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## ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

### Electrical Engineering

#### Test-4 : Electrical Machines + Analog Electronics + Control Systems

Name : .....

Roll No :

##### Test Centres

##### Student's Signature

Delhi ☒ Bhopal ☐ Jaipur ☐  
Pune ☐ Kolkata ☐ Hyderabad ☐

##### Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

##### FOR OFFICE USE

| Question No.                | Marks Obtained |
|-----------------------------|----------------|
| Section-A                   |                |
| Q.1                         | 45             |
| Q.2                         | 49             |
| Q.3                         |                |
| Q.4                         | 54             |
| Section-B                   |                |
| Q.5                         | 44             |
| Q.6                         | 35             |
| Q.7                         |                |
| Q.8                         |                |
| <b>Total Marks Obtained</b> | <b>227</b>     |

Signature of Evaluator

Cross Checked by

Sourabh  
Kumar

## IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

### DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

### DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.



## Section A : Electrical Machines + Analog Electronics + Control Systems

- (a) A 4-pole, 3- $\phi$  Slip Ring Induction Motor (SRIM) is used as a frequency changer. Its stator is excited from 3-phase, 50 Hz supply. A load requiring 3-phase, 20 Hz supply is connected to the star-connected rotor through three slip rings of SRIM.

(i) At what two speeds the prime mover should drive the rotor of this SRIM?

(ii) Find the ratio of two voltages available at the slip rings at the two speeds.

[12 marks]

(i)  $f = 50 \text{ Hz}$

$$f_r = s f$$

For  $f_r = 20 \text{ Hz}$

$$s = \frac{20}{50} = 0.4$$

$$N_r = N_s (1-s) \quad (\text{rotor speed})$$

For positive slip

$$N_r = N_s (1-0.4)$$

$$N_r = 1500 (1-0.4)$$

$$\boxed{N_r = 900 \text{ rpm}}$$

$$N_s = \frac{120f}{P}$$

$$N_s = \frac{120 \times 50}{4}$$

$$(N_s = 1500 \text{ rpm})$$

For negative slip

$$N_r = 1500 (1+0.4)$$

$$\boxed{N_r = 2100 \text{ rpm}}$$

So, Prime mover should drive the rotor of this SRIM at 900 rpm or 2100 rpm to get the rotor frequency of 20 Hz.

(ii) At slip ring voltage available is given by

$$E_r = s E_s$$

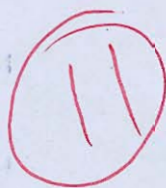
$$\text{So, } E_{s1} = 0.4 E_s$$

$$E_{s2} = -0.4 E_s$$

$$\text{Hence, } \frac{E_{s1}}{E_{s2}} = \frac{0.4}{-0.4}$$

$$\boxed{\frac{E_{s1}}{E_{s2}} = -1}$$

i.e., opposite polarity of voltages for same frequency at two different speeds.



Good  
Approach



- (b) The open-loop transfer function of a unity feedback ac position control system is

$$G(s) = \frac{10K}{s(1+0.1s)}$$

Find the minimum value of the amplifier gain  $K$  so that when the input shaft rotates at  $\frac{1}{2}$  revolution per second, the steady-state velocity error is  $0.2^\circ$ . With that value of  $K$ , what will be the value of damping factor and natural frequency?

[12 marks]

$$\begin{aligned} \text{Given input} \Rightarrow R(s) &= \frac{A}{s^2} \quad (\text{ramp input}) \\ &= \frac{1/2}{s^2} \quad (A = 1/2) \end{aligned}$$

For Ramp input

$$e_{ss} = \frac{A}{K_v} \quad \text{and} \quad K_v = \lim_{s \rightarrow 0} s G(s)$$

$$\Rightarrow K_v = \lim_{s \rightarrow 0} s \times \frac{10K}{s(1+0.1s)} = K$$

$$\therefore e_{ss} = \frac{1/2}{K} = \frac{1}{2K} = \frac{0.2}{360}$$

$$\Rightarrow \boxed{K = 900}$$

Now Characteristic Equation of the System is

$$1 + G(s) = 0$$

$$s + 0.1s^2 + 10K = 0$$

$$\Rightarrow s^2 + 10s + 100K = 0$$

$$\Rightarrow s^2 + 10s + 100 \times 900 = 0$$

Comparing it with standard 2<sup>nd</sup> order characteristic Equation  $\Rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$$\omega_n = \sqrt{900 \times 100}$$

$$\boxed{\omega_n = 300 \text{ rad/s}} \Rightarrow \text{natural frequency.}$$

$$2\zeta\omega_n = 10$$

$$2 \times \zeta \times 300 = 10$$

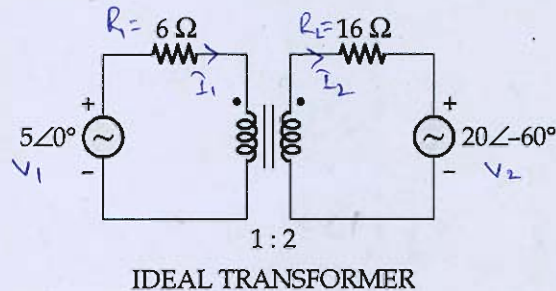
$$\boxed{\zeta = \frac{1}{60} = 0.0167}$$

↳ damping factor.

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(c) In the figure shown below:

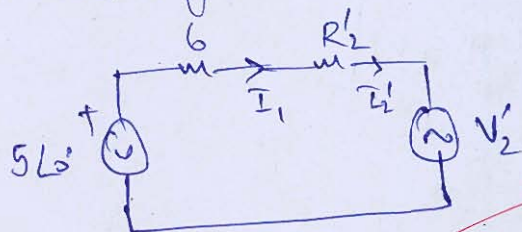


Calculate :

- The power delivered by each source.
- The power dissipated in each resistor.

[12 marks]

(i) Referring the circuit on primary side



$$V_2' = \frac{1}{2} 20 \angle -60^\circ$$

$$V_2' = 10 \angle -60^\circ$$

$$\text{or, } \left( \frac{V_1}{V_2} = \frac{N_1}{N_2} \right)$$

and  $R_2' = \frac{R_2}{\left(\frac{1}{2}\right)^2}$

$$R_2' = \frac{1}{4} \times 16 = 4 \Omega$$

$$\text{So, } I_1 = \frac{5 \angle 0^\circ - 10 \angle -60^\circ}{6 + 4} = \frac{\sqrt{3}}{2} \angle 90^\circ \text{ A}$$

Power delivered by Source 1

$$P_1 = V_1 \times I_1 \times \cos(\text{angle between } V_1 \text{ and } I_1)$$

$$= 5 \times \frac{\sqrt{3}}{2} \times \cos(90^\circ) = 0$$

$$\boxed{P_1 = 0 \text{ W}}$$

Power delivered by Source 2

$$P_2 = V_2' \times I_2' \times \cos(\text{angle between } V_2' \text{ and } I_2')$$

$$= 10 \times \frac{\sqrt{3}}{2} \times \cos(90 + 60^\circ) = -15/2 \quad (I_1 = I_2')$$

Since the current is going inside the source 2  
Hence this gives the Power absorbed

$\therefore$  Power delivered by Source 2 is

$$P_2 = 15/2 = 7.5 \text{ W}$$

(ii) Power dissipated in  $R_1$

$$P_{R_1} = I_1^2 \times R_1$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 \times 6 \Rightarrow P_{R_1} = 4.5 \text{ W}$$

and for  $R_2' \Rightarrow$  same for  $R_2$

$$P_{R_2} = I_1^2 \times R_2'$$

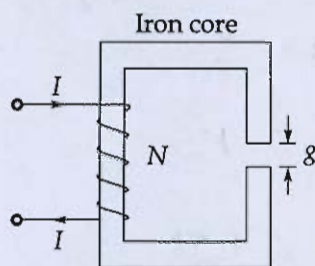
$$= \left(\frac{\sqrt{3}}{2}\right)^2 \times 4$$

$$P_{R_2} = 3 \text{ W}$$

Good  
Approach



- (d) For the magnetic circuit shown below:



Length of iron path = 120 cm,  $g = 0.5$  cm, area of cross-section of iron =  $5 \times 5$  cm<sup>2</sup>,  $\mu_r = 1500$ ,  $I = 2$  A,  $N = 1000$  turns.

Calculate and compare the field - energy stored and field energy density in iron as well as in air gap. Neglect fringing and leakage flux.

[12 marks]

$$\text{Field energy stored} = \frac{1}{2} LI^2$$

$$\text{and } L = \frac{N^2}{R_c}$$

$$\text{from the given figure} \Rightarrow R_c = R_{\text{core}} + R_{\text{airgap}} = R_{\text{core}} + R_g$$

$$\text{So, } R_{\text{core}} = \frac{1}{\mu_0 \mu_r} \times \frac{l_c}{A_c}$$

$$= \frac{1}{4\pi \times 10^{-7} \times 1500} \times \frac{120 \times 10^{-2}}{5 \times 5 \times 10^{-4}}$$

$$R_{\text{core}} = 254647.91 \text{ AT/Wb} \quad \text{--- (1)}$$

Now

$$R_g = \frac{1}{\mu_0} \times \frac{l_g}{A_g} \quad \left( \text{neglecting fringing effect, } A_g = A_c \right)$$

$$= \frac{1}{4\pi \times 10^{-7}} \times \frac{0.5 \times 10^{-2}}{25 \times 10^{-4}} = 1591549.4 \text{ AT/Wb} \quad \text{--- (2)}$$

$$R_c = 254647.91 + 1591549.4 = 1846197.34 \text{ AT/Wb}$$

$$X = \frac{N^2}{L_c} = \frac{(1000)^2}{1846197.34}$$

$$L = 0.5416 \text{ H}$$

$$\text{field energy} = \frac{1}{2} \times L I^2$$

$$W_f = \frac{1}{2} \times 0.5416 \times 2^2$$

$$W_f = 1.083 \text{ J} \quad \leftarrow \text{Total field energy}$$

Now field energy stored in iron

$$W_{fi} = \frac{1}{2} \times \frac{1000^2}{254647.91} \times 2^2$$

$$W_{fi} = 7.854 \text{ J}$$

$$W_{fg} = \frac{1}{2} \times \frac{1000^2}{1591549.4} \times 2^2$$

$$W_{fg} = 1.2566 \text{ J}$$

(air gap)

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Energy density

$$E_i = \frac{W_{fi}}{A_i \times l_i} = \frac{7.854}{25 \times 154 \times 12 \times 10^{-2}}$$

$$E_i = 2618 \text{ J/m}^3$$

$$E_g = \frac{W_{fg}}{A_g \times l_g} = \frac{1.2566}{25 \times 154 \times 0.5 \times 10^{-2}}$$

$$E_g = 100528 \text{ J/m}^3$$

On comparing field energy and energy density of iron core with air gap we can say that air gap can store more energy than iron.



- (e) The short-circuited tests on two single-phase transformer gave the following results:  
 200 kVA : 3% rated voltage ; rated current at 0.25 power factor lagging  $\Rightarrow A$   
 500 kVA : 4% rated voltage ; rated current at 0.3 power factor lagging  $\Rightarrow B$

These two transformers are connected in parallel. How do they share a load of 560 kW at 0.8 power factor lagging?

[12 marks]

For transformer A

$$Z_A = \frac{0.03}{1.0} \angle \cos^{-1}(0.25)$$

$$(Z_{pu} = \frac{V}{I} \angle \cos^{-1} \phi)$$

$$Z_A = 7.5 \times 10^{-3} + j0.029 \text{ pu} \rightarrow \text{on their own base}$$

and similarly

$$Z_B = \frac{0.04}{1.0} \angle \cos^{-1} 0.3$$

$$Z_B = 0.012 + j0.038 \text{ pu} \rightarrow \text{on their own base}$$

Taking  $S_{base} = 500 \text{ kVA}$

$$\text{So, } Z_{new} = Z_{old} \times \frac{S_{new}}{S_{old}}$$

$$\text{So, } Z_A = (7.5 \times 10^{-3} + j0.029) \times \frac{500}{200}$$

$$Z_A = 0.01875 + j0.0725$$

For parallel operation

$$S_A^* = \frac{S_L^*}{Z_A \left( \frac{1}{Z_A} + \frac{1}{Z_B} \right)} = \frac{S_L^*}{\left( 1 + \frac{Z_A}{Z_B} \right)}$$

$$S_L = \frac{P}{\cos \phi} \angle \cos^{-1} \phi = \frac{560}{0.8} \angle \cos^{-1} 0.8$$

$$= 560 + j420$$

$$S_A^* = \frac{560 + j420}{1 + \frac{0.01875 + j0.0721}{0.012 + j0.038}}$$

$$S_A^* = 189.417 - j152.54 \text{ KVA}$$

$$\boxed{\begin{aligned} S_A &= 189.417 + j152.54 \text{ KVA} \\ P_A &= 189.417 \text{ KW} \end{aligned}}$$

$$S_B = S_L - S_A$$

$$= 560 + j420 - 189.417 - j152.54$$

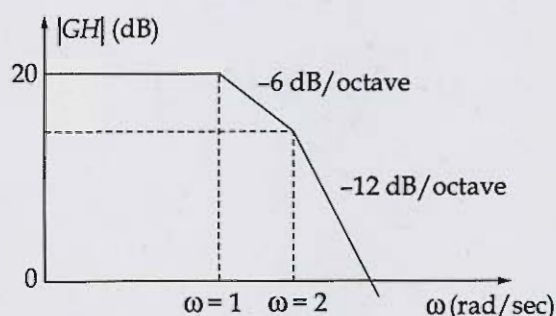
$$\boxed{\begin{aligned} S_B &= 370.58 + j267.46 \text{ KVA} \\ P_B &= 370.58 \text{ KW} \end{aligned}}$$

$$\boxed{\begin{aligned} S_B &= 370.58 + j267.46 \text{ KVA} \\ P_B &= 370.58 \text{ KW} \end{aligned}}$$

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- (a) The asymptotic approximation to the log-magnitude versus frequency plot (Bode plot) of a unity feedback control system is shown in the figure. The system is a minimum phase system.



Determine :

- Gain crossover frequency in rad/sec.
- Phase crossover frequency.
- Gain margin in dB.
- Phase margin in degrees.

[20 marks]

From the given Bode plot open loop transfer function will be,

$$G_H(s) = \frac{K}{(1+s)(1+s/2)} = \frac{2K}{(s+1)(s+2)}$$

and,  $y = mx + c$

$$20 = 0 + 20 \log K \Rightarrow K = 10$$

$$G_H(s) = \frac{20}{(s+1)(s+2)}$$

At  $(\omega = \omega_{gc}) \Rightarrow |G_H(j\omega)| = 1$

$$\Rightarrow |G_H(j\omega)| = \frac{20}{\sqrt{\omega^2+1} \sqrt{\omega^2+4}} = 1$$

$$\Rightarrow 400 = (\omega^2+1)(\omega^2+4)$$

$$\Rightarrow \omega^4 + 5\omega^2 + 4 - 400 = 0$$

$$\omega^4 + 5\omega^2 - 396 = 0$$

Solving this, we get

$$\omega = 4.19 \text{ rad/s}$$

$$\boxed{\omega = \omega_{gc} = 4.19 \text{ rad/s}}$$

(iii) At  $\omega = \omega_{pc}$

$$\angle G H(\omega) = -180^\circ$$

$$\Rightarrow \cancel{180^\circ} = 90^\circ$$

$$-\tan^{-1}(\omega) - \tan^{-1}(\omega/2) = -180^\circ$$

$$\tan^{-1}\left(\frac{\omega + \omega/2}{1 - \omega^2/2}\right) = 180^\circ$$

Since the system is 2nd order, its phase will not cross  $-180^\circ$  till  $\infty$

Hence,

$$\boxed{\omega_{pc} = \infty}$$

$$(iv) G.M = \frac{1}{|G H(\omega)|_{\omega = \omega_{pc}}}$$

$$\text{At } \omega = \omega_{pc} \Rightarrow |G H(\omega)| = 0$$

Hence,

$$\boxed{G.M = \infty}$$

$$(iv) P.M = 180^\circ + \phi_{at\ wge}$$

$$\begin{aligned}\phi_{at\ wge} &= -\tan^{-1}(w_g) - \tan^{-1}(w_{ge}/2) \\ &= -\tan^{-1}(4.19) - \tan^{-1}(4.19/2) \\ &= -141.06\end{aligned}$$

$$P.M = 180^\circ - 141.06$$

$$P.M = 38.94^\circ$$

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Good  
Approach

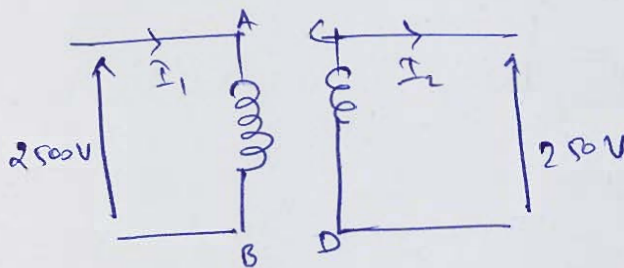


Q.2(b) A 150 kVA, 2500/250 V, single-phase two winding transformer is to be used as an auto transformer for stepping up the voltage from 2500 V to 2750 V. At rated load, the two winding transformer has 2.5% loss, 3% voltage regulation and 4% impedance. For the auto transformer, determine the followings:

- Voltage and current rating.
- kVA rating.
- Efficiency.
- Percentage impedance.
- Regulation, and
- Short circuit current on each side.

[20 marks]

Given single phase transformer



$$I_1 = \frac{S}{V_1} = \frac{150 \times 10^3}{2500} = 60 \text{ A}$$

$$I_2 = \frac{S}{V_2} = 600 \text{ A}$$

$$2.5\% \text{ loss} \Rightarrow R_{pu} = 0.025$$

$$\text{and } Z_{pu} = 4\% \Rightarrow Z_{pu} = 0.04$$

$$X_{pu} = \sqrt{Z_{pu}^2 - R_{pu}^2} = \sqrt{0.04^2 - 0.025^2}$$

$$X_{pu} = 0.03122 \text{ pu}$$

$$\text{and } V.R = 3\% = [R_{pu} \cos \phi + X_{pu} \sin \phi] \times 100$$

↳ full loading load

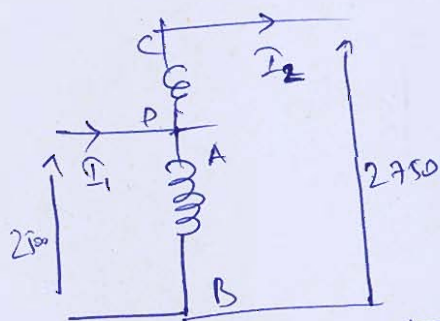
$$V.R = Z_{pu} \times \cos(\theta_{eq} - \phi)$$

$$\theta_{eq} = \tan^{-1}(X/R) = \tan^{-1}\left(\frac{0.03122}{0.025}\right) = 51.32^\circ$$

$$0.03 = 0.04 \times \cos(51.32 - \phi)$$

$$\boxed{\phi = 9.91^\circ}$$

(i) Now 2 winding transformer is converted into auto transformer



$$I_2 = I_{CO} = \frac{150 \times 10^3}{250} = 600 \text{ A}$$

$$I_2 = 600 \text{ A}$$

Voltage ratings  $V_1 = 2500 \text{ V}$ ,  $V_2 = 2750 \text{ V}$

$$\text{Hence as, } V_1 I_1 = V_2 I_2$$

$$\text{So, } 2500 \times I_1 = 2750 \times 600$$

$$I_1 = 660 \text{ A}$$

(ii) KVA rating  $= V_2 I_2 = 2750 \times 600$

$$\text{KVA rating} = 1650 \text{ KVA}$$

(iii)  $\eta = \frac{P_{out}}{P_{out} + \text{loss}} \times 100$

$$= \frac{1650 \times \cos \phi \times 100}{1650 \times \cos \phi + \frac{2.5}{100} \times 150} = \frac{1650 \times \cos(9.91) \times 100}{1650 \times \cos(9.91) + \frac{2.5}{100} \times 150}$$

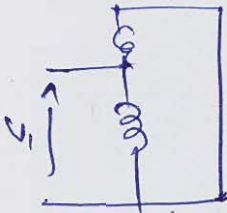
$$\eta = 99.77 \%$$

$$(iv) (Z_{pu})_{auto} = (Z_{pu})_{2\text{wgs}} = 0.04 \text{ pu}$$

$$(Z_{pu})_{auto} = 4\%$$

(v)

$$(vi) I_{sc1} = \frac{V_1}{Z_1 \parallel Z_2}$$

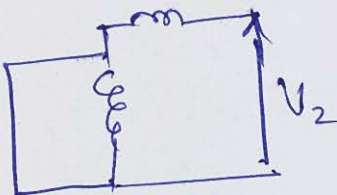


$$I_{sc1} = \frac{2750}{0.5} = 2 \text{ pu}$$

$$I_{sc1} = 2 \times 660$$

$$I_{sc1} = 1320 \text{ A}$$

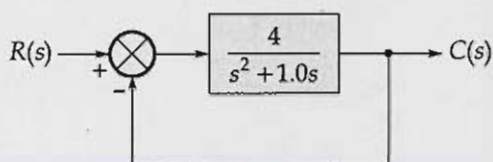
$$\text{and } I_{sc2} = \frac{2750}{Z_2}$$



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- (c) A closed-loop control system with unity feedback is shown in figure below. By using derivative control, the damping ratio is to be made 0.75. Determine the value of  $T_d$ . Also determine the rise time, peak time and peak overshoot without derivative control and with derivative control. The input to the system is a unit-step.



[20 marks]

without derivative control

Characteristic equation  $\Rightarrow 1 + G(s) = 0$

$$\Rightarrow 1 + \frac{4}{s^2 + s} = 0 \Rightarrow s^2 + s + 4 = 0$$

On comparing it with standard 2<sup>nd</sup> order  
Characteristic equation,  $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

we get

$$\omega_n = 2$$

$$\text{and } 2\zeta \times 2 = 1 \Rightarrow \zeta = 0.25$$

$$t_r = \frac{\pi - \theta}{\omega_n} = \frac{\pi - \cos^{-1}(0.25)}{2}$$

$$t_r = 0.9117 \text{ sec} \rightarrow \text{rise time}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$= \frac{\pi}{2\sqrt{1 - 0.25^2}}$$

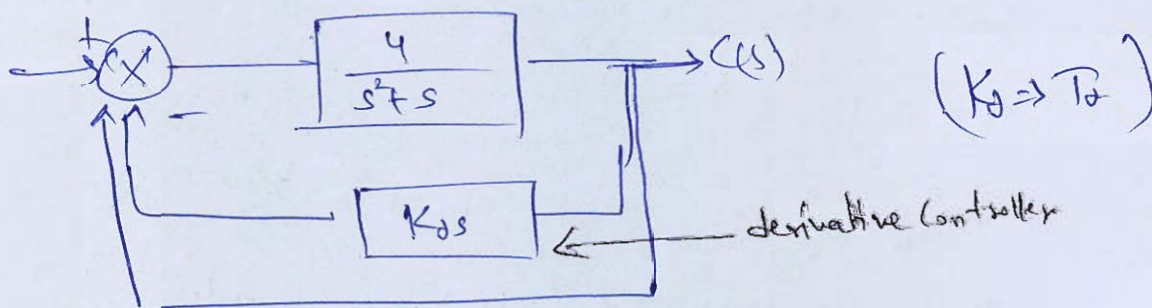
$$t_p = 1.622 \text{ sec} \rightarrow \text{Peak time}$$

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$$= e^{-0.25\pi/\sqrt{1-0.25^2}} = 0.444$$

$$\boxed{\text{maximum overshoot} = 0.444}$$

Now with derivative Controller ( $\zeta_c = 0.75$ )



$$\text{OLTF} \Rightarrow \frac{\frac{4}{s(s+1)}}{1 + \frac{4 \times K_d s}{s(s+1)}} = \frac{4}{s^2 + (4K_d + 1)s}$$

Characteristic Equation  $\Rightarrow$

$$1 + \text{OLTF} = 0$$

$$\Rightarrow \cancel{s^2 + 4 + K_d} \quad s^2 + (4K_d + 1)s + 4 = 0$$

On Comparing with standard 2nd order  
Characteristic equation,

$$n = 2$$

$$\cancel{2\zeta\omega_n} + 2\zeta\omega_n = 4K_d + 1$$

$$2 \times 0.75 \times 2 = 4K_d + 1$$

$$\boxed{K_d = T_d = 0.5}$$

Now Characteristic Equation with derivative control

$$\cancel{s^2 \times 2} \quad s^2 + 2 \times 0.75 \times 2 s + 2^2 = 0$$

$$\Rightarrow \cancel{s^2 \times}$$

$$t_p = \frac{\pi - \theta}{\omega_n} = \frac{\pi - \cos^{-1}(0.75)}{2}$$

$$\boxed{t_p = 1.21 \text{ sec.}}$$

$$t_d = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = \frac{\pi}{2 \sqrt{1 - 0.75^2}}$$

$$\boxed{t_d = 2.375 \text{ sec.}}$$

(15)

$$M_p = e^{-\zeta \omega_n t_p / \sqrt{1 - \zeta^2}}$$

$$= \frac{e^{-0.75 \pi / \sqrt{1 - 0.75^2}}}{e}$$

$$\cancel{M_p = 0.0284}$$

$$\boxed{M_p = 0.0284}$$





- 3 (a) When the primary of a transformer is energized at rated voltage of 11000 V and at rated frequency of 50 Hz, it takes 3.2 A and 2400 watt at no-load. Another transformer has all its core dimension  $\sqrt{2}$  times the corresponding core dimension of the first transformer. Number of primary turns, type of core material and lamination thickness are the same in both the transformers. If the primary of the second transformer is energized from 22000 V, 50 Hz supply, calculate the no-load current and power drawn by it.

[20 marks]







Q.3 (b) A 460 V, 25 hp, 60 Hz, 4-pole, Y-connected wound rotor induction motor has the following impedances per phase referred to stator side is  $a$  :

$$R_1 = 0.641 \, \Omega, R_2 = 0.332 \, \Omega$$

$$X_1 = 1.106 \, \Omega, X_2 = 0.464 \, \Omega, X_m = 26.3 \, \Omega$$

- (i) What is maximum torque of this motor? At what slip and speed does it occur?
- (ii) What is the starting torque of this motor?
- (iii) When the rotor resistance is doubled, what is the speed at which the maximum torque now occurs? What is the new starting torque of the motor?

[20 marks]







- (c) A 440 V, 50 Hz, 6 pole, Y-connected induction motor running at 950 rpm has the following parameters referred to the stator :  $R_s = 0.5 \, \Omega$ ,  $R'_r = 0.4 \, \Omega$ ,  $X_s = X'_r = 1.2 \, \Omega$ ,  $X_m = 50 \, \Omega$ . Motor is driving a fan load, the torque of which is given by  $T_L = 0.0123 \, \omega_m^2$ . Now one phase of the motor falls, calculate the motor speed and current. Will it be safe to allow the motor to run for a long period? (Solve using approximate circuit)

[20 marks]







Q.4 (a) A 4-pole compound generator has armature, series-field and shunt-field resistance of  $1\ \Omega$ ,  $0.5\ \Omega$  and  $100\ \Omega$  respectively. This generator delivers  $4\ \text{kW}$  at a terminal voltage of  $200\ \text{V}$ . Allowing  $1\ \text{V}$  per brush for contact drop, calculate for both short-shunt and long-shunt connections.

(i) The generated emf, and

(ii) The flux per pole if the armature has 200 lap-connected conductors and is driven at  $750\ \text{rpm}$ .

[20 marks]

$$\cancel{R_a + R_{se} = 1.5\ \Omega}, \quad R_{sn} = 100\ \Omega, \quad R_a = 1\ \Omega, \quad R_{se} = 0.5\ \Omega$$

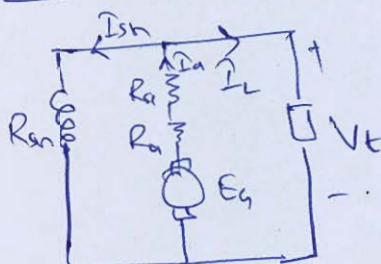
$$V_t = 200\ \text{V}, \quad P_{out} = 4\ \text{kW}$$

$$P_{out} = V_t \times I_L \Rightarrow 4000 = 200 \times I_L$$

$$I_L = 20\ \text{A}$$

$$V_{brush} = 1\ \text{V/brush}$$

(i) Long shunt



$$I_{sn} = \frac{V_t}{R_{sn}} = \frac{200}{100} = 2\ \text{A}$$

$$I_a = I_L + I_{sn} = 20 + 2 = 22\ \text{A}$$

$$E_g = V_t + I_a (R_a + R_{se}) + V_{brush} \times \# \text{ Brush}$$

$$= 200 + 22 \times (1.5) + 1 \times 2 \quad (\# \text{ brush} = 2)$$

$$E_g = 235\ \text{V}$$

(ii)

$$E_g = \frac{NP\phi Z}{60A}$$

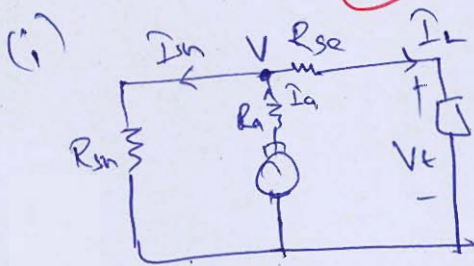
for lap winding  
( $A = P$ )

$$235 = \frac{750}{60} \times \phi \times 200$$



flux per pole  $\boxed{\phi = 0.094 \text{ Wb}}$

For short shunt



$$\begin{aligned} V &= V_t + I_L \times R_{se} \\ &= 200 + 20 \times 0.5 \\ &= 210 \text{ V} \end{aligned}$$

Now  $I_{sh} = \frac{V}{R_{sh}} = \frac{210}{100} = 2.1 \text{ A}$

$$I_a = I_L + I_{sh} = 20 + 2.1 = 22.1 \text{ A}$$

$$\begin{aligned} E_g &= V_t + I_a \cdot R_a + \text{Brush drop} \\ &= 210 + 22.1 \times 1 + 2 \end{aligned}$$

$\boxed{E_g = 234.1 \text{ V}}$

18

Good  
Approach

(ii)  $E_g = \frac{NP \phi Z}{60 A}$

$$234.1 = \frac{750}{60} \times \phi \times 200$$

flux per pole  $\boxed{\phi = 0.09364 \text{ Wb}}$



(b) Obtain the time response of the system described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

with the initial conditions  $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ,  $y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ .

[20 marks]

Using state equation given above,

$$\dot{x}_1 = x_2 + u$$

Applying Laplace transform,

$$sX_1(s) - X_1(0) = X_2(s) + U(s)$$

$$\cancel{sX_1(s) = X_2(s) + U(s)} \quad (1)$$

$$\cancel{X_1(0) = 0}$$

Now  $\cancel{X_2' = -x_1}$

$$sX_1(s) = X_2(s) + U(s) \quad (\because X_1(0) = 0)$$

$$\cancel{(1)}$$

and,

$$\dot{x}_2 = -x_1 - 2x_2 - u$$

Applying Laplace transform

$$sX_2(s) - X_2(0) = -X_1(s) - 2X_2(s) - U(s)$$

$$sX_2(s) = 1 - X_1(s) - 2X_2(s) - U(s) \quad (2)$$

Using (1) in (2)

$$sX_2(s) + 2X_2(s) = 1 - U(s) - \frac{X_2(s)}{s} - \frac{U(s)}{s}$$

$$X_2(s) \left[ s + 2 + \frac{1}{s} \right] = 1 - U(s) \left[ 1 + \frac{1}{s} \right] \quad (3)$$

also,

$$y = x_2$$



$$Y(s) = X_2(s) = \frac{1 \times s}{s^2 + 2s + 1} - \frac{s+1}{s^2 + 2s + 1} \cdot u(s)$$

Due to initial condition

$$Y(s) = \frac{s}{s^2 + 2s + 1} \quad \text{--- (4)}$$

and due to input

$$\frac{Y(s)}{U(s)} = \frac{s+1}{s^2 + 2s + 1} = -\frac{1}{s+1} \quad \text{--- (5)}$$

For ZIR

$$Y(s) = \frac{s}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2}$$

$$\Rightarrow s = A(s+1) + B$$

Comparing coefficient of 's' and constant terms.

$$A = 1, B = -1$$

$$Y(s) \Big|_{\text{ZIR}} = \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

Taking inverse Laplace,

$$Y(t) \Big|_{\text{ZIR}} = [e^{-t} - te^{-t}]u(t)$$

For ZSR and assuming unit step input

$$Y(s) = -\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$= -\left[ \frac{1}{s} - \frac{1}{s+1} \right]$$

Taking inverse Laplace,

$$y(t)|_{zsr} = [e^{-t} - 1] u(t)$$

So time response for ~~z~~ unit step  
will be

$$y(t) = y(t)|_{zsr} + y(t)|_{zsr}$$

$$= [e^{-t} - te^{-t} + e^{-t} - 1] u(t)$$

$$y(t) = [2e^{-t} - 1 - te^{-t}] u(t)$$

(18)

Good  
Approach

- Q.4 (c) A 10 kVA, 380 V, 4-pole, 50 Hz, 3- $\phi$ , star-connected cylindrical rotor alternator has a stator resistance and synchronous reactance of 1 ohm and 15 ohms respectively. It supplies a load of 8 kW at rated voltage and 0.8 lagging power factor.
- Draw a phasor diagram of operation.
  - Express the resistance and synchronous reactance in per unit values with the machine rating as the base.
  - Calculate the percentage regulation.
  - What is the terminal voltage if the load is suddenly removed (with the speed and excitation unaltered)?

[20 marks]

$$R_a = 1 \Omega, \quad X_s = 15 \Omega, \quad V_t = 380 \text{ V}$$

$$\cos \phi = 0.8 = 36.87^\circ$$

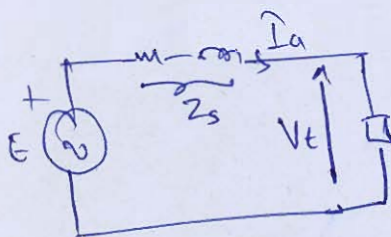
$$P = 8 \text{ kW}$$

$$\text{as, } P = \sqrt{3} V_t I_L \cos \phi$$

$$8 \times 10^3 = \sqrt{3} \times 380 \times I_L \times 0.8$$

$$I_L = 15.19 \text{ A} = I_a$$

$$E = V_t + I_a \cdot Z_s$$



$$E = \frac{380}{\sqrt{3}} \angle 0^\circ + 15.19 \angle -\cos^{-1} 0.8 \times (1 + j15)$$

$$[E = 406.94 \angle 25.184^\circ \text{ V}_{ph.}]$$

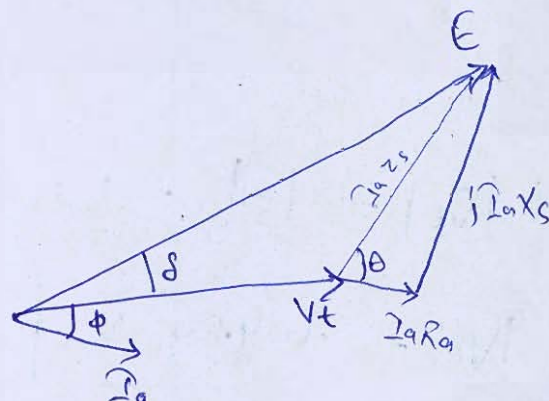
(i) ~~Phasor dia~~

$$I_a R_a = 15.19 \times 1 = 15.19 \text{ V}$$

$$I_a X_s = 15.19 \times 15 = 227.9 \text{ V}$$



(i) Phasor diagram of operation  
( $V_t$  as reference phasor)



$$\phi = 36.87^\circ$$

$$\delta = 25.18^\circ$$

$$\theta = \tan^{-1}(X_s/R_a)$$

(ii)  $S_{base} = 10 \text{ kVA}$ ,  $V_D = 380 \text{ V}$

$$Z_{base} = \frac{V_B^2}{S_B} = \frac{380 \times 380}{10 \times 10^3} = 14.44 \Omega$$

$$R_{pu} = \frac{R_a}{Z_B} = \frac{1}{14.44}$$

$$R_{pu} = 0.06925$$

$$X_{pu} = \frac{X_s}{Z_B} = \frac{15}{14.44}$$

$$X_{pu} = 1.0388 \text{ pu}$$

(iii) % Regulation =  $\frac{E - (V_t)_{pu}}{(V_t)_{pu}} \times 100$

$$= \frac{406.94 - 380/\sqrt{3}}{380/\sqrt{3}} \times 100$$

$$\% \text{ Regulation} = 85.5 \%$$

(iv) Now load is suddenly removed hence,

$$P = 0$$

$$\text{Hence, } I_a = 0$$

as the excitation is unaltered

$$\text{So, } |V_t| = |E|_u = 406.94 \text{ V}$$

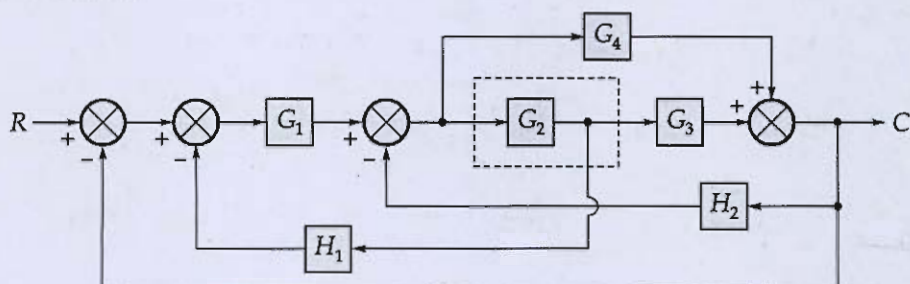
$$\boxed{|V_t| = 704.84 \text{ V}}$$

Good  
Approach

18

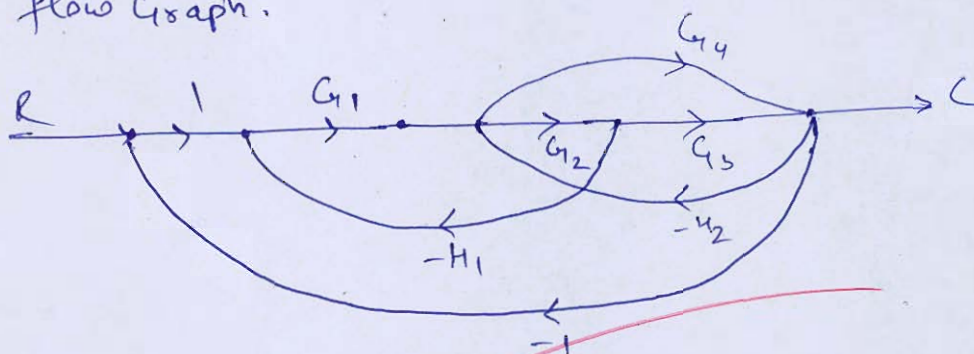
## Section B : Electrical Machines + Analog Electronics + Control Systems

- 5 (a) Obtain the transfer function of the feedback control system shown by using signal flow graph method.



[12 marks]

Converting the given block diagram into signal flow graph.



Mason gain's formula,

$$\frac{C(s)}{R(s)} = \frac{\sum P_k \Delta_k}{\Delta}$$

$P_k \Rightarrow k^{\text{th}}$  forward path

$\Delta = 1 - \text{Sum of all loop gains} + \text{Sum of product of 2 non touching loop gains} \dots$

$\Delta_k \Rightarrow \text{value of } \Delta \text{ excluding } k^{\text{th}} \text{ forward path.}$

Forward Paths

$$P_1 = G_1 G_2 G_3$$

$$P_2 = G_1 G_4$$

Loop gains

$$L_1 = -G_1 G_2 H_1$$

$$L_2 = -G_2 G_3 H_2$$

$$L_3 = -G_4 H_2$$

$$L_4 = -G_1 G_2 G_3$$

$$L_5 = -G_1 G_4$$

$$L_6 = G_1 G_4 G_2 H_1 H_2$$



Hence,

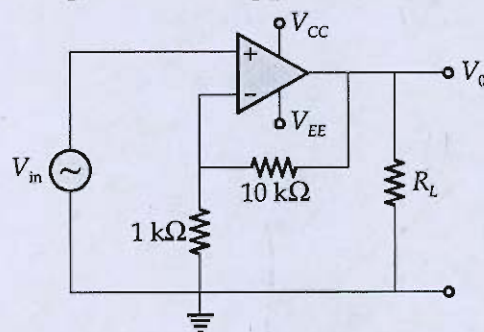
$$\frac{C}{R} = \frac{G_1 G_2 G_3 \times 1 + G_1 G_4 \times 1}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_2 + G_1 G_2 G_3 + G_1 G_4 - G_1 G_2 G_4 H_1 H_2}$$

Since ~~an~~ there is no non-touching loop

$$\text{and also, } \Delta_K \Rightarrow \Delta_1 = \Delta_2 = 1$$

8

2.5 (b) The 741 C Op-Amp having the following parameters is connected as shown in the figure.



$A = 20000$ ,  $R_i = 2 \text{ M}\Omega$ ,  $R_o = 75 \Omega$ ,  $f_0 = 5 \text{ Hz}$ , supply voltage =  $\pm 15 \text{ V}$ , output voltage swing =  $\pm 13 \text{ V}$ . Identify the circuit.

Compute the values of  $A_F$ ,  $R_{iF}$ ,  $R_{oF}$  and  $V_{OUT}$ .

[12 marks]

$$V_F = V_o \times \frac{1}{1+10} = \frac{V_o}{11} \quad \text{as } V_o = \beta V_F$$

$$\beta = 1/11$$

$$\text{So, } A_F = \frac{A}{1+A\beta} = \frac{20000}{1+20000 \times 1/11}$$

$$\boxed{A_F = 11}$$

$$\cancel{R_{iF}} = \cancel{R_i} \times \cancel{(1+A\beta)} = \frac{2 \times 10^6}{1+20000 \times 1/11}$$

$$R_{iF} = R_i (1+A\beta) = 2 \times 10^6 \times \left(1 + \frac{2 \times 10^4}{11}\right)$$

$$\boxed{R_{iF} = 3638.4 \text{ M}\Omega}$$

$$R_{oF} = \frac{R_o}{1+A\beta} = \frac{75}{1+2 \times 10^4 / 11}$$

$$\boxed{R_{oF} = 0.041 \Omega}$$

$$V_{out} = A_F \times V_i$$

$$V_{out} = 15 \times 11$$

$$V_{out} = 165 \text{ V}$$

4



5 (c) The open-loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{s(1+sT)}$$

- (i) By what factor the amplifier gain  $K$  should be multiplied so that the damping ratio is increased from 0.2 to 0.8?
- (ii) By what factor the time constant  $T$  should be multiplied so that the damping ratio is reduced from 0.9 to 0.3?

[12 marks]

(i) Initially,  $\xi_c = 0.2$

Characteristic Equation of the system is

$$1 + G(s) = 0 \Rightarrow s^2 T + s + K = 0$$

$$\Rightarrow s^2 + \frac{1}{T}s + \frac{K}{T} = 0$$

On Comparing it with standard 2nd order Characteristic Equation  $\Rightarrow s^2 + 2\xi_c \omega_n s + \omega_n^2$

$$\Rightarrow 2 \times 0.2 \times \sqrt{\frac{K_1}{T}} = \frac{1}{T} \quad \text{--- (1)}$$

Now with  $\xi_c = 0.8$

$$\Rightarrow 2 \times 0.8 \times \sqrt{\frac{K_2}{T}} = \frac{1}{T} \quad \text{--- (2)}$$

from (1) & (2) we get

$$\sqrt{\frac{K_2}{K_1}} = \frac{0.2}{0.8}$$

$$K_2 = \frac{1}{16} K_1$$

So,  $K$  should be multiplied by  $\frac{1}{16}$

$\frac{1}{16}$  to increase damping ratio from 0.2 to 0.8

(11)  
NowInitially,  $\epsilon_0 = 0.9$ 

$$\Rightarrow 2 \times 0.9 \times \sqrt{\frac{k}{T_1}} = \frac{1}{T_1}$$

$$\Rightarrow 2 \times 0.9 \sqrt{k} = \frac{1}{\sqrt{T_1}} \quad \text{--- (3)}$$

Similarly for  $\epsilon_0 = 0.3$ 

$$\Rightarrow 2 \times 0.3 \sqrt{k} = \frac{1}{\sqrt{T_2}} \quad \text{--- (4)}$$

from (3) and (4), we get

$$\sqrt{\frac{T_2}{T_1}} = \frac{2 \times 0.9}{2 \times 0.3}$$

$$\frac{T_2}{T_1} = 9$$

$$T_2 = 9 T_1$$

So, deduce,  $\epsilon_0$  from 0.9 to 0.3,~~the~~ 'T' should be multiplied by 9(11)  
Good  
Approach



- 5 (d) Consider a negative feedback system having the characteristic equation,

$$1 + \frac{K}{(1+s)(1.5+s)(2+s)} = 0.$$

It is desired that all the roots of the characteristic equation have real parts less than  $-1$ .  
Extend the Nyquist stability criterion to find the largest value of  $K$  satisfying the condition.

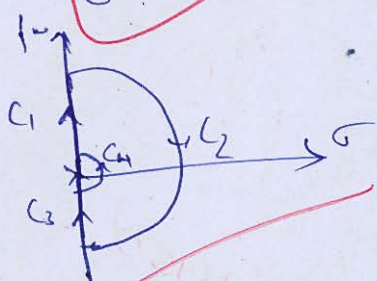
[12 marks]

Replacing the 's' by 's-1'

we get,

$$1 + \frac{K}{s(s+0.5)(s+1)} = 0 \Rightarrow G(s) = \frac{K}{s(s+0.5)(s+1)}$$

Considering the Nyquist Contour



Along  $C_1$   $s = j\omega$ ,  $\omega \Rightarrow 0$  to  $\infty$

$$G(j\omega) = \frac{K}{j\omega(j\omega+0.5)(j\omega+1)}$$

|          |             |              |
|----------|-------------|--------------|
| $\omega$ | 0           | $\infty$     |
| $M$      | $\infty$    | 0            |
| $\phi$   | $-90^\circ$ | $-270^\circ$ |

Along  $C_2$   $s = \lim_{R \rightarrow \infty} R e^{j\theta}$   $\theta \Rightarrow \pi/2$  to  $-\pi/2$

$$G(s) = \lim_{R \rightarrow \infty} \frac{K}{R e^{j\theta} (R e^{j\theta} + 0.5) (R e^{j\theta} + 1)} \approx 0$$

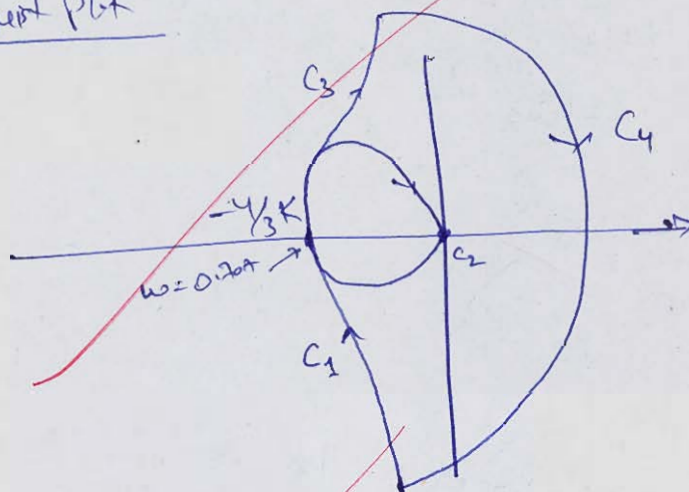


Along  $C_3$   $s = j\omega$   $\omega = -\infty$  to  $0$   
it will be mirror image of  $C_1$

Along  $C_4$   $s = \lim_{\lambda \rightarrow 0} \lambda e^{j\theta}$   $\theta \Rightarrow -\pi/2$  to  $\pi/2$

$$G(s) = \lim_{\lambda \rightarrow 0} \frac{1}{\lambda e^{j\theta}} = \infty \Rightarrow \pi/2 \text{ to } -\pi/2$$

Root Plot



10

Good Approach

At  $-180^\circ$  axis

$$\phi = -180^\circ = -90^\circ - \tan^{-1}\left(\frac{\omega}{0.5}\right) - \tan^{-1}(\omega)$$

$$\Rightarrow 1 - \frac{\omega^2}{0.5} = 0 \Rightarrow \omega = 0.707 \text{ rad/s}$$

$$\begin{aligned} |G(\omega)|_{\omega=0.707} &= \frac{K}{0.707 \times \sqrt{(0.707)^2 + 0.5^2} \times \sqrt{(0.707)^2 + 1}} \\ &= \frac{4}{3} K \end{aligned}$$

So, for all roots to have real part less than  $-1$

$$-\frac{4}{3} K > -1$$

$$K < \frac{3}{4}$$

$$\Rightarrow \boxed{K_{\max} = 0.75}$$

- 5 (e) Sketch the polar plots of the transfer function  $G(s) = \frac{1}{s(1+s)(1+2s)}$ . Determine whether the polar plots cross the real axis. If so, determine the frequency at which the plots cross the real axis and the corresponding magnitude  $|G(j\omega)|$ .

[12 marks]

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+j2\omega)}$$

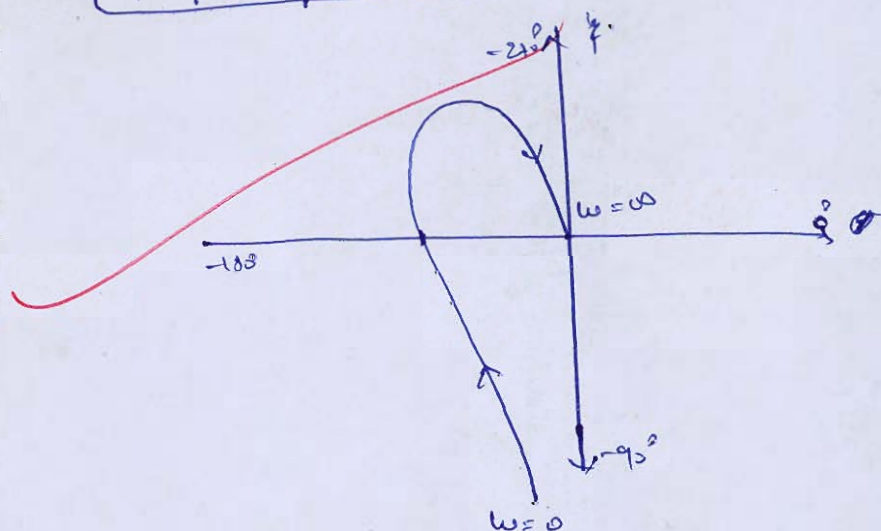
$$|G(j\omega)| = \frac{1}{\omega \sqrt{1+\omega^2} \times \sqrt{1+4\omega^2}}$$

$$\phi = \angle G(j\omega) = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

~~At~~ when plot

|                |             |              |
|----------------|-------------|--------------|
| $\omega$       | 0           | $\infty$     |
| $ G(j\omega) $ | $\infty$    | 0            |
| $\phi$         | $-90^\circ$ | $-270^\circ$ |

$\Rightarrow$  using this Bode plot of given  $G(s)$  will be



As the plot cut at real axis,

$$\phi = -180^\circ = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

$$\Rightarrow \tan^{-1}(\omega) + \tan^{-1}(2\omega) = 90^\circ$$

$$\tan^{-1} \left( \frac{\omega + 2\omega}{1 - 2\omega^2} \right) = 90^\circ$$

$$\Rightarrow \text{So, } 1 - 2\omega^2 = 0$$

$$\omega^2 = \frac{1}{2}$$

$$\Rightarrow \boxed{\omega = \frac{1}{\sqrt{2}} = 0.707 \text{ rad/s}}$$

frequency at plot cross  
real axis.

$$|G(\omega)|_{\omega = 1/\sqrt{2}} = \frac{1}{\frac{1}{\sqrt{2}} \times \sqrt{1 + \frac{1}{2}} \times \sqrt{1 + 4 \times \frac{1}{2}}}$$

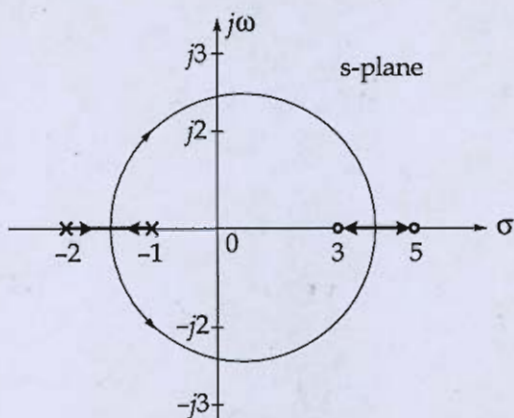
$$\boxed{|G(\omega)| = \frac{2}{3} = 0.6667}$$

11

Good  
Approach

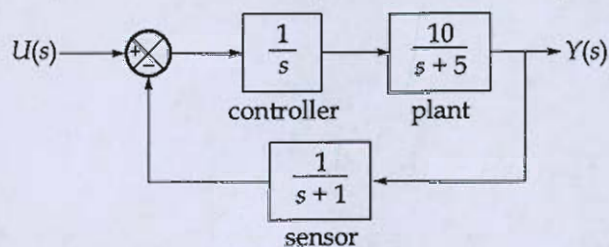


- 6 (a) (i) The root locus plot for the certain control system is shown below:



Find the break-away and break-in points for the above root locus plot.

- (ii) Obtain a state-space model of the system shown in figure below:



[10 + 10 marks]

(i) From the given plot

$$G(s) = \frac{K}{(s+1)(s+2)(s-3)(s-5)}$$

Characteristic equation

$$1 + G(s) = 0 \Rightarrow (s+1)(s+2)(s-3)(s-5) + K = 0$$

$$K = -[(s+1)(s+2)(s-3)(s-5)]$$

$$K = -[(s^2 + 3s + 2)(s^2 - 8s + 15)]$$

$$= -[s^4 - 8s^3 + 15s^2 + 3s^3 - 24s^2 + 45s + 2s^2 - 16s + 30]$$

$$= -[s^4 - 5s^3 - 7s^2 + 29s + 30]$$

for break away and breakin points  $\frac{dK}{ds} = 0$

$$\frac{dK}{ds} = 4s^3 - 15s^2 - 14s + 29 = 0$$

Solving this,

$$s = 4.1724, s = 1.124, s = -1.546$$

As, ~~Break~~ Breakaway Point lies between '-1' and '-2'

So, Valid Breakaway Point is

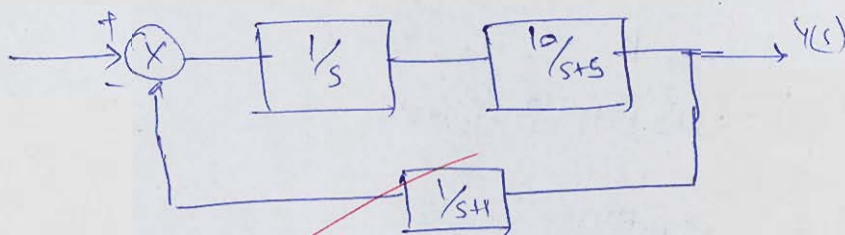
$$s = -1.546$$

and Valid Breakin Point lies between '3' and '5'

$$s = 4.1724$$

4

(ii)



$$\frac{10}{s+5} = \frac{10/s}{1+5/s} \Rightarrow$$

```

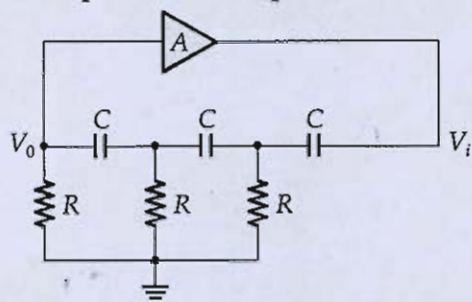
graph LR
    Input(( )) --> Sum((+/-))
    Sum --> B1[1/s]
    B1 --> B2[10]
    B2 --> Output((Y(s)))
    Output --> B3[1/2]
    B3 --> Sum
  
```





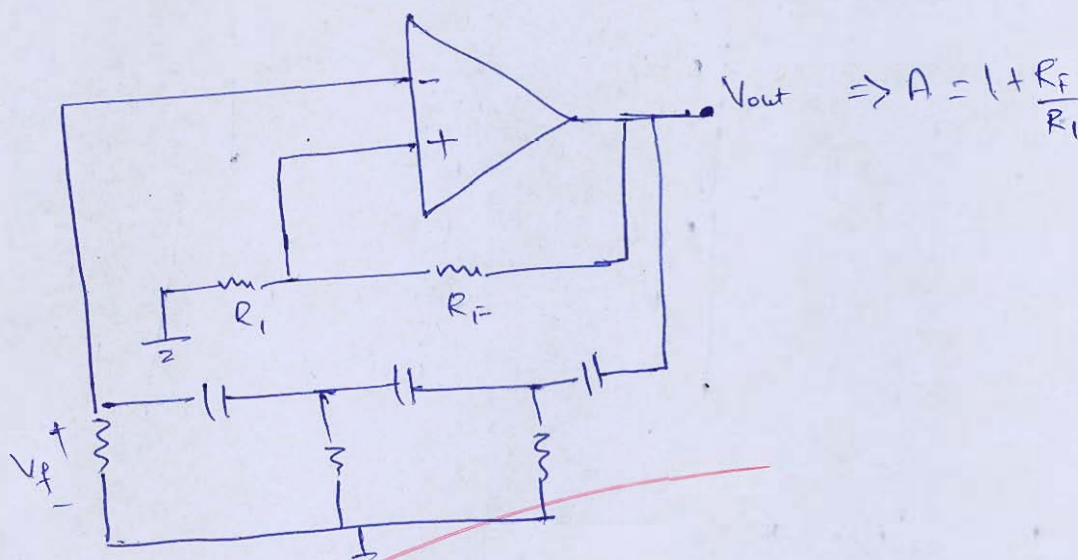


- 6 (b) Derive the condition of oscillation and the expression for the frequency of oscillations for the circuit shown. (Use mesh analysis and Barkhausen's criteria). Draw actual oscillator circuit with one operational amplifier and minimum number of RC elements.

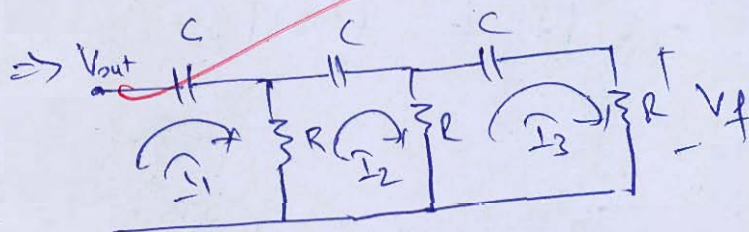


[20 marks]

Actual oscillator circuit with operational amplifier



$$\Rightarrow A = 1 + \frac{R_f}{R_i}$$



Using KVL in loop (1)

$$V_{out} = (R + X_C) \hat{I}_1 - R \hat{I}_2 \quad \text{--- (1)}$$

KVL in loop (2)

$$-R \hat{I}_1 + (2R + X_C) \hat{I}_2 - R \hat{I}_3 = 0 \quad \text{--- (2)}$$

KVL in loop (3)

$$-R \hat{I}_2 + (2R + X_C) \hat{I}_3 = 0 \quad \text{--- (3)}$$

$$A_3, V_f = I_3 \cdot R$$

So, finding  $I_3$  using Cramer's Rule

$$\underline{I_3} = \frac{A_3}{A}$$

using ①, ② and ③

$$\begin{bmatrix} R+X_c & -R & 0 \\ -R & 2R+X_c & -R \\ 0 & \cancel{2R+X_c} & 2R+X_c \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_{out} \\ 0 \\ 0 \end{bmatrix}$$

$$\text{So, } I_3 = \frac{\begin{vmatrix} R+X_c & -R & V_{out} \\ -R & 2R+X_c & 0 \\ 0 & -R & 0 \end{vmatrix}}{\begin{vmatrix} R+X_c & -R & 0 \\ -R & 2R+X_c & -R \\ 0 & -R & 2R+X_c \end{vmatrix}}$$

$$I_3 = \frac{V_{out} R^2}{(R+X_c)[(2R+X_c)^2 - R^2] + R[-R(2R+X_c)]}$$

Solving the denominator.

$$(R+X_c)[4R^2 + X_c^2 + 4RX_c - R^2] - 2R^3 - R^2X_c$$

$$\Rightarrow 3R^3 + RX_c^2 + 4R^2X_c + 3R^2X_c + X_c^3 + 4RX_c^2 - 2R^3 - R^2X_c$$

$$\Rightarrow R^3 + X_c^3 + 6R^2X_c + 5RX_c^2$$



$$V_f = I_3 \cdot R = \frac{V_{out} \cdot R^3}{R^3 + X_C^3 + 6R^2X_C + 5RX_C^2}$$

$$\frac{V_f}{V_{out}} = \beta = \frac{1}{1 + \left(\frac{X_C}{R}\right)^3 + 6\left(\frac{X_C}{R}\right) + 5\left(\frac{X_C}{R}\right)^2}$$

Now putting  $X_C = \frac{1}{j\omega C}$

$$\beta = \frac{1}{1 + \frac{1}{(j\omega RC)^3} + \frac{6}{j\omega RC} - \frac{5}{(\omega RC)^2}}$$

For ~~sustained~~ oscillation only Barkhausen Criteria.

$\angle -A\beta = 0^\circ \Rightarrow$  Hence equating imaginary term to zero, we get

$$\frac{6}{\omega RC} = \frac{1}{(\omega RC)^3}$$

$$\omega = \frac{1}{\sqrt{6}RC} \Rightarrow f = \frac{1}{2\pi\sqrt{6}RC}$$

$\hookrightarrow$  frequency of oscillation

and  $A\beta \geq 1$

$$A\beta \left(1 + \frac{R_f}{R_1}\right) \times \left(\frac{1}{1 - \frac{5}{(\omega RC)^2}}\right) \geq 1$$

$$A\beta \left(1 + \frac{R_f}{R_1}\right) \frac{1}{1-30} \geq 1$$

So,  $A \geq 29$  as  $\beta = \frac{1}{29}$

$\hookrightarrow$  Condition for sustained oscillation.

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- Q.6 (c) (i) Using the Routh criterion, check whether the system represented by the following characteristic equation is stable or not. Comment on the location of the roots. Determine the frequency of sustained oscillations if any,

$$s^4 + 2s^3 + 6s^2 + 8s + 8 = 0$$

[10 marks]

Using R-H Criterion,

|       |   |   |   |
|-------|---|---|---|
| $s^4$ | 1 | 6 | 8 |
| $s^3$ | 2 | 8 |   |
| $s^2$ | 2 | 8 |   |
| $s^1$ | 0 | 0 |   |

forming auxiliary equation

$$A(s) = 2s^2 + 8$$

$$\frac{dA(s)}{ds} = 4s$$

|       |   |   |
|-------|---|---|
| $s^2$ | 2 | 8 |
| $s^1$ | 4 |   |
| $s^0$ | 8 |   |

Since there are no sign changes, there is no pole in RHS of s-plane,

but as the auxiliary equation is formed means that there are poles lying symmetrically

along the imaginary axis.

And they will be roots of auxiliary equation.



Hence the System is marginally stable

Root lying on  $j\omega$  axis

$$2s^2 + 8 = 0$$

$$s = \pm j4$$

and frequency of sustain oscillation

$$s = \pm j\omega$$

$$\Rightarrow$$

$$\omega = 4 \text{ rad/s}$$

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Q.6 (c) (ii) A control system with open loop transfer function is represented by

$G(s)H(s) = \frac{K}{(s+2)^2(s+3)}$ . Determine the range of value of  $K$  for which value of gain margin (GM)  $\geq 4$  and position error constant is  $K_p > 2$  when unit step input is applied.

[10 marks]

For unit step input

$$\text{For } K_p = \lim_{s \rightarrow 0} G(s) \cdot 1(s)$$

$$K_p = \lim_{s \rightarrow 0} \frac{K}{(s+2)^2(s+3)} = \frac{K}{12}$$

$$\text{So, as } K_p > 2$$

$$\text{So, } \frac{K}{12} > 2 \Rightarrow K > 24 \quad \text{--- (1)}$$

Now

Phase Crosses frequency ( $\omega_{pc}$ )

$$\angle G(s)H(s) = -180^\circ = -2 \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3)$$

$$\Rightarrow 2 \tan^{-1}(\omega/2) + \tan^{-1}(\omega/3) = +180^\circ$$

Solving this we get,

$$\omega = 4 \text{ rad/s}$$

Now

$$\left| G(j\omega)H(j\omega) \right|_{\omega=4} = \frac{K}{(\omega^2+4)(\omega^2+9)}$$

$$= \frac{K}{20 \times \sqrt{25}} = \frac{K}{100}$$

$$GM = \frac{1}{|G(j\omega)H(j\omega)|}$$

$$G.M = \frac{100}{K} \geq 4$$

$$\Rightarrow K \leq 25$$

$$\text{So, } \boxed{24 < K \leq 25} \text{ Range of } K$$

for satisfying  $K_p > 2$

$$G.M \geq 4$$

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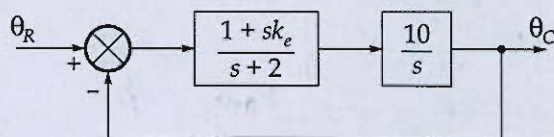
Good  
Approach

Q.7 (a) The control system shown in figure employs proportional plus error rate control. Determine the value of error rate constant  $K_e$  so that the damping ratio is 0.6.

(i) Determine the value of setting time and maximum overshoot.

Find the steady-state error if the input is a unit-ramp.

(ii) What will be the those values (as calculated in part-i) without error rate control?



[20 marks]







- (b) A negative unity feedback control system is provided with compensator in cascade with system, for system to be stable. The transfer function of plant and compensator are respectively  $\frac{1}{s(s+2)(s+4)}$  and  $\frac{(s+a)}{s+1}$ . Calculate the range of value of 'a' for system to be stable and also represent complete system in form of block diagram. At critical stability condition, what will be the nature of compensator?

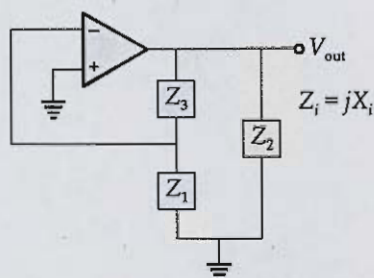
[20 marks]







Q.7 (c) In the figure shown below:



The op-amp in the circuit has a finite open loop gain ( $A_v$ ), finite output resistance ( $R_o > 0$ ) and it is ideal in all other aspects.  $Z_1$ ,  $Z_2$  and  $Z_3$  are purely reactive elements with magnitudes  $|X_1|$ ,  $|X_2|$  and  $|X_3|$ . Prove that  $X_1$  and  $X_2$  must be of the same type of reactance (i.e., both must be either capacitive or inductive) to produce sustained oscillations.

[20 marks]



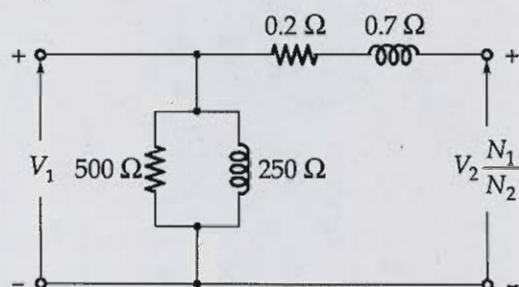








- Q.8 (a) The equivalent circuit referred to the low-tension side of a 250/2500 V single phase transformer is shown in figure.



The load impedance connected to the high-tension terminals is  $380 + j230\ \Omega$ . For a primary voltage of 250 V,

Find:

- The secondary terminal voltage.
- Primary current and power factor, and
- Power output and efficiency.

[20 marks]







9) Given the transfer function,

$$\frac{Y(s)}{U(s)} = \frac{1}{(s+5)(s+4)}$$

Obtain the state equation using :

(i) Cascade decomposition

(ii) Direct decomposition

It is desired that the closed loop poles are to be placed at  $s = (-1 \pm j2)$ . Determine the feedback gain matrix  $K$  for part (i) and (ii).

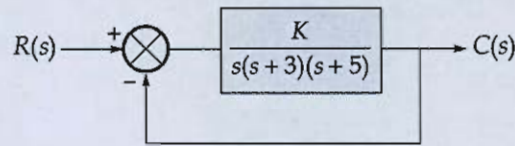
[20 marks]







- Q.8 (c) Using the Nyquist criterion, find the range of  $K$  for stability for the system shown in figure. Also find the value of gain  $K$  and frequency of oscillation for marginal stability.



[20 marks]







**Space for Rough Work**

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**Space for Rough Work**

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$$\frac{1}{2} \times \frac{B^2 \times V_c}{2\mu_0}$$

$$\boxed{\frac{1}{2} L I^2}$$

(Energy)

E/volun



$$\frac{4}{s^2 + s}$$

$$1 + \frac{4 \times K_0 s}{s^2 + s}$$

$$\frac{4}{s^2 + s + K_0 + 1}$$

$$\frac{10/s}{1 + 10/s \times 1/2}$$

$$\frac{(s^2 + s) \cdot s(s)}{1 + 2sK - s^2 - 1}$$

$$\frac{s^2(s^2 + s)}{(s^2 + 1)^2}$$