MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Civil Engineering Test No : 7

Section - A

Q.1 (a) Solution:

$$\alpha_s = 1.2 \times 10^5 \text{ N/mm}^2, \quad \alpha_c = 17.5 \times 10^{-6}/°\text{C},$$

$$\delta L = 0.2 \text{ mm}$$

To find: (i) Maximum temperature rise 't'

(ii) Stresses in bar σ_s and σ_c

$$\text{Area of steel bar, } A_s = \frac{\pi}{4} \times 30^2 = 706.86 \text{ mm}^2,$$

$$\text{Area of copper bar, } A_c = \frac{\pi}{4} \times 20^2 = 314.16 \text{ mm}^2$$

Maximum temperature rise which will not produce stress in bar

Temperature stresses are induced only when there is restriction to free expansion of bar. Stresses in the bar will not be produced if free expansion is equal to gap available.

$\therefore$ Free expansion = Gap

$\Rightarrow$ $(\alpha t L)_s + (\alpha t L)_c = \text{Gap}$

$\Rightarrow$ $(12 \times 10^{-6} \times 300 + 17.5 \times 10^{-6} \times 200)\, t = 0.2$

$\Rightarrow$ $t = 28.169°\text{C}$ **Ans.**

The maximum temperature rise is 28.169°C which will not produce any stress in bar.

Stresses in the bar when temperature rises to 40°C.

Let 'P' be the reaction exerted by the supports.

Free expansion due to temperature – Gap = Deformation due to support reaction.

$$\Rightarrow \quad (\alpha\, t\, L)_s + (\alpha\, t\, L)_c - \text{Gap} = \left(\frac{PL}{AE}\right)_s + \left(\frac{PL}{AE}\right)_c$$

$$\Rightarrow \quad 12 \times 10^{-6} \times 40 \times 300 + 17.5 \times 10^{-6} \times 40 \times 200 - 0.2$$

$$= \frac{P \times 300}{706.86 \times 2 \times 10^5} + \frac{P \times 200}{314.16 \times 1.2 \times 10^5}$$

$$\Rightarrow \quad 0.084 = P\,(7.427 \times 10^{-6}),$$

$$\Rightarrow \quad P = 11310.08$$

$$\text{Stress in steel } \sigma_s = \frac{P}{A_s} = \frac{11310.08}{706.86}$$

$$= 16 \text{ N/mm}^2$$ **Ans.**

$$\text{Stress in copper } \sigma_c = \frac{P}{A_c} = \frac{11310.08}{314.16}$$

$$= 36 \text{ N/mm}^2$$ **Ans.**

Q.1 (b) Solution:

Let the mass of fine aggregates be x kg/m^3.

Given:

FM of fine aggregate = 2.89

FM of coarse aggregate = 7.8

Desired FM of blended mix = 6.5

Coarse aggregate mass = 1538 kg/m^3

Using weighted average formula,

$$6.5 = \frac{2.9x + 7.8 \times 1538}{x + 1538}$$

$$\Rightarrow \quad 6.5\,(x + 1538) = 2.9x + 7.8\,(1538)$$

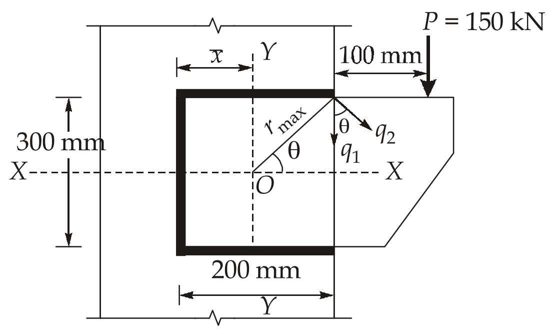
$$\Rightarrow \quad x = 555.389 \text{ kg/m}^3 \approx 555.4 \text{ kg/m}^3$$

Fineness modulus is a numerical index of fineness indicating the average size of the particles in the aggregate. It is calculated by passing aggregate through a standard set of sieves ranging from 80 mm to 150 µm. The cumulative percentage retained is noted for each sieve and their sum is divided by 100 to obtain FM.

Higher FM aggregate results in harsh concrete mixes i.e. less water-cement ratio. Lower FM means it is a fine mix i.e. more cement required which results in more shrinkage and deems it uneconomical.

It is important to control FM to ensure balanced FM for adequate strength, workability and economy.

Q.1 (c) Solution:



Direct shear stress due to direct load

$$q_1 = \frac{P_u}{(h+2b)t} = \frac{150\times10^3}{(2\times200+300)t} = \frac{214.28}{t} \text{ N/mm}^2$$

Where t = throat thickness = kS = 0.7 S

Shear stress due to twisting moment

$$q_2 = \frac{(P.e)r_{\max}}{J}$$

Distance of CG of weld from vertical weld = $\bar{x} = \dfrac{(300\times t)\times0+(200t)\left(\dfrac{200}{2}\right)\times2}{300t+2(200\times t)}$

$\Rightarrow$ $\bar{x}$ = 57.14 mm

$$r_{\max} = \sqrt{150^2+(200-57.14)^2} = 207.14 \text{ mm}$$

$$e = 300 - 57.14 = 242.86 \text{ mm}$$

Polar moment of inertia (J) $= (I_{xx} + I_{yy})$

$$I_{xx} = \frac{300^3\times t}{12}+2\cdot(200\times t)(150)^2 = 11.25t\times10^6 \text{ mm}^4$$

$$I_{yy} = (300\times t)(57.14)^2+2\left[\frac{t(200)^3}{12}+(200\times t)\left(\frac{200}{2}-57.14\right)^2\right]$$

$$= 3.0476t\times10^6 \text{ mm}^4$$

$\therefore$ $$J = I_{xx} + I_{yy}$$
$$J = 14.29t \times 10^6 = 1.429 \times 10^7 \cdot t$$

Now, $$q_2 = \frac{(P.e)r_{max}}{J} = \frac{150\times10^3\times242.86\times207.14}{1.429\times10^7 t} = \frac{528.05}{t}$$

$$\cos\theta = \frac{200-57.14}{207.14} = 0.6897$$

Now, resultant stress $$q_R = \sqrt{q_1^2+q_2^2+2q_1q_2\cos\theta}$$
$$= \sqrt{\left(\frac{214.28}{t}\right)^2+\left(\frac{528.05}{t}\right)^2+\left(\frac{214.28}{t}\right)\left(\frac{528.05}{t}\right)0.6897}$$

$\Rightarrow$ $$q_R = \frac{693.42}{t} \le \frac{f_u}{\sqrt{3}\gamma_{mw}}$$

For weld to be safe

$$\frac{693.42}{t} \le \frac{f_u}{\sqrt{3}\gamma_{mw}}$$

$\Rightarrow$ $$\frac{693.42}{t} \le \frac{410}{\sqrt{3}\times1.25} \quad \text{(shop welding)}$$

$\Rightarrow$ $$t \ge 3.66 \quad [t = ks = 0.7S]$$

$\Rightarrow$ $$0.7S \ge 3.66$$

$\Rightarrow$ $$S \ge 5.23 \text{ mm}$$

$\therefore$ Provide 6 mm weld size

Q.1 (d) Solution:

D.L. of the beam = 0.25 × 0.45 × 24 = 2.7kN/m = 0.0027 kN/mm

M.I. of the beam section $$I = \frac{250\times450^3}{12} = 1.8984 \times 10^9 \text{ mm}^4$$

(i) Initial prestressing force

$$P_o = 2 \times 510 \times 1500 = 1530 \times 10^3 \text{ N} = 1530\text{kN}$$

Downward deflection at the centre due to DL of the beam

$$= \frac{5}{384}\frac{w_d l^4}{EI}$$

$$= \frac{5}{384}\frac{0.0027\times8000^4}{40\times1.8984\times10^9} = 1.90 \text{ mm}$$

Upward deflection due to prestressing force

$$= \frac{P_o e l^2}{8EI} = \frac{1530 \times 90 \times 8000^2}{8 \times 40 \times 1.8984 \times 10^9} = 14.51 \text{ mm}$$

Net upward deflection at the centre

$$= 14.51 - 1.90 = 12.61 \text{ mm}$$

(ii) Upward deflection due to final prestressing force,

$$= 0.8 \times 14.52 = 11.62\text{mm}$$

Downward deflection due to imposed loads

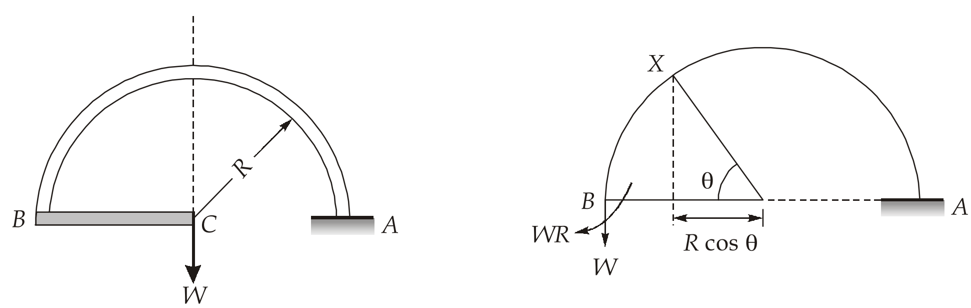
$$= \frac{15}{2.7} \times 1.90 = 10.56 \text{ mm}$$

∴ Net downward deflection at the centre

$$= 1.90 + 10.56 - 11.62 = 0.84 \text{ mm}$$

Q.1 (e) Solution:

The bracket *BC* is rigid, hence *EI* for *BC* is infinite. The strain energy for *BC* will be zero. Free body of strip *AB* is shown in figure given below.



B.M. at any section *X* at angle θ to centre of semi-circle is

$$M = WR - WR(1 - \cos\theta)$$

$$= WR\cos\theta$$

$$\frac{\partial M}{\partial W} = R\cos\theta$$

$$ds = Rd\theta$$

$$\frac{\partial U}{\partial W} = \Delta_C = \frac{1}{EI}\int_0^{\pi} M\frac{\partial M}{\partial W}ds$$

$$= \frac{1}{EI}\int_0^{\pi} WR\cos\theta \times R\cos\theta \times R\,d\theta$$

$$= \frac{WR^3}{EI}\int_0^{\pi}\cos^2\theta\, d\theta = \frac{WR^3}{EI}\int_0^{\pi}\frac{(1+\cos 2\theta)}{2}\, d\theta$$

$$= \frac{WR^3}{2EI}\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\pi} = \frac{WR^3}{2EI}\times\pi = \frac{\pi WR^3}{2EI}$$

Q.2 (a) Solution:

(i)

Rheology may be defined as the science of the deformation and flow of materials, and is concerned with relationships between stress, strain, rate of strain and time.

Following factors affect rheological properties of concrete:

1. Mix proportions

(a) A concrete mix having an excess amount of coarse aggregates will lack sufficient mortar to fill the voids, resulting in a loss of cohesion and mobility.

(b) A high fine aggregate content increases the surface area of particles, which increases the amount of paste required to coat these surfaces to have the same amount of mobility. This, in turn, can result in increased drying shrinkage and cracking.

2. Consistency

The consistency of concrete, as measured by the slump test, is an indicator of the relative water content in the concrete mix. An increase in the water content or slump more than that required to achieve a workable mix produces greater fluidity and decreased internal friction. The reduction in the cohesion within the mixture increases the potential for segregation and excessive bleeding.

3. Hardening and stiffening

Elevated temperature, use of rapid-hardening cement, cement deficient in gypsum, and use of accelerating admixtures, increase the rate of hardening which reduces the mobility of concrete. The dry and porous aggregate will rapidly reduce workability by absorbing water from the mixture.

4. Aggregate shape and texture

The shape of the aggregate particles and aggregate texture influences the rheology of concrete appreciably. The rough and highly angular aggregate particles will result in higher percentage of voids being filled by mortar, requiring higher fine aggregate contents and correspondingly higher water content. Similarly, angular fine aggregates will increase internal friction in the concrete mixture and require higher water content than well-rounded natural sand.

5. **Aggregate grading**

A well-graded aggregate gives good workability. The absence of a particular size of aggregate (gap-graded) or a change in the size distribution may have appreciable effect on the void system and workability. These effects are more in the fine aggregates than in coarse aggregates.

6. **Maximum aggregate size**

An increase in the maximum size of aggregate will reduce the fine aggregate content required to maintain a given workability and will thereby reduce the surface area to be wetted and hence the cement content necessary for a constant water-cement ratio.

7. **Admixture**

Out of the large number of admixtures used in concrete to obtain improved performance characteristics, the admixtures which have significant effect on the rheology of concrete are plasticizers and super-plasticizers, air-entraining agents, accelerators and retarders.

(ii)

Pozzolanas are finely ground siliceous materials which as such, do not possess cementing property in themselves, but react chemically with calcium hydroxide ($Ca(OH)_2$) released from the hydration of portland cement at normal temperature to form compounds of low solubility having cementing properties. The action is termed as pozzolanic action.

The pozzolanic materials can be divided into two groups namely, natural pozzolanas and artificial pozzolanas.

Examples:

Natural pozzolanas: Clay, shales, opaline, cherts, diatomaceous earth and volcanic tuffs and pumice.

Artificial pozzolanas: Fly ash, blast furnace slag, silica fume, rice husk ash, metakaolin, and surkhi.

Various implications seen on application of pozzolana in cement concrete:

(a) Improved workability with lesser amount of water.
(b) Lower heat of hydration and thermal shrinkage.
(c) Improved resistance to attack from salts and surfaces from soils and sea water.
(d) Reduced susceptibility to dissolution and leaching of calcium hydroxide.
(e) Reduced permeability
(f) Lower costs (economical) of concrete manufacture

(g) Reduction in the rate of development of strength

(h) An increase in drying shrinkage.

(i) Reduction in durability.

(j) Longer curing periods in case of use of fly ash concrete.

Q.2 (b) Solution:

Thickness of wall shall not be less than

(i) 150 mm

(ii) 30 mm per metre depth + 50 mm = 30 × 3 + 50 = 140 mm

(iii) 60 mm per metre length of side = 60 × 5 = 300 mm

∴ Provide wall thickness of 300 mm

Effective span of the slab = 5 + 0.3 = 5.3 m

Consider a level 1 m above the tank bottom.

Water pressure intensity at this level = 9.81 × (3 – 1)= 19.62 kN/m^2

B.M. per meter height at this level at corners = $\frac{19.62 \times 5.3^2}{12}$ kN-m = 45.93 kN-m

Bending moment at mid span = $\frac{19.62 \times 5.3^2}{16}$ kN-m = 34.45 kN-m

Pull in the wall per metre height at this level, $P = \frac{19.62 \times 5}{2}$ kN = 49.05 kN

Design of the corner section

$$\text{Effective cover} = 25 + \frac{18}{2} = 34 \text{ mm}$$

$$\therefore \quad \text{Effective thickness of wall} = 300 - 34 = 266 \text{ mm}$$

$$\text{Effective B.M. at corners} = M - P.x.$$

$$= 45.93 - 49.05\left(266 - \frac{300}{2}\right) \times 10^{-3}$$

$$= 40.24 \text{ kN-m}$$

Since this bending moment produces tension near the water face, the stresses and the design coefficients to be adopted are

$$\sigma_{st} = 115 \text{ N/mm}^2, \sigma_{cbc} = 7 \text{ N/mm}^2$$

$$\because \quad m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

$$x_{u,\text{balance}} = \frac{d}{1+\dfrac{\sigma_{st}}{m\,\sigma_{cbc}}} = \frac{266}{1+\dfrac{115}{13.33\times 7}}$$

$$= 119.15 \text{ mm}$$

Steel required for bending moment,

$$A_{st_1} = \frac{40.24\times 10^6}{115\times\left(266-\dfrac{119.15}{3}\right)} = 1546.35 \text{ mm}^2$$

Steel required for pull, $A_{st_2} = \dfrac{49.05\times 10^3}{115} = 426.52 \text{ mm}^2$

∴ Total steel required, $A_{st} = 1546.35 + 426.52 = 1972.87 \text{ mm}^2$

∴ Spacing of 18 mm diameter bars $= \dfrac{\dfrac{\pi}{4}\times 18^2\times 1000}{1972.87} = 128.98 \text{ mm c/c}$

Provide 18 mmϕ @ 120 mm c/c.

Design of mid-span section

Bending moment = 34.45 kN-m

Pull, P = 49.05 kN

∴ Effective B.M. $= 34.45 - 49.05\times\left(266-\dfrac{300}{2}\right)\times 10^{-3} = 28.76$ kN-m

Since this bending moment produces tension away from the water face, the stresses and the design coefficients to be adopted are:

$$\sigma_{st} = 125 \text{ N/mm}^2,\ \sigma_{cbc} = 7 \text{ N/mm}^2,\ m = 13.33$$

$$x_{u,\text{balance}} = \frac{d}{1+\dfrac{\sigma_{st}}{m\cdot\sigma_{cbc}}} = \frac{266}{1+\dfrac{125}{13.33\times 7}} = 113.69 \text{ mm}$$

Steel required for bending moment

$$A_{st_1} = \frac{28.76\times 10^6}{125\times\left[266-\dfrac{113.69}{3}\right]} = 1008.67 \text{ mm}^2$$

Steel required for pull, $A_{st_2} = \dfrac{49.05\times 10^3}{125} = 392.4 \text{ mm}^2$

Total steel, $A_{st} = 1008.67 + 392.4 = 1401.07 \text{ mm}^2$

$$\text{Spacing of 18 mm dia. bars} = \frac{\frac{\pi}{4}\times 18^2 \times 1000}{1401.07} = 181.62 \text{ mm}$$

Provide 18 mmϕ @ 180 c/c.

Design of the bottom 1 m height of wall

This will be designed as a cantilever of 1 m height and subjected to a triangular load ranging from zero at the top of the cantilever to 9.81 × 3 = 29.43 kN/m^2 at the bottom.

∴ Maximum pressure force on the cantilever (for 1 m width) = $\frac{1}{2}\times 29.43\times 1 = 14.715$ kN

acting at $\frac{1}{3}$ m from the base

$$\therefore \quad \text{B.M. for the cantilever} = 14.715\times\frac{1}{3} = 4.905 \text{ kN-m}$$

This bending moment produces tension near the water face.

Effective depth upto centre of 10 mm ϕ bars = 300 – 25 – 18 – 5 = 252 mm

$$x_{ub} = \frac{252}{1+\frac{115}{13.33\times 7}} = 112.88 \text{ mm}$$

$$\text{Steel required for B.M.} = \frac{4.905\times 10^6}{115\times\left(252-\frac{112.88}{3}\right)} = 198.96 \text{ mm}^2$$

$$\text{But 0.3\% of gross area} = \frac{0.3}{100}\times 300\times 1000 = 900 \text{ mm}^2 > 198.96 \text{ mm}^2$$

$$\text{Spacing of 10 mm}\phi\text{ bars} = \frac{\frac{\pi}{4}\times 10^2\times 1000}{900} = 87.266 \text{ mm}$$

Provide 10 mmϕ bars @ 80 mm c/c.

Also provide 10 mmϕ vertical bars @ 160 mm c/c near each face. These vertical bars near the water face are enough to resist cantilever moment.

Q.2 (c) Solution:

For Fe410, $f_u = 410$ N/mm^2

$f_y = 250$ N/mm^2

Outstand of flange element,

$$b = \frac{b_f}{2} = \frac{165}{2} = 82.5 \text{ mm}$$

Section classification:

Depth of web, $d = h - 2\,(t_f + R)$

$= 350 - 2\,(11.4 + 16)$

$= 295.2$ mm

$$\frac{b}{t_f} = \frac{82.5}{11.4} = 7.24 < 9.4 \in = 9.4 \qquad \left(\because \in = \sqrt{\frac{250}{f_y}} = 1\right)$$

$$\frac{d}{t_w} = \frac{295.2}{7.4} = 39.89 < 84 \in = 84$$

Hence the section is plastic,

Check for shear capacity:

Design shear force, $V = 210$ kN

Design shear strength of the section,

$$V_d = \frac{f_y}{\sqrt{3}\gamma_{m0}} \times h t_w = \frac{250}{\sqrt{3} \times 1.1} \times 350 \times 7.4 \times 10^{-3} \text{ kN}$$

$= 339.85$ kN > 210 kN

Which is OK.

Check for high/low shear:

$0.6\ V_d = 0.6 \times 339.85$

$= 203.91$ kN < 210 kN

∴ This case is of high shear since

$$V > 0.6\ V_d$$

Check for design bending strength:

$$M_{dv} = M_d - \beta(M_d - M_{fd}) \le 1.2 \frac{Z_e f_y}{\gamma_{m0}}$$

$$\beta = \left(\frac{2V}{V_d} - 1\right)^2 = \left(\frac{2 \times 210}{339.85} - 1\right)^2 = 0.0556$$

$$M_d = Z_{Pz} \times \frac{f_y}{\gamma_{m0}} = 851.11 \times 10^3 \times \frac{250}{1.1} \times 10^{-6} \text{ kNm}$$

$= 193.434$ kNm

$$Z_{fd} = Z_{pz} - A_w y_w$$

$$= 851.11 \times 10^3 - 350 \times 7.4 \times \left(\frac{350}{4}\right)$$

$$= 624485 \text{ mm}^3 = 624.485 \times 10^3 \text{ mm}^3$$

$$M_{fd} = \frac{Z_{fd} f_y}{\gamma_{m0}} = 624.485 \times 10^3 \times \frac{250}{1.1} \times 10^{-6} = 141.93 \text{ kNm}$$

$$M_{dv} = M_d - \beta(M_d - M_{fd})$$

$$= 193.434 - 0.0556\,(193.434 - 141.93)$$

$$= 190.57 \text{ kNm} > 150 \text{ kNm} \quad \text{(OK)}$$

$$\leq 1.2 \times \frac{Z_e f_y}{\gamma_{m0}}$$

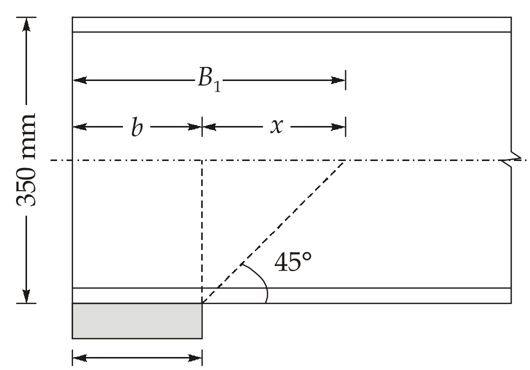
$$= 1.2 \times 751.9 \times 10^3 \times \frac{250}{1.1} = 205.06 \text{ kNm} \quad \text{(OK)}$$

Check for web buckling (at support):

Stiff bearing length, $b = 100$ mm

$$A_b = B_1 t_w = (b + x)\, t_w$$

$$= (100 + 175) \times 7.4 = 2035 \text{ mm}^2 \left\{x = \frac{h}{2} = \frac{350}{2} = 175 \text{ mm}\right\}$$



Effective length of web, $kL = 0.7 \times d = 0.7 \times 295.2 = 206.64$ mm

$$I_{eff} \text{ of web} = \frac{b t_w^3}{12} = \frac{100 \times 7.4^3}{12} = 3376.87 \text{ mm}^4$$

$$A_{eff} \text{ of web} = b t_w = 100 \times 7.4 = 740 \text{ mm}^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{3376.87}{740}} = 2.1362 \text{ mm}$$

$$\lambda = \frac{kL}{r} = \frac{206.64}{2.1362} = 96.733$$

For buckling curve '*c*'

Interpolating the value from given table,

$$\frac{100-90}{107-121} = \frac{96.733-90}{f_{cd}-121}$$

$$\Rightarrow \quad f_{cd} = 111.574 \text{ N/mm}^2$$

Capacity of web section,

$$F_{wb} = A_b f_{cd}$$

$$= 2035 \times 11.574 \times 10^{-3} \text{ kN}$$

$$= 227.05 \text{ kN} > 210 \text{ kN} \qquad \text{(OK)}$$

Check for web bearing:

$$F_w = (b+n_1)t_w \times \frac{f_y}{\gamma_{m0}}$$

$$n_1 = 2.5(t_f + R) = 2.5\ (11.4 + 16) = 68.5 \text{ mm}$$

(Dispersion ratio is 1 : 2.5)

Stiff bearing length, $b = 100$ mm

$$F_w = (100+68.5)\times 7.4 \times \frac{250}{1.1} \times 10^{-3}$$

$$\Rightarrow \quad F_w = 283.386 \text{ kN} > 210 \text{ kN} \qquad \text{(OK)}$$

Beam section ISLB 350@495 N/m is suitable for given loading conditions.

Q.3 (a) Solution:

The load of 8 kN will give rise to a clockwise couple of 8 × 0.5 = 4 kNm at *C*.

The point *C* will be displaced by an amount δ.

Slope deflection equations:

There are two unknowns θ_C and δ. We shall measure θ_C to be positive as δ for CA is assumed positive and that for CB, is assumed negative.

$$M_{AC} = \frac{2EI}{3}\left(\theta_C - \frac{3\delta}{3}\right) \qquad \text{...(i)}$$

$$M_{CA} = \frac{2EI}{3}\left(2\theta_C - \frac{3\delta}{3}\right) \qquad \text{...(ii)}$$

$$M_{CB} = \frac{2EI}{2}\left(2\theta_C + \frac{3\delta}{2}\right) \qquad \text{...(iii)}$$

$$M_{BC} = \frac{2EI}{2}\left(\theta_C + \frac{3\delta}{2}\right) \qquad \text{...(iv)}$$

Equilibrium equations:

As there are two unknowns, two equations will be required for finding out the values of unknowns. One equation will be provided by the fact that the clockwise couple at *C*, causes clockwise moments in *CA* and *CB*.

$$\therefore \quad M_{CA} + M_{CB} = 4$$

$$\Rightarrow \quad \frac{2EI}{3}\left(2\theta_C - \frac{3\delta}{3}\right) + \frac{2EI}{2}\left(2\theta_C + \frac{3\delta}{2}\right) = 4$$

$$\Rightarrow \quad \frac{4}{3}EI\theta_C - \frac{2EI\delta}{3} + 2EI\theta_C + \frac{3}{2}EI\delta = 4$$

$$\Rightarrow \quad 20EI\theta_C + 5EI\delta = 24$$

$$\Rightarrow \quad 20\theta_C + 5\delta = \frac{24}{EI} \quad \text{...(v)}$$

Shear equation

The couple acting at *C* also gives rise to horizontal reaction at *A* and *B*, the two being equal in magnitude but opposite in direction.

Now, horizontal reaction at *A* = $\frac{M_{AC} + M_{CA}}{3}$ and horizontal reaction at *B* = $\frac{M_{CB} + M_{BC}}{2}$.

As the two are equal so, $\frac{M_{AC} + M_{CA}}{3} = \frac{M_{CB} + M_{BC}}{2}$

$$\Rightarrow \quad \frac{\frac{2EI}{3}(\theta_C - \delta) + \frac{2EI}{3}(2\theta_C - \delta)}{3} = \frac{\frac{2EI}{2}\left(2\theta_C + \frac{3}{2}\delta\right) + \frac{2EI}{2}\left(\theta_C + \frac{3}{2}\delta\right)}{2}$$

$$\Rightarrow \quad \frac{4}{3}EI\theta_C - \frac{4}{3}EI\delta + \frac{8}{3}EI\theta_C - \frac{4}{3}EI\delta = 6EI\theta_C + \frac{9}{2}EI\delta + 3EI\theta_C + \frac{9}{2}EI\delta$$

$$\Rightarrow \quad 30\,EI\theta_C = -70\,EI\delta$$

$$\Rightarrow \quad \theta_C = -\frac{7}{3}\delta$$

Substituting θ_C in equation (v), we get

$$-\frac{140}{3}\delta + 5\delta = \frac{24}{EI}$$

$$\Rightarrow \quad -\frac{125\delta}{3} = \frac{24}{EI}$$

$$\Rightarrow \quad \delta = -\frac{0.576}{EI} \quad \text{...(vi)}$$

$$\therefore \qquad \theta_C = -\frac{7}{3}\delta = -\frac{7}{3}\times\frac{-0.576}{EI} = +\frac{1.344}{EI}$$

Final moments

The values of moments may now be found out by substituting the values of θ_C and δ in equations (i) to (iv).

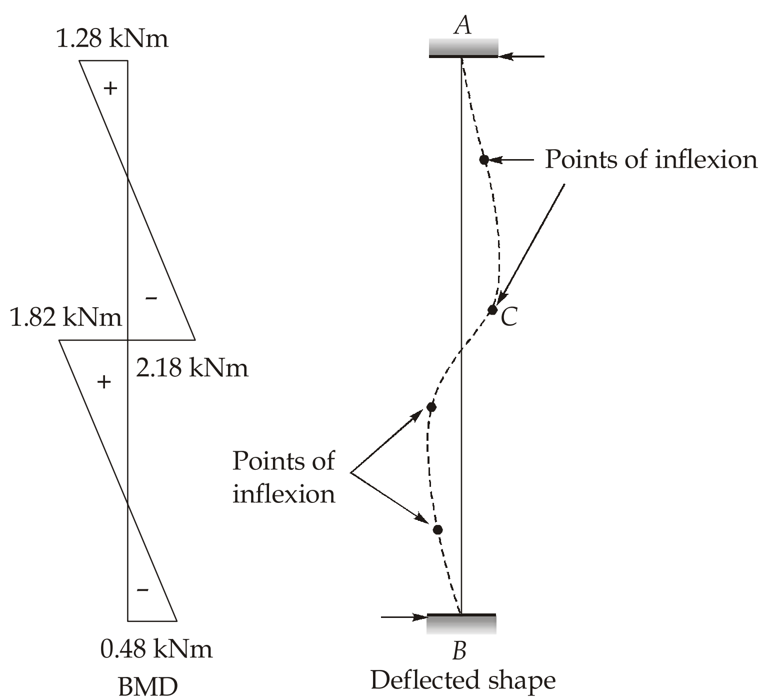
Thus,
$$M_{AC} = \frac{2EI}{3}\left(\frac{1.344}{EI}+\frac{0.576}{EI}\right) = 1.28 \text{ kNm}$$

$$M_{CA} = \frac{2EI}{3}\left(\frac{2\times1.344}{EI}+\frac{0.576}{EI}\right) = 2.18 \text{ kNm}$$

$$M_{CB} = \frac{2EI}{2}\left(\frac{2\times1.344}{EI}-\frac{3\times0.576}{2EI}\right) = 1.82 \text{ kNm}$$

and
$$M_{CB} = \frac{2EI}{2}\left(\frac{1.344}{EI}-\frac{3}{2}\times\frac{0.576}{EI}\right) = 0.48 \text{ kNm}$$

The BM diagram and deflected shape are shown below:



Q.3 (b) Solution:

(i)

1. **Scrap Value**

   Scrap value is the value of dismantled materials. For a building when the life is over at the end of its utility period, the dismantled materials such as steel, bricks, timber, etc., will fetch a certain amount which is the scrap value of the building. In the case of machine, the scrap value is the value of the metal only or the value of the dismantled parts. The scrap value of a building may be about 10 per cent of its total cost of construction. The cost of dismantling and removal of the rubbish material is deducted from the total receipt from the sale of the usable materials to get the scrap value.

2. **Salvage value**

   It is the value at the end of the utility period without being dismantled. A machine after the completion of its usual span of life or when it becomes uneconomical, may be sold and one may purchase the same for use for some other purpose, the sale value of the machine is the salvage value. It does not include the cost of removal, sale, etc.

3. **Book Value**

   Book value is the amount shown in the account book after allowing necessary depreciations. The book value of a property at a particular year is the original cost minus the amount of depreciation up to the previous year. The book value depends on the amount of depreciation allowed per year and will be gradually reduced year by year and at the end of the utility period of the property the book value will be only scrap value.

4. **Annuity**

   Annuity is the annual periodic payments for repayments of the capital amount invested by a party. These annual payments are either paid at the end of the year or at the beginning of the year, usually for a specified number of years.

5. **Capitalized Value**

   The capitalized value of a property is the amount of a money whose annual interest at the highest prevailing rate of interest will be equal to the net income from the property. To determine the capitalized value of a property it is required to know the net income from the property and the highest prevailing rate of interest.

(ii)

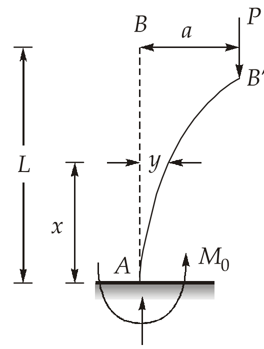
Consider a column AB of length L fixed at end A and free at end B as shown in figure.

Let the column buckle due to crippling load P.

a = Deflection at top end B.

M_o = Moment at fixed ends.

Consider a section at a distance x from fixed end A and y is the deflection at these section.



Bending moment at this section is,

$$EI\frac{d^2y}{dx^2} = +P(a-y)$$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} + Py = Pa$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{P}{EI}a$$

The solution of the above differential equation is,

$$y = C_1\cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sin\left(x\sqrt{\frac{P}{EI}}\right) + a \quad \text{...(i)}$$

Where C_1 and C_2 are integration constants.

Applying boundary conditions,

(i) At. A, $x = 0, y = 0$

$\therefore$ $0 = C_1 + 0 + a$ $\therefore$ $C_1 = -a$

(ii) At A, $x = 0, \frac{dy}{dx} = 0$

Slope at any section is obtained by differentiating Equation (i),

$$\frac{dy}{dx} = -C_1\sqrt{\frac{P}{EI}}\sin\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sqrt{\frac{P}{EI}}\cos\left(x\sqrt{\frac{P}{EI}}\right)$$

$$\therefore \quad 0 = 0 + C_2\sqrt{\frac{P}{EI}}$$

$$\Rightarrow \qquad C_2 = 0$$

At free end B, deflection $y = a$

(i) At B, $x = L$, $y = a$

Putting in Equation (i), $a = -a\cos\left(L\sqrt{\frac{P}{EI}}\right) + 0 + a$

$$\therefore \qquad \cos\left(L\sqrt{\frac{P}{EI}}\right) = 0, \qquad \Rightarrow \quad L\sqrt{\frac{P}{EI}} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

Considering the least practical value,

$$L\sqrt{\frac{P}{EI}} = \frac{\pi}{2}$$

$$\Rightarrow \qquad \sqrt{\frac{P}{EI}} = \frac{\pi}{2L}$$

$$\Rightarrow \qquad P = \frac{\pi^2 EI}{4L^2}$$

Q.3 (c) Solution:

(i)

Gross diameter of rivets = 22 + 1.5 = 23.5 mm

1. For section along *abc*

   Deduction in width for hole = 1 × 23.5 = 23.5 mm

2. For section along *abde*,

$$\text{Deduction in width for hole} = nd - \frac{s^2}{4g} = 2\times 23.5 - \frac{50^2}{4\times 75} = 38.67\ \text{mm}$$

∴ Maximum deduction for hole, in 200 mm leg = 38.67 mm

Now one leg (long leg) of the angle is connected to gusset plate.

$$\therefore \qquad A_{net} = A_1 + A_2 k$$

$$\text{where,} \qquad k = \frac{3A_1}{3A_1 + A_2}$$

$$A_1 = \text{net area of connected leg} = \left(200 - 38.67 - \frac{t}{2}\right)t$$

$$A_2 = \text{Area of outstanding leg} = \left(100 - \frac{t}{2}\right)t$$

Assuming t = 10 mm for computation of net width of leg, we get

$$A_1 = \left(200 - 38.67 - \frac{10}{2}\right)t = 156.33\,t \text{ mm}^2$$

$$A_2 = \left(100 - \frac{t}{2}\right)t = 95t\,\text{mm}^2$$

$$k = \frac{3 \times 156.33t}{3 \times 156.33t + 95t} = 0.8316$$

$$A_{net} = A_1 + kA_2 = 156.33t + 95t \times 0.8316 = 235.33t \text{ mm}^2$$

Now, $P = \sigma_{at} \cdot A_{net.}$

$\Rightarrow$ $350 \times 10^3 = 180 \times 235.33\text{ t}$

From which t = 8.26 mm

Hence adopt t = 10 mm

Thus provide ISA 200 × 100 × 10 mm angle section.

(ii)

1. Design strength due to yielding of cross section

$$T_{dg} = \frac{A_g f_y}{\gamma_{m0}}$$

A_g = 100 × 8 + 92 × 8 = 800 + 736 = 1536 mm²

$$f_y = 250 \text{ N/mm}^2$$

$$T_{dg} = 1536 \times \left(\frac{250}{1.1}\right) = 349.09 \text{ kN}$$

2. Design strength due to rupture of critical section

$$T_{dn} = 0.9\frac{A_{nc} f_u}{\gamma_{m1}} + \beta\frac{A_{g0} f_y}{\gamma_{m0}}$$

$$\beta = 1.4 - 0.076\left(\frac{w}{t}\right)\left(\frac{f_y}{f_u}\right)\left(\frac{b_s}{L_c}\right) \le \frac{f_u m_0}{f_y ml} \times 0.9 \ge 0.7$$

where, w = Outstanding leg width = 100 mm; t = Thickness of leg = 8 mm; b_s = Shear lag width (as shown in figure 1); $b_s = w + w_1 - t = 100 + 60 - 8 = 152$ mm; L_c = Length of connection = 160 mm (as shown in figure 2).

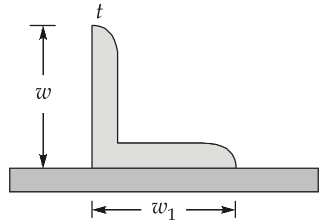


***Figure 1** : Shear leg width*

Note : w_1 is the gauge distance to centre of bolt as per 'g' of steel table.

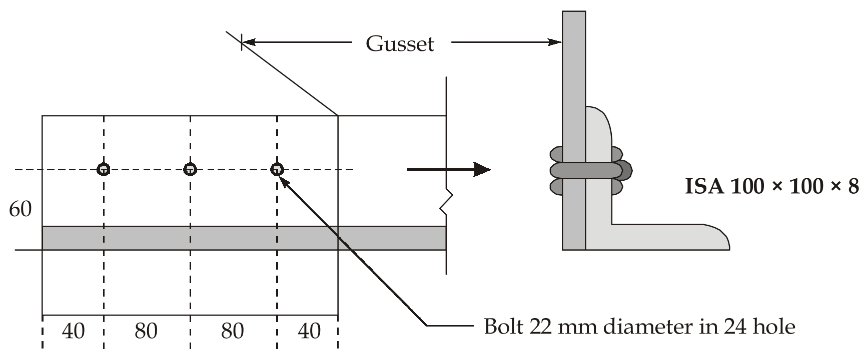


***Figure 2** : End connections of angle section*

Therefore, $$\beta = 1.4 - 0.076\left(\frac{100}{8}\right)\times\left(\frac{250}{410}\right)\times\left(\frac{152}{160}\right) = 0.85$$

$$0.9\times\frac{f_u\gamma_{m0}}{f_y\gamma_{ml}} = \frac{0.9\times410\times1.1}{250\times1.25} = 1.298$$

Therefore, $\beta = 0.85 < 1.298 > 0.7$ (OK)

Net area of connected leg, $A_{nc} = (100 - 4)\times 8 - 24\times 8 = 576\ \text{mm}^2$

Gross area of outstanding leg,

$$A_{go} = (100 - 4)\times 8 = 768\ \text{mm}^2$$

$$T_{dn} = 0.9\times576\times\left(\frac{410}{1.25}\right) + 0.85\times\left(\frac{250}{1.1}\right)\times768$$

$$= 170.035 + 148.364 = 318.399\ \text{kN} = 318.4\ \text{kN}$$

Q.4 (a) Solution:

(i)

Given, $L = 4$ m

$$a = 0.75L = 3 \text{ m}$$

$$\therefore \quad L - a = 1 \text{ m}$$

$$m = 20 \text{ kN} = 2038.736 \text{ kg}$$

Stiffness of beam is given by

$$K_b = \frac{3LEI}{a^2(L-a)^2}$$

$$= \frac{3 \times 4 \times 2.64 \times 10^6}{3^2(4-3)^2} = 3.52 \times 10^6 \text{ N/m}$$

Natural period of vibration is given by, $T = \dfrac{2\pi}{\omega_n}$

where ω_n = Natural frequency = $\sqrt{\dfrac{K_b}{m}} = \sqrt{\dfrac{3.52 \times 10^6}{2038.736}}$

$$= 41.55 \text{ rad/s}$$

Therefore, $$T = \frac{2\pi}{41.55} = 0.1512 \text{ sec}$$

(ii)

The following assumptions are made in the theory of bending:

1. The material of beam is homogeneous and isotropic.
2. The beam is straight before loading.
3. The beam is of uniform cross section throughout its length.
4. Transverse sections, which are plane before bending, remain plane even after bending.
5. The material is elastic and Hooke's law is applicable.
6. The effect of shear is neglected. Therefore the analysis is meant for pure bending.
7. The modulus of elasticity, E has same value in tension and compression.
8. Each layer is free to expand or contract having no influence of the neighbouring layers for their extension or contraction.
9. The beam is initially straight and all longitudinal filament bend into circular arcs with a common centre of curvature.

Q.4 (b) Solution:

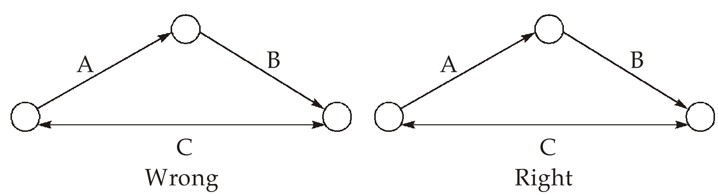
(i)

Rules of drawing a network

1. There can be only one initial and one final event.
2. An event can not occur unless all preceding activities are completed.
3. An event can not occur twice.
4. Number of arrows should be equal to number of activities.
5. Time should always flow from left to right.
6. Length of arrow does not show any magnitude. Straight arrows should be taken as far as possible
7. Arrows should normally not cross each other. If it is necessary to cross, one should be bridged over the other.
8. No activity can start until its tail event has occurred.

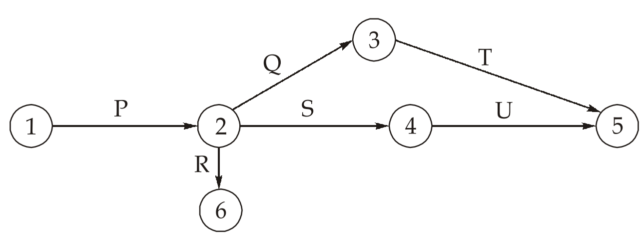
Errors in network

1. **Looping error:** Loops should not get formed.



Looping Error

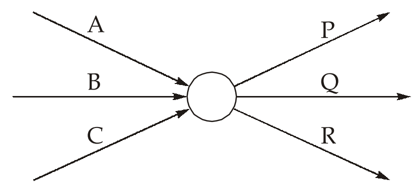
2. **Dangling error:** Project is complete only when all its activities are complete but the duration of activity '*R*' has no effect on the project time as shown in figure. i.e., in a network, there should be only one final event.



Dangling Error

To avoid dangling error, the network must be examined in such a manner that all events except initial and final events must have at least one activity entering and one activity leaving them.

3. **Wagon wheel error:** As shown in figure, each of the activities *P*, *Q* and *R* cannot start until all the three activities *A*, *B* and *C* are completed. But in reality, this may not be the situation. There is no error visible in the construction of diagram but logical error has crept into it.



Wagon Wheel Error

(ii)

1. **Soundness of aggregates:** The soundness indicates the ability of the aggregate to resist excessive changes in volume due to changes in environmental conditions, e.g., freezing and thawing, thermal changes, and alternating wetting and drying. The aggregate is said to be unsound when volume changes result in the deterioration of concrete. This may appear in the form of local scaling to extensive surface cracking or to disintegration over a considerable depth, and thus vary from an impaired appearance to a structurally dangerous situation.

   IS: 2386 (Part-V)–1963 describes method to determine the resistance to disintegration of aggregates by saturated solution of sodium sulphate (Na_2SO_4) or magnesium sulphate ($MgSO_4$). According to IS: 383-1970, the average loss of weight after ten cycles should not exceed 12 and 18 percent when tested with sodium sulphate and magnesium sulphate, respectively.

2. **Alkali-aggregate reaction (AAR):** The alkali-aggregate reaction (AAR) or alkali-silica reactivity (ASR) is the reaction between active silica constituents of the aggregate and alkalies, i.e., Na_2O and K_2O present in the cement. The reactive forms of silica generally occur in the aggregates obtained from traps, opaline or chalcedonic cherts, andestite and andesite tuffs, rhyolites and rhylotic tuffs, siliceous limestones and certain types of sandstones.

   The soluble alkalies in the cement dissolve in the mixing water turning it into a highly caustic liquid which reacts with the reactive silica present in the reactive aggregates to form highly expansive alkali-silica gel altering the boundaries of aggregate. The expansive growth due to continuous supply of water and correct temperature results from unabated formation of silica gel. As the silica gel is confined by the surrounding paste, the continuous growth of silica gel exerts internal hydraulic pressure generated through osmosis on the surrounding and set cement gel to cause pattern cracking with subsequent loss in strength and elasticity particularly in thinner sections like pavements. The formation of cracks due to alkali-aggregate reaction accelerates other processes of deterioration like carbonation.

However, only such aggregates which contain reactive silica in particular proportion and in particular fineness are found to exhibit tendencies for alkali-aggregate reaction. The factors promoting the alkali-aggregate reaction are: reactive type of aggregate; high alkali content in cement; availability of moisture and optimum temperature conditions.

IS: 2386 (Part-VII)-1963 describes two methods namely the **mortar bar expansion test** and the **chemical test** for the determination of the potential reactivity of the aggregate.

Q.4 (c) Solution:

(i)

1. Unbraced building without any masonry infill

   As per Cl. 7.6.2 (a) of IS : 1893 (Part-1) - 2016 (for RC framed building)

   Fundamental period of vibration

$$T = 0.075\ H^{0.75} = 0.075 \times 70^{0.75} = 1.82 \text{ sec}$$

2. Braced building with infilled brick masonry wall

   Fundamental period of vibration

$$T = \frac{0.09H}{\sqrt{D}}$$

In short-direction, $$T = \frac{0.09 \times 70}{\sqrt{15}} = 1.63 \text{ sec}$$

In long direction, $$T = \frac{0.09 \times 70}{\sqrt{30}} = 1.15 \text{ sec}$$

(ii)

Clear height of wall above opening = 6 – 2.75 = 3.25 m

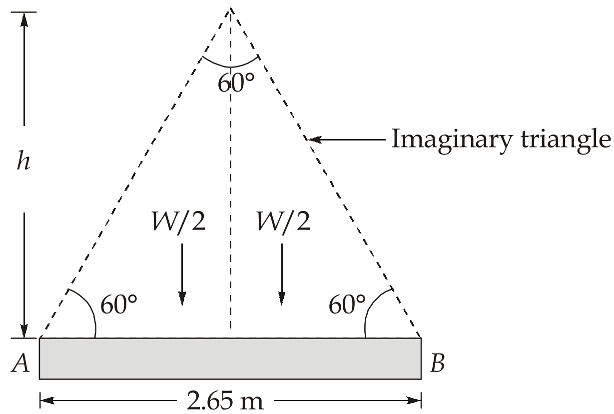
Let overall depth of lintel = 170 mm and effective cover = 20 mm

∴ Effective depth of lintel = 170 – 20 = 150 mm

∴ Effective span = Clear span + Effective depth of lintel

= 2.5 + 0.15 = 2.65 m

Height of wall above the top of lintel is 3.25 m. (Ignoring the depth of lintel to be on conservative side)



$$h = \frac{2.65}{2} \times \tan 60° = 2.295 \text{ m}$$

∴ Height of wall above the top of lintel is 3.25 m > 1.25h (= 1.25 × 2.295 = 2.87 m), so arch action is possible.

Total load on lintel, $$W = \frac{1}{2} \times 2.65 \times 2.295 \times 0.4 \times 19 = 23.11 \text{ kN}$$

Maximum BM at mid-span due to triangular load W is

$$= R_A \cdot \frac{l}{2} - \frac{W}{2} \times \frac{1}{3} \times \frac{l}{2}$$

$$\because \quad R_A = R_B = \frac{W}{2}$$

$$\therefore \quad \text{Maximum BM} = \frac{23.11}{2} \times \frac{2.65}{2} - \frac{23.11}{2} \times \frac{1}{3} \times \frac{2.65}{2}$$

$$= 10.21 \text{ kNm}$$

$$\text{Self weight of lintel} = 0.17 \times 0.4 \times 25$$

$$= 1.7 \text{ kN/m}$$

Maximum bending moment at mid-span due to uniformly distributed load (self weight of lintel)

$$\text{BM} = \frac{wl^2}{8} = \frac{1.7 \times 2.65^2}{8} = 1.49 \text{ kNm}$$

Total factored BM, $$\text{BM}_u = 1.5\,(10.21 + 1.49) = 17.55 \text{ kN-m}$$

$$\therefore \quad M = 0.36 f_{ck} B x_{u,\lim} \left(d - 0.42 x_{u,\lim}\right)$$

$$\because \quad x_{u,\lim} = 0.48d \qquad \text{(Fe415 steel)}$$

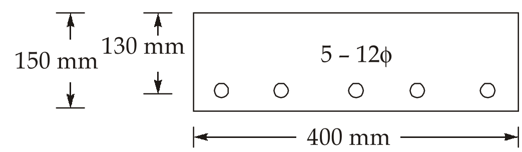
$$\therefore \quad 17.55 \times 10^6 = 0.36 \times 20 \times 400 \times 0.48\,(1 - 0.42 \times 0.48)d^2$$

$$\Rightarrow \quad d = 126.099 \text{ mm}$$

Adopt overall depth as 150 mm and effective depth as 130 mm.

$$A_{st} = \frac{0.36 \times 20 \times 400 \times 0.48 \times 130}{0.87 \times 415} = 497.75 \text{ mm}^2$$

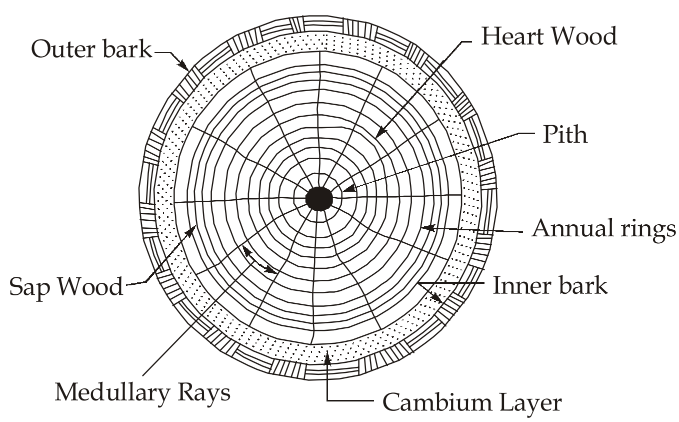
$\therefore$ Provide 5 nos – 12ϕ bars at bottom of lintel.



Section - B

Q.5 (a) Solution:

The structure of wood visible to the naked eye or at a small magnification is called the macrostructure of wood.



Cross-section of an exogeneous tree

1. **Pith:** The innermost central portion or core of the tree is called the pith or medulla.
2. **Heart Wood:** The inner annual rings surrounding the pith is known as heart wood. It is usually dark in colour. It does not take active part in the growth of tree. But it imparts rigidity to tree and hence, it provides strong and durable timber for various engineering purposes.
3. **Sap Wood:** The outer annual rings between heart wood and cambium layer is known as sap wood. It is usually light in colour and light in weight. It indicates recent growth and it contains sap.

It takes active part in the growth of tree and sap moves in an upward direction through it. Sap wood is also known as alburnum.

4. **Annual Rings**: Annual rings consist of closed cells of woody fibres and tissues arranged in distinct approximately concentric circles around pith. Every year, one such ring is formed. Hence, the total number of annual rings indicates the age of the tree. The wood near the bark is the youngest.
5. **Cambium Layer:** The thin layer of sap between sap wood and inner bark is known as cambium layer. It indicates sap which has yet not been converted into sap wood.
6. **Inner Bark:** It gives protection of cambium layer from any injury.
7. **Outer Bark:** It consists of cells of wood fibres and is also known as cortex.
8. **Medullary Rays:** The thin radial fibres extending from pith to cambium layer are known as *medullary rays*. The function of these rays is to hold together the annual rings of heart wood and sap wood.

Q.5 (b) Solution:

Given: diameter of bar *ABC*, $D = 70$

Diameter of bar *MN* and *PQ*, $d = 15$ mm

Twisting moment, $T = 5000$ Nm

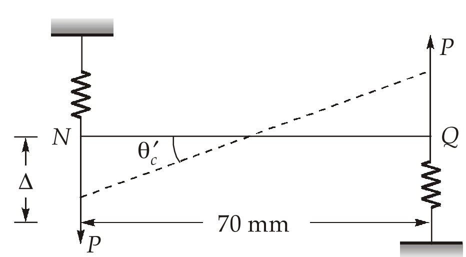
Let us consider that bars MN and PQ are temporarily disconnected from bar ABC, then the angle of twist at *B* relative to *A* is

$$\theta_B = \frac{TL}{GJ} = \frac{5000 \times 0.75}{(80 \times 10^9) \times \left(\frac{\pi}{32} \times 0.07^4\right)}$$

$$\Rightarrow \quad \theta_B = 0.0199 \text{ rad}$$

Since no additional twisting moments act between B and C and thus this same angle of twist exists at *C*,

$$\therefore \quad \theta_B = \theta_C = 0.0199 \text{ rad}$$



As there is rotation at *C*, so there would be extension Δ of each of the vertical bar, which is accompanied by an axial force *P* in each bar.

For a small angle of rotation θ,

$$\Delta = 0.035\theta_C'$$

The axial force P constitute a couple of magnitude

$$P(0.07) = T_C$$

This couple must act in a sense opposite to the 5000 Nm torque since the elastic vertical bars tend to restrain angular rotation.

The elongation of each vertical bar may be found to be

$$\Delta = \frac{PL}{AE} = \frac{P \times 1.5}{\frac{\pi}{4} \times (0.015)^2 \times E}$$

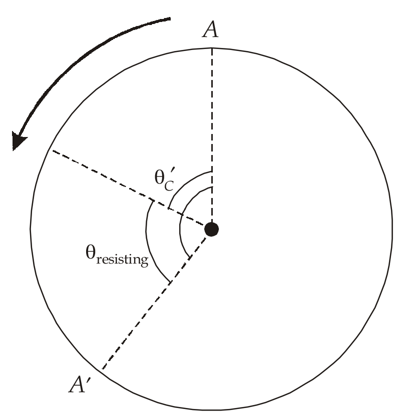
$$\Rightarrow \quad \Delta = \frac{\left(\frac{T_C}{0.07}\right) \times 1.5}{\frac{\pi}{4} \times (0.015)^2 \times 200 \times 10^9}$$

$$\Rightarrow \quad \Delta = 6.063 \times 10^{-7} T_c$$

Now,
$$\theta'_C = \frac{\Delta}{D/2} = \frac{6.063 \times 10^{-7} T_c}{0.035}$$

$$\Rightarrow \quad \theta'_C = 1.732 \times 10^{-5} T_c \text{ rad}$$

The angular rotation at end C is the due to twisting moment is θ_C and angular rotation caused by the axial force P in vertical bars is θ'_C.



By compatibility equation,

$$\theta_C - \theta'_C = \theta_{resisting}$$

$$\Rightarrow \quad \theta_C - \theta'_C = \frac{T_C \times 1.5}{GJ}$$

$$0.0199 - 1.732 \times 10^{-5}T_C = \frac{T_C \times 1.5}{80 \times 10^9 \times \frac{\pi}{32} \times (0.07)^4}$$

$$\Rightarrow \quad 0.0199 - 0.1732 \times 10^{-5}T_C = 7.954 \times 10^{-6}T_C$$

$$\Rightarrow \quad T_C = 787.37 \text{ Nm}$$

$$\therefore \quad P = \frac{T_C}{0.07} = 11248.14 \text{ N}$$

The twisting moment between B and C is 787.37 Nm and between A and B is

$$5000 - 787.37 = 4212.63 \text{ Nm}$$

The peak torsional shearing stress occurs at the outer most fibres at all points between A and B.

$$\therefore \quad \tau_{max} = \frac{4213.24 \times 0.035}{\frac{\pi}{32} \times (0.07)^4}$$

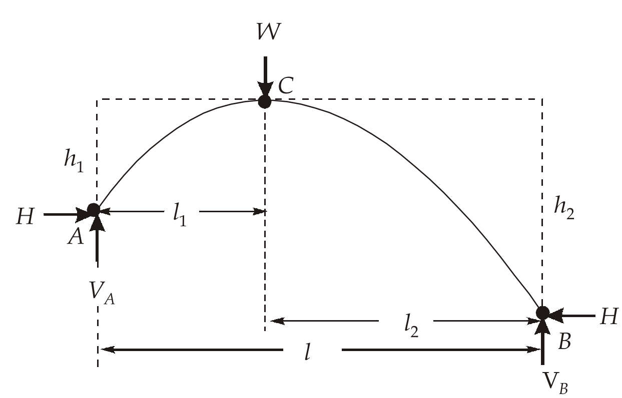
$$\Rightarrow \quad \tau_{max} = 62.55 \text{ MPa}$$

The axial stress in each of the vertical bars is

$$\sigma = \frac{11248.14}{\pi (0.0075)^2} = 63.65 \text{ MPa}$$

Q.5 (c) Solution:

Let l_1 and l_2 be the horizontal distances of the lower hinges A and B from the crown C respectively



We have,

$$\frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} = \frac{l_1 + l_2}{\sqrt{h_1} + \sqrt{h_2}} = \frac{l}{\sqrt{h_1} + \sqrt{h_2}}$$

$$l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} \text{ and } l_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$$

Taking moments about C of the forces on the left-hand side of C, we have:

$$V_A l_1 = Hh_1 \quad \text{...(i)}$$

$$\Rightarrow \quad V_A = H\frac{h_1}{l_1}$$

Similarly, taking moments about C of the forces on the right-side of C, we have:

$$V_B l_2 = Hh_2 \quad \text{...(ii)}$$

$$\Rightarrow \quad V_B = H\frac{h_2}{l_2}$$

Adding equations (i) and (ii), we have:

$$V_A + V_B = H\left(\frac{h_1}{l_1}+\frac{h_2}{l_2}\right)$$

But $\quad V_A + V_B = W$

$$\therefore \quad W = H\left(\frac{h_1}{l_1}+\frac{h_2}{l_2}\right)$$

$$\Rightarrow \quad H = \frac{W}{\frac{h_1}{l_1}+\frac{h_2}{l_2}}$$

But $\quad l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}$

and $\quad l_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$

$$\therefore \quad H = \frac{W}{\frac{h_1\left(\sqrt{h_1}+\sqrt{h_2}\right)}{l\sqrt{h_1}}+\frac{h_2\left(\sqrt{h_1}+\sqrt{h_2}\right)}{l\sqrt{h_2}}}$$

$$\Rightarrow \quad = \frac{Wl}{\frac{h_1\left(\sqrt{h_1}+\sqrt{h_2}\right)}{\sqrt{h_1}}+\frac{h_2\left(\sqrt{h_1}+\sqrt{h_2}\right)}{\sqrt{h_2}}}$$

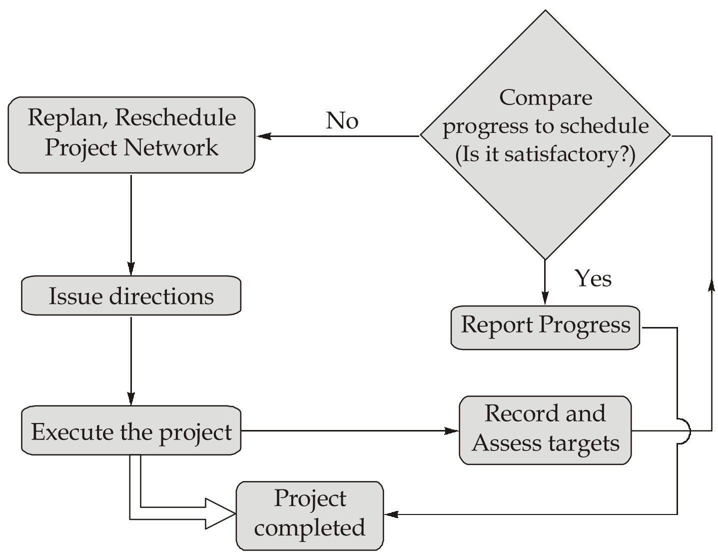
$$\Rightarrow \quad H = \frac{Wl}{\left(\sqrt{h_1}+\sqrt{h_2}\right)^2}$$

Q.5 (d) Solution:

Methods of management of a large construction project:

1. Bar charts or Gantt charts.
2. Milestone charts.
3. Network Analysis: CPM is used for construction project
4. Work breakdown structure (WBS)

The control of large projects involves close monitoring of resources, cost, quality and targets as well as budget. Feedback loop is used to revise the project plan. Resources are shifted, where they are needed most to meet time, cost and quality demands. Updating is the process of rescheduling the activities on the basis of actual prevailing conditions at the given point to time.



Updating Cycle of a project

Q.5 (e) Solution:

For Fe415, $x_{u,\,max} = 0.48 \times 500 = 240$ mm

$M_{u,\,lim}$ for singly reinforced section for Fe415 is given by;

$$M_{u,\,lim} = 0.138 f_{ck} bd^2 = 0.138 \times 20 \times 250 \times (500)^2$$

$$= 172.5 \times 10^6 \text{ Nmm} = 172.5 \text{ kNm} > 225 \text{ kNm}$$

∴ Doubly reinforced beam section is required.

For M20 grade and Fe415 grade steel

$$A_{st1} = \frac{172.5 \times 10^6}{0.87 \times 415(500 - 0.42 \times 0.48 \times 500)}$$

$$= 1196.83 \simeq 1200 \text{ mm}^2$$

$\because$ $M_u > M_{u,\,lim}$, additional moment of resistance required is

$$M_{ua} = (225 - 172.5) = 52.5 \text{ kNm}$$

Additional tension reinforcement is obtained as follows:

$$M_{ua} = 0.87 f_y A_{st2}(d - d')$$

$$A_{st2} = \frac{M_{ua}}{0.87 f_y (d-d')} = \frac{52.5\times10^6}{0.87\times415(500-50)}$$

$$= 323.13 \text{ mm}^2$$

Total tensile steel, $A_{st} = A_{st1} + A_{st2}$

$$= (1200 + 323.13) = 1523.13 \text{ mm}^2$$

Compression steel is given by relation,

$$M_{ua} = A_{sc}(f_{sc} - f_{cc})(d - d')$$

$$\Rightarrow \quad A_{sc} = \frac{M_{ua}}{(f_{sc}-f_{cc})(d-d')}$$

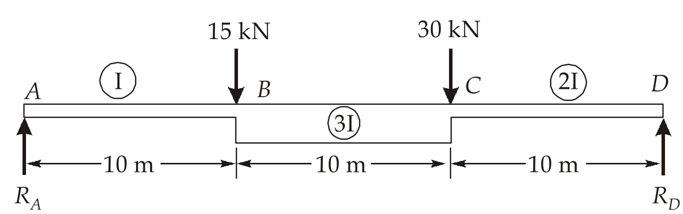
Given, $f_{sc} = 351.93$ MPa

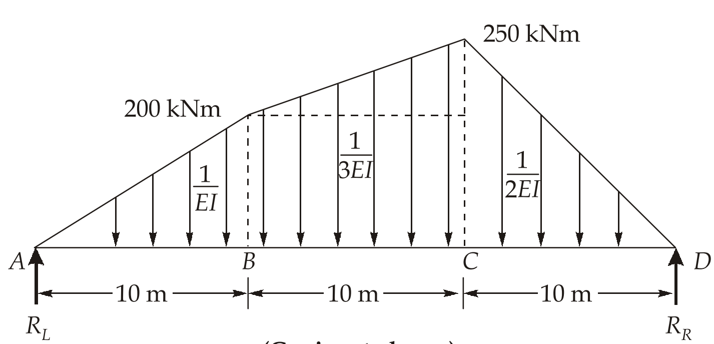
$$f_{cc} = 0.446 f_{ck} = 0.446 \times 20 = 8.92 \text{ MPa}$$

$$\therefore \quad A_{sc} = \frac{52.5\times10^6}{(351.93-8.92)(500-50)} = 340.13 \text{ mm}^2$$

Q.6 (a) Solution:

(i)





(Conjugate beam)

Real beam

Reactions: $\Sigma V = 0$

$$\Rightarrow \quad R_A + R_D = 45 \text{ kN} \quad \text{...(i)}$$

$$\Sigma M_A = 15 \times 10 + 30 \times 20 - R_D \times 30 = 0$$

$$\Rightarrow \quad R_D = 25 \text{ kN} \uparrow$$

$$\therefore \quad R_A = 45 - R_D = 45 - 25 = 20 \text{ kN} \uparrow$$

Bending moment calculation:

$$M_A = M_D = 0$$

$$M_B = R_A(10) = 20 \times 10 = 200 \text{ kNm}$$

$$M_C = R_D(10) = 25 \times 10 = 250 \text{ kNm}$$

Conjugate beam:

Reactions: Let R_L and R_R be the reactions for conjugate beam at A and D respectively.

$$\Sigma V = 0$$

$$\Rightarrow \quad R_L + R_R = \frac{1}{2}\times 10 \times 200 \times \frac{1}{EI} + 10 \times 200 \times \frac{1}{3EI} + \frac{1}{2} \times 10 \times 50 \times \frac{1}{3EI} + \frac{1}{2} \times 10 \times 250 \times \frac{1}{2EI}$$

$$\Rightarrow \quad R_L + R_R = \frac{1000}{EI} + \frac{666.67}{EI} + \frac{83.33}{EI} + \frac{625}{EI}$$

$$\Rightarrow \quad R_L + R_R = \frac{2375}{EI}$$

$$\Rightarrow \quad \Sigma M_A = 0$$

$$\Rightarrow \quad 30R_R = \frac{1000}{EI} \times \left(\frac{2}{3}\times 10\right) + \frac{666.67}{EI} \times 15 + \frac{83.33}{EI} \times \left(10 + \frac{2}{3} \times 10\right) + \frac{625}{EI}\left(20 + \frac{10}{3}\right)$$

$$\Rightarrow \quad R_R = \frac{1087.96}{EI} \uparrow$$

$$\therefore \quad R_L = \frac{2375}{EI} - R_R = \frac{1287.04}{EI} \uparrow$$

Slope at A = S.F at A in conjugate beam.

$$\therefore \quad \theta_A = R_L = \frac{1287.04}{EI} = \frac{1287.04}{4 \times 10^6}$$

$$= 3.2176 \times 10^{-4} \text{ rad } (\curvearrowright)$$

$$\text{S.F at } B = \text{Slope at } B = -R_R$$

$$= \frac{(-)1087.96}{4\times10^6} = 2.7199 \times 10^{-4} (\circlearrowleft)$$

$$\text{Deflection at } B = \text{Bending moment at } B \text{ in conjugate beam}$$

$$= R_L \times 10 - \frac{1}{2}\times10\times200\times\frac{1}{EI}\times\frac{1}{3}\times10$$

$$= \frac{1287.04}{EI}\times10 - \frac{1}{2}\times10\times200\times\frac{1}{EI}\times\frac{10}{3}$$

$$= \frac{9537.07}{EI} = \frac{9537.07}{4\times10^6} = 2.384 \times 10^{-3} \text{ m } (\downarrow)$$

$$= 2.384 \text{ mm } (\downarrow)$$

(ii)

Given: $S = 50 \text{ kN} = 50 \times 10^3 \text{ N}$

To find: Shear stress distribution across the depth of section.

$$\text{Position of neutral axis, } \bar{y} = \frac{60}{2} = 30\,\text{mm}$$

$$\text{Moment of inertia, } I = \frac{BD^3}{12} - 4\frac{bh^3}{12}$$

$$\Rightarrow \quad I = \frac{40\times60^3}{12} - \frac{4\times10\times10^3}{12}$$

$$\Rightarrow \quad I = 716.67 \times 10^3 \text{ mm}^4$$

Shear stress at 10 mm from top,

$$\tau = \frac{SA\bar{y}}{bI} = \frac{50\times10^3\times10\times40\times25}{40\times716.67\times10^3} = 17.44 \text{ N/mm}^2$$

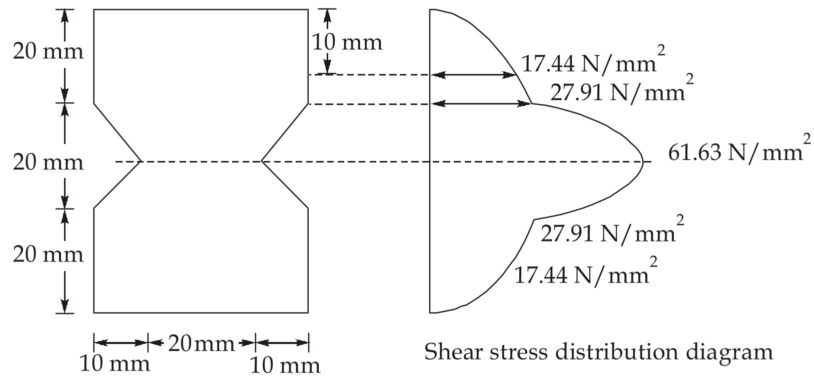
Shear stress at 20 mm from top,

$$\tau = \frac{SA\bar{y}}{bI} = \frac{50\times10^3\times20\times40\times20}{40\times716.67\times10^3} = 27.91 \text{ N/mm}^2$$

Shear stress at neutral axis,

$$\tau_{NA} = \frac{SA\bar{y}}{bI} = \frac{50\times10^3\times\left(30\times40\times15-\frac{1}{2}\times10\times10\times2\times\frac{10}{3}\right)}{20\times716.67\times10^3}$$

$$\Rightarrow \quad \tau_{NA} = 61.63 \text{ N/mm}^2$$

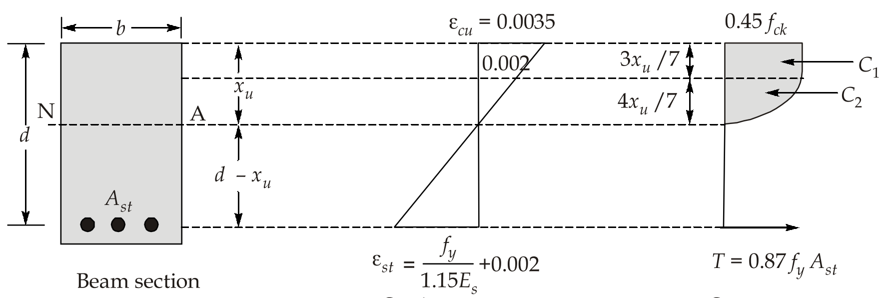


Cross-section of beam

Shear stress distribution diagram

Q.6 (b) Solution:

(i)



Beam section

Strains

Stresses

From strain diagram,
$$\frac{0.0035}{x_u} = \frac{\frac{0.87 f_y}{E_s} + 0.002}{d - x_u}$$

$$\Rightarrow \quad \frac{d - x_u}{x_u} = \frac{\frac{0.87 f_y}{E_s} + 0.002}{0.0035}$$

Given
$$f_y = 415 \text{ MPa}$$
$$E_s = 2 \times 10^5 \text{ MPa}$$

$$\therefore \quad \frac{d}{x_u} - 1 = \frac{\frac{0.87 \times 415}{2 \times 10^5} + 0.002}{0.0035}$$

$$\Rightarrow \quad \frac{d}{x_u} = 2.0872$$

$$\Rightarrow \quad x_u = 0.479 \text{ d}$$

Ans. 1

$$\text{Limiting BM, } M_{u,\,\text{lim}} = C(\text{Lever arm})$$

$$C = C_1 + C_2$$

$$= \left(0.45 f_{ck} \frac{3}{7} x_u b\right) + \left(\frac{2}{3} \times 0.45 f_{ck} \times \frac{4}{7} x_u b\right)$$

$$= 0.19 f_{ck}\, b x_u + 0.17\, f_{ck}\, b x_u = 0.36 f_{ck}\, b x_u$$

$\therefore$
$$C\bar{x} = C\bar{x}_2 + C_2\bar{x}_2$$

$\Rightarrow$
$$0.36 f_{ck} b x_u \bar{x} = 0.19 f_{ck} b x_u \times \left(\frac{3}{14} x_u\right) + 0.17 f_{ck} b x_u \times \left(\frac{3}{8} \times \frac{4}{7} + \frac{3}{7}\right) x_u$$

$\Rightarrow$
$$\bar{x} = 0.42 x_u$$

$\therefore$
$$\text{Lever arm} = d - 0.42 x_u$$

$$M_{u\,\text{lim}} = 0.36 f_{ck}\, b x_u (d - 0.42 x_u)$$

$$= 0.36 f_{ck} b \times 0.479d(d - 0.42 \times 0.479 d)$$

$$= 0.138 f_{ck} b d^2$$

$$C = T$$

$\Rightarrow$
$$0.36 f_{ck}\, b x_u = 0.87 f_y A_{st}$$

$\Rightarrow$
$$0.36 f_{ck} b (0.479 x_u) = 0.87 \times 415 \times A_{st}$$

$\Rightarrow$
$$A_{st} = 4.78 \times 10^{-4} f_{ck}\, bd$$

(ii)

$$\text{Percentage area of steel, } p = \frac{100 A_s}{bd} = \frac{100 \times 4 \times \frac{\pi}{4} \times 25^2}{500 \times 350} = 1.12\%$$

$$\text{Shear strength of concrete, } \tau_c = 0.6688 \text{ N/mm}^2 \text{ (from table)}$$

$$\text{Nominal shear stress, } \tau_v = \frac{V_u}{bd} = \frac{350 \times 1000}{350 \times 500}$$

$\Rightarrow$
$$\tau_v = 2 \text{ N/mm}^2$$

$$\text{Maximum shear stress, for M20 } \tau_{c,\,\max} = 0.625\sqrt{f_{ck}} = 2.8 \text{ N/mm}^2$$

$\therefore$
$$\tau_c < \tau_v < \tau_{c\,\max} \quad \text{(Shear reinforcement required)}$$

Shear strength of shear reinforcement required.

$$V_{us} = V_u - \tau_c bd$$

$$= 350 \times 1000 - 0.6688 \times 350 \times 500 = 232960 \text{ N}$$

Adopt 8 mm 2 legged vertical stirrups.

$$A_{sv} = \frac{2\times\pi\times 8^2}{4} = 100.5 \text{ mm}^2$$

$$\text{Spacing of shear reinforcement, } x = \frac{0.87 f_y A_{sv} d}{V_u} = \frac{0.87\times 415\times 100.5\times 500}{232960}$$

$$A_{sv\,min} = 77.88 \text{ mm c/c}$$

Provide minimum spacing of 100 mm

$$\therefore \quad \frac{A_{sv\,min}}{bS_v} \geq \frac{0.4}{f_y} \quad \Rightarrow \quad A_0 \geq \frac{0.4\times 350\times 100}{0.87\times 415} = 38.8 \text{ mm}^2$$

$$\text{Also,} \quad A_{sv} = \frac{V_{us}x}{0.87 f_y d} = \frac{232960\times 100}{0.87\times 415\times 500} = 129.05 \text{ mm}^2 > A_{sv\,min}$$

$$\text{Area of one leg} = \frac{129.05}{2} = 64.525 \text{ mm}^2$$

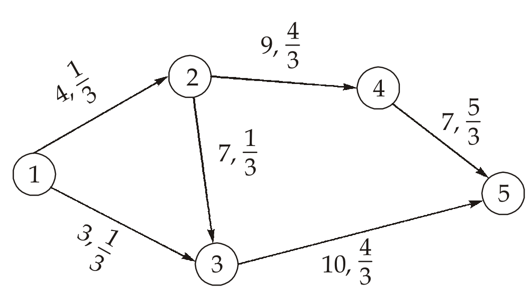
$$\text{Now area of 10 mm bar} = \frac{\pi}{4}\times 10^2 = 78.5 \text{ mm}^2 > 64.525 \text{ mm}^2$$

$\therefore$ Use 10 mm 2 legged stirrups @ 100 mm c/c.

Q.6 (c) Solution:

Given: $T_s = 24$ days

Activity	Time duration in days				Standard deviation
	Optimistic (t_o)	Most likely (t_m)	Pessimistic (t_p)	Expected $t_E = \frac{t_o + 4t_m + t_p}{6}$	$\sigma = \frac{t_p - t_o}{6}$
1 - 2	3	4	5	4	1/3
1 - 3	2	3	4	3	1/3
2 - 3	6	7	8	7	1/3
2 - 4	5	9	13	9	4/3
3 - 5	8	9	16	10	4/3
4 - 5	2	7	12	7	5/3



From network diagram:

1. Critical path is 1 – 2 – 3 – 5

Critical path duration, T_c = 21 days

Standard deviation of critical path, $\sigma = \sqrt{\sigma_{1-2}^2 + \sigma_{2-3}^2 + \sigma_{3-5}^2}$

$$\Rightarrow \quad \sigma = \sqrt{\frac{1}{9}+\frac{1}{9}+\frac{16}{9}} = \sqrt{2} \text{ days} \simeq 1.414 \text{ days}$$

2. For scheduled time of 24 days,

$$\text{Probability factor, } z = \frac{T_s - T_c}{\sigma} = \frac{24-21}{1.414}$$

$$\Rightarrow \quad z = 2.122$$

From table,

when, $z = 2.1, P = 98.21\%$

$z = 2.2, P = 98.61\%$

$$\text{Probability of completion, } P = 98.21 + \frac{(98.61-98.21)}{(2.2-2.1)} \times (2.122 - 2.1)$$

$$= 98.298\% \simeq 98.30\%$$

3. For probability of completion = 98.8%

From table,

when, $P = 98.93\%, z = 2.3$

$P = 98.61\%, z = 2.2$

$$\text{Probability factor, } z = 2.2 + \frac{98.8-98.61}{98.93-98.61} \times (2.3-2.2)$$

$$= 2.259374 \simeq 2.2594$$

Duration for 98.8% probability

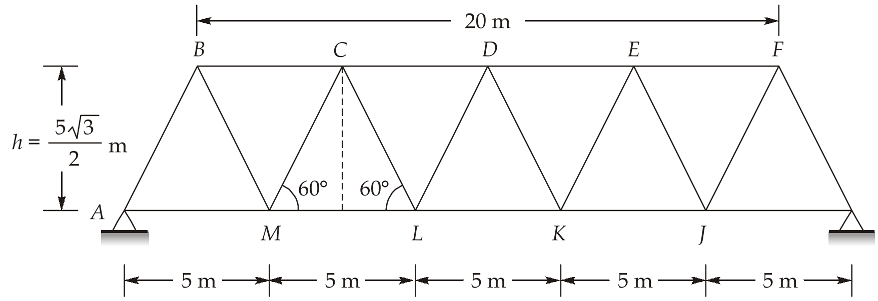
$$T = T_c + z\sigma$$

$$= 21 + (2.2594)(1.414) = 24.195 \text{ days}$$

Q.7 (a) Solution:

(i)

Let the Warren truss be of height h and span 25 m as shown below:



By geometry, $\sin 60° = \frac{h}{5} \Rightarrow h = \frac{5\sqrt{3}}{2}$ m

The I.L.D for the bottom member should be drawn by making some amendments.

The condition that $P_{ML} = \frac{M_C}{h}$ is valid only for the position of the unit load on bottom chord on left side of M or on the right side of L. When the unit load is between M and L, the force in ML will follow a different law.

The I.L.D is drawn as follows:

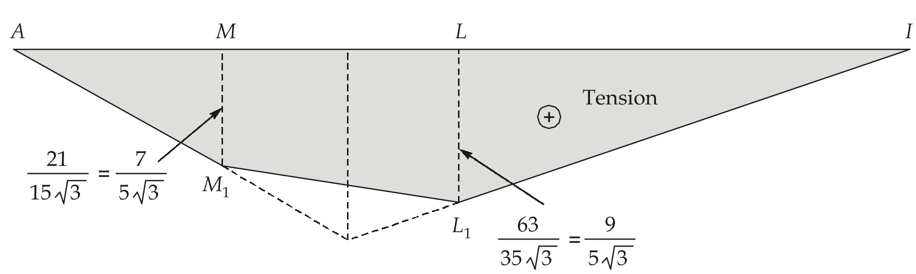
Following the law that $P_{ML} = \frac{M_C}{h}$, the I.L.D is first drawn. The diagram will have height of $\frac{a(l-a)}{lh}$ corresponding to C.

$$= \frac{7.5\times(25-7.5)\times 2}{25\times 5\sqrt{3}} = \frac{21}{10\sqrt{3}}$$

On this diagram, points M_1 and L_1 are marked projecting from M and L respectively. Now M_1L_1 is joined. The actual I.L.D for force in the member ML is given by AM_1L_1H.

$$\text{Ordinate } MM_1 = \frac{5}{7.5}\times\frac{21}{10\sqrt{3}} = \frac{50}{75}\times\frac{21}{10\sqrt{3}} = \frac{21}{15\sqrt{3}}$$

$$\text{Ordinate } LL_1 = \frac{15}{17.5}\times\frac{21}{10\sqrt{5}} = \frac{3\times 21}{35\sqrt{3}} = \frac{63}{35\sqrt{3}}$$



The maximum tensile force due to live load of 20 kN/m will occur when it occupies whole span like total dead load of 10 kN/m.

$$\text{Total udl} = (10 + 20) = 30 \text{ kN/m}$$

$\therefore$ Maximum tensile force = 30 × Area under loads

$$= 30\left[\left\{\frac{1}{2}\left\{\frac{21}{15\sqrt{3}}+\frac{9}{5\sqrt{3}}\right\}\times 5+\frac{1}{2}\times\frac{7}{5\sqrt{3}}\times 5+\frac{1}{2}\times\frac{9}{5\sqrt{3}}\times 15\right\}\right]$$

$$= 30\left\{2.5\times\frac{16}{5\sqrt{3}}+\frac{1}{10\sqrt{3}}(35+135)\right\}$$

$$= 30\left\{\frac{8}{\sqrt{3}}+\frac{17}{\sqrt{3}}\right\} = \frac{30\times 25}{\sqrt{3}}$$

$$= 10\sqrt{3}\times 25 = 250\sqrt{3} \text{ (tension)}$$

(ii)

Given:

$$\text{Major tensile stress, } \sigma_1 = 600 \text{ N/mm}^2$$

$$\text{Minor tensile stress, } \sigma_2 = 300 \text{ N/mm}^2$$

$$\text{Shear stress, } \tau = 450 \text{ N/mm}^2$$

The normal and tangential stresses are to be calculated on the two planes which are equally inclined to the planes of major tensile stress. This mean θ = 45° and 135°.

Normal stress, (σ_n) is given by equation,

$$\sigma_n = \left(\frac{\sigma_1+\sigma_2}{2}\right)+\left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta+\tau\sin 2\theta$$

When θ = 45°

$$\therefore \quad \sigma_n = \left(\frac{600+300}{2}\right)+\left(\frac{600-300}{2}\right)\cos(2\times 45^\circ)+450\sin(2\times 45^\circ)$$

$$= 450 + 150\cos 90^\circ + 450\sin 90^\circ$$

$$= 450 + 0 + 450 \times 1 = 900 \text{ N/mm}^2$$

When θ = 135°,

$$\sigma_n = \left(\frac{600+300}{2}\right)+\left(\frac{600-300}{2}\right)\cos(2\times 135^\circ)+450\sin(2\times 135^\circ)$$

$$= 450 + 150\cos 270^\circ + 450\sin 270^\circ$$

$$= 450 + 150 \times 0 + 450 = 900 \text{ N/mm}^2$$

Tangential stress (σ_t) is given by

$$\sigma_1 = \left(\frac{\sigma_1 - \sigma_2}{2}\right)\sin 2\theta - \tau\cos 2\theta$$

When $\theta = 45°$,

$$\sigma_t = \left(\frac{600-300}{2}\right)\sin 90° - 450\cos 90°$$

$$= 150 \times 1 - 450 \times 0 = 150 \text{ N/mm}^2$$

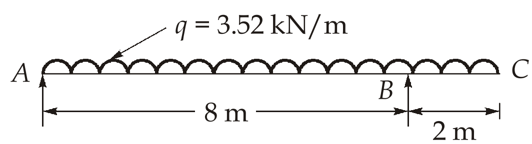
When $\theta = 135°$,

$$\sigma_t = \left(\frac{600-300}{2}\right)\sin 270° - 450\cos 270°$$

$$= 150 \times (-1) - 450 \times 0 = -150 \text{ N/mm}^2$$

Q.7 (b) Solution:

The single overhang beam *ABC* supporting uniformly distributed load is shown below.



Prestressing force in the cable, $P = 500$ kN

Self-weight of the beam = $(0.3 \times 0.9 \times 24) = 6.48$ kN/m

Live load on the beam = 3.52 kN/m

$\therefore$ Total load on beam = 10 kN/m

The reactions at A and B are obtained as,

$$R_A = 37.5 \text{ kN and } R_B = 62.5 \text{ kN}$$

$$M_B = (0.5 \times 10 \times 2^2) = 20 \text{ kNm}$$

The bending moment at a distance x from A is

$$M_x = 37.5x - 0.5 \times 10 \times x^2$$

For maximum bending moment, $(dM_x/dx) = 0$

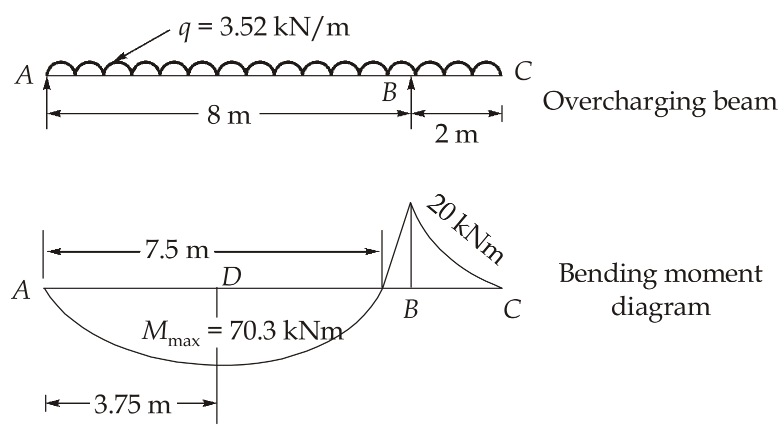
$$\therefore \quad 37.5 - 10x = 0$$

Hence, $x = 3.75$ m from A

$$\therefore \quad M_{max} = (37.5 \times 3.75 - 0.5 \times 10 \times 3.75^2) = 70.3 \text{ kNm}$$

$$M_x = 0 \text{ when } 5x^2 = 37.5x$$

$$\therefore \quad x = 7.5 \text{ m}$$



Hence, the eccentricity of the cable at the position of maximum bending moment is computed as,

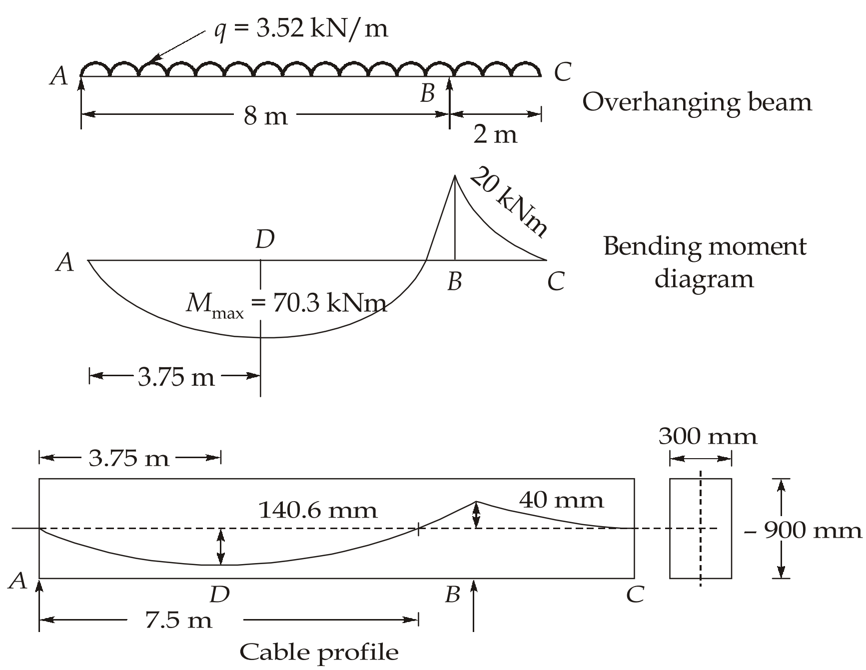
$$e = \left(\frac{M_{max}}{P}\right) = \frac{(70.3 \times 10^6)}{(500 \times 10^3)} = 140.6 \text{ mm}$$

Eccentricity of the cable at B is calculated as,

$$e = \left(\frac{M_B}{P}\right) = \frac{(20 \times 10^6)}{(500 \times 10^3)} = 40 \text{ mm}$$

Since the bending moment at *A* is zero, the cable is concentric at this point.

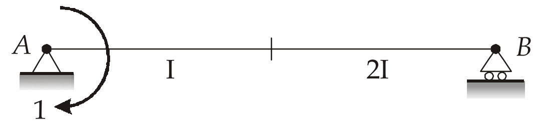
The cable profile is parabolic with eccentricities of 140.6 mm below the centroidal axis at *D* and 40 mm above the centroidal axis at the support section *B* and with zero eccentricities at *A* and *C* as shown in figure.

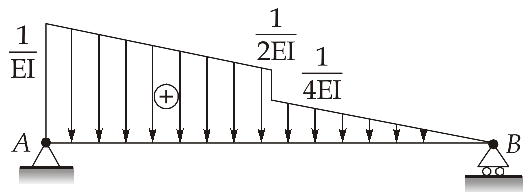


Prestressed concrete single overhang beam

Q.7 (c) Solution:

Apply unit moment in co-ordinate direction (1).





$\frac{M}{EI}$ diagram

Using conjugate beam theorem:

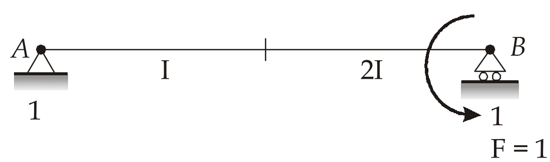
$$f_{11} = R_A = \frac{\left[\left(\frac{1}{2}\times\frac{L}{2}\times\frac{1}{2EI}\times\left(\frac{L}{2}+\frac{2}{3}\frac{L}{2}\right)+\frac{1}{2EI}\times\frac{L}{2}\left(\frac{L}{2}+\frac{L}{4}\right)+\left(\frac{1}{2}\times\frac{1}{4EI}\times\frac{L}{2}\times\frac{2}{3}\times\frac{L}{2}\right)\right)\right]}{L}$$

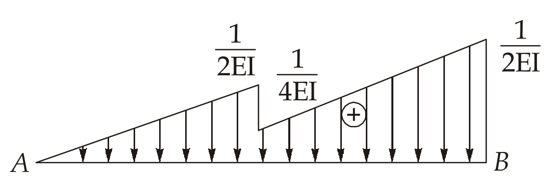
$$= \frac{5L}{48EI}+\frac{3L}{16EI}+\frac{L}{48EI}=\frac{5L}{16EI}$$

$$f_{21} = R_B = \left[\frac{1}{2}\times\frac{L}{2}\times\frac{1}{2EI}+\frac{1}{2EI}\times\frac{L}{2}+\frac{1}{2}\times\frac{1}{4EI}\times\frac{L}{2}\right]-R_A$$

$$= \frac{L}{8EI}+\frac{L}{4EI}+\frac{1}{16EI}-\frac{5L}{16EI}=\frac{L}{8EI}$$

Apply unit moment in co-ordinate direction (2).





$\frac{M}{EI}$ diagram

Using conjugate beam theorem:

$$f_{12} = R_A = \frac{\frac{1}{2}\times\frac{1}{2EI}\times\frac{L}{2}\times\left(\frac{L}{2}+\frac{1}{3}\times\frac{L}{2}\right)+\frac{1}{4EI}\times\frac{L}{2}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4EI}\times\frac{L}{2}\times\frac{1}{3}\times\frac{L}{2}}{L}$$

$$\Rightarrow \quad R_A = \frac{L}{12EI} + \frac{L}{32EI} + \frac{L}{96EI} = \frac{L}{8EI} = f_{21}$$

$$f_{22} = R_B = \frac{1}{2} \times \frac{1}{2EI} \times \frac{L}{2} + \frac{1}{4EI} \times \frac{L}{2} + \frac{1}{2} \times \frac{1}{4EI} \times \frac{L}{2} - R_A$$

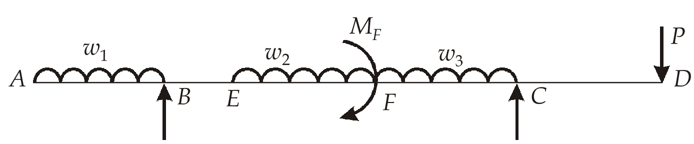
$$= \frac{L}{8EI} + \frac{L}{8EI} + \frac{L}{16EI} - \frac{L}{8EI} = \frac{3L}{16EI}$$

Hence the flexibility metric is:

$$[f] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} = \frac{L}{16EI} \begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix}$$

Q.8 (a) Solution:

Wherever BMD is 2°, loading will be UDL



$$M_B = -\frac{w_1 \times 4^2}{2} = -24 \text{ kN-m}$$

$$\Rightarrow \quad w_1 = \frac{24 \times 2}{16} = 3 \text{ kN/m}$$

$$M_E = 0$$

$$\Rightarrow \quad 12 \times 4 = R_B \times 2$$

$$\Rightarrow \quad R_B = 24 \text{ kN}$$

$$M_F(\text{Just to left}) = 24 \text{ kN-m}$$

$$\Rightarrow \quad -3 \times 4 \times 8 + 24 \times 6 - w_2 \times 4 \times 2 = 24 \text{ kN-m}$$

$$\Rightarrow \quad -96 + 144 - w_2 \times 8 = 24$$

$$\Rightarrow \quad w_2 \times 8 = 3 \text{ kN/m}$$

$$M_F(\text{Just to right}) = 0$$

$$\Rightarrow \quad +24 + M_F = 0$$

$$\Rightarrow \quad M_F = -24 \text{ kN-m}$$

$$M_C = -24 \text{ kN-m}$$

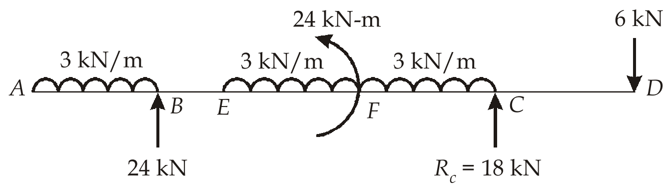
$$\Rightarrow \quad -3 \times 4 \times 12 + 24 \times 10 - 3 \times 4 \times 6 - 24 - w_3 \times 4 \times 2 = -24 \text{ kN}$$

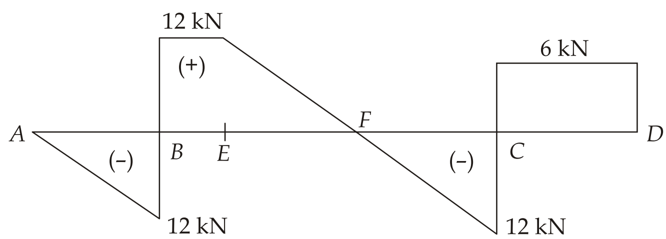
$$\Rightarrow \quad w_3 = 3 \text{ kN/m}$$

$$M_C(\text{Just to right}) = -24 \text{ kN-m}$$

$\Rightarrow \qquad -P \times 4 = -24$ kN-m

$\Rightarrow \qquad P = 6$ kN

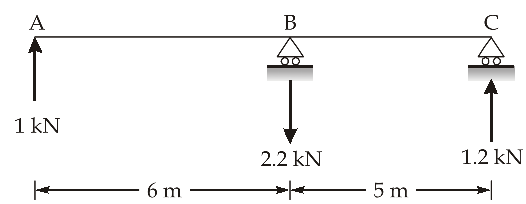




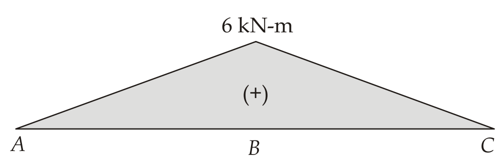
Shear force diagram

Q.8 (b) Solution:

ILD of R_A: Remove restraint offered by reaction at end A and apply unit displacement in the direction of reaction R_A.

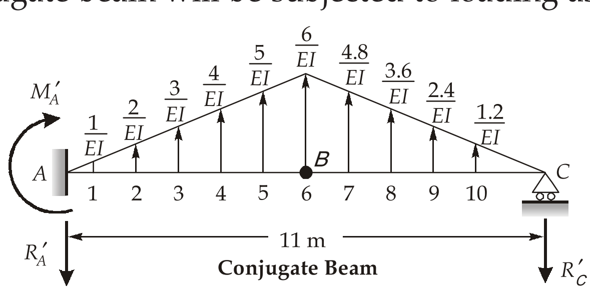


The BMD for above loading is shown below.



BMD

Corresponding conjugate beam will be subjected to loading as M/EI diagram.



Conjugate Beam

In conjugate beam:

Reactions:

$\Sigma F_y = 0;$ $$R_A' + R_C' = \frac{1}{2}\times 11\times\frac{6}{EI} = \frac{33}{EI}$$

$\Sigma M_B = 0;$ $$R_C' + (5) = \frac{1}{2}\times 5\times\frac{6}{EI}\times\frac{5}{3}$$

$$\therefore \quad R_C' = \frac{5}{EI}$$

$$\therefore \quad R'_A = \frac{33}{EI} - R_C' = \frac{33}{EI} - \frac{5}{EI} = \frac{28}{EI}$$

Also, $\Sigma M_A = 0$

$$\Rightarrow \quad M'_A + \frac{5}{EI}\times 11 - \frac{1}{2}\times 6\times\frac{6}{EI}\left(\frac{2}{3}\times 6\right) - \frac{1}{2}\times 5\times\frac{6}{EI}\left(6+\frac{5}{3}\right) = 0$$

$$\Rightarrow \quad M_A' = \frac{132}{EI}$$

We know, the bending moment in conjugate beam shows deflection in real beam. Hence,

$$\delta_1 = M_1 = \frac{132}{EI} - \frac{28}{EI}\times 1 + \frac{1}{2}\times 1\times\frac{1}{EI}\times\frac{1}{3} = \frac{104.167}{EI}$$

$$\delta_2 = M_2 = \frac{132}{EI} - \frac{28}{EI}\times 2 + \frac{1}{2}\times 2\times\frac{2}{EI}\times\frac{2}{3} = \frac{77.33}{EI}$$

$$\delta_3 = M_3 = \frac{132}{EI} - \frac{28}{EI}\times 3 + \frac{1}{2}\times 3\times\frac{3}{EI}\times 1 = \frac{52.5}{EI}$$

$$\delta_4 = M_4 = \frac{132}{EI} - \frac{28}{EI}\times 4 + \frac{1}{2}\times 4\times\frac{4}{EI}\times\frac{4}{3} = \frac{30.67}{EI}$$

$$\delta_5 = M_5 = \frac{132}{EI} - \frac{28}{EI}\times 5 + \frac{1}{2}\times 5\times\frac{5}{EI}\times\frac{5}{3} = \frac{12.83}{EI}$$

$$\delta_6 = M_B = 0 \text{ (Hinged)}$$

$$\delta_{10} = M_{10} = -\frac{5}{EI}\times 1 + \frac{1}{2}\times 1\times\frac{1.2}{EI}\times\frac{1}{3} = -\frac{4.8}{EI}$$

$$\delta_9 = M_9 = -\frac{5}{EI}\times 2 + \frac{1}{2}\times 2\times\frac{2.4}{EI}\times\frac{2}{3} = -\frac{8.4}{EI}$$

$$\delta_8 = M_8 = -\frac{5}{EI}\times 3 + \frac{1}{2}\times 3\times\frac{3.6}{EI}\times 1 = -\frac{9.6}{EI}$$

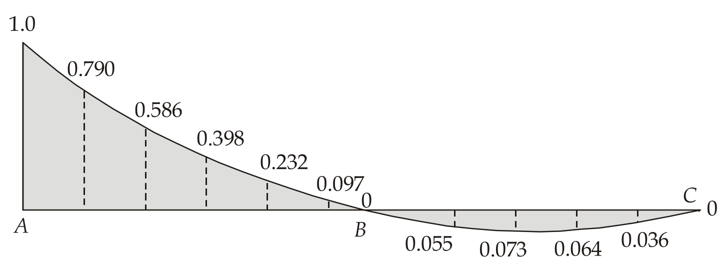
$$\delta_7 = M_7 = -\frac{5}{EI}\times 4+\frac{1}{2}\times 4\times\frac{4.8}{EI}\times\frac{4}{3} = -\frac{7.2}{EI}$$

The moments calculated above represents deflection in real beam. It is also noted that deflection at A in real beam will be unity.

But, $$M_A' = \frac{132}{EI}(\uparrow)$$

Hence, divide value of bending moment in conjugate beam by $\frac{132}{EI}$.

Point	A	1	2	3	4	5	B	7	8	9	10	C
B.M. in conjugate beam	1	0.790	0.586	0.398	0.232	0.097	0	–0.055	–0.073	–0.064	–0.036	0
Deflection in real beam	1	0.790	0.586	0.398	0.232	0.097	0	–0.055	–0.073	–0.064	–0.036	0



ILD for R_A

Q.8 (c) Solution:

(i)

Advantages of high strength friction grip bolts: High strength friction grip (HSFG) bolts have replaced the rivet because of their distinct advantages as discussed below:

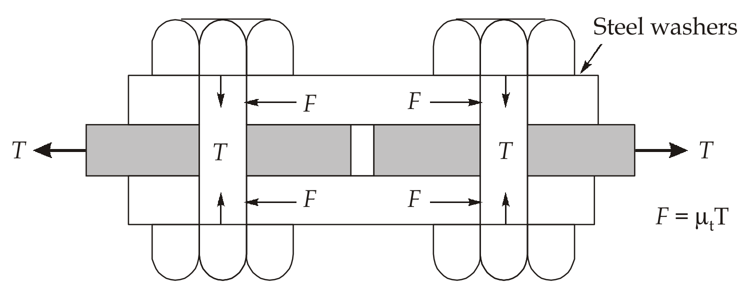
- These provide a rigid joint. There is no slip between the elements connected.
- Large tensile stresses are developed in the bolts, which in turn provide large clamping force to the elements connected.
- Because of clamping action, load is transmitted by friction only and the bolts are not subjected to shear and bearing.
- The frictional resistance is effective outside the hole and therefore lesser load is through the net section. Thus, possibility of failure at the net section is minimized.
- The tension in the bolts is uniform. Also, the bolts are tensioned upto proof load and hence, the nuts are prevented from loosening.

- For some strength, lesser number of bolts are required as compared to rivets which brings an overall economy.

Principle of load transfer in HSFG bolts: The connections made with high strength bolts may be **slip resistant** or **bearing type**.

In slip resistant connections, a very dependable high tension is developed in high strength bolts resulting in large clamping force and large amount of frictional resistance to slipping. The entire forces are resisted by friction and the joints are not really subjected to shear or bearing. The bolts are first brought to snug-tight condition and then are further tightened. Joint with so tightened bolts are also called pre-tensioned joints. However, when the load exceeds the frictional resistance there will be slippage and consequently the bolts will be subjected to shear and bearing and will behave like bearing type joints.

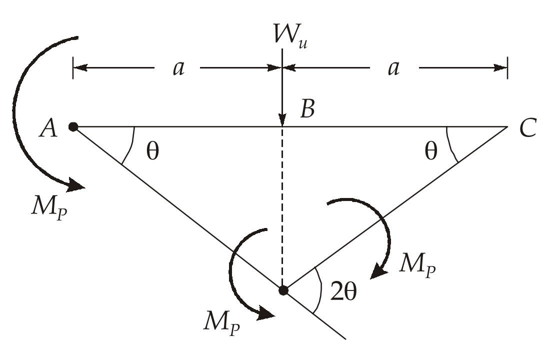
Also, when the high strength bolts are not tightened sufficiently so as to significantly squeeze the plates together, there will be negligible friction between the plates. On the application of load, the plates slip a little and the load will tend to shear the bolts off at the interface and press or bear against the side of the bolts. The load transfer in this case will be as that for bearing type connection.



Slip resistance
High strength bolted connection

(ii)

Cantilever beam *ABC*,

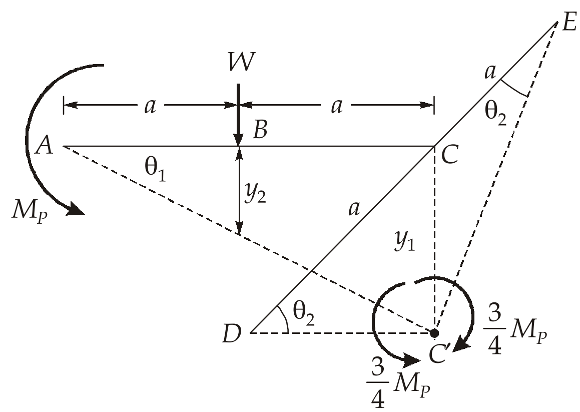


Total work done by internal force = Total work done by external force

$$\Rightarrow \quad W_u a\theta = M_p\theta + M_p(2\theta)$$

$$\Rightarrow \quad W_u = \frac{3M_P}{a} \quad \text{...(i)}$$

Simply supported beam *DCE*,



In ΔACC′, $\theta_1 = \dfrac{y_1}{2a}$

In ΔDCC′, $\theta_2 = \dfrac{y_1}{a}$

$$\therefore \quad 2\theta_1 = \theta_2$$

Total workdone by internal force = Total workdone by external force

$$\Rightarrow \quad M_P\theta_1 + \frac{3}{4}M_P\theta_2 + \frac{3}{4}M_P\theta_2 = Wy_2$$

$$\Rightarrow \quad M_P\theta_1 + \frac{3}{2}M(2\theta_1) = W(a\theta_1)$$

$$\Rightarrow \quad 4M_p\theta_1 = Wa\theta_1$$

$$\Rightarrow \quad W = W_u = \frac{4M_P}{a} \quad \text{...(ii)}$$

The collapse load is minimum of (*i*) and (*ii*) i.e.

$$W_u = \frac{3M_P}{a}$$

■■■■