac{1}{D^2 - 4D + 3}\sin(5x) + \frac{1}{D^2 - 4D + 3}\sin(x)\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{-25 - 4D + 3}\sin(5x) + \frac{1}{-1 - 4D + 3}\sin(x)\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{-4D - 22}\sin(5x) + \frac{1}{-4D + 2}\sin(x)\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{4D + 22}\sin(5x) + \frac{1}{4D - 2}\sin(x)\right\}$$

$$= \frac{1}{2}\left\{\frac{-(4D-22)}{16D^2-484}\sin(5x)-\frac{(4D+2)}{16D^2-4}\sin(x)\right\}$$

$$= \frac{1}{2}\left\{\frac{-(4\times 5\cos(5x)-22\sin(5x))}{16\times 25-484}-\frac{(4\cos x+2\sin x)}{16\times -1-4}\right\}$$

$$= \frac{1}{2}\left\{\left[\frac{22\sin(5x)-20\cos(5x)}{-884}\right]-\left[\frac{(2\sin x+4\cos x)}{-20}\right]\right\}$$

$$= \frac{1}{2}\left\{\left[\frac{20\cos(5x)-22\sin(5x)}{884}\right]+\frac{(\sin x+2\cos x)}{20}\right\}$$

$\therefore$ The general solution is

$$y = C_1e^{x}+C_2e^{3x}+\frac{1}{884}\{10\cos(5x)-11\sin(5x)\}+\frac{(\sin x+2\cos x)}{20}$$

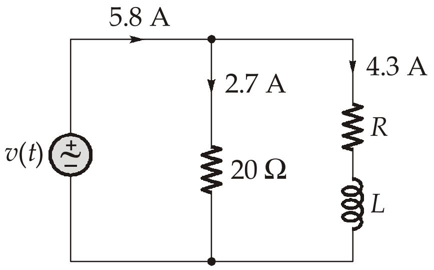
Q.1 (b) Solution:

Here, $f = 50$ Hz, $R = 20\ \Omega$,

$I_L = 4.3$ A; $I_R = 2.7$ A;

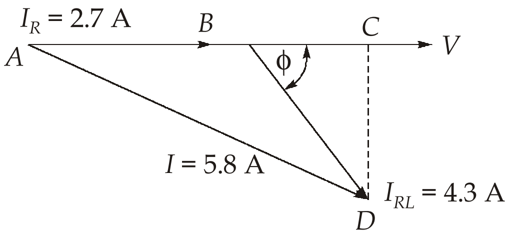
$I = 5.8$ A

The circuit is shown below,



Supply voltage, $V = I_R \times R = 2.7 \times 20 = 54$ V

The phasor diagram is shown in figure below,



Impedance of the *RL* branch, $Z = \frac{V}{I_{RL}} = \frac{54}{4.3} = 12.56\ \Omega$

Now, from the phasor diagram, the phase angle of the impedance of the *RL* branch is obtained as

$$AD^2 = AB^2 + BD^2 + 2AB \times BD \cos\phi$$

$$\Rightarrow \qquad \cos\phi = \frac{AD^2 - AB^2 - BD^2}{2AB \times BD} = \frac{5.8^2 - 2.7^2 - 4.3^2}{22.7 \times 4.3} = 0.3385$$

∴ resistance of the *RL* branch, $R = Z\cos\phi = 12.56 \times 0.3385 = 4.25\ \Omega$

∴ reactance of the *RL* branch, $X = \sqrt{Z^2 - R^2} = \sqrt{12.56^2 - 4.25^2} = 11.82\ \Omega$

(i) Power absorbed by the inductive branch

$$= I_{RL}^2 \times R = 4.3^2 = 4.25 = 78.6\text{ W}$$

(ii) Inductance, $L = \dfrac{X}{2\pi f} = \dfrac{11.82}{2\pi \times 50} = 37.6\text{ mH}$

(iii) Power of the combined circuit is obtained as

$$pf = \frac{AC}{AD} = \frac{AB + BC}{AD}$$

$$= \frac{2.7 + BD\cos\phi}{5.8} = \frac{2.7 + 4.3 \times 0.3385}{5.8} = 0.716\text{ (lagging)}$$

Q.1 (c) Solution:

Using Clausius - Mossotti relation, we have

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\varepsilon_0}$$

At optical frequencies, $\varepsilon_r = n_r^{\,2}$,

We have,

$$\frac{n_r^2 - 1}{n_r^2 + 2} = \frac{N\alpha_e}{3\varepsilon_0} \quad \text{since at optical range of frequencies } \alpha_i = 0$$

We get,

$$\frac{\alpha_e}{\alpha_e + \alpha_i} = \frac{n_r^2 - 1}{n_r^2 + 1} \times \frac{\varepsilon_r + 2}{\varepsilon_r - 1}$$

$$\left(1 - \frac{\alpha_e}{\alpha_e + \alpha_i}\right) \times 100 = \left(1 - \frac{n_r^2 - 1}{n_r^2 + 2} \times \frac{\varepsilon_r + 2}{\varepsilon_r - 1}\right) \times 100$$

$$\frac{\alpha_i}{\alpha_e+\alpha_i}\times 100 = \left(1-\frac{n_r^2-1}{n_r^2+2}\times\frac{\varepsilon_r+2}{\varepsilon_r-1}\right)\times 100$$

Substituting the values, we have

$$= \left(1-\frac{1.5^2-1}{1.5^2+2}\times\frac{5.6+2}{5.6-1}\right)\times 100 = 51.4\%$$

Q.1 (d) Solution:

Given, K [meter constant] = 80 Rev/unit energy

Load current = 30 A

Load voltage = 230 V

Load power factor = 0.6

Power consumed by load = (30) (230) (0.6)

= 4140 Watts = 4.14 kW

Energy consumed (in one hour) = (4.14) kWh = 4.14 kWh

$\because$ For unit energy (1 kWh), revolutions made by meter are 80 so

For 4.14 kWh,

Number of revolutions, (N_T) = 80 × 4.14 = 331.2 revolutions

Now, the meter actually makes 330 revolutions,

Therefore, N_m = 330 revolutions

$$\%\text{error} = \frac{N_m-N_T}{N_T}\times 100 = \frac{330-331.2}{331.2}\times 100\%$$

$$= -0.3622\%$$

Meter is making less number of revolutions, compared to required revolutions. So the meter is running slow.

Q.1 (e) Solution:

In the matched transistors, the reverse saturation currents are equal applying KVL with

$$V_1 \approx 0,$$

$$V_2 = V_{BE2} - V_{BE1} \quad \text{(By virtual ground, } V_2 = V^+\text{)} \qquad \text{...(i)}$$

A grounded BJT can also be utilized as diode, since its emitter current and base to emitter voltage are related by

$$I_E = I_s\, e^{V_{BE}/V_T} \qquad \text{...(ii)}$$

Taking the logarithm of both sides of equation (ii), we get

$$V_{BE} = V_T \ln\frac{I_E}{I_S} \quad \text{...(iii)}$$

Using equation (i) and (iii), with $I_C \approx I_E$,

$$V_2 = V_T \ln\frac{I_{E2}}{I_S} - V_T \ln\frac{I_{E1}}{I_S} = -V_T \ln\frac{I_{C1}}{I_{C2}}$$

According to equation (i), V_2 is the difference between two small voltages. If V_R is several voltages in magnitude, then $V_S << V_R$

$$I_{C2} \approx I_{E2} = \frac{V_R - V_2}{R_2} \approx \frac{V_R}{R_2}$$

$$I_{C1} \approx I_{E1} = \frac{V_S - V_1}{R_1} \approx \frac{V_S}{R_1}$$

$$V_0 = \frac{-R_4}{R_3} V_2$$

$$V_0 = \frac{R_4}{R_3} \times V_T \ln\left(\frac{V_s}{V_R} \times \frac{R_2}{R_1}\right) \quad \text{...(iv)}$$

Selecting $\left(\frac{R_1}{R_2}\right) V_R = 1$

Also, R_3 can be selected with a temperature sensitivity similar to that V_T

Therefore, $V_0 \approx V_T \frac{R_4}{R_3} \ln V_S$

Q.2 (a) (i) Solution:

Instruction read cycle time:

$\Rightarrow H_1T_1 + (1 - H_1)\ H_2(T_2 + T_1) + (1 - H_1)\ (1 - H_2)\ H_3\ (T_m + T_2 + T_D)$

$\Rightarrow (0.8 \times 2) + (1 - 0.8)\ 0.9\ (8 + 2) + (1 - 0.8)\ (1 - 0.9)\ (90 + 8 + 2)$

$\Rightarrow 5.4$ ns

$$T_{\text{avg read (inst)}} = (\text{Frequency of instruction} \times \text{Read cycle time})$$
$$= 60\% \times 5.4 \text{ ns} = 3.24 \text{ ns}$$

Data read cycle time:

$\Rightarrow H_1T_D + (1 - H_1)\ H_2\ (T_2 + T_D) + (1 - H_1)\ (1 - H_2)\ H_3\ (T_m + T_2 + T_D)$

$\Rightarrow (0.9 \times 2) + (1 - 0.9)\ (0.9)\ (8 + 2) + (1 - 0.9)\ (1 - 0.9)\ (90 + 8 + 2)$

$\Rightarrow 3.7$ ns

$$T_{\text{avg read (data)}} = (\text{Frequency of data} \times \text{Read cycle time})$$
$$= 40\% \times 3.7 \text{ ns} = 1.48 \text{ ns}$$
$$\text{Total time} = (3.24 + 1.48) = 4.72 \text{ ns}$$

Q.2 (a) (ii) Solution:

Cache memory size = 16 kB

Block size = 16 B

Main memory address = 32 bit

$$\text{Number of lines (N)} = \frac{16\text{K}}{16} \Rightarrow \frac{2^{14}}{2^4} = 2^{10}$$

Fully associative cache memory (N-way)

So, $$\text{number of sets (S)} = \frac{\text{N}}{\text{P-way}} \Rightarrow \frac{2^{10}}{2^{10}} = 1$$

∴ Address format:

32 bit

TAG	WO
28 bit	$\log_2 16 = 4$ bit

So, TAG = 28 bit

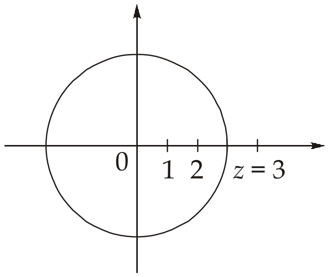
Index = 0 bit (No address)

Q.2 (b) (i) Solution:

Let, $$f(z) = \frac{\cos(\pi z^2)}{(z-2)(z-1)}$$

Then the singular points of $f(z)$ are given by $(z - 2)(z - 1) = 0$

$z = 1$ and $z = 2$



Here, the two singular points, $z = 1$ and $z = 2$

lie inside the circle C:

$$|z| = 3$$

Now, $$f(z) = \cos(\pi z^2)\left[\frac{1}{(z-2)(z-1)}\right] = \cos(\pi z^2)\left[\frac{1}{z-2} - \frac{1}{z-1}\right]$$

$$f(z) = \frac{\cos(\pi z^2)}{z-2} - \frac{\cos(\pi z^2)}{z-1}$$

∴ By Cauchy's integral formula, we have

$$\oint_C f(z)dz = \oint_C \frac{\cos(\pi z^2)}{z-2}dz - \oint_C \frac{\cos(\pi z^2)}{z-1}dz$$

$$= 2\pi i\left[\cos(\pi z^2)\right]_{z=2} - 2\pi i\left[\cos(\pi z^2)\right]_{z=1}$$

$$= 2\pi\, i(1) - 2\pi\, i(-1) = 4\pi\, i$$

Q.2 (b) (ii) Solution:

Here the subsidiary equations are

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy} \qquad \text{...(i)}$$

Each of these equations

$$\frac{dx - dy}{x^2 - y^2 - (y-x)z} = \frac{dy - dz}{y^2 - z^2 - x(z-y)}$$

$$\frac{d(x-y)}{(x-y)(x+y+z)} = \frac{d(y-z)}{(y-z)(x+y+z)}$$

$$\frac{d(x-y)}{x-y} = \frac{d(y-z)}{y-z}$$

Integrating on both sides, we get

$$\ln(x - y) = \ln(y - z) + \log C$$

$$\frac{x-y}{y-z} = C \qquad \text{...(ii)}$$

Each of the subsidiary equation (i),

$$= \frac{x\,dx + y\,dy + z\,dz}{x^3 + y^3 + z^3 - 3x\,yz}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{(x+y+z)(x^2 + y^2 + z^2 - yz - zx - xy)} \qquad \text{...(iii)}$$

Also each of the subsidiary equations

$$= \frac{dx + dy + dz}{x^2 + y^2 + z^2 - yz - zx - xy} \qquad \text{...(iv)}$$

From equation (iii) and (iv),

$$\frac{x\,dx + y\,dy + z\,dz}{x+y+z} = dx + dy + dz$$

$$x\,dx + y\,dy + z\,dz = (x+y+z)\,d(x+y+z)$$

Integrating on both sides, we get

$$\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2} = \frac{(x+y+z)^2}{2}+C'$$

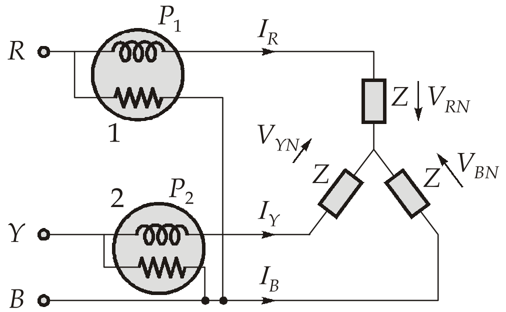
$$x^2 + y^2 + z^2 = (x+y+z)^2 + 2C'$$

$$xy + yz + zx + C' = 0 \qquad \text{...(v)}$$

Combining equation (ii) and (v), we get

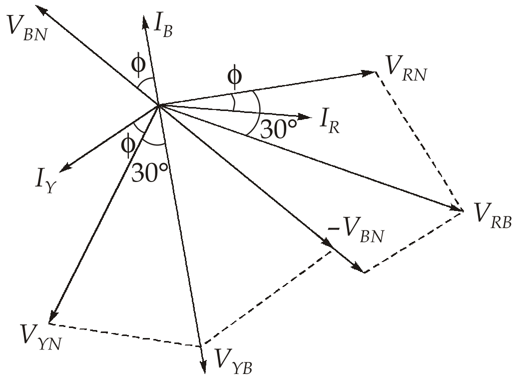
$$\frac{x-y}{y-z} = f(xy+yz+zx)$$

Q.2 (c) Solution:



The connection diagram of a 3-phase balanced load connected as star connection is shown below.

Phasor Diagram :



Power measured by Wattmeter 1 :

$$P_1 = V_{RB}I_R \cos \text{(angle between } V_{RB} \text{ and } I_R)$$

$$P_1 = V_{RB}I_R \cos (30 - \phi) \quad ...(i)$$

Similarly, power measured by wattmeter 2 :

$$P_2 = V_{YB} \times I_Y \cos \text{(angle between } V_{YB} \text{ and } I_Y)$$

$$= V_{YB} \times I_Y \cos(30 + \phi) \quad ...(ii)$$

Since the load is in balanced condition.

Hence, $I_R = I_Y = I_B = I_L$

and $V_{RY} = V_{YB} = V_{BR} = V_L$

So, $P_1 = V_L I_L \cos(30 - \phi)$ ...(iii)

$$P_2 = V_L I_L \cos(30 + \phi) \quad ...(iv)$$

From (iii) and (iv),

$$P_1 + P_2 = V_L I_L[\cos(30 - \phi) + \cos(30 + \phi)]$$

$$= \sqrt{3} V_L I_L \cos\phi \quad ...(v)$$

$$P_1 - P_2 = V_L I_L[\cos(30 - \phi) - \cos(30 + \phi)] = V_L I_L \sin \phi \quad ...(vi)$$

From (v) and (vi), $\tan \phi = \dfrac{\sqrt{3}(P_1 - P_2)}{(P_1 + P_2)}$

So, power factor of the load is given by

$$\cos \phi = \cos\left(\tan^{-1} \frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2}\right)$$

or $\cos \phi = \dfrac{1}{\sec\phi} = \dfrac{1}{\sqrt{1 + \tan^2 \phi}}$ ...(vii)

By putting $\tan \phi = \dfrac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2}$ in (vii),

$$\cos \phi = \frac{1}{\sqrt{1 + 3\left(\frac{P_1 - P_2}{P_1 + P_2}\right)^2}}$$

Hence Proved.

Q.3 (a) Solution:

The characteristic equation is

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$(1 - \lambda)[(5 - \lambda)\,(1 - \lambda) - 1] - 1[1 - \lambda - 3] + 3[1 - 3(5 - \lambda)] = 0$$

$$(1 - \lambda)[(5 - 6\lambda + \lambda^2 - 1] - 1[- \lambda - 2] + 3[-14 + 3\lambda] = 0$$

$$\lambda^2 - 6\lambda + 4 - \lambda^3 + 6\lambda^2 - 4\lambda + \lambda + 2 - 42 + 9\lambda = 0$$

$$-\lambda^3 + 7\lambda^2 - 36 = 0$$

$$\lambda^3 - 7\lambda^2 + 36 = 0$$

By solving above equation, we get

$$\lambda = -2, 3, 6$$

Eigen vectors :

$$[A - \lambda I]X = 0$$

$$\begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = -2$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3x + y + 3z = 0$$

$$x + 7y + z = 0$$

$$\begin{array}{cccc} & x & y & z \\ 1 & 3 & 3 & 1 \\ 7 & 1 & 1 & 7 \end{array}$$

$$\frac{x}{1-21} = \frac{y}{3-3} = \frac{z}{21-1}$$

$$\frac{x}{-20} = \frac{y}{0} = \frac{z}{20}$$

$$\frac{x}{-1} = \frac{y}{0} = \frac{z}{1}$$

$\therefore$ Eigen vector is $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

For $\lambda = 3$

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x + y + 3z = 0$$

$$x + 2y + z = 0$$

$$\begin{array}{cccc} & \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \\ 1 & 3 & -2 & 1 \\ 2 & 1 & 1 & 2 \end{array}$$

$$\frac{x}{1-6} = \frac{y}{3+2} = \frac{z}{-4-1}$$

$$\frac{x}{-5} = \frac{y}{5} = \frac{z}{-5}$$

$$\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$$

$\therefore$ Eigen vector is $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

For $\lambda = 6$,

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-5x + y + 3z = 0$$

$$x - y + z = 0$$

$$\begin{array}{cccc} \boldsymbol{x} & & \boldsymbol{y} & \boldsymbol{z} \\ 1 & 3 & -5 & 1 \\ -1 & 1 & 1 & -1 \end{array}$$

$$\frac{x}{1-3} = \frac{y}{3+5} = \frac{z}{5-1}$$

$$\frac{x}{4} = \frac{y}{8} = \frac{z}{4}$$

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

∴ Matrix which convert given matrix into diagonal form is

$$P = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|P| = -1(-1-2) - 1(-2) + 1(1) = 3 + 2 + 1 = 6$$

$$P^{-1} = \frac{\text{adj (A)}}{|P|} = \frac{(\text{cofactor of } A)^T}{|P|}$$

$$\text{cofactor of } A = \begin{bmatrix} -3 & 2 & 1 \\ 0 & -2 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} -3 & 0 & 3 \\ 2 & -2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{\text{adj } A}{|P|} = \frac{1}{6}\begin{bmatrix} -3 & 0 & 3 \\ 2 & -2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\therefore \quad D = P^{-1} AP = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$D^4 = \begin{bmatrix} (-2)^4 & 0 & 0 \\ 0 & 3^4 & 0 \\ 0 & 0 & 6^4 \end{bmatrix} = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 1296 \end{bmatrix}$$

$$A^4 = PD^4 P^{-1}$$

$$A^4 = \frac{1}{6}\begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 16 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 1296 \end{bmatrix}\begin{bmatrix} -3 & 0 & 3 \\ 2 & -2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -8 & 0 & 8 \\ 27 & -27 & 27 \\ 216 & 512 & 216 \end{bmatrix}$$

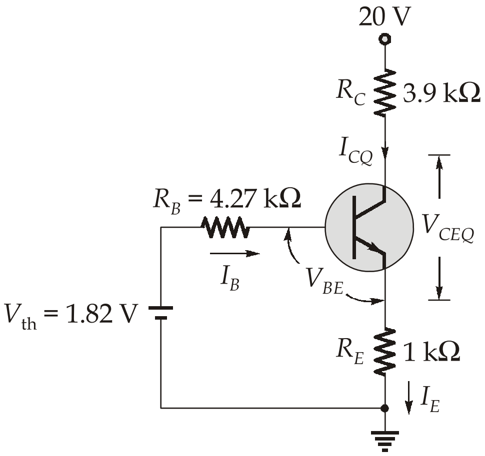
$$= \begin{bmatrix} 251 & 485 & 235 \\ 485 & 1051 & 485 \\ 235 & 485 & 251 \end{bmatrix}$$

Q.3 (b) Solution:

In order to determine the percent change in DC bias point due to change in temperature, we must obtain the DC bias points at the two temperatures.

To obtain the bias point (Q-point) at $T = 30°$ C

The Thevenin's equivalent voltage,



V_{Th} is given as,

$$V_{Th} = \frac{R_2}{(R_1 + R_2)} V_{CC}$$

$$\therefore \quad V_{Th} = 1.82 \text{ V}$$

and the effective resistance R_B is given by,

$$R_B = R_1 \,||\, R_2 = \frac{R_1 R_2}{(R_1 + R_2)} = \frac{4.7 \times 47}{51.7} = 4.27 \text{ k}\Omega$$

$$V_{Th} = I_B R_B + V_{BE} + I_E R_E$$

$$= I_BR_B + (1 + \beta_{dc})I_BR_E$$

As $$I_E = (1 + \beta_{dc})I_B$$

$\therefore$ $$I_B = \frac{V_{Th} - V_{BE}}{R_B + (1+\beta_{dc})R_E}$$

Substituting the values we get,

$$I_B = \frac{1.82 - 0.7}{(4.27 \times 10^3) + (1+30) \times 1 \times 10^3} = 31.75\ \mu A$$

$\therefore$ $$I_{CQ}(30°) = \beta_{dc}(30°) \times I_B$$

$$= 30 \times 31.75 \times 10^{-6} = 0.952\text{ mA}$$

Now apply KVL to the collector circuit

$$V_{CC} = I_{CQ}R_C + V_{CEQ} + I_ER_E$$

$$I_E = (1 + \beta_{dc})I_B = 31 \times 31.75 \times 10^{-6}$$

$$= 0.984\text{ mA}$$

$\therefore$ $$V_{CEQ} = V_{CC} - I_ER_E - I_{CQ}R_C$$

$$= 20 - (0.984 \times 1) - (0.952 \times 3.9)$$

$$= 15.3\text{ V}$$

Thus the bias point or Q point at $T = 30°$ C is

$$I_{CQ}(30°) = 0.952\text{ mA},$$

$$V_{CEQ}(30°) = 15.3\text{ V}$$

To obtain the bias point (Q-point) at $T = 70°$ C

Now we will use,

$$\beta_{dc} = 70$$

The new value of base current is

$$I_B = \frac{1.82 - 0.7}{(4.27 \times 10^3) + (1+70) \times 1 \times 10^3} = 14.87\ \mu A$$

$\therefore$ $$I_{CQ}(70°) = 14.87 \times 70 \times 10^{-6}$$

$$= 1.04\text{ mA}$$

$$I_E = 14.87 \times 10^{-6} \times 71$$

$$= 1.056\text{ mA}$$

$$V_{CEQ}(70°\text{ C}) = 20 - (1.056 \times 1) - (1.04 \times 3.9)$$

$$= 14.88\text{ V}$$

To obtain the percent change in the bias point,

$$\%\text{change in } I_{CQ} = \frac{I_{CQ}(70°) - I_{CQ}(30°)}{I_{CQ}(30°)} \times 100$$

$$= \frac{1.04 - 0.952}{0.952} \times 100 = 9.24\%$$

$$\therefore \quad \%\text{change in } I_{CQ} = 9.24\% \text{ (increase)}$$

$$\%\text{ change in } V_{CEQ} = \frac{V_{CEQ}(70°) - V_{CEQ}(30°)}{V_{CEQ}(30°)} \times 100$$

$$= \frac{14.88 - 15.3}{15.3} \times 100$$

$$\therefore \quad \%\text{change in } V_{CEQ} = -2.74\% \text{ (decreases)}$$

Q.3 (c) (i) Solution:

Hall effect is observed when a potential difference (Hall voltage) is generated across an electric material, that is transverse to an electric current in the material and to an applied magnetic field perpendicular to current.

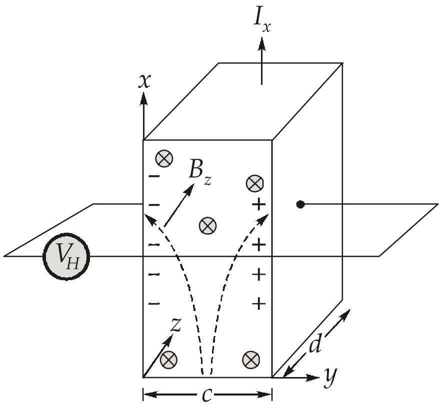
In given specimen when magnetic field is imposed in positive *z*-direction, the resulting force brought to bear on the charge carrier will cause them to be deflected in the *y*-direction (for holes in right direction in specimen and for electrons to the left in specimen).

Where V_H = Hall voltage

B_z = Magnetic field

I_x = Current (*x*-direction)

$$V_H = \frac{R_H I_x B_z}{d}$$



R_H is Hall coefficient,

n is number of charge carrier.

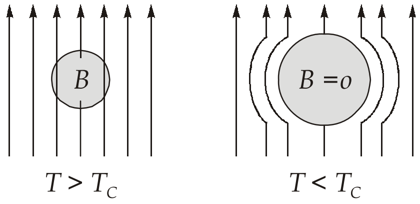
$$R_H = \frac{1}{n|e|}$$

Electron mobility, $\mu_e = \dfrac{\sigma}{n|e|}$

or $\mu_e = |R_H|\sigma$

Q.3 (c) (ii) Solution:

Superconductors exhibit zero resistance but also spontaneously expel all magnetic flux when cooled through the superconducting transition, and behave as perfect diamagnetic material. This phenomena is called "Meissner effect" represented by figure below,



Meissner effect

It has been observed that when a long superconductor is cooled in a longitudinal magnetic field as shown in figure, below the transition temperature, the lines of induction are pushed out of material hence magnetic field becomes zero inside the specimen.

We know from magnetic properties of materials that

$$B = \mu_0(H + M),$$

$$\text{for } B = 0,\ H = -M$$

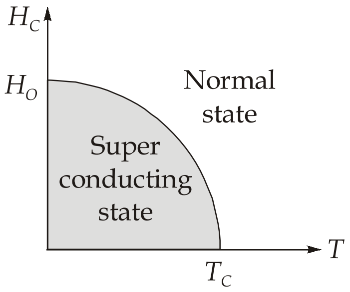
$$\Rightarrow \qquad \chi_m = \frac{M}{H} = -1$$

Now, we may state that magnetic susceptibility in a superconductor is negative. This is referred to as perfect diamagnetism is justified by Meissner effect.

With only small deviations, the critical field H_c varies with temperature according to the parabolic law,

$$H_c = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

H_0 is the critical field at absolute zero and T_c is the transition temperature. For any particular superconductor the shape of variation of H_c with temperature is shown in figure below.



Factors affecting transition temperature of superconductor are as follows:

- **Magnetic field:**
  Transition temperature can be reduced with increment in critical magnetic field value.
- **Isotropic mass:**
  Transition temperature varies in materials found in isotopic form having different isotopic mass.

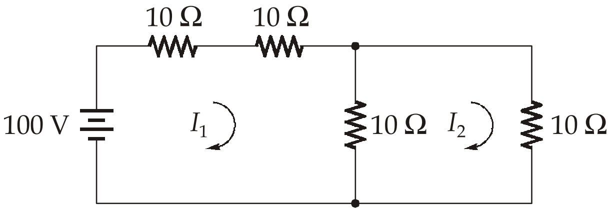
$$T_C \propto \frac{1}{\sqrt{m}}$$

- **Mechanical stress or pressure:**
  Transition temperature can be varied with application of mechanical stress or pressure.

Q.4 (a) Solution:

When the switch K is opened, under steady state condition, two inductors behave as short circuits. Therefore, the initial currents flowing through the indicators can be found out by writing the KVL equations for the circuit at $t = 0^-$.

By KVL for the two meshes,



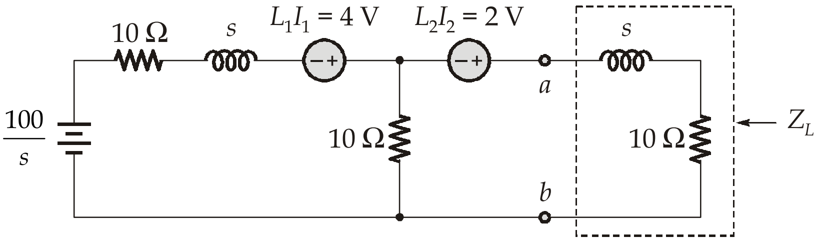
$$30I_1 - 10I_2 = 100$$

and $$-10I_1 + 20I_2 = 0$$

Solving, $$I_1 = 4\text{ A},$$

$$I_2 = 2\text{ A}$$

Hence, the transform network for $t > 0$ is shown in figure below,



Thevenin equivalent impedance with respect to the terminals a and b is given as

$$Z_{\text{th}} = \frac{(s+10)\times 10}{s+10+10} = \frac{10(s+10)}{(s+20)}$$

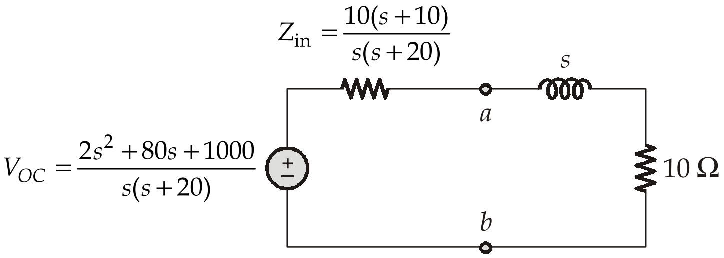
To find the open-circuit voltage across the terminals *a* and *b*, we have the current flowing in the left mesh

$$I(s) = \frac{\frac{100}{s} + 4}{s + 10 + 10} = \frac{4s + 100}{s(s+20)}$$

$$\therefore \qquad V_{OC(s)} = I(s) \times 10 + 2 = \frac{4s+100}{s(s+20)} \times 10 + 2$$

$$= \frac{2s^2 + 80s + 1000}{s(s+20)}$$

Therefore, the Thevenin's equivalent circuit is shown in figure



Hence, the current through the resistor *R* = 10 Ω is given as,

$$I_L(s) = \frac{V_{oc}(s)}{Z_{th} + R} = \frac{2s^2 + 80s + 1000}{s(s+20)\left[\frac{10(s+10)}{(s+20)} + (s+10)\right]}$$

$$= \frac{2s^2 + 80s + 1000}{s(s+10)(s+30)}$$

By partial fraction expansion, let

$$I_L(s) = \frac{2s^2 + 80s + 1000}{s(s+10)(s+30)} = \frac{K_1}{s} + \frac{K_2}{s+10} + \frac{K_3}{s+30}$$

$$\therefore \qquad K_1 = s\left[\frac{2s^2 + 80s + 1000}{s(s+10)(s+30)}\right]_{s=0} = \frac{10}{3}$$

$$\therefore \qquad K_2 = (s+10)\left[\frac{2s^2 + 80s + 1000}{s(s+10)(s+30)}\right]_{s=-10} = -2$$

$$\therefore \qquad K_3 = (s+30)\left[\frac{2s^2+80s+1000}{s(s+10)(s+30)}\right]_{s=-30} = \frac{2}{3}$$

$$I_L(s) = \frac{\frac{10}{3}}{s} - \frac{2}{s+10} + \frac{\frac{2}{3}}{s+30}$$

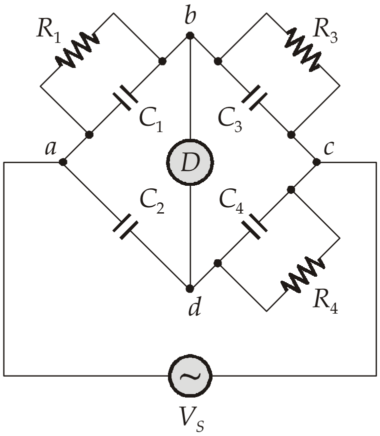
Taking inverse Laplace transform, we get

$$i_L(t) = \frac{10}{3} - 2e^{-10t} + \frac{2}{3}e^{-30t}$$

$$= (3.33 - 2e^{-10t} + 0.67e^{-30t}) \text{ A}; \quad t \geq 0$$

Q.4 (b) (i) Solution:

The voltage circuit is shown below :



For balance, $$Y_1Y_4 = Y_2Y_3$$

or, $$\left(\frac{1}{R_1} + j\omega C_1\right)\left(\frac{1}{R_4} + j\omega C_4\right) = (j\omega C_2)\left(\frac{1}{R_3} + j\omega C_3\right)$$

or, $$\left(\frac{1}{R_1R_4} - \omega^2 C_1C_4\right) + j\omega\left(\frac{C_4}{R_1} + \frac{C_1}{R_4}\right) = j\omega\frac{C_2}{R_3} - \omega^2 C_2C_3$$

Equating the real and imaginary parts, we have

$$\frac{1}{R_1R_4} - \omega^2 C_1C_4 = -\omega^2 C_2C_3 \qquad \text{...(i)}$$

and $$\frac{C_4}{R_1} + \frac{C_1}{R_4} = \frac{C_2}{R_3} \qquad \text{...(ii)}$$

From (i) and (ii), we have

$$C_1 = \frac{\dfrac{C_2R_4}{R_3} + \omega^2 C_2C_3C_4R_4^2}{1+\omega^2 C_4^2R_4^2}$$

Now $\omega^2 C_2C_3C_4R_4^2 << \dfrac{C_2R_4}{R_3}$

and $\omega^2 C_4^2R_4^2 <<1$

Hence we can write, $C_1 = \dfrac{C_2R_4}{R_3}$

When the capacitor C_1 is without specimen dielectric let its capacitance be C_0 .

$\therefore$ $$C_0 = \frac{C_2R_4}{R_3} = 150\times\frac{5000}{5000} = 150 \text{ pF}$$

When the specimen is inserted as dielectric, let the capacitance be C_S .

$\therefore$ $$C_S = \frac{C_2R_4}{R_3} = 900\times\frac{5000}{5000} = 900 \text{ pF}$$

Now, $C_0 = \dfrac{\varepsilon_0 A}{d}$ and $C_S = \dfrac{\varepsilon_r\varepsilon_0 A}{d}$

Hence relative permittivity of specimen

$$\varepsilon_r = \frac{C_S}{C_0} = \frac{900}{150} = 6$$

Q.4 (b) (ii) Solution:

Deflection, $D = \dfrac{Ll_dE_d}{2dE_a}$

$\therefore$ Voltage applied to deflecting plates,

$$E_d = \frac{2dE_aD}{Ll_d} = \frac{2\times5\times10^{-3}\times2000\times3\times10^{-2}}{0.3\times2\times10^{-2}} = 100 \text{ V}$$

$\therefore$ Input voltage required for a deflection of 3 cm

$$= \frac{E_d}{\text{gain}} = \frac{100}{100} = 1 \text{ V}$$

Q.4 (c) Solution:

We first evaluate the dc operating point,

$$I_D = \frac{1}{2}K_n'\left(\frac{W}{L}\right)(V_{GS}-V_t)^2$$

$$V_{GS} = V_{DS} = V_D \qquad ...(\because V_S = 0 \text{ and } V_D = V_G \text{ as } I_G = 0)$$

$$= \frac{1}{2}\times 0.25(V_D-1.5)^2 \text{ mA}$$

Also,

$$V_D = 15 - I_D R_D = 15 - 10I_D$$

$$8I_D = [15 - 10I_D - 1.5]^2$$

$$= 182.25 + 100I_D{}^2 - 270I_D$$

$$100\, I_D{}^2 - 278\, I_D + 182.25 = 0$$

$$I_D = 1.06 \text{ mA}, 1.72 \text{ mA}$$

So,

$$I_D = 1.06 \text{ mA}$$

$$V_D = 15 - 10 \times 1.06 = 4.4 \text{ V}$$

The value of g_m is given by

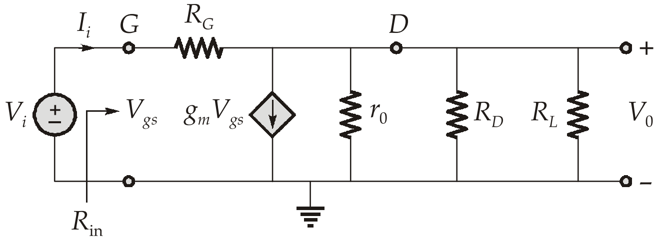
$$g_m = K_n'\left(\frac{W}{L}\right)(V_{GS}-V_t)$$

$$= 0.25(4.4 - 1.5) = 0.725 \text{ mA/V}$$

The output resistance r_0 is given by

$$r_0 = \frac{V_A}{I_D} = \frac{50}{1.06} = 47 \text{ k}\Omega$$

The small signal model of amplifier circuit is given as



$$V_0 \approx -g_m V_{gs}\left[r_0 \parallel R_D \parallel R_L\right]$$

Since,

$$V_{gs} = V_i$$

$$A_V = -g_m\left[r_0 \parallel R_D \parallel R_L\right]$$

$$= -0.725\left[10 \parallel 10 \parallel 47\right] = -3.27$$

To evaluate the input resistance R_{in} ,

$$I_i = \frac{(V_{\text{in}} - V_0)}{R_G} = \frac{V_i}{R_G}\left(1 - \frac{V_0}{V_i}\right)$$

$$I_i = \frac{V_i}{R_G}[1 - (-3.27)] = \frac{4.27V_i}{R_G}$$

$$R_{\text{in}} = \frac{V_i}{I_i} = \frac{R_G}{4.27} = \frac{10}{4.27} = 2.34\ \text{M}\Omega$$

The largest allowable input signal $\hat{V}_i$ is determined by the need to keep the MOSFET in saturation at all time ; that is

$$V_{DS} \geq V_{GS} - V_t$$

$$V_{DS\,(\text{min})} = V_{GS\,(\text{max})} - V_t$$

$$V_{DS} - |A_V|\hat{V}_i = V_{GS} + \hat{V}_i - V_t$$

$$4.4 - 3.27\hat{V}_i = 4.4 + \hat{V}_i - 1.5$$

$$\hat{V}_i = 0.35\ \text{V}$$

So maximum allowable input signal peak is 0.34 V.

Section-B

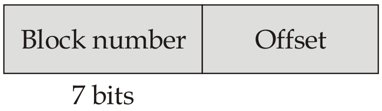
Q.5 (a) Solution:

Number of bits requires for 128 main memory block

$$= \log_2(128)$$

$$= 7\ \text{bits}$$

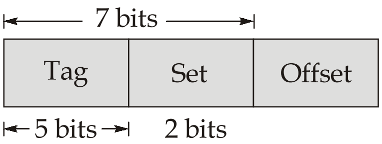
Main memory format:



Since main memory and cache memory offset will be same.

Number of bits for set $= \log_2(4) = 2$

So, cache memory format will be:



So, each set can maps 32 blocks and at one time can hold only 2 different block.

	Block 1st	Block 2nd
Set 0	0	16
Set 1	5	9
Set 2		
Set 3	~~3~~ 55	7

For block $0 = 00000\underline{00}$ = set 0
For block $5 = 00001\underline{01}$ = set 1
For block $3 = 00000\underline{11}$ = set 3
For block $9 = 00010\underline{01}$ = set 1
For block $7 = 00001\underline{11}$ = set 3
For block $0 = 00000\underline{00}$ = set 0
For block $16 = 00100\underline{00}$ = set 0
For block $55 = 01101\underline{11}$ = set 3

So, for block 55, block 3 will be replaced since 3 is come before block 7.
So, memory block 0, 5, 7, 9, 16 and 55 will be present in cache.

Q.5 (b) Solution:

Let $u = xy + z; \quad V = x^2 + y^2 - z^2$

and $u = \phi(V)$

$$xy + z = \phi(x^2 + y^2 - z^2)$$

$$z = \phi(x^2 + y^2 - z^2) - xy$$

Differentiating z w.r.t. x

$$\frac{\partial z}{\partial x} = \phi'(x^2 + y^2 - z^2)\left(2x - 2z\frac{\partial z}{\partial x}\right) - y$$

$$p = \phi'(x^2 + y^2 - z^2)(2x - 2zp) - y$$

$$2(x - zp)\phi'(x^2 + y^2 - z^2) = p + y \qquad \text{...(i)}$$

Again, differentiating z w.r.t. y,

$$\frac{\partial z}{\partial y} = \phi'(x^2 + y^2 - z^2)\left(2y - 2z\frac{\partial z}{\partial y}\right) - x$$

$$q = \phi'(x^2 + y^2 - z^2)(2y - 2zq) - x$$

$$2(y - zq)\phi'(x^2 + y^2 - z^2) = q + x \qquad \text{...(ii)}$$

Divide equation (i) and (ii),

$$\frac{p+y}{q+x} = \frac{x-pz}{y-qz}$$

$$\Rightarrow \quad py - pqz + y^2 - qyz = xq - pqz + x^2 - pxz$$

$$\Rightarrow \quad p(y + xz) - q(yz + x) = x^2 - y^2$$

So, partial differential equation is

$$(y + xz)\frac{\partial z}{\partial x} - (x + yz)\frac{\partial z}{\partial y} = x^2 - y^2$$

Q.5 (c) Solution:

Total energy stored in a capacitor,

$$W_E = \frac{1}{2}CV^2$$

Before the switch is opened.

$$E_1 \text{ energy in } C_1 \text{ capacitor} = \frac{1}{2}C_1V^2 \text{ and } E_2 \text{ energy in } C_2 \text{ capacitor} = \frac{1}{2}C_2V^2$$

After the switch is opened.

Dielectric $\in_R = 2$ being introduced

$$E_1' \text{ energy in } C_1 \text{ capacitor} = \frac{1}{2}2C_1V^2 = C_1V^2$$

As capacitance doubles due to dielectric.

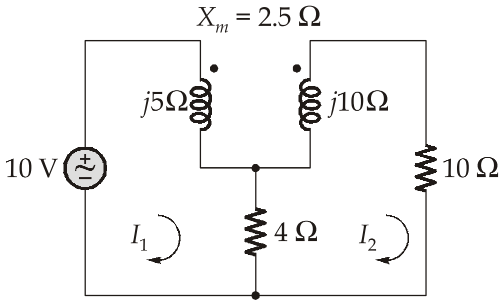
E_2' energy in C_2 capacitor has to be same before and after dielectric is introduced (law of conservation of energy)

$$E_2' = E_2 = \frac{1}{2}C_2V^2$$

Ratio of the energies,
$$\frac{E_2'}{E_1'} = \frac{1}{2}\frac{C_2V^2}{C_1V^2} = \frac{C_2}{2C_1}$$

Q.5 (d) Solution:

(i) We consider the two loop currents as I_1 and I_2 as shown in figure,



Applying KVL for the two meshes, we get

$$(4 + j5)I_1 - (4 + j2.5)I_2 = 10 \quad \text{...(i)}$$

$$-(4 + j2.5)I_1 + (14 + j10)I_2 = 0 \quad \text{...(ii)}$$

Solving (i) and (ii), we get the current through the 10 Ω resistance as

$$I_2 = \frac{\begin{vmatrix} (4+j5) & 10 \\ -(4+j2.5) & 0 \end{vmatrix}}{\begin{vmatrix} (4+j5) & -(4+j2.5) \\ -(4+j2.5) & (4+j5) \end{vmatrix}}$$

$$= \frac{10(4+j2.5)}{(4+j5)(4+j5)-(4+j2.5)^2} = 0.523\angle{-60.4°}\text{ (A)}$$

(ii) If the direction of winding of one of the coils is reversed, the sign of mutual inductance will be positive.

Here, the KVL equations will become

$$(4+j5)I_1 - (4-j2.5)I_2 = 10 \quad \text{...(iii)}$$

$$-(4-j2.5)I_1 + (14+j10)I_2 = 0 \quad \text{...(iv)}$$

Solving (iii) and (iv), we get the current through the 10 Ω resistance as

$$I_2 = \frac{\begin{vmatrix} (4+j5) & 10 \\ -(4-j2.5) & 0 \end{vmatrix}}{\begin{vmatrix} (4+j5) & -(4-j2.5) \\ -(4-j2.5) & (4+j5) \end{vmatrix}}$$

$$= \frac{10(4+j2.5)}{(4+j5)(4+j5)-(4-j2.5)^2}$$

$$= 0.362\angle{-123.65°}\text{ (A)}$$

Q.5 (e) Solution:

Initially, $$I_C = \frac{V_{CC}-V_{CE}}{R_C} = \frac{10-3.2}{6.8} = 1\text{ mA}$$

(i) For, $$V_{CE} = 0.3\text{ V}$$

$$I_C = \frac{10-0.30}{6.8} = 1.4264\text{ mA}$$

As we know, $$I_C = I_S\left(e^{V_{BE}/V_T}\right)$$

$$\frac{I_{C2}}{I_{C1}} = e^{\frac{\Delta V_{BE}}{V_T}}$$

Here, $$I_{C2} = 1.4264\text{ mA and } I_{C1} = 1\text{ mA}$$

To increase I_C from 1 mA to 1.4264 mA,

V_{BE} must be increased by

$$\Delta V_{BE} = V_T \ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

$$\Delta V_{BE} = 25 \times \ln\left(\frac{1.4264}{1}\right) = 8.87 \text{ mV}$$

(ii) For $V_0 = 0.99$,

$$V_{CE} = 9.9 \text{ V}$$

$$I_C = \frac{V_{CC} - V_0}{R_C} = \frac{10 - 9.9}{6.8} = 0.0147 \text{ mA}$$

To decrease I_C from 1 mA to 0.0147 mA, V_{BE} must change by

$$\Delta V_{BE} = V_T \ln\left[\frac{0.0147}{1}\right] = -105.50 \text{ mV}$$

Q.6 (a) Solution:

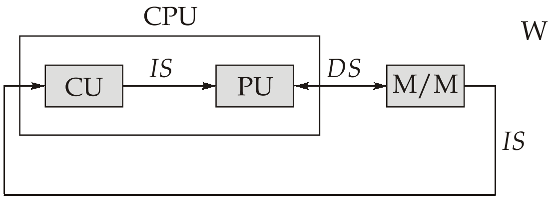
(i)

Reduced Instruction Set Computer (RISC)		Complex Instruction Set Computer (CISC)
1.	Emphasis on software/less load on hardware.	Emphasis on hardware/less load on programmer.
2.	Single-clock, reduced instruction only.	Includes multi-clock complex instructions.
3.	Register to register 'LOAD' and 'STORE' are independent instructions.	Memory to memory: 'LOAD' and 'STORE' incorporated in instruction.
4.	Low cycles per second (CPI), large code size.	Small code sizes, high cycles per second (CPI).
5.	Spends more transistors on memory 'registers'.	Transistors used for storing complex instructions (memory).
6.	Simple addressing modes.	Complex addressing modes.
7.	Highly pipelined processors.	Less pipelined or not.

(ii) Flynn's Classification:

1. Single instruction stream, single data stream (SISD):

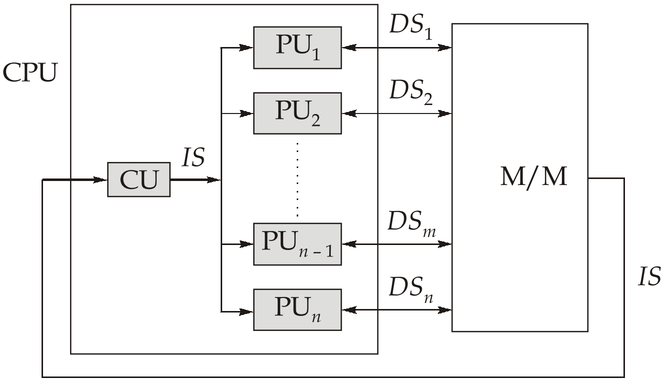
   It represents a computer organization containing a single control unit, a processor unit and a memory unit. Instructions are executed sequentially.



Where, CU : Control unit
PU : Processor units
M/M: Memory module
IS : Instruction stream
DS : Data stream

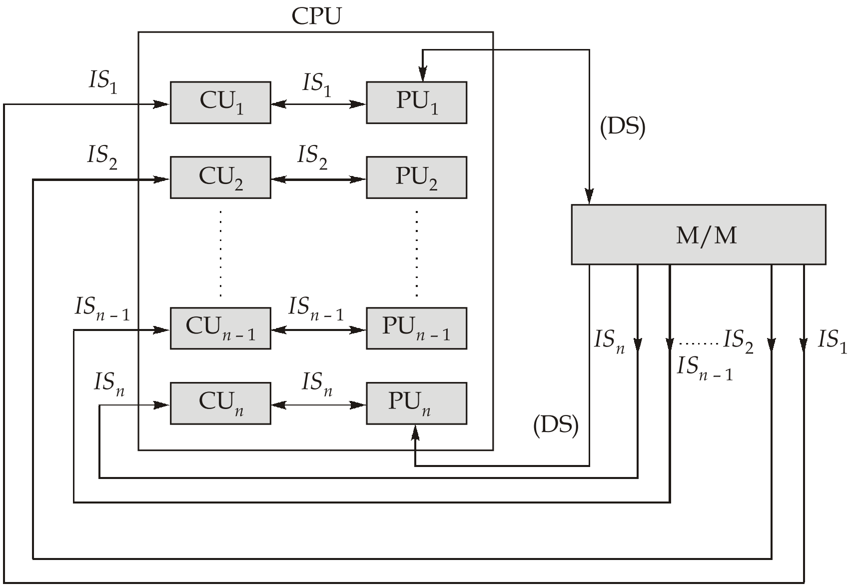
2. Single instruction stream Multiple data stream (SIMD):

   It represents an organization which has multiple processors under the supervision of a common control unit. It is mainly dedicated to array processing machines.



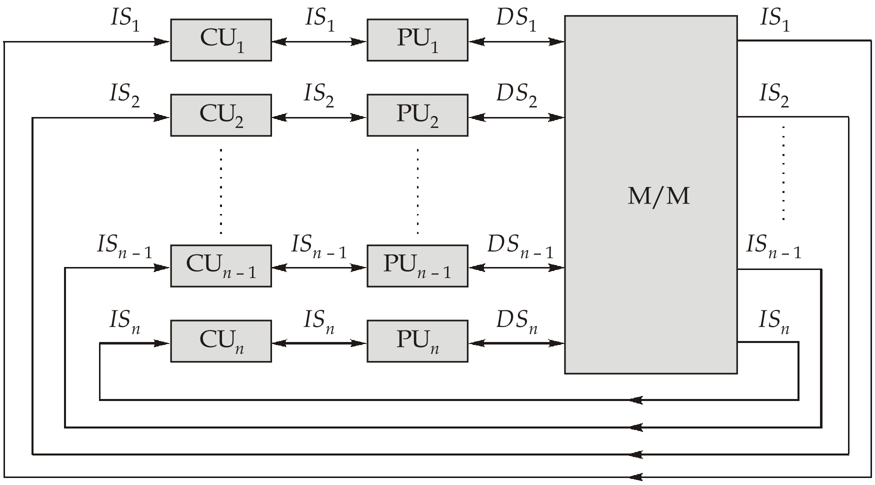
3. Multiple instruction steam, single data stream (MISD):

   It refers to the computer organization in which several instructions manipulate the same data stream concurrently. It execute operations by different instruction on the same data set.



4. Multiple instruction stream, multiple data stream (MIMD):

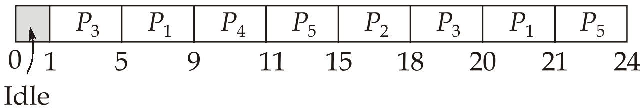
   Its organization refers to system capable of processing several programs at a time. Most multiprocessor systems and multiple computer system fall in this category.



(iii)

P_{id}	AT	BT	CT	TAT = CT – AT	WT = TAT – BT
1	2	5	21	19	14
2	4	3	18	14	11
3	1	6	20	19	13
4	2	2	11	9	7
5	3	7	24	21	14

Gantt chart:



$$\text{Average Turn Around Time (TAT)} = \frac{19+14+19+9+21}{5} = 16.4 \text{ quanta}$$

$$\text{Average Wait Time (WT)} = \frac{14+11+13+7+14}{5} = 11.8 \text{ quanta}$$

Q.6 (b) Solution:

We know the fourier series of $f(x)$ as

$$f(x) = a_o + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x) \qquad \left(\because \omega_n = \frac{2\pi}{2} = \pi\right)$$

where,
$$a_o = \frac{1}{2}\int_0^2 f(x)dx$$

$$= \frac{1}{2}\int_0^1 \pi x dx + \int_1^2 \pi(1-x)dx$$

$$= \frac{1}{2}\left[\left(\frac{\pi x^2}{2}\right)_0^1 + \left(\pi\left(x-\frac{x^2}{2}\right)\right)_1^2\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2}+\pi\left(0-\frac{1}{2}\right)\right] = 0$$

$$a_n = \frac{2}{2}\int f(x)\cos n\pi x\, dx$$

$$= \int_0^1 \pi x \cos n\pi x dx + \int_1^2 \pi(1-x)\cos n\pi x dx$$

$$= \left(\frac{\pi x \sin n\pi x}{n\pi}\right)_0^1 - \int_0^1 \frac{\pi \sin n\pi x}{n\pi}dx + \left(\pi(1-x)\frac{\sin n\pi x}{n\pi}\right)_1^2 + \int_1^2 \frac{\pi \sin n\pi x}{n\pi}dx$$

$$= 0 - \frac{1}{n^2\pi}[-\cos n\pi x]_0^1 + 0 + \frac{1}{n^2\pi}[-\cos n\pi x]_1^2$$

$$= -\frac{1}{n^2\pi}\left[-(-1)^n+1\right] + \frac{1}{n^2\pi}\left[-1+(-1)^n\right] = \frac{2}{n^2\pi}\left[(-1)^n-1\right]$$

$$= -\frac{4}{n^2\pi}, \text{ for } n = 1, 3, 5$$

$$b_n = \frac{2}{2}\int_0^2 f(x)\sin n\pi x\, dx$$

$$= \int_0^1 \pi x \sin n\pi x\, dx + \int_1^2 \pi(1-x)\sin n\pi x\, dx$$

$$= \left[-\pi x\frac{\cos n\pi x}{n\pi}\right]_0^1 + \int_0^1 \pi\frac{\cos n\pi x}{n\pi}dx + \left[-\pi(1-x)\frac{\cos n\pi x}{n\pi}\right]_1^2$$

$$- \int_1^2 \pi\frac{\cos n\pi x}{n\pi}dx$$

$$= \frac{-(-1)^n}{n} + \left[\frac{\sin n\pi x}{n^2\pi}\right]_0^1 + \frac{1}{n} - \left[\frac{\sin n\pi x}{n^2\pi}\right]_1^2$$

$$= \frac{1}{n}\left[1-(-1)^n\right]$$

$$= \frac{2}{n}, \text{ for n} = 1, 3, 5, 7$$

Fourier series of $f(x)$ is

$$f(x) = 0 - \frac{4}{\pi}\left[\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + ...\right]$$

$$+\ 2\left[\frac{\sin \pi x}{1} + \frac{\sin 3\pi x}{3} + \frac{\sin 5\pi x}{5} + ...\right]$$

Now put $x = 1$ in above expression

$$f(1) = 0 - \frac{4}{\pi}\left[\frac{-1}{1^2} + \frac{(-1)}{3^2} + \frac{(-1)}{5^2} + ...\right] + 2\times 0$$

$$\pi = \frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...\right]$$

$$\Rightarrow \qquad \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{4}$$

Q.6 (c) (i) Solution:

Due to polarisation, the electrical field inside the atom is given as the Lorentz field,

i.e. $$E = \frac{P}{3\varepsilon_0}$$

The dipole moment of the polarized atom is therefore,

$$\mu = \frac{4\pi}{3}R^3 P$$

Since, $\mu = \alpha E$, therefore we have

$$\alpha = 4\pi\ \varepsilon_0 R^3$$

Substituting the value, we get

$$\alpha = \frac{4\times 3.14\times 8.854\times 10^{-12}\times (0.53\times 10^{-10})^3}{1}$$

$$\alpha = 1.66 \times 10^{-41} \text{ F-m}^2$$

$$\varepsilon_r = 1 + 4\pi\ NR^3$$

$$= 1 + 4\pi \times 9.81 \times 10^{25}(0.53 \times 10^{-10})^3$$

$$= 1.0018$$

Q.6 (c) (ii) Solution:

Let susceptibility and magnetization in copper and Fe_2O_3 be X_{m1} , M_1 and X_{m2} , M_2 respectively. We have

$$\chi_{m1} = \frac{M_1}{H}$$

and $$\chi_{m2} = \frac{M_2}{H}$$

or $$M_1 = \chi_{m1} \text{ and } M_2 = \chi_{m2}\text{H}$$

Substituting the values,

$$\chi_{m1} = -0.5 \times 10^{-5}$$

$$\chi_{m2} = 1.4 \times 10^{-3}$$

and $$H = 10^6$$

$$M_1 = -0.5 \times 10^{-5} \times 10^6 = 5 \text{ Am}^{-1}$$

$$M_2 = 1.4 \times 10^{-3} \times 10^6 = 1.4 \times 10^3 = 1400 \text{ Am}^{-1}$$

Similarly, let the flux densities in copper and Fe_2O_3 be B_1 and B_2 respectively,

We have: $$B_1 = \mu_0 H + \mu_0 M_1$$

$$= \mu_0(H + M_1)$$

$$B_2 = \mu_0 H + \mu_0 M_2$$

$$= \mu_0(H + M_2)$$

Substituting the values, we have

$$B_1 = 4\pi \times 10^{-7} (10^6 + 5)$$

$$= 1.256 \text{ Wbm}^{-2}$$

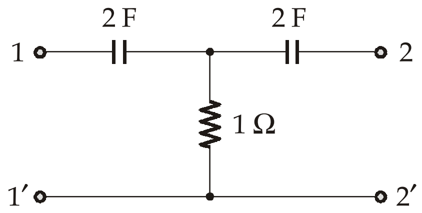
$$B_2 = 4\pi \times 10^{-7} (10^6 + 1400)$$

$$= 1.256 \text{ Wbm}^{-2}$$

Q.7 (a) Solution:

The given network is the parallel combination of the two networks:

For the network (a),



$$z_{11a} = \left(\frac{1}{2s}+1\right) = \frac{1+2s}{2s},$$

$$z_{12a} = z_{21a} = 1$$

$$z_{22a} = \left(\frac{1}{2s}+1\right) = \frac{1+2s}{2s}$$

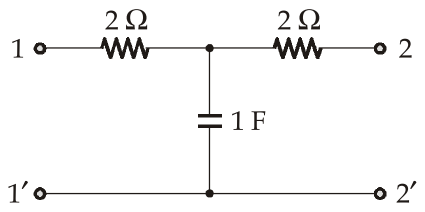
$$\therefore \quad \Delta z_a = \frac{1+4s}{4s^2}$$

$$\therefore \quad y_{11a} = \frac{z_{22a}}{\Delta z_a} = \frac{2s(1+2s)}{(1+4s)};$$

$$y_{12a} = y_{21a} = -\frac{z_{12a}}{\Delta z_a} = -\frac{4s^2}{(1+4s)};$$

$$y_{22a} = \frac{z_{11a}}{\Delta z_a} = \frac{2s(1+2s)}{(1+4s)}$$

For network (b),



$$z_{11b} = \left(\frac{1}{s}+2\right) = \frac{1+2s}{s},$$

$$z_{12b} = z_{21b} = \frac{1}{s};$$

$$z_{22b} = \left(\frac{1}{s}+2\right) = \frac{1+2s}{s}$$

$$\therefore \quad \Delta z_b = \frac{4(s+1)}{s}$$

$$\therefore \quad y_{11b} = \frac{z_{22b}}{\Delta z_b} = \frac{(1+2s)}{4(s+1)}$$

$$y_{12b} = y_{21b} = -\frac{z_{12b}}{\Delta z_b} = -\frac{z_{12b}}{\Delta z_b} = -\frac{1}{4(s+1)}$$

$$y_{22b} = \frac{z_{11b}}{\Delta z_b} = \frac{(1+2s)}{4(s+1)}$$

Thus, the overall y-parameter are,

$$y_{11} = y_{22} = (y_{11a} + y_{11b})$$

$$= \frac{2s(1+2s)}{1+4s} + \frac{(1+2s)}{4+4s} = \frac{(1+2s)(8s^2+12s+1)}{4(s+1)(4s+1)}$$

and

$$y_{12} = y_{21} = (y_{12a} + y_{12b})$$

$$= -\frac{4s^2}{1+4s} - \frac{1}{4(s+1)} = -\frac{16s^3+16s^2+4s+1}{4(4s+1)(s+1)}$$

Q.7 (b) (i) Solution:

1. For intrinsic Silicon,

$$\rho = n = n_i = 1.5 \times 10^{10} \text{ cm}/^3$$

Thus,

$$\rho = \frac{1}{q(p\mu_p + n\mu_n)}$$

$$= \frac{1}{1.6\times10^{-19}(1.5\times10^{10}\times480 + 1.5\times10^{10}\times1350)}$$

$$= 2.27 \times 10^5 \ \Omega\text{-cm}$$

2. For the p-type Silicon,

$$p_p \approx N_A = 10^{16}/\text{cm}^3$$

$$n_\rho \approx \frac{n_i^2}{N_A} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25 \times 10^4/\text{cm}^3$$

Thus,

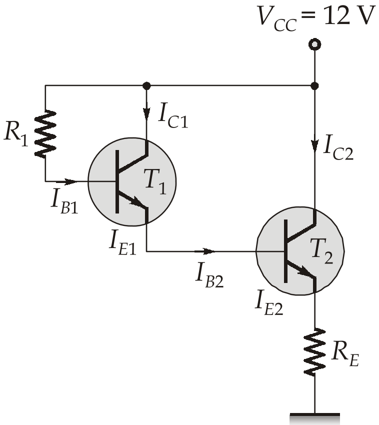
$$\rho = \frac{1}{q(p\mu_p + n\mu_n)}$$

$$= \frac{1}{1.6\times10^{-19}(10^{16}\times400 + 2.25\times10^4\times1110)}$$

$$\approx \frac{1}{1.6\times10^{-19}\times10^{16}\times400} = 1.56 \ \Omega\text{-cm}$$

Observe that the resistivity of the p-type Silicon is determined almost entirely by the doping concentration. Also observe that doping the Silicon reduces its resistivity by a factor of about 10^5, a truly remarkable change.

Q.7 (b) (ii) Solution:



1.

$$\beta = 30$$

$$R_E = 1.5 \text{ k}\Omega$$

$$(V_{CE})_{Q2} = 5 \text{ V}$$

$$I_{E1} = I_{B2}$$

$$\Rightarrow \quad (\beta + 1)I_{B1} = \frac{I_{E2}}{(1+\beta)}$$

$$(1 + \beta)^2 \times I_{B1} = I_{E2} \qquad \text{...(i)}$$

Applying KVL in loop (1)

$$I_{B1} \times R_1 + V_{BE1} + V_{BE2} + I_{E2} \times R_E = 12 \text{ V}$$

as $V_{BE1} = V_{BE2} = 0.7$ V

$$I_{B1} \times R_1 + (1 + \beta)^2 \times I_{B1} \times R_E = 12 - 1.4 \text{ V}$$

$$I_{B1}[R_1 + (1 + \beta)^2 \times R_E] = 10.6 \text{ V} \qquad \text{...(ii)}$$

KVL in loop (2)

$$I_{E2} \times R_E = (12 - V_{CE})$$

$$\Rightarrow \quad I_{E2} \times 1.5 \text{ k}\Omega = 7 \text{ V}$$

$$I_{E2} = \frac{7}{1.5} \text{ mA} = 4.67 \text{ mA}$$

From (i),

$$I_{B1} = \frac{4.67}{(1+\beta)^2} = \frac{4.67}{(1+30)^2} \text{ mA}$$

$$\frac{4.67}{(31)^2} \times [R_1 + 31^2 \times R_E] = 10.6 \text{ V}$$

From (ii), $R_1 = 741.34 \text{ k}\Omega$

2. Applying KVL in loop (3)

$$V_{CC} = (V_{CE})_{Q1} + V_{BE2} + I_{E2} \times R_E$$

$$12 = (V_{EE})_{Q1} + 0.7 \text{ V} + 7 \text{ V}$$

$$(V_{CE})_{Q1} = 4.3 \text{ V}$$

Q.7 (c) (i) Solution:

We hve,

$$\Delta f = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2 - y^2 + 2z^2)$$

$$= 2x\hat{i} - 2y\hat{j} + 4z\hat{k}$$

$$= 2\hat{i} - 4\hat{j} + 12\hat{k} \text{ at } P(1, 2, 3)$$

Also, $$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= (5\hat{i} + 0\hat{j} + 4\hat{k}) - (i + 2j + 3\hat{k})$$

$$= 4i - 2j + \hat{k} = \vec{A} \text{ (Say)}$$

∴ Unit vector of $\vec{A} = \hat{A} = \frac{A}{|\vec{A}|}$

$$= \frac{4i - 2j + \hat{k}}{\sqrt{16 + 4 + 1}} = \frac{4\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{21}}$$

Thus the directional derivative of f in the direction of $\overrightarrow{PQ}$.

$$\nabla f \cdot \hat{A} = \frac{(2i - 4j + 12\hat{k}) \cdot (4i - 2j + \hat{k})}{\sqrt{21}}$$

$$= \frac{(8 + 8 + 12)}{\sqrt{21}} = \frac{28}{\sqrt{21}}$$

The directional derivative of its maximum in the direction of the normal to the surface i.e., in the direction of Δf.

Hence, maximum value of this directional derivative

$$= |2\hat{i} - 4\hat{j} + 12\hat{k}|$$

$$= \sqrt{(4 + 16 + 144)} = \sqrt{164}$$

Q.7 (c) (ii) Solution:

Phantom loading:

- In order to avoid the wastage of power, in case the meter under testing has high current rating with actual loading arrangements, such type of fictions loading is done.
- It consists of supplying the normal voltage to pressure circuit and rated current from a separate circuit. With this, the total power supplied for the test is that due to the small pressure coil current at normal voltage, plus that due to the current circuit current supplied at low voltage. The total power, therefore, required for testing the meter is very low.

Q.8 (a) Solution:

(i) The characteristic equation of A is,

$$\begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = 0$$

$$(1 - \lambda)(3 - \lambda) - 8 = 0$$

$$3 - \lambda - 3\lambda + \lambda^2 - 8 = 0$$

$$\lambda^2 - 4\lambda - 5 = 0 \qquad \text{...(i)}$$

By Cayley-Hamilton theorem, A must satisfy its characteristic equation (1), so that,

$$A^2 - 4A - 5I = 0 \qquad \text{...(ii)}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

This verifies the theorem, multiplying (ii) by A^{-1}, we get

$$A - 4I - 5A^{-1} = 0$$

$$A^{-1} = \frac{1}{5}(A - 4I)$$

$$= \frac{1}{5}\left\{\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\} = \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$$

Now dividing the polynomial, $\lambda^5 - 4\lambda^4 - 7\lambda^3 + 11\lambda^2 - \lambda - 10I$ by the polynomial, $\lambda^2 - 4\lambda - 5$, we obtain

$$\lambda^5 - 4\lambda^4 - 7\lambda^3 + 11\lambda^2 - \lambda - 10I = (\lambda^2 - 4\lambda - 5)(\lambda^3 - 2\lambda + 3) + \lambda + 5$$

$$= \lambda + 5$$

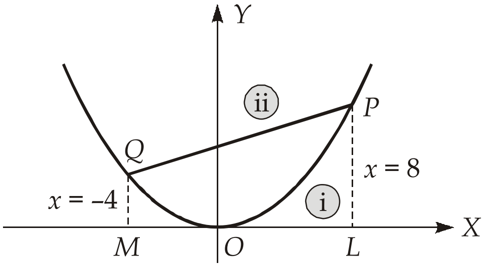
Hence, $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I = A + 5I$

Which is a linear polynomial in A.

(ii) Given parabola is, $x^2 = 8y$ ...(i)

and the straight line is,

$$x - 2y - 8 = 0 \qquad \text{...(ii)}$$



Substituting the value of y from equation (ii) in equation (i) we get,

$$x^2 = 4(x+8)$$

or, $$x^2 - 4x - 32 = 0$$

$$(x-8)(x+4) = 0$$

∴ $$x = 8, -4$$

Thus, equation (i) and (ii) intersect at P and Q where $x = 8$ and $x = -4$.

∴ Required area $POQ = \int_{-4}^{8} y\,dx$ from equation (ii) – $\int_{-4}^{8} y\,dx$ from equation (i),

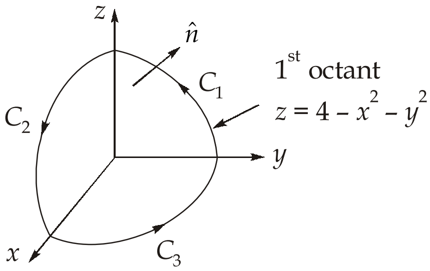
$$= \int_{-4}^{8} \frac{x-8}{2}dx - \int_{-4}^{8} \frac{x^2}{8}dx$$

$$= \frac{1}{2}\left|\frac{x^2}{2}+8x\right|_{-4}^{8} - \frac{1}{8}\left|\frac{x^3}{3}\right|_{-4}^{8}$$

$$= \frac{1}{2}\{(32+64)-(-24)\} - \frac{1}{24}(512+64)$$

$$= 36$$

Q.8 (b) Solution:



$$\vec{F} = yz\hat{i} - xz\hat{j} + \hat{k}$$

(i)
$$\int_C \vec{F}\cdot\overrightarrow{dr} = \int_{C_1} \vec{F}\cdot\overrightarrow{dr} + \int_{C_2} \vec{F}\cdot\overrightarrow{dr} + \int_{C_2} \vec{F}\cdot\overrightarrow{dr}$$

C_1 lies in the yz-plane :

$$x = 0,\ y = t,\ z = 4 - t^2 \Rightarrow dz = -2t\,dt,\ t \text{ goes from 2 to 0.}$$

$$\int_{C_1} \vec{F}\cdot\overrightarrow{dr} = \int_{C_1} 1\,dz = \int_2^0 (-2t)dt = 4$$

C_2 lies in the xz-plane:

$$y = 0,\ x = t,\ z = 4 - t^2,\ dz = (-2t)\ dt,\ t \text{ goes from 0 to 2.}$$

$$\int_{C_2} \vec{F}\cdot\overrightarrow{dr} = \int_{C2} 1\,dz = \int_0^2 (-2t)dt = -4$$

C_3 lies in the xy-plane :

$$z = 0,\ dz = 0$$

$$\int_{C_3} \vec{F}\cdot\overrightarrow{dr} = 0$$

$$\Rightarrow \qquad \int_C \vec{F}\cdot\overrightarrow{dr} = 4 - 4 + 0 = 0$$

(ii)
$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ yz & -xz & 1 \end{vmatrix} = (x)\hat{i} + (y)\hat{j} + (-2z)\hat{k} = <x, y, -2z>$$

$$dS\hat{n} = \left(-\frac{\partial}{\partial x}(4 - x^2 - y^2)\hat{i} - \frac{\partial}{\partial y}(4 - x^2 - y^2)\hat{j} + \hat{k}\right)dA$$

$$= \left(2x\hat{i} + 2y\hat{j} + \hat{k}\right)dA$$

$$\iint(\vec{\nabla}\times\vec{F})\cdot dS\hat{n} = \iint(2x^2 + 2y^2 - 2z)dA$$

$$= \iint(2x^2 + 2y^2 - 2(4 - x^2 - y^2))dA \text{ (Substitute for } z\text{)}$$

$$= \iint 4(x^2 + y^2 - 2)dA$$

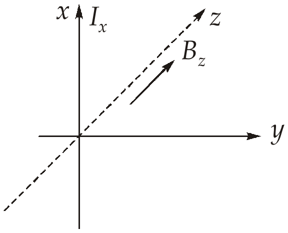
$$= 4\int_0^{\pi/2}\int_0^2 (r^2 - 2)r\,dr\,d\theta \text{ (Change to polar coordinates)}$$

$$= 4\cdot\frac{\pi}{2}\left(\frac{r^4}{4} - r^2\right)\Bigg|_0^2 = 0$$

Hence, $\iint(\vec{\nabla}\times\vec{F})\cdot dS\hat{n} = \int_C \vec{F}\cdot\vec{dr} = 0$ is verified

Q.8 (c) Solution:

(i) For metals, R_H is negative and is given by:



Hall coefficient,

$$R_H = \frac{-\mu_e}{\sigma} = -\frac{0.0012\text{ m}^2/\text{V-s}}{3.8\times10^7(\Omega\text{-m})^{-1}} = -3.16\times10^{-11}\text{ V-m/A-T}$$

$$V_H = \frac{R_H I_x B_z}{d} = \text{Hall voltage (established in } y\text{-direction)}$$

$$= \frac{(-3.16\times10^{-11}\text{ V-m/A-T})\,(25\text{ A})\,(0.6\text{ T})}{15\times10^{-3}\text{ m}} = -3.16\times10^{-8}\text{ V}$$

(ii) Barium Titanate ($BaTiO_3$) belongs to pervoskite family ABO_3. An important transformation occur on heating $BaTiO_3$ above 130°C (Curie temperature) when its dielectric property changes from ferroelectric-tetragonal structure with a net dipole moment to paraelectric cubic structure. Also, the ferroelectric behavior of $BaTiO_3$ ceases above its ferroelectric curie temperature because the unit cell transforms from tetragonal geometry to cubic.

(iii) The types of polarization are:

- **Electronic polarization:** It results from a displacement of the center of negatively charged electron cloud relative to the positive nucleus of an atom by the electric field. This polarization type is found in all dielectric materials and exists only while an electric field is present.

- **Ionic polarization:** It occurs only in ionic materials. An applied electric field acts to displace cations in one direction and anions in the opposite direction, which gives rise to net dipole moment.
- **Orientation polarization:** It is found only in substances that process permanent moments. It results due to rotation of the permanent moments into the direction of the applied field.
- **Space charge polarization:** It occurs due to multiphase defects, vacancies, trapped charges. It is also known as interfacial polarization.

Gaseous argon – Electronic polarization

Solid LiF – Ionic polarization

Liquid H_2O – Orientation polarization

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