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Detailed Solutions

ESE-2025 Mains Test Series

Mechanical Engineering Test No : 7

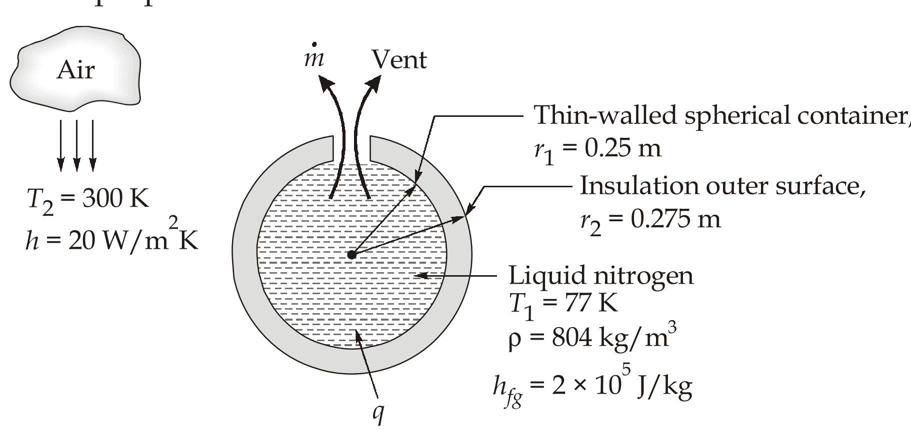
Full Syllabus Test (Paper-I)

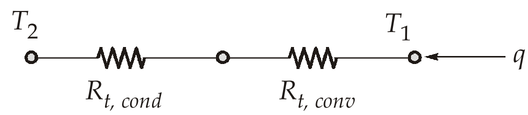
Section : A

1. (a) Solution:

Assumptions:

1. Steady-state heat transfer.
2. One-dimensional transfer in the radial direction.
3. Negligible resistance to heat transfer through the container wall and from the container to the nitrogen.
4. Negligible radiation exchange between outer surface of insulation and surroundings.
5. Constant properties.





Thermal circuit representing conduction through the insulation and convection on the outer surface

(i)

$$R_{t,\text{ cond}} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$R_{t,\text{ conv}} = \frac{1}{h4\pi r_2^2}$$

The rate of heat transfer to the liquid nitrogen is then

$$q = \frac{T_2 - T_1}{\left(\frac{1}{4\pi k}\right)\left[\left(\frac{1}{r_1}\right) - \left(\frac{1}{r_2}\right)\right] + \left(\frac{1}{h4\pi r_2^2}\right)}$$

$$q = \frac{(300-77)K}{\left[\frac{1}{4\pi(0.0017)}\left[\left(\frac{1}{0.25}\right) - \left(\frac{1}{0.275}\right)\right] + \left(\frac{1}{(20)4\pi(0.275)^2}\right)\right]}$$

$$q = 13.06$$

(ii) The heat transfer to the liquid nitrogen provides energy to vaporize the liquid nitrogen by boiling

$$q = \dot{m}h_{fg}$$

and the mass rate of nitrogen boil-off is

$$\dot{m} = \frac{q}{h_{fg}} = \frac{13.089}{2\times 10^5} = 6.545\times 10^{-5}$$

The mass rate per day is $\dot{m} = 6.545\times 10^{-5} \times 3600 \times 24 = 5.65 \text{ kg/day}$

or on a volumetric flow rate basis

$$\frac{\dot{m}}{\rho} = \frac{5.65}{804} = 7.03\times 10^{-3} = 7.03\times 10^{-3}\times 1000$$

$$= 7.03 \text{ litre/day}$$

1. (b) Solution:

(i) As we know circulation around a circle of radius R is

$$\Gamma = \int_0^{2\pi} VRd\theta = (2\pi R)V$$

(a) For $V = \frac{C}{R}$

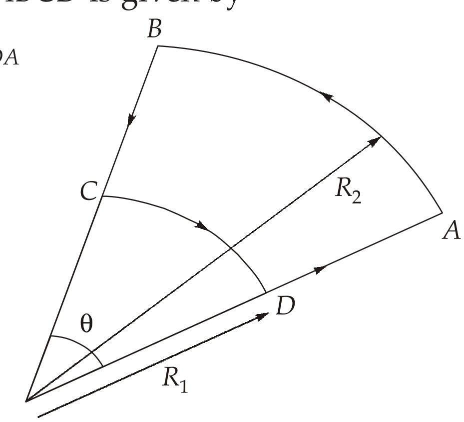
$\therefore \quad \Gamma = 2\pi C$

(b) For $V = CR$

$\therefore \quad \Gamma = 2\pi CR^2$

(ii) Let $ABCD$ be the closed path formed by the arcs of two circles of radii R_1 and R_2 and the two radius vectors with an angle θ between them, then $AB = R_1\theta$ and $CD = R_2\theta$. The total circulation around the path $ABCD$ is given by

$\therefore \quad \Gamma = \Gamma_{AB} + \Gamma_{BC} + \Gamma_{CD} + \Gamma_{DA}$



(a) for $\left(V = \frac{C}{R}\right)$ $\quad \Gamma_{AB} = 2\pi C\,;\ \Gamma_{CD} = -2\pi C;$

$\Gamma_{BC} = \Gamma_{DA} = 0$

$\therefore \quad \Gamma = 0$

(b) for $V = CR$ $\quad \Gamma_{AB} = 2\pi C R_1^2\ ;$

$\Gamma_{CD} = -2\pi C R_2^2\,;$

$\Gamma_{BC} = \Gamma_{DA} = 0$

$\therefore \quad \Gamma = -2\pi C\left(R_1^2 - R_2^2\right)$

In cylindrical polar coordinates the vorticity may be expressed as

$$\Omega = \frac{\partial\left(rV_\theta\right)}{r\partial r} - \frac{\partial V_r}{r\partial\theta}$$

in which V_θ and V_r are the velocity components in the tangential and radial directions respectively.

(a) $\quad V_\theta = V = \left(\frac{C}{r}\right); V_r = 0$

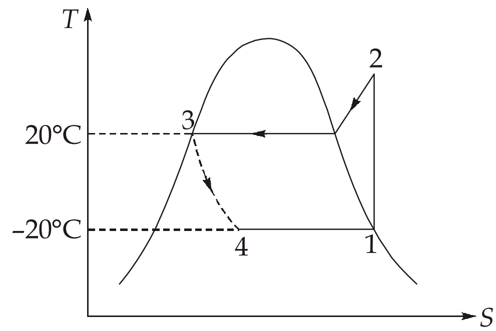
$\therefore \quad \Omega = 0$

(b) $\quad V_\theta = V = Cr;\ V_r = 0$

$\therefore \quad \Omega = 2C$

1. (c) Solution:

The cycle is represented on T-S diagram as shown in figure.



$$\text{Refrigeration capacity of the plant} = \frac{20 \times 1000 \times 335}{24 \times 3600} = 77.55\ \text{kJ/s} = 77.55\ \text{kW}$$

From given table: $h_1 = 1419.05$ kJ/kg

$$h_2 = 1461.58 + 2.8\,(50 - 20)$$
$$= 1545.58\ \text{kJ/kg}$$

$$\text{Theoretical COP} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{1419.05 - 274.98}{1545.58 - 1419.05} = 9.04$$

$$\text{Actual COP} = 0.7 \times 9.04 = 6.328$$

$$\text{Compressor power} = \frac{\text{Plant capacity}}{\text{Actual COP}} = \frac{77.55}{6.328} = 12.255\ \text{kW}$$

$$\text{Mass flow rate} = \frac{\text{Actual compressor power}}{(h_2 - h_1)}$$

$$= \frac{12.255}{(1545.58 - 1419.05)} = 0.097\ \text{kg/s}$$

$$\text{Volumetric efficiency, } (\eta_v) = \frac{\dot{m} \times V}{\frac{\pi}{4} D^2 \times L \times \frac{N}{60}}$$

$$0.8 = \frac{0.097 \times 0.624}{\frac{\pi}{4} \times \frac{240}{60} \times D^3}$$

$$D^3 = 0.024$$

$$D = 0.288\ \text{m} = 28.8\ \text{cm}$$

$$L = D = 28.8\ \text{cm}$$

1. (d) Solution:

As we know Joule-Thomson coefficient is given by

$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h \quad \text{...(i)}$$

Consider $s = f(T, p)$

Then
$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp \quad \text{...(ii)}$$

From Tds relations,

$$dh = Tds + vdp$$

Substituting the value of ds from Eq. (ii) in above equation.

$$dh = T\left(\frac{\partial s}{\partial T}\right)_p dT + T\left(\frac{\partial s}{\partial p}\right)_T dp + vdp$$

$$= T\left(\frac{\partial s}{\partial T}\right)_p dT + \left[v + T\left(\frac{\partial s}{\partial p}\right)_T\right]dp$$

where
$$T\left(\frac{\partial s}{\partial T}\right)_p = c_p$$

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p \quad \text{from Maxwell's relation}$$

∴
$$dh = c_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

For throttling process, $dh = 0$

∴
$$0 = c_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

$$-c_p dT = \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

or
$$\left(\frac{\partial T}{\partial p}\right)_h = \frac{1}{c_p}\left[-v + T\left(\frac{\partial v}{\partial T}\right)_p\right] \quad \text{...(iii)}$$

and
$$\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p = v \times \left(\frac{-1}{T^2}\right) + \frac{1}{T}\left(\frac{\partial v}{\partial T}\right)_p$$

$$\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p = \frac{1}{T^2}\left[-v + T\left(\frac{\partial v}{\partial T}\right)_p\right]$$

or $$-v+T\left(\frac{\partial v}{\partial T}\right)_p = T^2\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p\right] \quad ...(iv)$$

From Eqs. (iii) and (iv), we get

$$\left(\frac{\partial T}{\partial p}\right)_h = \frac{T^2}{c_p}\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p\right]$$

or $$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h = \frac{T^2}{c_p}\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p\right]$$

1. (e) Solution:

The main purpose is to ensured quick and complete combustion. Fuel injector increase the surface area of the fuel droplets resulting in better mixing and subsequent combustion by means of atomizing the fuel into very fine droplets. Atomization is done by forcing the fuel through a small orifice under high pressure. The fuel injector is a small nozzle into which liquid fuel is injected at high pressure. It works like spay nozzle on a pressure washer.

For a proper running and good performance from the engine, the following requirements must be met by the injection system:

(i) Accurate metering of the fuel injected per cycle. This is very critical due to the (act that very small quantities of fuel being handled. Metering errors may cause drastic variation From the desired output The quantity of the fuel metered should vary to meet changing speed and bad requirements of the engine.

(ii) Timing the injection of the fuel correctly In the cycle so that maximum power is obtained ensuring fuel economy and clean burning

(iii) Proper control of rate of injection so that the desired beat-release pattern is achieved during combustion.

(iv) Proper atomization of fuel into very fine droplets.

Proper spray pattern to ensure rapid mixing of fuel and air

(vi) Uniform distribution of fuel droplets throughout the combustion chamber.

(vii) To supply equal quantities of metered fuel to all cylinders in case of multi cylinder engines.

(viii) No lag during beginning and end of injection i.e., to eliminate dribbling of fuel droplets into the cylinder.

The injector assembly mainly consists of

(i) a needle valve (ii) a compression spring
(iii) a nozzle (iv) an injector body
(v) Nozzle body (vi) Nozzle valve
(vii) Spindle (viii) End cap

2. (a) Solution:

$$V = \frac{0.02}{\frac{\pi}{4}(0.15)^2} = 1.132 \text{ m/s}$$

(i) For horizontal pipe from Hagen-Poiseuille equation, we have

$$p_1 - p_2 = \frac{32\mu VL}{D^2}$$

$$p_1 - p_2 = \frac{32 \times 12.066 \times 10^{-2} \times 1.132 \times 300}{(0.15)^2} = 58277.17 \text{ N/m}^2$$

$$\tau_0 = \left(-\frac{\partial p}{\partial x}\right)\frac{R}{2}$$

$$= \frac{58277.17}{300} \times \frac{0.15}{2 \times 2} = 7.285 \text{ N/m}^2$$

Power required to maintain the flow is

$$P = Q(p_1 - p_2)$$

$$= 0.02 \times 58277.17 = 1166 \text{ W} = 1.166 \text{ kW}$$

(ii)

1. For inclined pipe with flow in upward direction, we have

$$\rho g(h_1 - h_2) = \frac{32\mu VL}{D^2}$$

$$h_1 - h_2 = \frac{32 \times 12.066 \times 10^{-2} \times 1.132 \times 300}{9810 \times 0.82 \times (0.15)^2}$$

or $$\left(\frac{p_1}{\rho g} + 0\right) - \left(\frac{p_2}{\rho g} + Z_2\right) = 7.245$$

Since $$\frac{Z_2}{300} = \sin 15° = 0.25882; \; Z_2 = 77.646 \text{ m}$$

∴ $$(p_1 - p_2) = (9810 \times 0.82)(7.245 + 77.646)$$

$$= 682880 \text{ N/m2} = 682.880 \text{ kN/m}^2$$

$$\tau_0 = w\left(\frac{\partial h}{\partial x}\right)\frac{R}{2}$$

$$= \frac{(9810 \times 0.82) \times 7.245}{300} \times \frac{0.15}{2 \times 2} = 7.285 \text{ N/m}^2$$

Power required to maintain the flow is

$$P = Q(p_1 - p_2)$$

$$= 0.02 \times 682.880 = 13.657 \text{ W} = 13.657 \text{ kW}$$

2. For inclined pipe with flow in downward direction, we have

$$\rho g(h_1 - h_2) = \frac{32\mu VL}{D^2}$$

$$(h_1 - h_2) = \frac{32 \times 12.066 \times 10^{-2} \times 1.132 \times 300}{9810 \times 0.82 \times (0.15)^2} = 7.245 \text{ m}$$

or $$\left(\frac{p_1}{\rho g} + Z_1\right) - \left(\frac{p_2}{\rho g} + 0\right) = 7.245$$

Since, $$\frac{Z_1}{300} = \sin 15° = 0.2588; \; Z_1 = 77.646 \text{ m}$$

$\therefore$ $$(p_1 - p_2) = (9810 \times 0.82)(7.245 - 77.646)$$

$$= -566319 \text{ N/m}^2 = -566.319 \text{ kN/m}^2$$

i.e. in this case the pressure increases in the direction of flow, or there is positive pressure gradient.

$$\tau_0 = w\left(-\frac{\partial h}{\partial x}\right)\frac{R}{2}$$

$$= \frac{(9810 \times 0.82) \times 7.245}{300} \times \frac{0.15}{2 \times 2} = 7.285 \text{ N/m}^2$$

In this case the resistance to flow is compensated by the excessive downward slope of the pipe and hence no external power is required to maintain the flow. Moreover, in this case the flow will have to be regulated by means of a regulating value to maintain the given flow rate.

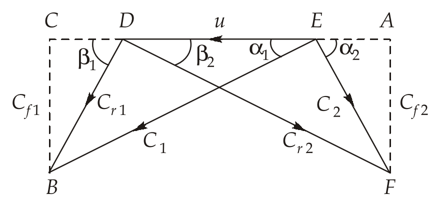
For pressure gradient along the pipe to be zero, $p_1 = p_2$.

Then from the two above noted cases of inclination of the pipe we have either, $-Z_2$ = 7.245 m, or Z_1 = 7.245 m, from which it may be concluded that point 1 is higher than point 2, so that the flow is in the downward direction. The slope required to be provided for the pipe in this case is given by

$$\sin\theta = \frac{Z_1}{300} = \frac{7.245}{300} = 0.0242$$

$$\therefore \quad \theta = 1°23'$$

2. (b) Solution:



The peripheral velocity of blades, $u = \frac{\pi DN}{60} = \frac{\pi D \times 400}{60} = 20.944\text{ D m/s}$

Actual mass of steam flowing through the blades for producing work.

$$\dot{m}_1 = 5(1 - 0.05) = 5 \times 0.95 = 4.75 \text{ kg/s}$$

$$\text{Power developed} = \frac{\dot{m}_1 \cdot C_w \cdot u}{1000}$$

$$P = \frac{4.75 \times C_w \times 20.944D}{1000} = 4.4$$

$$C_w = \frac{4.4 \times 1000}{4.75 \times 20.944D} = \frac{44.228}{D} \text{ m/s}$$

For Parson's reaction turbine,

$$C_f = C_{f1} = C_{f2} = C_1 \sin\alpha_1 = CB = 0.72 \text{ u}$$

$$C_1 = \frac{0.72 \times 20.944D}{\sin 20°} = 44\text{D m/s}$$

Also,

$$C_w = CD + DA$$

$$= (C_1\cos\alpha_1 - u) + C_{r2}\cos\beta_2$$

$$= C_1\cos\alpha_1 - u + C_1\cos\alpha_1 [\beta_2 = \alpha_1, C_{r2} = C_1]$$

$$C_w = 2C_1\cos\alpha_1 - u$$

$$\frac{44.228}{D} = 2 \times 44D \times \cos 20° - 20.944D$$

$$= D(88\cos 20° - 20.94)$$

$$\frac{44.228}{D} = 61.749\ D$$

$$D^2 = 0.716255 \ \text{ or } D = 0.8463 \text{ m}$$

Mean diameter, D = 84.63 cm

Specific volume at 2 bar and 0.96 dry = xv_g = 0.96 × 0.8851 = 0.8497 m^3/kg

Using the continuity equation,

$$\pi D l \cdot C_f = \dot{m}_1 \times v$$

$$C_f = 0.72u = 0.72 \times 20.944D = 0.72 \times 20.944 \times 0.8463$$

$$= 12.762 \text{ m/s}$$

Now, we have

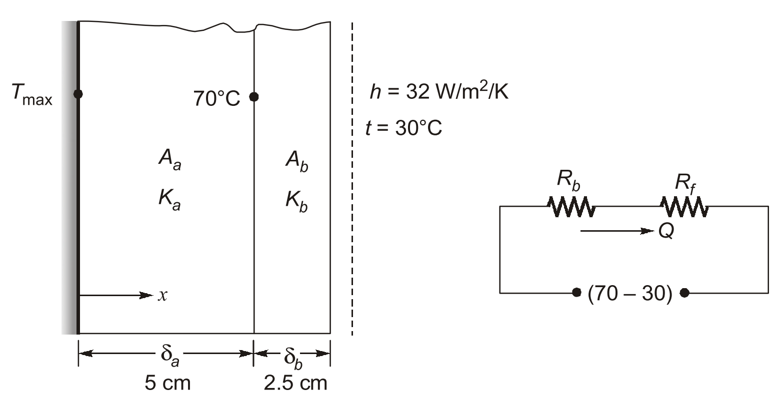
$$\pi \times 0.8463 \times l \times 12.762 = 4.75 \times 0.8497$$

$$l = \frac{4.75 \times 0.8497}{\pi \times 0.8463 \times 12.762} = 0.11895 \text{ m or } 11.895 \text{ cm}$$

Drum diameter, $D_r = D - l$

$$= 84.63 - 11.895 = 72.735 \text{ cm}$$ Answer

2. (c) Solution:



Thermal resistance of layer B:

$$R_b = \frac{\delta_b}{K_b A} = \frac{2.5 \times 10^{-2}}{25 \times A} = \frac{0.001}{A}$$

$$R_f = \frac{1}{hA} = \frac{1}{32 \times A} = \frac{0.03125}{A}$$

$$\Sigma R = R_b + R_f = \frac{0.001 + 0.03125}{A} = \frac{0.03225}{A}$$

Rate of heat flux from the slab B to the fluid:

$$Q = \frac{70-30}{\frac{0.03225}{A}}$$

$$\frac{Q}{A} = 1240.3 \text{ W/m}^2 \qquad \text{Ans. (1)}$$

The temperature would be maximum at insulated face of slab A.

$$Q = -kA\frac{dt}{dx}$$

$$Q = -0.4(1+0.07t)A\frac{dt}{dx}$$

$$\frac{Q}{A}\int dx = -\int 0.4(1+0.07t)dt$$

For this integral equation, we have to apply following boundary conditions:

(a) At $x = 0$; $t = t_{max}$

(b) At $x = \delta_a = 0.05$ m, $t = 70°C$ (interface temperature)

$$\frac{Q}{A}\int_0^{0.05} dx = -\int_{t_{max}}^{70} 0.4(1+0.07t)dt$$

$$\frac{Q}{A}\times 0.05 = 0.4\left[(t_{max}-70)+\frac{0.07}{2}(t_{max}^2-70^2)\right]$$

$$\frac{Q}{A} = 8(t_{max} - 70) + 0.28\ (t_{max}^2 - 70^2)$$

$$1240.3 = 8\ t_{max} - 560 + 0.28\ t_{max}^2 - 1372$$

$$0.28\ t_{max}^2 + 8\ t_{max} - 3172.3 = 0$$

$$t_{max} = \frac{-8 \pm \sqrt{64+4\times 0.28\times 3172.3}}{2\times 0.28}$$

$$= \frac{-8\pm 60.14}{0.56} = 93.1°C \text{ (–ve value discarded)} \qquad \text{Ans. (2)}$$

Now for the third part of question, boundary condition can be written as

$$x = x;\ t_x = 80°C$$

Then
$$\frac{Q}{A}\int_0^{x} dx = -\int_{t_{max}}^{t_x} 0.4(1+0.07t)dt$$

$$\frac{Q}{A}x = 0.4\left[(t_{max} - t_x) + \frac{0.07}{2}(t_{max}^2 - t_x^2)\right]$$

or
$$1240.3\,x = 0.4\left[(93.1 - 80) + \frac{0.07}{2}(93.1^2 - 80^2)\right]$$

$$1240.3\,x = 0.4[13.1 + 79.366]$$

$$x = 0.02982 \text{ m or } 2.982 \text{ cm} \qquad \text{Ans. (3)}$$

Thus the temperature would be 80°C at a distance of 2.982 cm from the insulated surface.

3. (a) Solution:

Given : m = 150 kg; P_1 = 6 bar; T_1 = 288 K; P_2 = 6 bar; T_2 = 348 K; P_0 = 1 bar; T_0 = 283 K

(i) The change availability of the water

$$\Delta\phi = m[(u_2 - u_1)] - T_0(s_2 - s_1) + P_0(v_2 - v_1)$$

From steam tables,

$$\Delta\phi = 150[(313.86 - 62.95) - 283(1.0154 - 0.22437) + 100(0.00102557) - 0.00100067)]$$

$$\Delta\phi = 4057.65 \text{ kJ}$$

(ii) In this case, there is no heat interaction. The work interactive is the sum of volume work and electric work. The change in the energy of the system is given by the first law as

$$\Delta U = U_2 - U_1 = -W = -W_{el} - P\Delta V$$

from which the electric work of the system W_{el} can be calculated as

$$W_{el} = -m(h_2 - h_1)$$
$$= -150 \times (314.48 - 63.55)$$
$$= -37639.5 \text{ kJ}$$

The change in availability of the water is $\Delta\phi$ = 4057.65 kJ

The work of the electric source W_{source} is equal in magnitude and opposite in sign to the electric work of the system.

Hence, the change in the availability of the source of the electricity is

$$\Delta\phi_{source} = -W_{source} = W_{el} = -37639.5 \text{ kJ}$$

The total change in availability is then

$$\Delta\phi_{total} = \Delta\phi_{source} + \Delta\phi$$
$$= -37639.5 + 4057.65$$
$$= -33581.85 \text{ kJ}$$

(iii) In this case, the heat interaction with the steam is

$$Q = U_2 - U_1 + P(V_2 - V_1)$$
$$= H_2 - H_1$$
$$= m(h_2 - h_1)$$
$$= 37639.5 \text{ kJ}$$

The change in the availability of the system is the same as the change in the availability of a reservoir at $T_R = 373$ K.

$$\Delta\phi_{steam} = Q\left(1 - \frac{T_0}{T_R}\right) = -37639.5\left(1 - \frac{283}{373}\right) = -9081.92 \text{ kJ}$$

$$\Delta\phi_{total} = 4057.65 - 9081.92 = -5024.27 \text{ kJ}$$

(iv) The irreversibility in the process is found by

$$I = T_0(\Delta s + \Delta s_E)$$

In this process there is no change of entropy in the environment, hence, $\Delta s_E = 0$

$$I = mT_0(s_2 - s_1)$$
$$= 150 \times 283(1.0154 - 0.22437)$$
$$= 33579.22 \text{ kJ}$$

(v) In this case the entropy change of the system plus the steam is

$$I = T_0(\Delta S + \Delta S_{steam})$$
$$= T_0\left(m\Delta s + \frac{-Q}{T_R}\right)$$
$$= 283\left(150 \times (1.0154 - 0.22437) - \frac{37639.5}{373}\right)$$
$$= 5021.64 \text{ kJ}$$

3. (b) Solution:

Given : $D_2 = 0.27$ m; $N = 1440$ rpm; $Q = 25 \times 10^{-3}$ m^3/s; $b_2 = 12 \times 10^{-3}$ m;
$\eta_{mech} = 0.85$; $\phi = 35°$

(i)
$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.27 \times 1440}{60}$$
$$u_2 = 20.36 \text{ m/s}$$

Area of flow at exit, $A = (\pi \times 0.27 - 10 \times 0.006) \times 0.012$

$$A = 0.00946 \text{ m}^2$$

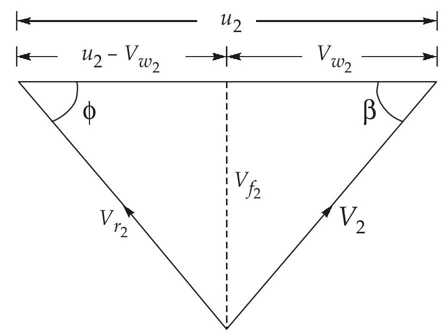
Discharge, $Q = AV_{f_2}$

$$V_{f_2} = \frac{Q}{A} = \frac{25 \times 10^{-3}}{9.46 \times 10^{-3}} = 2.643 \text{ m/s}$$

Actual difference in pressure head across the impeller

$\Rightarrow$ $$H_m = 16 - (-4) = 20 \text{ m}$$

From outlet velocity triangle,



Outlet velocity triangle

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w_2}}$$

$$\tan 35° = \frac{2.643}{20.36 - V_{w_2}}$$

$$V_{w_2} = 16.585 \text{ m/s}$$

Theoretical total head, $H = \dfrac{u_2 V_{w_2}}{g} = \dfrac{20.36 \times 16.585}{9.81} = 34.421$ m

(ii) Actual manometric efficiency,

$$\eta_m = \frac{H_m}{H} = \frac{20}{34.421} = 0.581$$

(iii) Absolute velocity at exit

$$V_2 = \sqrt{V_{f2}^2 + V_{w_2}^2}$$

$$= \sqrt{(2.643)^2 + (16.585)^2} = 16.79 \text{ m/s}$$

Kinematic energy per unit weight of liquid (velocity head) at impeller exit

$$= \frac{V_2^2}{2g} = \frac{(16.79)^2}{2 \times 9.81} = 14.37 \text{ m}$$

$$\Delta KE = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\Delta KE = 14.37 - \frac{V_1^2}{2g}$$

Head recovery = 50% of kinetic energy head at outlet

$$= 0.5 \times 14.37$$
$$= 7.185$$

Unrecovered kinetic energy head at outlet,

$$(\Delta KE)_{\text{un}} = \left(14.37 - \frac{V_1^2}{2g} - 7.185\right)$$

$$= 7.185 - \frac{V_1^2}{2g}$$

Now, $$H = H_m + (\Delta KE)_{un} + \text{Losses in impeller}$$

$$34.421 = 20 + 7.185 - \frac{V_1^2}{2g} + \text{Losses in impeller}$$

Considering $\left(-\frac{V_1^2}{2g} + \text{Losses in impeller}\right)$ as total losses in the impeller.

$$\text{Losses in impeller} = H_{iL} = 7.236 \text{ m}$$

(iv) Overall efficiency, $\eta_0 = \eta_m \times \eta_{\text{mech}}$

$$= 0.581 \times 0.85 = 0.494$$

$$\text{Brake power} = \frac{\rho g Q H}{\eta_0} = \frac{1000 \times 9.81 \times 0.025 \times 34.421}{0.494}$$

$$= 17.094 \text{ kW}$$

3. (c) Solution:

The rate of heat transfer for all finite lengths is given as

$$Q = \sqrt{hPkA}\,\theta_b \frac{\sin h(mL) + \left(\frac{h}{mk}\right)\cos h(mL)}{\cos h(mL) + \left(\frac{h}{mk}\right)\sin h(mL)}$$

$$= \sqrt{hPkA}\,\theta_b \frac{\tan h(mL) + \left(\frac{h}{mk}\right)}{1 + \left(\frac{h}{mk}\right)\tan h(mL)}$$

$$\frac{P}{A} = \frac{\pi D}{\left(\frac{\pi D^2}{4}\right)} = \frac{4}{D} = \frac{4}{0.01} = 400 \text{ m}^{-1}$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{10\times 400}{370}} = 3.288\ \text{m}^{-1}$$

and, $$\sqrt{hPkA} = \sqrt{\frac{10\times\pi(0.01)\times 370\times\pi\times(0.01)^2}{4}} = 0.0955\ \text{W/K}$$

$$\theta_b = 120° - 20° = 100°\text{C}$$

For $L = 0.02$ m

$$Q = 0.0955\times 100\frac{0.0082+\tan h(3.288\times 0.02)}{1+0.0082\times \tanh(3.288\times 0.02)}$$

$$= 0.704\ \text{W}$$

Repeating the calculations for lengths of 0.04, 0.08, 0.2, 0.4, 0.8, 1 and 10 m, we obtain the results which are shown below:

x(m)	0.02	0.04	0.08	0.2	0.4	0.8	1	10
Q(W)	0.704	1.336	2.530	5.561	8.286	9.450	9.524	9.550

Ans. ..(i)

For an infinitely long rod,

$$Q_{L\to\infty} = \sqrt{hPkA}\,\theta_b$$

$$= 0.0955 \times (120 - 20) = 9.55\ \text{W}$$

We can observe that since k is large, there is significant difference between the finite length and the infinite length cases. However, when the length of the rod approaches 1 m (or more), the result becomes almost same to that of the infinite length.

4. (a) Solution:

Given: t_{db4} = 28°C, ϕ_4 = 50%, t_{db1} = 40°C, ϕ_1 = 40%, RSH = 34 kW,

RLH = 45000 kJ/h = 12.5 kW, t_{db2} = 17°C.

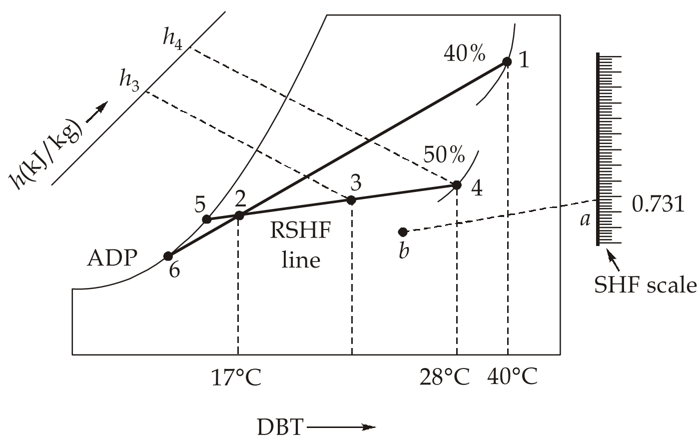
(i) Room sensible heat factor, RSHF = $\frac{RSH}{RSH+RLH} = \frac{34}{34+12.5} = 0.731$

(ii) Condition of air entering the auditorium:

- Locate point 1 at the intersection of 40°C DBT and 40% RH.
- Locate point 4 at the intersection of 28°C DBT and 50% RH.
- Mark the calculated vale of RSHF = 0.731 on the SHF scale as point a and join with the alignment circle (i.e. 26°C DBT and 50% RH) i.e. point *b*.
- From point 4, draw a line 4-5 (known on RSHF line) parallel to the line a-b.

- Since the condition of air leaving the coil is 17°C, therefore, locate point 2 such that db_2 = 17°C.
- Join point 1 and 2 and produce upto point 6 on the saturation curve. The line 1-2-6 is the GSHF line.
- It is given that 65% of the air from the auditorium is recirculated and mixed with 35% of the make up air after the cooling coil. The mixing condition of air is shown at point 3 such that

$$\text{Length 2-3} = 0.65 \times \text{Length 2-4}$$



The point 3 gives the condition of air entering the auditorium.

From Psychrometric chat, we find.

Dry bulb temperature, t_{db3} = 23.1°C

Wet bulb temperature, t_{wb3} = 18.6°C

Relative humidity, ϕ_3 = 63%

(iii) Amount of make up air.

From psychrometric chart we find,

h_4 = 60 kJ/kg of dry air

h_3 = 53 kJ/kg of dry air

$$\text{Mass of supply air to the auditorium} = \frac{\text{Room latent heat}}{h_4 - h_3} = \frac{RSH - RLH}{h_4 - h_3}$$

$$= \frac{34 + 12.5}{(60 - 53)} = 6.643 \text{ kg/s}$$

Since the make up air is 35% of the supply air, therefore mass of make up air

$$= 0.35 \times 6.643 = 2.325 \text{ kg/s}$$

(iv) Apparatus dew point, ADP

From Psychrometric chart,ADP = t_{db} = 12.5°C

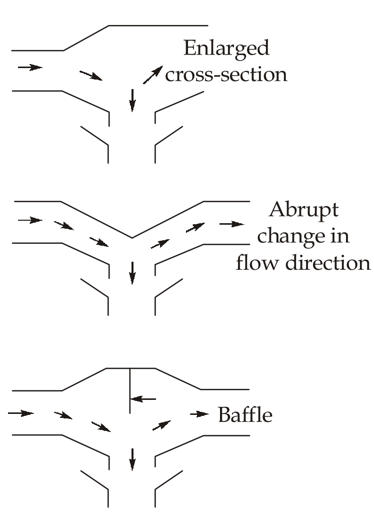
(v) By pass factor of the cooling coil, BF:

$$\text{B.F.} = \frac{t_{db_2} - ADP}{T_{db_1} - ADP} = \frac{17 - 12.5}{40 - 12.5} = 0.164$$

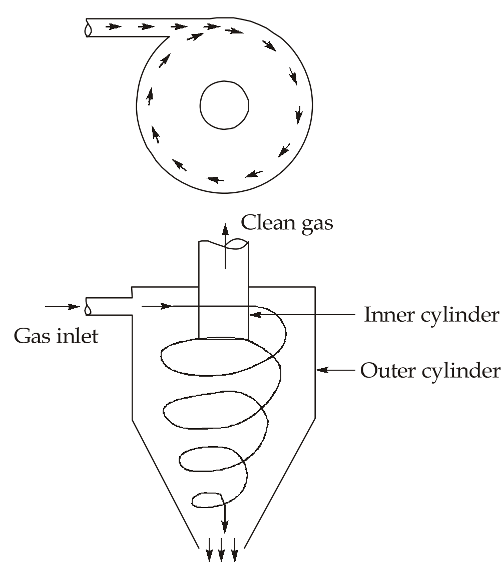
4. (b) Solution:

Cyclone Separator: In this type of separator, the flue gases rotate inside a cylinder causing the generation of centrifugal forces inside it. This is shown in figure. The gas enters tangentially and produces a cyclonic motion of flue gas. The heavier particle goes outward and separates on the wall because of the baffling action by gravitation. The particles fall down in the cylinder and are collected at the bottom. There is another inner cylinder in the cyclone separator through which clean gas escapes out. In this separator, the particles having greater than 10 μ size can be separated from the flue gas. If m is the amount of particles which enters the cylinder and m is the amount of particles leaving the separator with clean gas, the collection efficiency can be expressed as

$$\eta_{\text{collection}} = \frac{m - m_1}{m}$$



Gravity separation methods



Cyclone separator

The collection efficiency becomes more if the conditions occur as follows:

1. Higher particle size.
2. Higher inlet gas velocity.
3. Reduction in diameter.
4. Larger length of cyclone.
5. More number of gas revolution.
6. Less cyclone diameter.
7. Less outlet diameter.

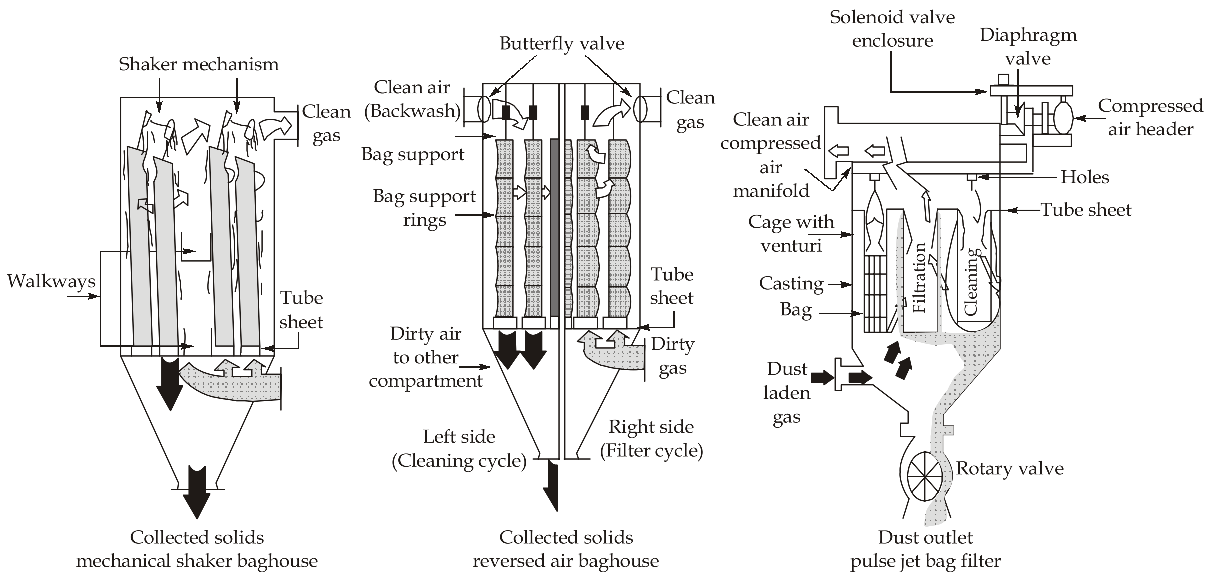
Bag Filter: A bag filter, also known as fabric filter, comprises of multiple compartments. Each compartment has several cloth or fabric bags of height 2-10 m and diameter 12-14 cm. The dust-laden gas passes through the bags which act as a filter. The bags are made of woven, felted nylon, synthetic, polyester, fibre glass, etc., depending upon the gas temperature. Dirty gas enters each bag at bottom and clean gas escapes out from the top of the bag. Particulate matter adheres to the surface of the cloth and gets accumulated. These accumulated particles can be cleaned by pulse jet or reversed jet of air and then the particles are collected at the bottom hopper. Its collection efficiency is more than 99% and can remove the fine particles.

The important factor considered during fixation of sizing of bag filter is air/cloth ratio. It is the ratio of flue gas flow rate to the cloth exposed surface area to flue gas ($m^3/s/m^2$). For the pulse jet bag filters, this ratio is larger as compared to reverse jet bag filters.

The factors that affect the performance of a bag filter are the material of filter which is able to withstand temperature and resist abrasion. Because of the dust absorbed by the bags, pressure across the filter drops, and hence it is required to clean the bag periodically. This can be done in both online or offline mode. Online cleaning is done without interrupting flow of flue gas. The following three methods are used for cleaning purpose:

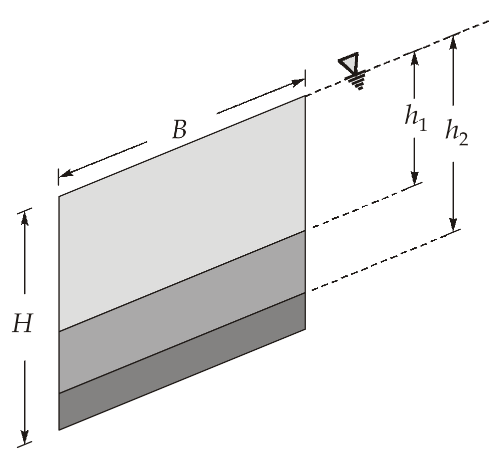
1. Mechanical shaker: Filter bags are shaken mechanically to remove the dust particle. This is offline cleaning.
2. Reverse air: It is also an offline cleaning arrangement. In this system, cleaning gas is sent in reverse direction of gas flow compartment wise through the bags.
3. Reverse jet: It is an online cleaning. It is popularly used in practice. In this system, flow of dust laden flue gas is normally allowed to pass from outside to inside the bag. Bags are cleaned by compressed air suddenly through a set of nozzles.

The three types of cleaning are shown in figure.



4. (c) Solution:

Let h_1 and h_2 be the depths of the two horizontal lines below the top of the gate, divide the gate into three portions.



The total pressures on the three portions of the gate are thus obtained as

$$P_1 = \rho g \times (B \times h_1) \times \frac{h_1}{2} = \frac{\rho g B h_1^2}{2} \quad \text{...(i)}$$

$$P_2 = \rho g \times [B \times (h_2 - h_1)] \times \frac{h_1 + h_2}{2} = \frac{\rho g B (h_2^2 - h_1^2)}{2} \quad \text{...(ii)}$$

$$P_3 = \rho g \times [B \times (H - h_2)] \times \frac{h_2 + H}{2} = \frac{\rho g B (H^2 - h_2^2)}{2} \quad \text{...(iii)}$$

Since, $P_1 = P_2 = P_3$; we obtain from equation (i) and (ii)

$$\frac{\rho g B h_1^2}{2} = \frac{\rho g B\left(h_2^2 - h_1^2\right)}{2}$$

or $$2h_1^2 = h_2^2 \quad \text{...(iv)}$$

and from equation (ii) and (iii),

$$\frac{\rho g B\left(h_2^2 - h_1^2\right)}{2} = \frac{\rho g B\left(H^2 - h_2^2\right)}{2}$$

or $$2h_2^2 = h_1^2 + H^2 \quad \text{...(v)}$$

From equation (iv) and (v), we obtain

$$h_1 = H\sqrt{\frac{1}{3}} \quad \text{...(vi)}$$

and $$h_2 = H\sqrt{\frac{2}{3}} \quad \text{...(vii)}$$

Hence, from these equations the following general equation may be obtained

$$h_n = H\sqrt{\frac{n}{N}}$$

Let $\bar{h}_1, \bar{h}_2$ and $\bar{h}_3$ be the depths of the centres of pressures below the top of the gate for the three portions of the gate. Thus,

$$\bar{h}_1 = \frac{h_1}{2} + \frac{\frac{1}{12}B \times h_1^3}{(B \times h_1) \times \frac{h_1}{2}} = \frac{2}{3}h_1 \quad \text{...(viii)}$$

Substituting the value of h_1 from equation (vi), we get

$$\bar{h}_1 = \frac{2}{3}H\sqrt{\frac{1}{3}} \quad \text{...(ix)}$$

Similarly, $$\bar{h}_2 = \left(\frac{h_1 + h_2}{2}\right) + \frac{\frac{1}{12} \times B(h_2 - h_1)^3}{B \times (h_2 - h_1) \times \left(\frac{h_1 + h_2}{2}\right)}$$

or $$\bar{h}_2 = \frac{3(h_1 + h_2)^2 + (h_2 - h_1)^2}{6(h_1 + h_2)} \quad \text{...(x)}$$

Substituting the values of h_1 and h_2 from equation (vi) and (vii), we get

$$\bar{h}_2 = \frac{2}{3}H\frac{\left(2^{3/2}-1\right)}{\sqrt{3}} \quad \text{...(xi)}$$

and

$$\bar{h}_3 = \left(\frac{h_2+H}{2}\right)+\frac{\frac{1}{12}\times B\times(H-h_2)^3}{B\times(H-h_2)\times\left(\frac{h_2+H}{2}\right)}$$

or

$$\bar{h}_3 = \frac{3(h_2+H)^2+(H-h_2)^2}{6(h_2+H)}$$

Substituting the value of h_2 from equation (vii), we get

$$\bar{h}_3 = \frac{2}{3}H\frac{\left(3^{3/2}-2^{3/2}\right)}{\sqrt{3}} \quad \text{...(xii)}$$

Hence from these equations the following general equation may be obtained

$$\bar{h}_n = \frac{2}{3}H\frac{\left(n^{3/2}-(n-1)^{3/2}\right)}{\sqrt{N}}$$

Section : B

5. (a) Solution:

Given : A = 4 km^2 = 4 × 10^6 m^2; R = 12 m; r = 3.6 m; η_{gen} = 0.72; ρ = 1027 kg/m^3

It is given that power generation is in flood cycle only.

Total potential energy of water stored in the basin is

$$W = \int_r^R \rho g A h dh = \rho g A\left[\frac{h^2}{2}\right]_r^R = \frac{\rho g A}{2}\left[R^2-r^2\right]$$

$$= \frac{1}{2}\times 1027\times 9.81\times 4\times 10^6\left[12^2-3.6^2\right]$$

= 2640421.93 MJ

As the time between consecutive high and low tides is 22350 s.

$$\text{Average power, } P_{avg} = \frac{2640421.93}{22350} = 118.14 \text{ MW}$$

Actual power generated = 118.14 × 0.72 = 85.06 MW

Energy available in single emptying process = 2640421.93 MJ

One ebb cycle duration = 12 hours and 25 minutes = 12.4166 hours

Number of ebb cycles in a year $= \frac{365 \times 24}{12.4166} = 705.5 \simeq 706$

Average annual energy generation = 706 × 0.72 × 2640421.93 MJ

$$= \frac{1342.179 \times 10^6 \times 3600}{3600} \text{MJ}$$

$$= 3.728 \times 10^{11} \text{ Wh} = 3.728 \times 10^8 \text{ kWh}$$

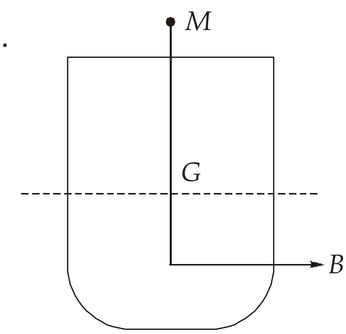
5. (b) Solution:

The periodic time is given by

$$T = 2\pi\left\{\frac{k_G^2}{gGM}\right\}^{1/2}$$

Let T_1 and T_2 be the periodic times for the ship before and after leaving the cargo respectively.

In the first case since the centre of gravity lies in the water line.



$$\overline{BG} = \frac{1.25}{2} = 0.625 \text{ m}$$

Now, if l is the length of the ship at the water level then

$$I = \frac{1}{12} \times l \times (7)^3 \text{m}^4$$

and $$V = (l \times 7 \times 1.25)\text{m}^3$$

$$\overline{BM} = \frac{I}{V} = \frac{\frac{1}{12} \times l \times (7)^3}{(l \times 7 \times 1.25)} = 3.267 \text{ m}$$

Thus, $$\overline{GM} = \overline{BM} - \overline{GM} = 3.267 - 0.625 = 2.642 \text{ m}$$

∴ $$T_1 = 2\pi\sqrt{\frac{k_G^2}{g \times 2.645}} \quad \text{...(i)}$$

Similarly after leaving the cargo,

$$\overline{BG} = \frac{1}{2} = 0.5 \text{ m}$$

$$I = \frac{1}{12} \times l \times (7)^3 \text{ m}^4$$

$$V = (l \times 7 \times 1)\text{m}^3$$

$$\therefore \qquad \overline{BM} = \frac{\frac{1}{12} \times l \times (7)^3}{(l \times 7 \times 1)} = 4.08 \text{ m}$$

Thus, $$\overline{GM} = \overline{BM} - \overline{BG} = 4.08 - 0.5 = 3.58 \text{ m}$$

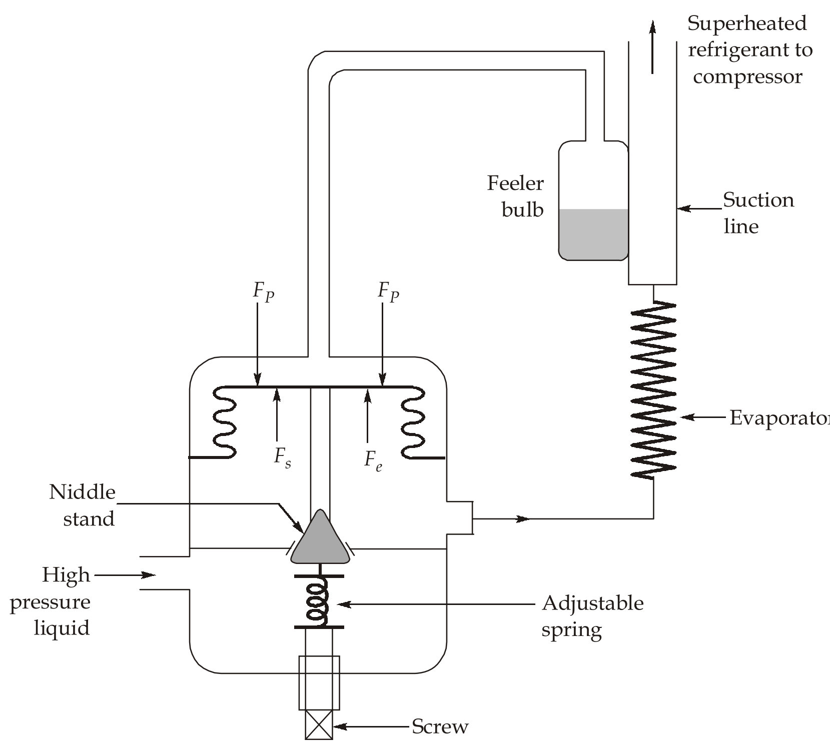
$$\therefore \qquad T_2 = 2\pi\sqrt{\frac{k_G^2}{g \times 3.58}} \qquad \text{...(ii)}$$

On dividing equation (i) by equation (ii),

$$\therefore \qquad \frac{T_1}{T_2} = \sqrt{\frac{3.58}{2.645}} = 1.164$$

5. **(c) Solution:**

Thermostatic expansion valve: A thermostatic expansion valve is used to maintain a constant degree of superheat at the exit of evaporator, hence it is most effective for dry evaporators in preventing the slugging of the compressors since it does not allow the liquid refrigerant to enter the compressor. The schematic diagram of the valve is given as below:



It consists of a feeler bulb that is attached to the evaporator exit tube so that it senses the temperature at the exit of evaporator. The feeler bulb and the narrow tube contains some fluid that is called power fluid. The power fluid may be same as the refrigerant or it may be different. In case if it is different from the refrigerant then the TEV is called TEV with cross charge. Let P_p is the pressure of power fluid, P_e is the saturation pressure corresponding to evaporator exit temperature and evaporator temperature is T_e then the purpose of TEV is to maintain a temperature $(T_e + \Delta T_s)$ at evaporator exit where ΔT_s is the degree of superheat. Feeler bulb senses the temperature $(T_e + \Delta T_s)$ and its pressure P_p is saturation pressure at this temperature. So force exerted on the top area A_b of bellows:

$$F_p = P_p A_b \quad \text{...(i)}$$

Force exerted by evaporator pressure from bottom side of bellows:

This is called external equalizer if the evaporator is large and has significant pressure drop otherwise it is known as TEV with internal equalizer.

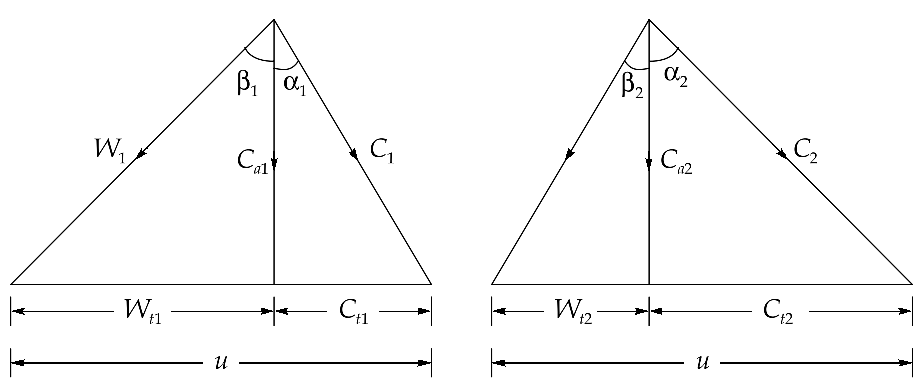
The difference of forces F_p and F_e is exerted on the top of the middle which controls the opening of orifice and is equal to spring force F_s i.e.

$$F_s = (P_p - P_e)A_b$$

Also $$\Delta T_s \propto (P_p - P_e)A_b$$

As the compressor starts, P_e decreases so a positive spring force is applied on middle which opens the orifice and refrigerant flow starts.

5. (d) Solution:



Inlet velocity triangle *Outlet velocity triangle*

Since , degree of reaction is 50%, the velocity triangles are symmetrical

$$\alpha_1 = \beta_2 = 15° \text{ and } \alpha_2 = \beta_1 = 45°$$

$$\text{Degree of reaction, } R = \frac{C_a}{2u}(\tan\beta_1 + \tan\beta_2)$$

Since, $$R = 0.5$$

$$C_a = \frac{u}{\tan\beta_1 + \tan\beta_2} = \frac{180}{\tan 45° + \tan 15°} = 141.96 \text{ m/s}$$

Temperature rise per stage is given by

$$\Delta T_s = \frac{\Omega u C_a}{c_p} \times (\tan\theta - \tan\phi) = \frac{\Omega u C_a}{c_p} \times (\tan 45° - \tan 15°)$$

$$= \frac{0.84 \times 180 \times 141.96}{1005}(1 - 0.268)$$

$$= 15.634 \text{ K}$$

$$\Delta T_{\text{overall}} = \frac{T_1}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right) = \frac{290}{0.82}\left(4^{0.286} - 1\right) \qquad \left[\frac{\gamma - 1}{\gamma} = 0.286\right]$$

$$= 172.084 \text{ K}$$

$$\text{Number of stages, } n = \frac{\Delta T_{overall}}{\Delta T_{stage}} = \frac{172.084}{15.634} = 11$$

5. (e) Solution:

Components of a photovoltaic (PV) system: A PV system is made up of different components. These include PV modules (groups of PV cells), which are commonly called PV panels; one or more batteries; a charge regulator or controller for a stand-alone system; an inverter for a utility-grid-connected system and when alternating current (ac) rather than direct current (dc) is required; wiring; and mounting hardware or a framework.

Longevity of Photovoltaic (PV) Systems : A PV system that is designed, installed, and maintained well will operate for more than 20 years. The basic PV module (interconnected, enclosed panel of PV cells) has no moving parts and can last more than 30 years. The best way to ensure and extend the life and effectiveness of your PV system is by having it installed and maintained properly. Experience has shown that most problems occur because of poor or sloppy system installation.

Difference between PV and other solar energy technologies: There are four main types of solar energy technologies: Photovoltaic (PV) systems, which convert sunlight directly to electricity by means of PV cells made of semiconductor materials. Concentrating solar power (CSP) systems, which concentrate the sun's energy using reflective devices such as troughs or mirror panels to produce heat that is then used to generate electricity. Solar water heating systems, which contain a solar collector that faces the sun and either heats water directly or heats a "working fluid" that, in turn, is used to heat water.

6. **(a) Solution:**

A fuel cell is an electrochemical energy conversion device that continuously converts chemical energy of a fuel directly into electrical energy. Continuous operation requires supply of fuel and oxidant and removal of water vapor, spent fuel, spent oxidant, inert residue and heat. It is known as a cell because of some similarities with a primary cell. Like a conventional primary cell it also has two electrodes and an electrolyte between them and produces AC power. It is also a static power conversion device. However, active materials are generally supplied from outside unlike a conventional cell where it is contained inside the cell. Fuel is supplied at the negative electrode, also known as fuel electrode or anode and the oxidant is supplied at positive electrode, also known as oxidant electrode or cathode. The only exhaust of a fuel cell, if pure hydrogen is used as fuel (and pure oxygen as oxidant), is water vapour, which is not a pollutant. In case of a hydrocarbon fuel, carbon dioxide is also produced. If air is used as oxidant, nitrogen (spent air) is also produced in the exhaust. No other pollutant such as particulate matter, NO_x and SO_x are produced. Some amount of heat is also produced. Some other pollutant also produced, which can be easily dissipated to the atmosphere or used locally for heating purposes. No cooling water is required unlike conventional thermal power-conversion devices where a substantial quantity of cooling water is required.

As the conversion of chemical energy of fuel to electrical energy occurs directly without intermediate thermal stage, the efficiency of conversion is better and not limited by Carnot efficiency of thermal stage. The efficiency of a practical fuel cell may be around 50%. The average cell voltage is typically about 0.7 V (on rated load) and several cells may be connected by series parallel connection of the required number of cells. A general large-scale use will require the development of a low-cost fuel cell with a reasonably long life.

The main advantages of a fuel cell are: (i) it is quiet in operation as it is a static device, (ii) it is less pollutant, (iii) its conversion efficiency is more due to direct single-stage energy conversion, (iv) fuel cell plant can be installed near the point of use, thus transmission and distribution losses are avoided, (v) no cooling water is needed as required in the condenser of a conventional steam plant. The heat generated can be easily removed and discharged to the atmosphere or used locally, (vi) because of modular nature, any voltage/current level can be realized and the capacity can be added later on as the demand grows, (vii) fuel-cell plants are compact and require less space, (viii) availability to choose from large number of possible fuels, (ix) can be used efficiently at part load from 50% to 100%, and no extra charging is required.

Classification of fuel cell:

(i) Based on the type of electrolyte

1. Phosphoric acid fuel cell (PAFC)
2. Alkaline fuel cell (AFC)
3. Polymer electrolytic membranes fuel cell (PEMFC) or solid polymer fuel
4. Molten carbonate fuel cell (MCFC)
5. Solid oxide fuel cell (SOFC)

(ii) Based on the types of the fuel and oxidant

1. Hydrogen (pure)– Oxygen (pure) fuel cell
2. Hydrogen rich gas– air fuel cell
3. Hydrazine–Oxygen/hydrogen peroxide fuel cell
4. Ammonia–air fuel cell
5. Synthesis gas–air fuel cell
6. Hydrocarbon (gas)– air fuel cell
7. Hydrocarbon (liquid)–air fuel cell

(iii) Based on operating temperature

1. Low temperature fuel cell (below 150°C)
2. Medium temperature fuel cell (150°C – 250°C)
3. High temperature fuel cell (250°C – 800°C)
4. Very high temperature fuel cell (800°C – 1100°C)

(iv) Based on application

1. Fuel cell for space applications
2. Fuel cell for vehicle propulsion
3. Fuel cell for submarines
4. Fuel cell for defense applications
5. Fuel cell for commercial applications

(v) Based on the chemical nature of electrolyte

1. Acidic electrolyte type
2. Alkaline electrolyte type
3. Neutral electrolyte type

6. (b) Solution:

Given : P = 150 kW; N = 1600 rpm; CV = 42000 kJ/kg; η_m = 78%; η_v = 83%; $\frac{L}{d} = 1.25$;

ρ_a = 1.26 kg/m^3; η_i = 37%

$$\text{Stoichiometric air/fuel ratio} = \left[0.85\times\frac{32}{12}+0.14\times\frac{8}{1}\right]\frac{1}{0.23} = 14.725$$

$$\text{Actual air/fuel ratio} = (1 + 1.15)(14.725) = 31.66$$

$$\text{Molecular weight of air} = (0.23 \times 32) + (0.77 \times 28) = 28.92 \text{ kg/kmol}$$

Let x kmol of air be supplied per kg of fuel. The combustion equation per kg of fuel can be written as

$$\frac{0.85}{12}C+\frac{0.14}{2}H_2+0.21xO_2+0.79xN_2 = aCO_2+bH_2O+cO_2+dN_2$$

From carbon balance, $a = \frac{0.85}{12} = 0.07083$

From hydrogen balance, $b = \frac{0.14}{2} = 0.07$

From oxygen balance, $0.21x = a+\frac{b}{2}+c$

$$= 0.07083+\frac{0.07}{2}+c$$

$$0.21x = 0.10583 + c \quad \text{...(i)}$$

$$\text{Number of kmol of air per kg of fuel} = \frac{31.66}{28.92} = 1.095$$

$$x = 1.095$$

Putting value of x in equation (i), we have

$$c = 0.21(1.095) - 0.10583$$
$$= 0.1241$$

from nitrogen balance, $d = 0.79 \times 1.095 = 0.865$

The volumetric composition of dry exhaust gas is given by

Constituent	kmol	Volume (%)
CO_2	0.07083	6.68
O_2	0.1241	11.71
N_2	0.865	81.61

Indicated power, $ip = \frac{bp}{\eta_m} = \frac{150}{0.78} = 192.3$ kW

$$\eta_i = \frac{ip}{\dot{m}_f \times CV}$$

$$\dot{m}_f = \frac{ip}{\eta_i \times CV} = \frac{192.3}{0.37 \times 42000}$$

$$\dot{m}_f = 0.01237 \text{ kg/s}$$

$$\dot{m}_a = \dot{m}_f \times \text{actual (A/F)}$$
$$= 0.01237 \times 31.66$$
$$\dot{m}_a = 0.3916 \text{ kg/s}$$

$\therefore$ $$\dot{V}_a = \frac{\dot{m}_a}{\rho_a} = \frac{0.3916}{1.26} = 0.3108 \text{ m}^3\text{/s}$$

Swept volume per second, $\dot{V}_s = \frac{\dot{V}_a}{\eta_v} = \frac{0.3108}{0.83} = 0.3745 \text{ m}^3\text{/s}$

Now, $$\dot{V}_s = \frac{\pi}{4} d^2 \times L \times \frac{N}{2 \times 60} \times n$$

$$0.3745 = \frac{\pi}{4} d^2 \times (1.25d) \times \frac{1600}{2 \times 60} \times 6$$

$$d = 168.31 \text{ mm}$$

and $L = 1.25d = 1.25 \times 168.31$

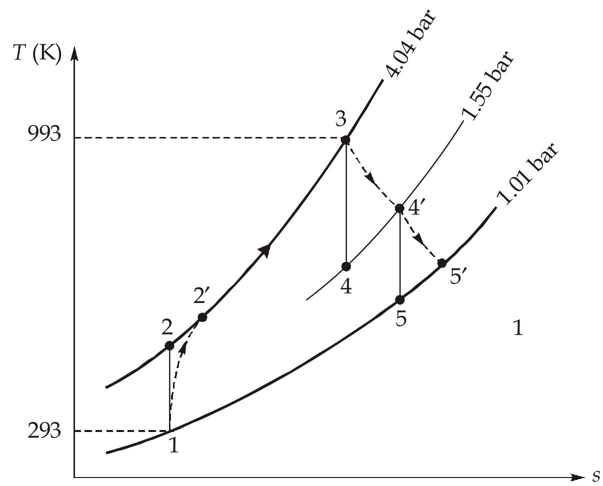
$$L = 210.4 \text{ mm}$$

6. (c) Solution:

Given: $T_1 = 20 + 273 = 293$ K; $p_1 = 1.01$ bar; $p_2 = 4.04$ bar; $T_3 = 720 + 273 = 993$ K

$$\eta_{compressor} = 80\%;\ \eta_{turbine} = 75\%;\ \eta_{nozzle} = 85\%$$

$$R_{air} = 0.287\ \text{kJ/kgK};\ \gamma = 1.4$$



$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4.04}{1.01}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

$\therefore$ $$T = 293 \times 1.486 = 435.4\ \text{K}$$

$$\eta_{compressor} = \frac{T_2 - T_1}{T_2' - T_1}$$

i.e. $$0.80 = \frac{435.4 - 293}{T_2' - 293}$$

$\therefore$ $$T_2' = 471\ \text{K}$$

$$c_p = R \times \left(\frac{\gamma}{\gamma - 1}\right) = 0.287 \times \frac{1.4}{(1.4-1)} = 1.0045\ \text{kJ/kgK}$$

(i) Power required to drive the compressor:

Power required to drive the compressor (per kg of air/sec)

$$= c_p(T_2' - T_1) = 1.0045(471 - 293) = 178.801\ \text{kW}$$

(ii) Air-fuel ratio

$$m_f \times C = (m_a + m_f) \times c_p \times (T_3 - T_2')$$

where, m_a = mass of air per kg of fuel, and

$$\therefore \quad m_a = \frac{m_f \times C}{c_p(T_3 - T_2')} - m_f$$

$$\therefore \quad \frac{m_a}{m_f} = \frac{C}{c_p(T_3 - T_2')} - 1 = \frac{42000}{1.0045(993 - 471)} - 1 = 79.1$$

i.e. air-fuel ratio = 79.10 : 1

(iii) Pressure of the gases leaving the turbine, p_4

Neglecting effect of fuel on mass flow,

Actual turbine work = Actual compressor work

i.e., $$c_p(T_2' - T_1) = c_p(T_3 - T_4')$$

or $$T_2' - T_1 = T_3 - T_4'$$

$$471 - 293 = 993 - T_4'$$

$$T_4' = 815 \text{ K}$$

Also, $$\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$0.75 = \frac{993 - 815}{993 - T_4}$$

$$\therefore \quad T_4 = 755.67 \text{ K}$$

$$\therefore \quad \frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$

or $$\frac{p_4}{p_3} = \left(\frac{T_4}{T_3}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{755.67}{993}\right)^{\frac{1.4}{1.4-1}} = 0.384$$

$$\Rightarrow \quad p_4 = 4.04 \times 0.384 = 1.55 \text{ bar}$$

(iv) Thrust pet kg of air per second

$$\frac{T_4'}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.55}{1.01}\right)^{\frac{1.4-1}{1.4}} = 1.13$$

$$\therefore \quad T_5 = \frac{815}{1.13} = 721.13 \text{ K}$$

$$\eta_{nozzle} = \frac{T_4' - T_5'}{T_4' - T_5}$$

$$0.85 = \frac{815 - T_5'}{815 - 721.13}$$

$$\therefore \quad T_5' = 735.21 \text{ K}$$

If V_j is the jet velocity, then

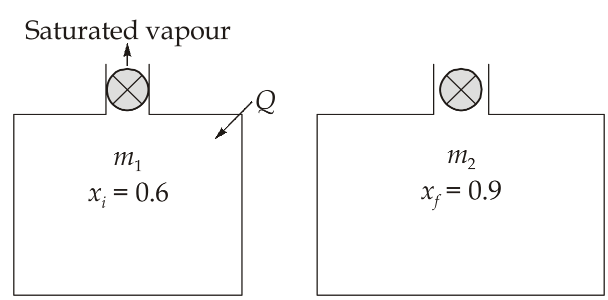
$$\frac{V_j^2}{2} = c_p(T_4' - T_5')$$

$$\therefore \quad V_j = \sqrt{2 \times c_p(T_4' - T_5')}$$

$$= \sqrt{2 \times c_p(815 - 735.21) \times 1000} = 400.3 \text{ m/s}$$

$\therefore$ Thrust per kg per second = 1 × 400.3 = 400.3 N **Ans.**

7. (a) Solution:



From unsteady flow energy equation

$$\frac{dE}{dt} = \frac{d}{dt}(m_i h_i + KE_i + PE_i + Q) - \frac{d}{dt}(m_e h_e + KE_e + PE_e + W)$$

There is no change in kinetic energy and potential energy.

$$\frac{du}{dt} = \frac{d}{dt}(m_i h_i + Q) - \frac{d}{dt}(m_e h_e + W)$$

For a given condition, there is no inlet of fluid so, $m_i = 0$, $h_i = 0$, $W = 0$

$$\frac{du}{dt} = \dot{Q} - \frac{d}{dt}(m_e h_e) = \dot{Q} - h_e \dot{m}_e$$

From conservation of mass, $\frac{dm}{dt} = \dot{m}_i - \dot{m}_e = m_2 - m_1$

$$= -\dot{m}_e = m_2 - m_1$$

$$\Rightarrow \quad \dot{m}_e = m_1 - m_2$$

$$\frac{du}{dt} = m_2u_2 - m_1u_1$$

$$\Rightarrow \quad m_2u_2 - m_1u_1 = \dot{Q} - h_e(m_1 - m_2)$$

$$x_i = 0.6$$

$$u_1 = u_f + x_i(u_g - u_f) = 1033.48 + 0.6(2602.4 - 1033.48)$$

$$= 1974.832 \text{ kJ/kg}$$

$$v_1 = v_f + x_i(v_g - v_f) = 0.001229 + 0.6(0.05965 - 0.001229)$$

$$= 0.03628 \text{ m}^3\text{/kg}$$

Initial mass, $m_1 = \frac{V}{v_1} = \frac{0.9}{0.03628} = 24.807 \text{ kg}$

$$x_f = 0.9$$

$$u_2 = u_f + x_f(u_g - u_f) = 1033.48 + 0.9(2602.4 - 1033.48)$$

$$= 2445.508 \text{ kJ/kg}$$

$$v_2 = v_f + x_f(v_g - v_f) = 0.001229 + 0.9(0.05965 - 0.001229)$$

$$= 0.0538 \text{ m}^3\text{/kg}$$

$$m_2 = \frac{V}{v_2} = \frac{0.9}{0.0538} = 16.7286 \text{ kg}$$

$$\Rightarrow \quad m_2u_2 - m_1u_1 = Q - h_e(m_1 - m_2)$$

$$\Rightarrow (16.7286 \times 2445.508) - (24.807 \times 1974.832) = Q - 2802.2\,(24.807 - 16.7286)$$

(Here, $h_e = h_g$ (Saturated steam is going out))

$$Q = 40909.925 - 48989.657 + 22637.29$$

$$= 14557.56 \text{ kJ} = 14.56 \text{ MJ}$$

7. (b) Solution:

The vortex tube or Range-Hilsch tube, consists of a straight length of a tube with a concentric orifice located in a diaphragm near one end and a nozzle located tangentially near the outer radius adjacent to the orifice plate. Compressed gas enters the tube tangentially through a nozzle forming a vortex which, therefore, travels towards the right hand side of the tube called the hot end. A hot stream at temperature T_h which is above the temperature of supply, say, T_3 ejects from the hot end through the throttle value, while the cold stream at temperature T_c below the temperature of supply is received at the cold end through the orifice. The throttle valve opening controls the temperature and proportion of the cold stream with respect to the hot stream. The larger the throttle valve opening the lower the temperature of the cold stream and the smaller its fraction and vice-versa. The throttle value is placed sufficiently distant from the nozzle and the diaphragm immediately close to it.

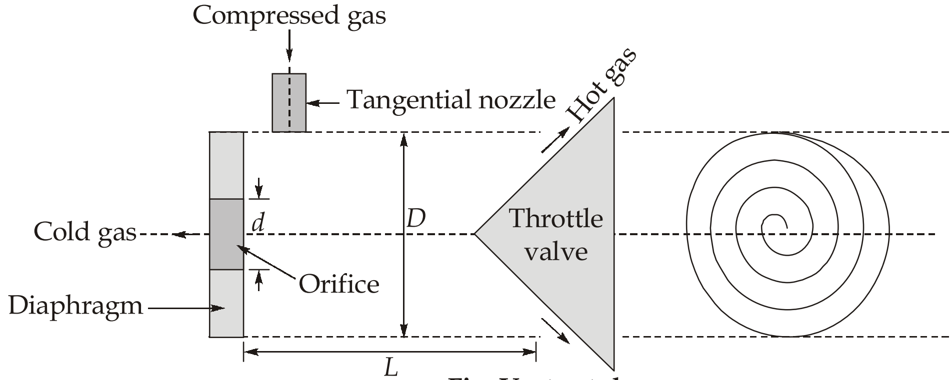
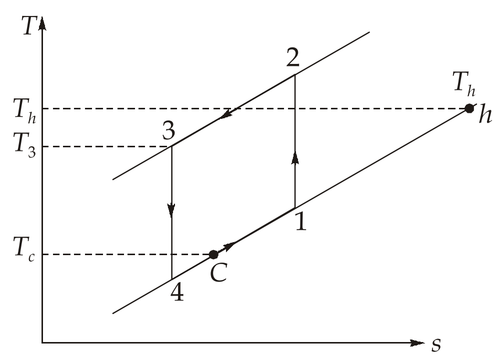


Fig. Vortex tube

The vortex tube system is a modification of the open-type air refrigeration system with the expander having been replaced by a vortex tube. In the Joule cycle, a temperature drop is obtained equal to the isentropic temperature drop (T_3 – T_4). The work of expansion is utilized to either run a cooling fan or a secondary compressor. The temperature drop obtained with the vortex tube, (T_3 – T_C) is smaller than the isentropic drop.



From the nozzle, the high-velocity gas travels from the periphery of the tube to the axis during which the separation of kinetic energy occurs. The kinetic energy is retained by the outer layers due to which they are heated and energy from the hot end of the tube at state h. The central core after having lost some kinetic energy emerges from the cold end at state C, i.e., at a temperature slightly above the static temperature of the expanded gas. The pressure of the cold gas stream is usually lowered further due to expansion in the vortex chamber.

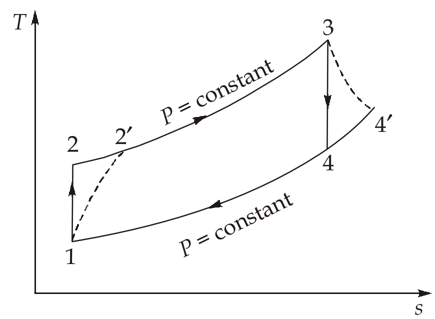
7. (c) Solution:

Given: $T_1 = 273 + 20 = 293$ K, $r_p = \frac{P_2}{P_1} = 5$, $T_3 = 780 + 273 = 1053$ K

Polytropic efficiency of compressor,

$$\eta_{P,C} = \frac{dT_S}{dT} = \frac{(\gamma-1)}{\gamma} \times \left(\frac{n}{n-1}\right)$$

Polytropic efficiency of turbine,



$$\eta_{P,T} = \frac{dT}{dT_s} = \frac{\gamma}{(\gamma-1)} \times \left(\frac{n-1}{n}\right)$$

For compressor, $\left(\frac{n-1}{n}\right) = \left(\frac{\gamma-1}{\gamma}\right) \times \left(\frac{1}{\eta_{P,C}}\right)$

For turbine, $\left(\frac{n-1}{n}\right) = \left(\frac{\gamma-1}{\gamma}\right) \times (\eta_{P,T})$

We know that,
$$\frac{T_2'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{P_2}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)\times\left(\frac{1}{\eta_{P,C}}\right)}$$

$$\frac{T_2'}{293} = (5)^{\left(\frac{1.4-1}{1.4}\right)\times\left(\frac{1}{0.88}\right)}$$

$$T_2' = 494.09 \text{ K}$$

We know that,

$$\frac{T_3}{T_4'} = \left(\frac{P_3}{P_4}\right)^{\frac{n-1}{n}} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{P_2}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)\times(\eta_{P,T})}$$

$$\frac{T_3}{T_4'} = (5)^{\left(\frac{1.4-1}{1.4}\right)\times(0.88)}$$

$$\frac{1053}{T_4'} = (5)^{(0.4\times0.88/1.4)}$$

$$T_4' = \frac{1053}{1.4988} = 702.56 \text{ K}$$

(i) Net work done = Turbine work – Compressor work

$= c_p\,(T_3 - T_4') - c_p(T_2' - T_1)$

[Neglecting the mass flow rate of fuel]

$= 1.005\,(1053 - 702.56) - 1.005(494.02 - 293)$

Specific output of plant $= 150.1671$ kJ/kg of air

Heat supplied $= c_p(T_3 - T_2') = 1.005\,(1053 - 494.02)$

$= 561.775$ kJ/kg of air

(ii) $$\text{Overall efficiency} = \frac{\text{Net work done}}{\text{Heat supplied}} = \frac{150.1671}{561.775}$$

$= 0.2673$ or 26.73%

(iii) $$\text{Fuel air ratio} = \frac{\text{Heat supplied in kJ per kg of air}}{\text{Calorific value of fuel in kJ per kg of fuel}}$$

$$= \frac{561.775}{42\times10^3} = 0.0133756$$

$$\text{Air fuel ratio} = \frac{1}{\text{Fuel air ratio}} = \frac{1}{0.0133756} = 74.763$$

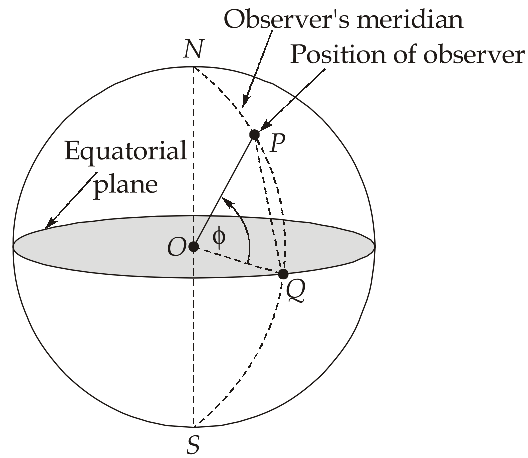
(iv) $$\text{Specific fuel consumption} = \frac{\text{Fuel air ratio}\times3600}{\text{Specific output per kg of air}}$$

$$= \frac{0.133756\times3600}{150.1671}$$

Specific fuel consumption = 0.32066 kg/kW-h

8. (a) (i) Solution:

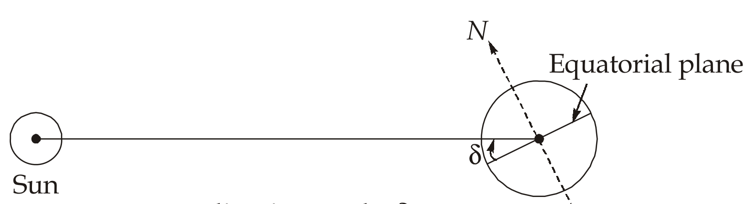
1. **Latitude (Angle of Latitude), (ϕ):** The latitude of a location on earth's surface is the angle made by radial line, joining the given location to the center of the earth, with its projection on the equator plane as shown below. The latitude is positive for northern hemisphere and negative for southern hemisphere.



2. **Declination,(δ):** It is defined as the angular displacement of the sun from the plane of earth's equator as shown in Fig. It is positive when measured above equatorial plane in the northern hemisphere. The declination δ can be approximately determined from the equation:

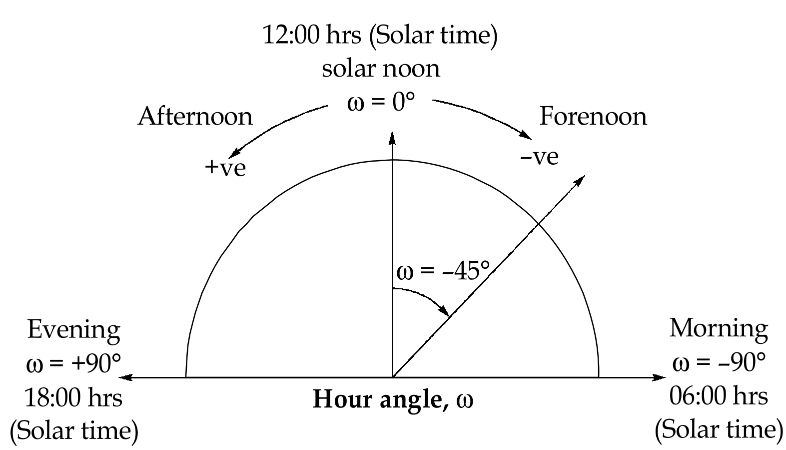
$$\delta = 23.45 \times \sin\left[\frac{360}{365}(284 + n)\right] \text{degrees}$$

where n is day of the year counted from 1st January.



Declination angle, δ

3. **Hour angle,(ω):** The hour angle at any moment is the angle through which the earth must turn to bring the meridian of the observer directly in line with sun's rays.



Hour angle, ω

In other words, at any moment, it is the angular displacement of the sun towards east or west of local meridian (due to rotation of the earth on its axis). The earth completes one rotation in 24 hours. Therefore, one hour corresponds to 15° of rotation. At solar noon, as sunrays are in line with local meridian, hour angle is zero. It is positive in the forenoon and negative in the afternoon. Thus at 06:00 hrs it is –90° and at 18:00 hrs it is +90° as shown above. We adopt the convention of measuring it from noon based on LAT, being positive in the morning and negative in the afternoon.

It can be calculated as:

$$\omega = [\text{Solar time} - 12{:}00] \text{ (in hours)} \times 15 \text{ degrees}$$

8. (a) (ii) Solution:

Dry matter produced by two cows = 2 × 2 = 4 kg/day

As dry matter content in cow dung is only 18%, cow dung produced

$$= \frac{4}{0.18} = 22.22 \text{ kg/day}$$

Equal amount of water is added to make the slurry.

The amount of slurry produced per day = 22.22 + 22.22 = 44.44 kg/day

$$\text{Slurry volume produced per day} = \frac{44.44}{1090} = 0.04077 \text{ m}^3/\text{day}$$

Total slurry in the digester = Retention time × Slurry volume produced per day

$$= 50 \times 0.04077 = 2.0385 \text{ m}^3$$

As 10% digester volume is occupied by the gas, the net digester size

$$= \frac{2.0385}{0.9} = 2.265 \text{ m}^3$$

Gas produced = Dry matter produced per day × Biogas yield

$$= 4 \times 0.22 = 0.88 \text{ m}^3/\text{day}$$

Thermal energy available = 0.88 × 22 × 0.6 = 11.616 MJ/day

$$\text{Continuous thermal power available} = \frac{11.616 \times 10^6}{24 \times 60 \times 60} = 134.44 \text{ W}$$

8. (b) Solution:

$$\text{Swept volume/min} = v_s \times \frac{N}{2}$$

$$V_s = 3000 \times 10^{-6} \times \frac{3500}{2} = 5.25 \text{ m}^3/\text{min}$$

Unsupercharged inducted volume

$= V_s \times \eta_V$

$= 5.25 \times 0.85 = 4.4625$ m^3/min

Blower delivery pressure $= (r_p) \times P_1$

$P_2 = 1.6 \times 1.013 = 1.6208$ bar

Temperature after isentropic compression

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 1.6^{\frac{1.4-1}{1.4}}$$

$$T_{2s} = T_1(1.6)^{2/7} = 300 \times (1.6)^{2/7} = 343.116 \text{ K}$$

Isentropic efficiency of compressor is 80%

$$\eta_{isen} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$0.8 = \frac{343.116 - 300}{T_2 - 300} \Rightarrow T_2 = 353.895 \text{ K}$$

Since it is given that after supercharging, the volume inducted is equal to the swept volume.

∴ The blower delivers 5.25 m^3/min at 1.6208 bar and 353.895 K

Equivalent volume at 1.013 bar and 300 K

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_1 = \frac{1.6208 \times 5.25}{353.895} \times \frac{300}{1.013} = 7.1207 \text{ m}^3\text{/min}$$

∴ Increase in the inducted volume

$= 7.1207 - 4.4625 = 2.6582$ m^3/min

∴ Increase in IP from air induced $= 14 \times 2.6582 = 37.2148$ kW

∴ Increase in IP due to increased induction pressure

$$= \Delta P \times V_s = (1.6208 - 1.013) \times \frac{5.25}{60} \times 10^2 = 5.31825 \text{ kW}$$

∴ Total increase in IP $= 37.2148 + 5.31825 = 42.533$ kW

∴ Increase in BP $= \eta_{mech} \times$ IP $= 0.9 \times 42.533 = 38.2797$ kW

Power required by blower

Mass of air delivered by the blower

$$\dot{m} = \frac{P_2\dot{V}}{RT_2} = \frac{1.6208 \times 100 \times 5.25}{0.287 \times 343.895 \times 60} = 0.13963 \text{ kg/s}$$

Power required by the blower

$$P_b = \frac{\dot{m}c_p\Delta T}{\eta_{mech}} = \frac{0.13963 \times 1.005 \times (353.895 - 300)}{0.9} = 8.403 \text{ kW}$$

$\therefore$ Net increase in BP = Increase in IP – Power required by blower

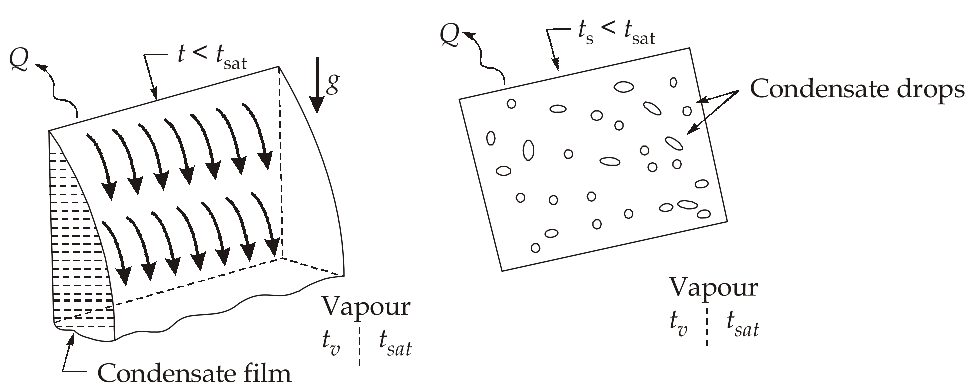
BP = 38.2797 – 8.403 = 29.8767 kW

8. **(c) (i) Solution:**

Depending upon the condition of cool surface, condensation may occur in two possible ways: Film condensation and dropwise condensation.

1. **Film Condensation:** If the condensate tends to wet the surface and thereby forms a liquid film, then the condensation process is known as 'film condensation'. Here, the heat from the vapour to the cooling medium is transferred through the film of the condensate formed on the surface. The liquid flows down the cooling surface under the action of gravity and the layer continuously grows in thickness because of newly condensing vapours. The continuous film offers thermal resistance and checks further transfer of heat between the vapour and the surface.

   Further, the heat transfer from the vapour to the cooling surface takes place through the film formed on the surface. The heat is transferred from the vapour to the condensate formed on the surface by 'convection' and it is further transferred from the condensate film to the cooling surface by the conduction'. This combined mode of heat transfer by conduction and convection reduces the rates of heat transfer considerably (compared with dropwise condensation). That is the reason that beat transfer rates of filmwise condensation are lower than dropwise condensation. Fig. (i) shows the film condensation on a vertical plate.



(i) Film condensation **(ii) Dropwise condensation**

2. **Dropwise condensation:** In 'dropwise condensation' the vapour condenses into small liquid droplets of various sizes which fall down the surface in random fashion.

The drops form in cracks and pits on the surface, grow in size, break away from the surface, knock off other droplets and eventually run off the surface, without forming a film under the influence of gravity, Fig. (ii) shows the dropwise condensation on a vertical plate.

In this type of condensation, a large portion of the area of solid surface is directly exposed to vapour without an insulating film of condensate liquid, consequently higher heat transfer rate are achieved. Dropwise condensation has been observed to occur either on highly polished surfaces, or on surfaces contaminated with impurities like fatty acids and organic compounds. This type of condensation gives coefficient of heat transfer generally 5 to 10 times larger than with film condensation. Although dropwise condensation would be preferred to filmwise condensation yet it is extremely difficult to achieve or maintain. This is because most surfaces become wetted' after being exposed to condensing vapours over a period of time. Dropwise condensation san be obtained under controlled conditions with the help of certain additives to the condensate and various surface coatings but its commercial viability has not yet been approved. For this reason the condensing equipment in use is designed on the basis of filmwise condensation.

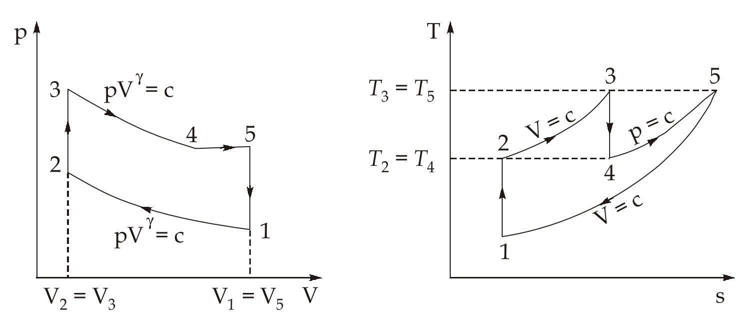
8. (c) (ii) Solution:

Given data: $T_2 = 555\,K$; $T_3 = 835\,K$; $T_4 = T_2 = 555\,K$; $T_5 = T_3 = 835\,K$

From p-V diagram,

$$\frac{V_2}{V_1} = \frac{V_3}{V_5}$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_5} \times \frac{V_4}{V_4}$$



$$\frac{V_2}{V_1} = \frac{V_3}{V_4} \times \frac{V_4}{V_5}$$

$$\left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = \left(\frac{T_4}{T_3}\right)^{\frac{1}{\gamma-1}} \times \frac{T_4}{T_5}$$

or $$\frac{T_1}{T_2} = \frac{T_4}{T_3} \times \left(\frac{T_4}{T_5}\right)^{\gamma-1}$$

$$\frac{T_1}{555} = \frac{555}{835} \times \left(\frac{555}{835}\right)^{1.4-1}$$

or $$T_1 = 313.28 \text{ K}$$

Net work done: $$w_{net} = w_{1-2} + w_{2-3} + w_{3-4} + w_{4-5} + w_{5-1}$$

$$= \frac{R(T_1 - T_2)}{\gamma - 1} + 0 + \frac{R(T_3 - T_4)}{\gamma - 1} + R(T_5 - T_4) + 0$$

$$= \frac{0.287(313.28 - 555)}{1.4 - 1} + \frac{0.287(835 - 555)}{1.4 - 1} + 0.287(835 - 555)$$

$$= -173.43 + 200.9 + 80.36 = 107.83 \text{ kJ/kg}$$

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