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ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-4 : Analog and Digital Communication + Control Systems

Name : ....

Roll No :

Test Centres				Student's Signature
Delhi ☐	Bhopal ☐	Jaipur ☐	Pune ☐	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	37
Q.2	38
Q.3	—
Q.4	37
Section-B	
Q.5	41
Q.6	—
Q.7	—
Q.8	37
Total Marks Obtained	190

Signature of Evaluator   Cross Checked by

Chaithra S M

* Good Performance
* Avoid silly mistakes.

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

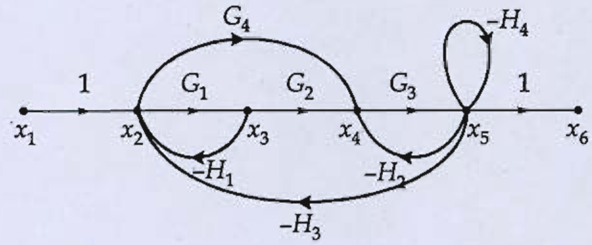
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Analog and Digital Communication + Control Systems

Q.1 (a) Obtain the transfer function $\frac{x_6}{x_1}$ for the signal flow graph shown in figure below:



[12 marks]

$$\frac{x_6}{x_1} = \sum_{k=1}^{2} \frac{P_k \Delta_k}{\Delta}$$

Here two forward path : $\rightarrow$ $P_1 = G_1 G_2 G_3$

$P_2 = G_4 G_3$

$\Delta = 1 - \left(\text{sum of individual loop gain}\right) + \left(\text{sum of product of two non touching loop gain}\right) - \left(\text{sop of 3 non touching loop gain}\right) + \ldots$

Here 5 loops $\rightarrow$ $-H_4$, $-G_3H_2$, $-G_1H_1$, $-G_1G_2G_3H_3$, $-G_4G_3H_3$

$$\therefore \frac{x_6}{x_1} = \frac{G_1G_2G_3(1-0) + G_4G_3(1-0)}{1 - \left[-H_4 - G_3H_2 - G_1H_1 - G_1G_2G_3H_3 - G_4G_3H_3\right] + \left[H_4G_1H_1 + G_3H_2G_1H_1\right]}$$

$$\frac{x_6}{x_1} = \frac{(G_1G_2G_3 + G_4G_3)}{\left[1 + H_4 + G_3H_2 + G_1H_1 + G_1G_2G_3H_3 + G_3G_4H_3 + G_1H_1H_4 + G_1G_3H_1H_2\right]}$$

Ans

Do not write in this margin

Q.1 (b) The instantaneous frequency of a sine wave is equal to $f_c + \Delta f$ for $|t| \le \frac{T}{2}$, and f_c for $|t| > \frac{T}{2}$. Determine the spectrum of this frequency-modulated wave.

[12 marks]

Do not write in this margin

Q.1 (c) A unit-step response test conducted on a second-order system yielded peak overshoot $M_p = 0.12$, and peak time $t_p = 0.2s$. Obtain the corresponding frequency response indices $(M_r, \omega_r, \omega_b)$ for the system.

[12 marks]

$m_p = 0.12$; $t_p = 0.2\,s$; $m_r = ?$ $\omega_r = ?$ $\omega_b = ?$

$\because$ $m_p = e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.12$ (for unit-step response)

$\Rightarrow \dfrac{1}{0.12} = \exp\left[\dfrac{\xi\pi}{\sqrt{1-\xi^2}}\right]$

$\Rightarrow 2.12 = \dfrac{\xi\pi}{\sqrt{1-\xi^2}}$

$\Rightarrow (2.12)^2(1-\xi^2) = \xi^2\pi^2$

$\Rightarrow 4.49 - 4.49\,\xi^2 = \xi^2\pi^2$

$\Rightarrow \xi^2 = \dfrac{4.49}{1+\pi^2} = 0.413$

$\Rightarrow \xi = 0.642$ (rejecting -ive value)

[here $\xi < \frac{1}{\sqrt{2}}$] underdamped for frequency response

again $t_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = 0.2$

$\Rightarrow \dfrac{\pi}{0.2\sqrt{1-0.642^2}} = \omega_n$

$\Rightarrow \omega_n = 20.487$ rad/sec.

$\because$ we know that $\omega_r = \omega_n\sqrt{1-2\xi^2}$

$\Rightarrow \omega_r = 20.487\sqrt{1-2(0.642)^2}$

$\Rightarrow \omega_r = 8.586$ rad/sec.

again $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$

$\Rightarrow M_r = \frac{1}{2 \times 0.642 \times \sqrt{1-0.642^2}}$

$\Rightarrow \boxed{M_r = 1.0157}$

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Bandwidth, $\omega_b = \omega_n \sqrt{(1-2\xi^2) + \sqrt{(2-4\xi^2+4\xi^4)}}$

⋮

$\omega_b = 22.76$ rad/s

Q.1 (d) Suppose that binary PSK is used for transmitting information over an AWGN channel with power-spectral density of $\frac{N_0}{2} = 10^{-10}$ W/Hz. The transmitted signal energy is $E_b = \frac{A^2T}{2}$ where T is the bit interval and A is the amplitude of signal. Determine the signal amplitude required to achieve an error probability of 10^{-6}, if the data rate is

1. 10 Kbps
2. 1 Mbps

(Assume $Q[4.74] = 10^{-6}$)

[12 marks]

$\because P_e = Q\left[\sqrt{\frac{d_{min}^2}{2N_0}}\right]$

for BPSK $\Rightarrow d_{min} = 2\sqrt{E_b}$

$\therefore P_e = Q\left[\sqrt{\frac{4E_b}{2N_0}}\right] = Q\left[\sqrt{\frac{2E_b}{N_0}}\right]$

$\because \quad P_e = 10^{-6} = Q[4.74] = Q\left[\sqrt{\frac{2E_b}{N_0}}\right]$

$\therefore \quad \sqrt{\frac{2E_b}{N_0}} = 4.74$

$\Rightarrow \quad E_b = (4.74)^2 \frac{N_0}{2}$ $\quad \left[\text{given } \frac{N_0}{2} = 10^{-10}\, W/Hz\right]$

$\Rightarrow \quad E_b = (4.74)^2 \times 10^{-10}$

$\Rightarrow \quad \boxed{E_b = 2.246 \times 10^{-9} \text{ Joule}}$

$\because \quad E_b = \frac{A^2 T}{2} = \frac{A^2}{2R_b}$ $\quad \left[\because T_b = \frac{1}{R_b}\right]$

(i) $R_b = 10 \times 10^3$ bps $= 10$ kbps

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$\frac{A^2}{2R_b} = 2.246 \times 10^{-9}$

$\Rightarrow \quad A = \sqrt{2.246 \times 10^{-9} \times 2 \times 10 \times 10^3}$

$\Rightarrow \quad \boxed{A = 6.702 \times 10^{-3} \text{ volt}}$

(ii) $R_b = 1$ mbps $= 10^6$ bps

$\frac{A^2}{2R_b} = 2.246 \times 10^{-9}$

$\Rightarrow \quad A = \sqrt{2.246 \times 10^{-9} \times 2 \times 10^6}$

$\Rightarrow \quad \boxed{A = 0.067 \text{ volt}}$

Q.1 (e) (i) The signal $m(t) = 6 \sin(2\pi t)$ volts is transmitted using a 4-bit binary PCM system. The quantizer is of the midrise type, with a step size of 1 volt. Sketch the resulting PCM wave for one complete cycle of the input. Assume a sampling rate of four samples per second, with samples taken at $t = \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{5}{8}, ...$ seconds.

(ii) Band-limited message signal $m(t)$ is encoded using PCM system which uses uniform quantizer and 12-bit encoding. If the bit rate is 64 Mb/sec, what is the maximum bandwidth of $m(t)$ for satisfactory operation?

Calculate signal to quantization noise ratio if $m(t)$ is full load single tone sinusoidal signal of frequency 1 MHz.

[6 + 6 marks]

(ii) $n = 12$; $R_b = 64$ Mb/sec.

$R_b = n f_s$ [f_s = Sampling frequency]

$\Rightarrow 64 \times 10^6 = 12 \times f_s$

$\Rightarrow$ $f_s = 5.33 \times 10^6$ sample/sec.

Nyquist rate $f_s \geq 2 f_m$

$$(f_m)_{max} = \frac{f_s}{2} = \frac{5.33 \times 10^6}{2}$$

$(f_m)_{max} = 2.66$ MHz

Signal to quantization noise ratio $= \frac{3}{2} \times 2^{2n}$

$\Rightarrow$ SQNR $= \frac{3}{2} \times 2^{2 \times 12}$ $(n = 12)$

6

$\Rightarrow$ SQNR $= 25165824$

$\Rightarrow$ SQNR $= 10\log_{10}(25165824)$ (or) $[= 1.76 + 6n \text{ dB}]$

$\Rightarrow$ **SQNR = 74.008 dB**

Q.2 (a) Design a unity feedback system, having the open loop transfer function with the following characteristics:

1. It must have one zero.
2. It is a type two system with one pole at –5.
3. On applying the input, $R(s) = \frac{3}{s^3}$; we get steady state error of 0.2.
4. The magnitude of open loop transfer function at $\omega = 1$ is 24.94 dB.

Calculate the output of the feedback system $Y(s)$ for the applied input $R(s) = \frac{3}{s^3}$.

[20 marks]

$$GH(s) = \frac{K(s+a)}{s^2(s+5)}$$ (assume zero at $s = -a$)

$$e_{ss} = \lim_{s \to 0} \frac{s\,R(s)}{1 + GH(s)}$$

if $R(s) = \frac{3}{s^3}$ then $e_{ss} = 0.2$

$$\Rightarrow e_{ss} = \lim_{s \to 0} \frac{s\left(3/s^3\right)}{1 + \frac{K(s+a)}{s^2(s+5)}} = 0.2$$

$$\Rightarrow \lim_{s \to 0} \frac{3}{s^2 + \frac{K(s+a)}{(s+5)}} = 0.2$$

$\Rightarrow \dfrac{3 \times 5}{ka} = 0.2$

$\Rightarrow \boxed{ka = 75}$ —(1)

magnitude of open loop transfer function →

$$GH(j\omega) = \frac{k(a + j\omega)}{(j\omega)^2 (5 + j\omega)}$$

$$\Rightarrow |GH(j\omega)| = \frac{k\sqrt{a^2 + \omega^2}}{\omega^2 \sqrt{25 + \omega^2}}$$

at $\omega = 1$; $|GH(j\omega)| = 10^{\frac{24.94}{20}} = 17.66$

$\therefore \dfrac{k\sqrt{a^2+1}}{1\sqrt{25+1}} = 17.66$

$\left[\because 20\log x = 24.94 \quad x = 17.66\right]$

$\Rightarrow k\sqrt{a^2+1} = 90.04$

$\Rightarrow \boxed{k^2(a^2+1) = 8107.201}$ —(2)

$\Rightarrow k^2a^2 + k^2 = 8107.201$

$\Rightarrow 75^2 + k^2 = 8107.201$

$\Rightarrow \boxed{k = 49.82}$; $\boxed{a = \dfrac{75}{49.82} = 1.505}$

$\dfrac{C(s)}{R(s)} = \dfrac{s^2(s+5) + k(s+a)}{s^2(s+5)}$ $\qquad \dfrac{C(s)}{R(s)} = \dfrac{k(s+a)}{s^2(s+5) + k(s+a)}$

$\Rightarrow Y(s) = \left[\dfrac{s^2(s+5) + 49.82(s+1.505)}{s^2(s+5)}\right]\dfrac{3}{s^3}$

$\Rightarrow \boxed{Y(s) = \dfrac{3(s^3 + 5s^2 + 49.82s + 75)}{s^5(s+5)}}$ Ans

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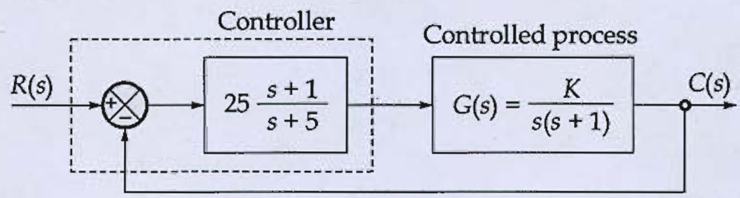
Q.2 (b) (i) Design a unity negative feedback system with following specifications:

1. The system must be a second order with $\xi < 1$.
2. The time gap between first peak overshoot and first peak undershoot is 0.785 sec.
3. Number of cycles completed before the output is settled within 5% of its final value is 0.6366 cycles.

(ii) Consider the feedback control system shown in the below figure. The normal value of process parameter K is 1. Calculate the sensitivity of transfer function $T(s) = C(s)/R(s)$ to variations in parameter K.



[15 + 5 marks]

(ii)

$$\frac{C(s)}{R(s)} = T(s) = \frac{\frac{25(s+1)}{(s+5)}\,\frac{K}{s(s+1)}}{1 + \frac{25(s+1)}{(s+5)}\,\frac{K}{s(s+1)}} \Rightarrow \frac{C(s)}{R(s)} = \frac{25K}{s(s+5)+25K}$$

$$T(s) = \frac{25K(s+1)}{s(s+5)(s+1) + 25K(s+1)} \qquad = \frac{25K}{s^2+5s+25K}$$

$$\Rightarrow \quad T(s) = \frac{25K(s+1)}{s^3+6s^2+(5+25K)s+25K}$$

Be careful while writing overall transfer function $\frac{C(s)}{R(s)}$ to Avoid mistakes

$$\therefore \quad S_K^T = \frac{\partial T/T}{\partial K/K} = \frac{K}{T}\,\frac{\partial T}{\partial K}$$

$$S_K^T = \frac{[s^3+6s^2+(5+25K)s+25K]}{25(s+1)}\,\frac{\partial}{\partial K}\left[\frac{25K(s+1)}{s^3+6s^2+(5+25K)s+25K}\right]$$

$$= \left(\frac{s^3+6s^2+(5+25K)s+25K}{25(s+1)}\right)\left[\frac{25K(s+1)\cdot(25s+25) - (s^3+6s^2+(5+25K)s+25K)(25s)}{[s^3+6s^2+(5+25K)s+25K]^2}\right]$$

$$S_K^T = \frac{s(s+5)}{s^2+5s+25K}, \quad \text{For } K=1: \quad S_K^T = \frac{s(s+5)}{s^2+5s+25}$$

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Q.2 (c) (i) In a DSB-SC system, the message signal $m(t)$ is multiplied with the carrier signal $c(t) = 5\cos(2\pi f_c t)$ to produce a modulated signal $s(t)$. If $m(t) = 3\,\text{sinc}(2t) - 2\,\text{sinc}^2(t)$ and $f_c = 100$ Hz, then determine and sketch the spectrum of the modulated signal $s(t)$. Assume that, $\sin c(t) = \dfrac{\sin(\pi t)}{\pi t}$.

(ii) The spectrum of the message signal $m(t)$ is shown below in figure (a). This signal is processed by the system shown below in figure (b).

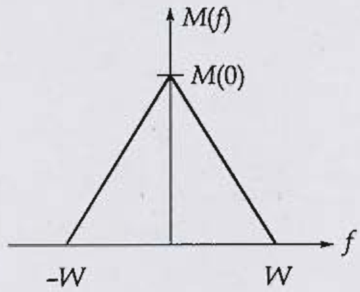


Fig. (a)

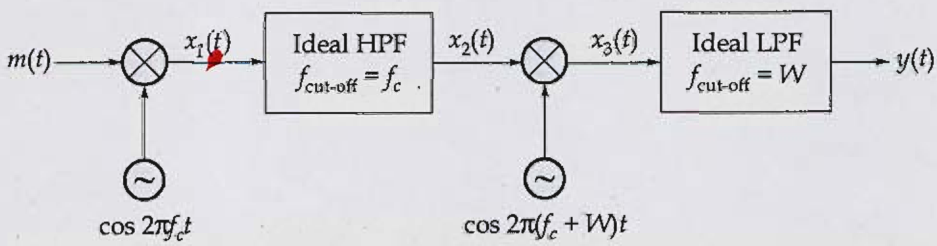


Fig. (b)

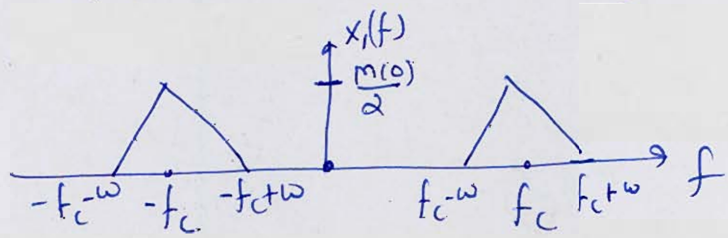
If each filter has a passband gain of 1, then sketch the spectrum of the output signal $y(t)$. Assume that $f_c >> W$.

[10 + 10 marks]

(ii) $x_1(t) = m(t)\cos 2\pi f_c t$

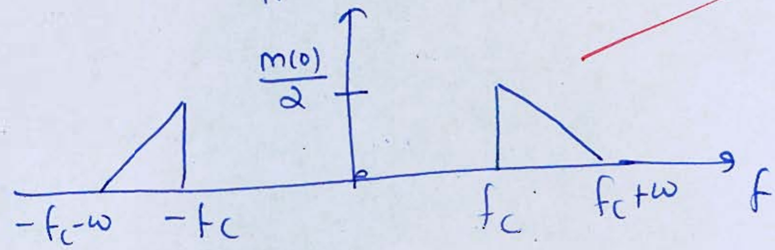
$\Rightarrow X_1(f) = \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right]$

$X_1(f) \Rightarrow$



after ideal HPF ; $f_{cutoff} = f_c$

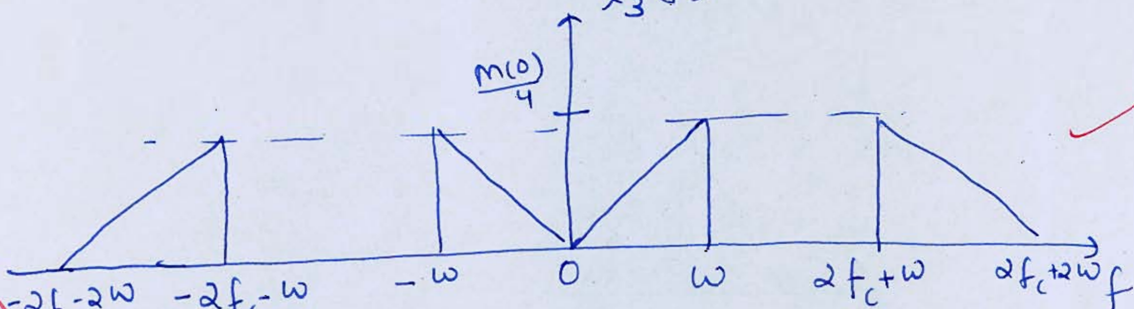
$X_2(f) =$



$\because \quad x_3(t) = x_2(t) \cos\left(2\pi(f_c+\omega)t\right)$

$\therefore \quad X_3(f) = \frac{1}{2}\left[X_2(f-f_c-\omega) + X_2(f+f_c+\omega) \right]$

~~after~~



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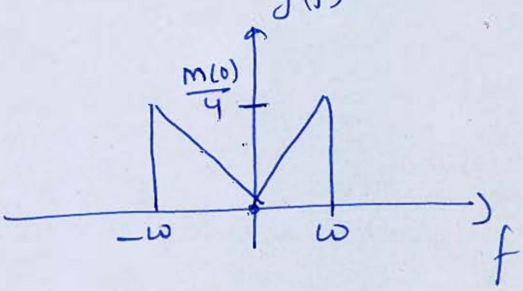
again passing after ideal low pass filter $f_{cutoff} = \omega$

$y(f) \Rightarrow$



(i) $m(t) = 3\,\text{sinc}(2t) - 2\,\text{sinc}^2(t)$; $\text{sinc}(t) = \frac{\sin \pi t}{\pi t}$

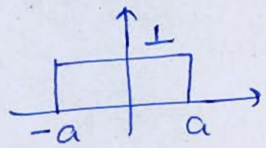
$c(t) = 5\cos(2\pi f_c t)$; $f_c = 100\text{ Hz}$

$s(t) = m(t)\, c(t)$

$$m(t) = 3\,\frac{\sin 2\pi t}{2\pi t} - 2\left(\frac{\sin \pi t}{\pi t}\right)\left(\frac{\sin \pi t}{\pi t}\right)$$

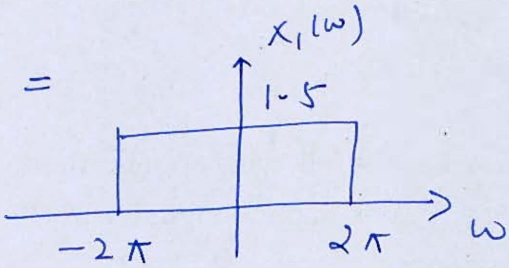
$$\Rightarrow m(t) = \underbrace{1.5\left[\frac{\sin 2\pi t}{\pi t}\right]}_{x_1(t)} - \underbrace{2\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin \pi t}{\pi t}\right]}_{x_2(t)}$$

[we know that $\frac{\sin at}{\pi t} \overset{FT}{\longleftrightarrow}$



]

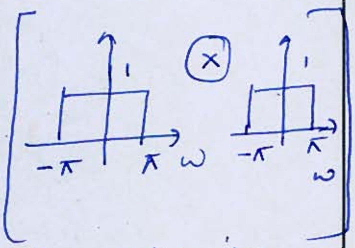
$x_1(t) \longleftrightarrow X_1(\omega) =$



— (1)

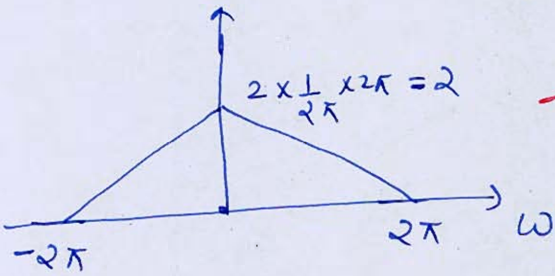
$x_2(t) = 2\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin \pi t}{\pi t}\right] \longleftrightarrow X_2(\omega) =$

$2 \times \frac{1}{2\pi}$



multiplication in time domain results Convolution in frequency domain

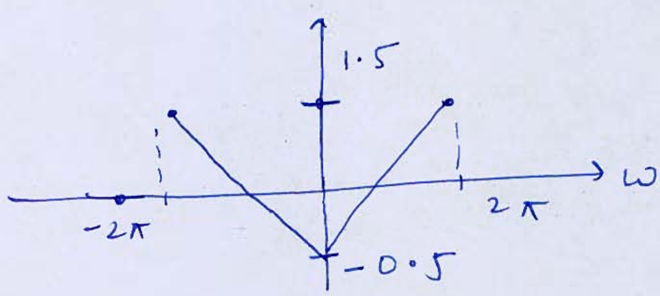
$\therefore\ X_2(\omega) =$



— (2)

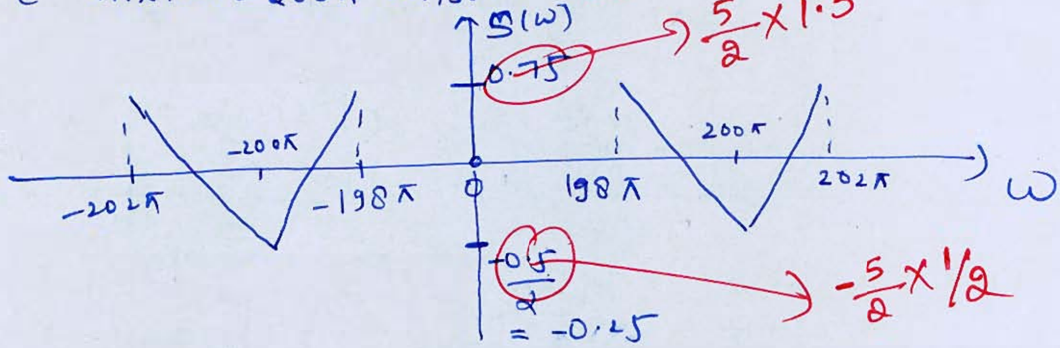
$M(\omega) = X_1(\omega) - X_2(\omega)$

$M(\omega) =$



$\therefore \quad s(t) = m(t)\, c(t) \;\; \xrightarrow{\mathcal{F}} \;\; S(\omega) = \frac{5}{2}\left[M(\omega - \omega_c) + M(\omega + \omega_c) \right]$

$\omega_c = 2\pi \times 100 = 200\pi$ rad/sec.



Q.3 (a) A second-order uncontrolled unity negative feedback system has a plant transfer function,

$$G_p(s) = \frac{4}{s(s+9)}$$

Select a controller to satisfy the following specifications:

(i) The steady-state error due to ramp input is zero.

(ii) The closed loop system has a zero at $s = -3$.

(iii) The complex poles corresponds to natural frequency 5 rad/sec.

[20 marks]

Q.3 (b) (i) A discrete memoryless source is described by the alphabet $X = \{x_1, x_2, ... x_8\}$ and the corresponding probability vector $P = \{0.2, 0.12, 0.06, 0.15, 0.07, 0.1, 0.13, 0.17\}$.

Design a Huffman code for this source; find $\overline{L}$, the average codeword length for the Huffman code; and determine the efficiency of the code.

(ii) Channel C_1 is an additive white Gaussian noise channel with a bandwidth W, average transmitter power P and noise power spectral density $\frac{1}{2}N_0$. Channel C_2 is an Gaussian noise channel with the same bandwidth and power as channel C_1 but with noise power spectral density $S_n(f)$. It is further assumed that the total noise power for both channels is the same i.e.,

$$\int_{-W}^{W} S_n(f)df = \int_{-W}^{W} \frac{1}{2}N_0 df = N_0 W$$

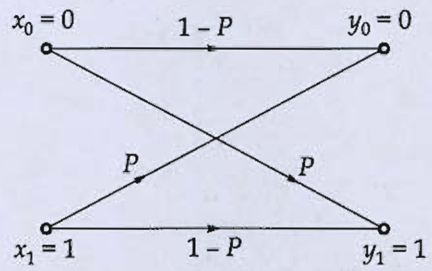
Which channel has a larger capacity? Give an intuitive reasoning.

[10 + 10 marks]

Q.3 (c) (i) A discrete-time stochastic process $X(n) \equiv X(nT)$ is obtained by periodic sampling of a continuous-time zero-mean stationary process $X(t)$, where T is the sampling interval; i.e., $f_s = \frac{1}{T}$ is the sampling rate.

1. Determine the relationship between the autocorrelation function of $X(t)$ and the autocorrelation sequence of $X(n)$.
2. Express the power spectral density of $X(n)$ in terms of the power spectral density of the process $X(t)$.
3. Determine the conditions under which the power spectral density of $X(n)$ is equal to the power spectral density of $X(t)$.

(ii) Consider the binary symmetric channel described in figure. Let P_0 denote the probability of sending binary symbol $x_0 = 0$, and let $P_1 = 1 - P_0$ denote the probability of sending binary sysmbol $x_1 = 1$. Let P denote the transition probability of the channel.



1. Show that the mutual information between the channel input and channel output is given by $I(x : y) = H(z) - H(P)$;

   where,

   $$H(z) = z\log_2\left(\frac{1}{z}\right) + (1-z)\log_2\left(\frac{1}{1-z}\right);$$

   $$z = P_0P + (1 - P_0)(1 - P)$$

   and $$H(P) = P\log_2\left(\frac{1}{P}\right) + (1-P)\log_2\left(\frac{1}{1-P}\right).$$

2. Show that the value of P_0 that maximizes $I(x : y)$ is equal to $\frac{1}{2}$.
3. Also, show that the channel capacity equals $C = 1 - H(P)$.

[10 + 10 marks]

Do not write in this margin

Q.4 (a) Sketch the root-locus plot and determine the approximate damping ratio for a value of K = 1.33 for a control system having a forward transfer function,

$$G(s) = \frac{K(s+2)}{(s+1)^2 + (\sqrt{2})^2}.$$

[20 marks]

$$G(s) = \frac{K(s+2)}{s^2+2s+3}$$

Open loop zero → −2

Open loop pole → $-1 \pm \sqrt{2}\, i$

$$q_1(s) = s^2+2s+3 + K(s+2) = 0$$

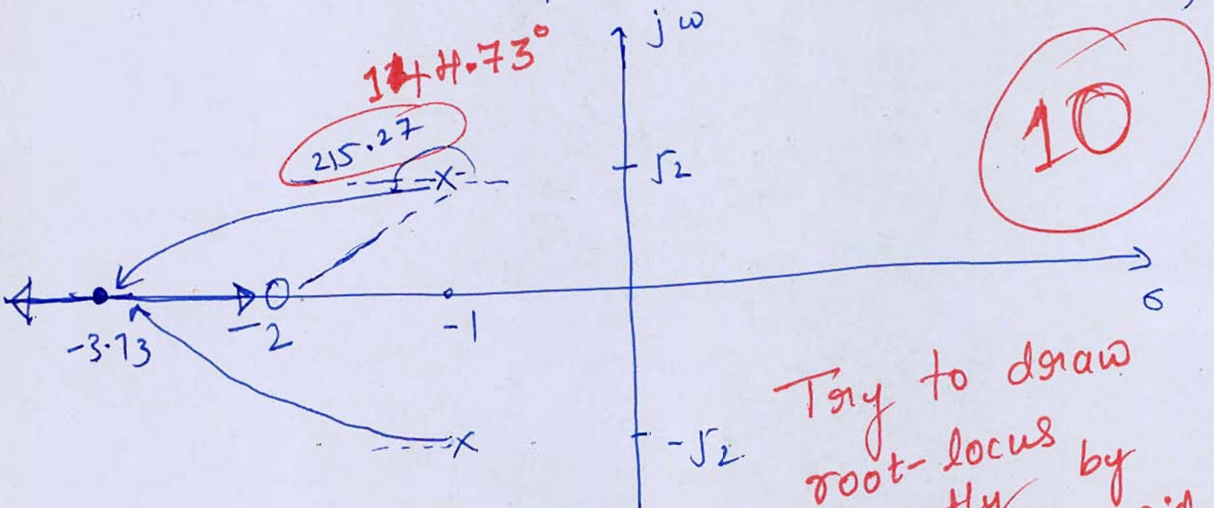
$$K = -\frac{(s^2+2s+3)}{s+2}$$

$$\Rightarrow \frac{dK}{ds} = -\left[\frac{(s+2)(2s+2) - (s^2+2s+3)}{(s+2)^2}\right] = 0$$

$$\Rightarrow 2s^2 + 4s + 2s + 4 - s^2 - 2s - 3 = 0$$

$$\Rightarrow s^2 + 4s + 1 = 0$$

⇒ saddle point $s = -2 \pm \sqrt{3}$ (after (−) −3.73)



Try to draw root-locus neatly by indicating centroid, angle of departure.

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$\phi_D = 180^\circ + \phi$

$\phi_{net} = \phi_z - \phi_p$

$\phi_z = \tan^{-1}\frac{\sqrt{2}}{1} = 54.73$

$\phi_p = 90^\circ$

$\phi_{net} = 90^\circ - 54.73^\circ$

$\phi_{net} = 35.27^\circ$

$\phi_D = 180^\circ + 35.27 = 215.27$

$\phi_D = 180^\circ - (\phi_z - \phi_p)$
$= 180^\circ - (90^\circ - \tan^{-1}\sqrt{2})$
$= 180^\circ - 90^\circ + 54.73^\circ$
$= 144.73^\circ$

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Q.4 (b) **(i)** A message signal $m(t) = A \tanh(\beta t)$ is applied to a delta modulator. Find the minimum step-size required by the delta modulator to eliminate the slope-overload distortion for the given message signal. Assume that A and β are real positive constants.

(ii) Eight telemetry signals, each with a bandwidth of 12 kHz, are to be transmitted by binary PCM with TDM. The maximum tolerable quantization error is 0.6 percent of the peak signal amplitude. The signals are sampled at 25 percent above the Nyquist rate. In TDM, framing and synchronization require an additional 0.5 percent extra bits. Determine the minimum transmission data rate and the minimum required channel bandwidth to transmit the multiplexed signal.

[10 + 10 marks]

(ii) No. of signals $N = 8$

$f_m = 12\text{ kHz}$; max quantization error $= \frac{\Delta}{2} = \frac{0.6}{100} V_m$

$f_s = (1.25)(NR) = (1.25)(2 \times 12\text{ kHz})$

$$\boxed{f_s = 30\text{ kHz}}$$

$\because \quad \frac{\Delta}{2} \leq \frac{0.6}{100} A_m$

$\Rightarrow \quad \left(\frac{2A_m}{2^n}\right)\frac{1}{2} \leq \frac{0.6}{100} A_m$

$\Rightarrow \quad \frac{1}{2^n} < \frac{0.6}{100}$

$\Rightarrow \quad 2^n > \frac{100}{0.6}$

$\Rightarrow \quad 2^n > 166.66$

$\therefore \quad \boxed{n = 8}$ (encoder)

Transmission data rate $R_b = Nnf_s + 0.5\%$ extra

$R_b = 1.005\, Nnf_s$

$\Rightarrow \quad R_b = 8 \times 8 \times 30 \times 10^3 \times 1.005$

$\Rightarrow \quad \boxed{R_b = 1.929 \times 10^6 \text{ bps}}$

10

minimum required channel Bandwidth (BW) $= \frac{R_b}{2}$

Good

$\boxed{BW = 0.964 \text{ MHz}}$

(i) $m(t) = A \tanh(\beta t) = A\left[\frac{e^{\beta t} - e^{-\beta t}}{e^{\beta t} + e^{-\beta t}}\right] = A\left[\frac{e^{2\beta t} - 1}{e^{2\beta t} + 1}\right]$

to avoid slop-overload error $\left|\frac{d}{dt} m(t)\right|_{max} \leq \frac{\Delta}{T_s}$

1

$\Rightarrow$ To avoid slope-overload distortion,

$$\left|\frac{dm(t)}{dt}\right|_{max} \leq \frac{\Delta}{T_s}$$

$$\left|\frac{d}{dt}(A \tanh \beta t)\right|_{max} \leq \Delta f_s$$

$$\left|A\beta \operatorname{sech}^2 \beta t\right|_{max} \leq \Delta f_s$$

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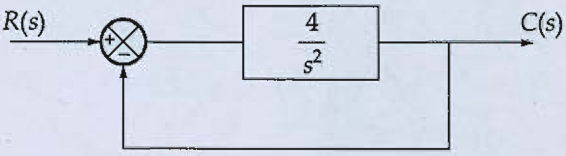
Q.4 (c) (i) System is described by the state equations.

$$X = \begin{bmatrix} 5 & 0 \\ 1 & -1 \end{bmatrix} X + \begin{bmatrix} 1 \\ -1 \end{bmatrix} U \text{ and } Y = [1 \ \ 1]X$$

Determine whether the system is controllable and observable.

(ii) The system shown in figure below is to be compensated by tachometer feedback such that the maximum overshoot is limited to 50%. Determine the value of tachometer feedback constant k_t.



Uncompensated system

[10 + 10 marks]

(i) $A = \begin{bmatrix} 5 & 0 \\ 1 & -1 \end{bmatrix}$; $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; $C = [1 \ \ 1]$

for Controllable $Q_c = [B \ \ AB]$

$AB = \begin{bmatrix} 5 & 0 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

$Q_c = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} \Rightarrow |Q_c| = 2+5 = 7 \neq 0$

∴ Controllable

10

for observable $Q_o = \begin{bmatrix} C \\ CA \end{bmatrix}$

$CA = [1 \ \ 1]\begin{bmatrix} 5 & 0 \\ 1 & -1 \end{bmatrix} = [6 \ \ -1]$

$Q_o = \begin{bmatrix} 1 & 1 \\ 6 & -1 \end{bmatrix} \Rightarrow |Q_o| = -1-6 = -7 \neq 0$

∴ Observable

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(ii) For tachometer we place $\boxed{K_D s}$ in feedback

now $GH(s) = \dfrac{4/s^2}{1 + \frac{4}{s^2} K_D s} = \dfrac{4}{s^2 + 4K_D s}$

$$GH(s) = \frac{4}{s^2 + 4K_D s}$$

$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 4K_D s + 4} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\boxed{\omega_n = 2 \text{ rad/sec}}$

$M_p = 0.05 \rightarrow 0.5$ ✓

$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.05 \rightarrow 0.5$

$\Rightarrow 20 = \exp\left[\dfrac{\xi\pi}{\sqrt{1-\xi^2}}\right]$ → 0.5

$\Rightarrow 2.99 = \dfrac{\xi\pi}{\sqrt{1-\xi^2}}$ → 0.693

$\Rightarrow (2.99)^2 (1-\xi^2) = \xi^2 \pi^2$

$\Rightarrow 8.94 - 8.94\xi^2 = \xi^2\pi^2$

$\Rightarrow \xi^2 = \dfrac{8.94}{8.94 + \pi^2}$

$\Rightarrow \boxed{\xi = 0.689}$ ✗ after neglecting (-ive) value

$\xi = 0.57$

6

Avoid calculation mistakes

$\because 4K_D = 2\xi\omega_n$

$\Rightarrow K_D = \dfrac{\xi\omega_n}{2} = \dfrac{0.689 \times 2}{2}$

$\Rightarrow \boxed{K_D = 0.689}$ ✗ $K_D = 0.57$

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Section B : Analog and Digital Communication + Control Systems

Q.5 (a) Sketch the polar plot of the transfer function given below. Determine whether the plot cross the real axis. If so, determine the frequency at which the plot cross the real axis and the corresponding values of $G(j\omega)$.

$$G(s) = \frac{(1+0.2s)(1+0.025s)}{s^3(1+0.005s)(1+0.001s)}$$

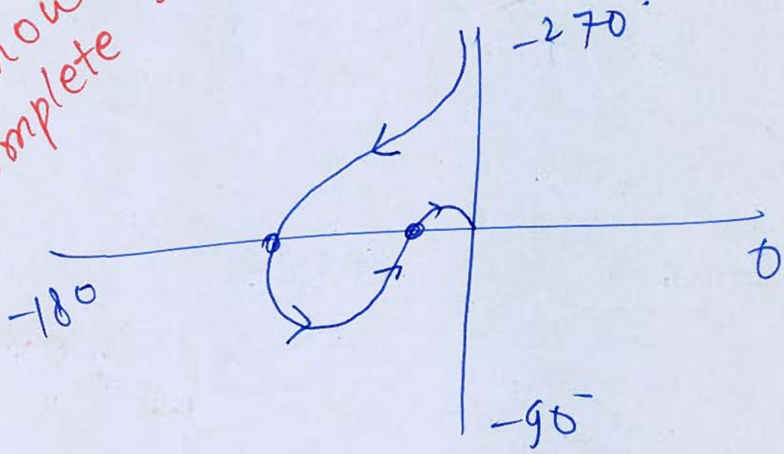
[12 marks]

$$|GH(j\omega)| = \frac{\sout{1+0.2j\omega)}\ \sqrt{1+0.04\omega^2}\ \sqrt{1+(0.025\omega)^2}}{\omega^3\sqrt{1+(0.005\omega)^2}\ \sqrt{1+(0.001\omega)^2}}$$

6

$$\phi = \tan^{-1}0.2\omega + \tan^{-1}0.025\,\omega - 270^\circ - \tan^{-1}0.005\omega - \tan^{-1}0.001\omega$$

Show complete steps



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Q.5 (b) (i) An FM signal with a frequency deviation of 10 kHz at a modulation frequency of 5 kHz is applied to two frequency multipliers connected in cascade. The first multiplier doubles the frequency and the second multiplier triples the frequency. Determine the frequency deviation and the modulation index of the FM signal obtained at the second multiplier output. What is frequency separation of the adjacent sidebands of this FM signal?

(ii) A certain sinusoid at a frequency f_m Hz is used as the modulating signal in both a conventional AM system and a FM system. When modulated, the peak frequency deviation of FM system is set to three times the bandwidth of the AM system. The magnitude of those sidebands spaced at $\pm f_m$ Hz from carrier in both systems are equal and the total average power is equal in both systems (Given $J_1(6) = 0.34$)

Determine the

1. Modulation index of the FM system.
2. Modulation index of the AM system.

[6 + 6 marks]

Q.5 (c) A unity negative feedback control system has an open-loop transfer function consisting of two poles, two zeros and a variable gain K. The zeros are located at –2 and –1; and poles at –0.1 and 1.

Using Routh Hurwitz stability criterion, determine the range of values of K for which the closed loop system has

(i) no poles in the right half of s-plane.

(ii) 1 pole in the right half of s-plane.

(iii) 2 poles in the right half of s-plane.

[12 marks]

Open loop Transfer function $GH(s) = \dfrac{K(s+1)(s+2)}{(s+0.1)(s-1)}$

$q(s) = 1 + GH(s) = 0$

$\Rightarrow q(s): (s+0.1)(s-1) + K(s+1)(s+2) = 0$

$\Rightarrow s^2 + 0.1s - s - 0.1 + Ks^2 + 3Ks + 2K = 0$

$\Rightarrow (1+K)s^2 + (3K-0.9)s + (2K-0.1) = 0$

(i) No poles in right half of s-plane (closed loop) means stable system;

$\therefore$ $K+1>0$ → $K > -1$ | $3K - 0.9 > 0$ → $K > 0.3$ | $2K - 0.1 > 0$ → $K > 0.05$

Now R-H array

s^2 $(1+K)$ $(2K-1)$

s^1 $(3K-0.9)$

s^0 $(2K-1)$

In first column there should be no sign change for stability

$\therefore$ $K > 0.3$

Ans

(ii) 1 Pole in the right half of s-plane :→

(means there should be one sign change in 1st column)

∴ $1+K>0$, $K>-1$ | $3K-0.9>0$, $K>0.3$ | $2K-0.1<0$, $K<0.05$

Hence there is no possible value of K for this option

(iii) 2-poles in right half of s-plane

(means there should be two sign change in 1st column)

∴ $1+K>0$, $K>-1$ | $3K-0.9<0$, $K<0.3$ | $2K-0.1>0$, $K>0.05$

∴ $\boxed{0.05 < K < 0.3}$

11

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Q.5 (d) An FSK system transmits binary data at the rate of 4.5×10^6 bits per second. During the course of transmission, white Gaussian noise of zero mean and power spectral density 10^{-20} watts per hertz is added to the signal. In the absence of noise, the amplitude of the received sinusoidal wave for digit 1 or 0 is 1.2 μ volt. Determine the average probability of symbol error, assuming coherent detection.

$$\left[\text{Assume } erfc(z) = \frac{e^{(-z^2)}}{\sqrt{\pi}z}\right]$$

[12 marks]

12

Good

$R_b = 4.5 \times 10^6$ bits/sec. ; $\frac{N_0}{2} = 10^{-20}$ watt/Hz

Amplitude of sinusoidal wave $\boxed{A = 1.2 \times 10^{-6} \text{ volt}}$ ✓

we know that

$$P_e = Q\left[\sqrt{\frac{d_{min}^2}{2N_0}}\right] = \frac{1}{2} erfc\left[\sqrt{\frac{d_{min}^2}{4N_0}}\right]$$

for BFSK $\boxed{d_{min} = \sqrt{2E_b}}$ ✓

$\therefore \quad P_e = \frac{1}{2} \text{erfc}\left[\sqrt{\frac{2E_b}{4N_0}}\right] = \frac{1}{2}\text{erfc}\left[\sqrt{\frac{E_b}{2N_0}}\right]$ —①

$E_b = \frac{A^2 T_b}{2} = \frac{A^2}{2R_b}$

$\therefore \quad E_b = \frac{(1.2\times10^{-6})^2}{2\times4.5\times10^6} = 1.6\times10^{-19}$ Joule

$\therefore \quad P_e = \frac{1}{2}\text{erfc}\left[\sqrt{\frac{1.6\times10^{-19}}{2\times2\times10^{-20}}}\right]$

$\Rightarrow \quad P_e = \frac{1}{2}\text{erfc}\left[\sqrt{\frac{16}{4}}\right]$

$\Rightarrow \quad \boxed{P_e = \frac{1}{2}\text{erfc}(2)}$

$\therefore \quad \text{erfc}(z) = \frac{e^{-z^2}}{\sqrt{\pi}\, z}$

$\therefore \quad \text{erfc}(2) = \frac{e^{-4}}{2\sqrt{\pi}} = 5.166\times10^{-3}$

$\therefore \quad P_e = \frac{5.166\times10^{-3}}{2}$

$\Rightarrow \quad \boxed{P_e = 2.583\times10^{-3}}$ Ans

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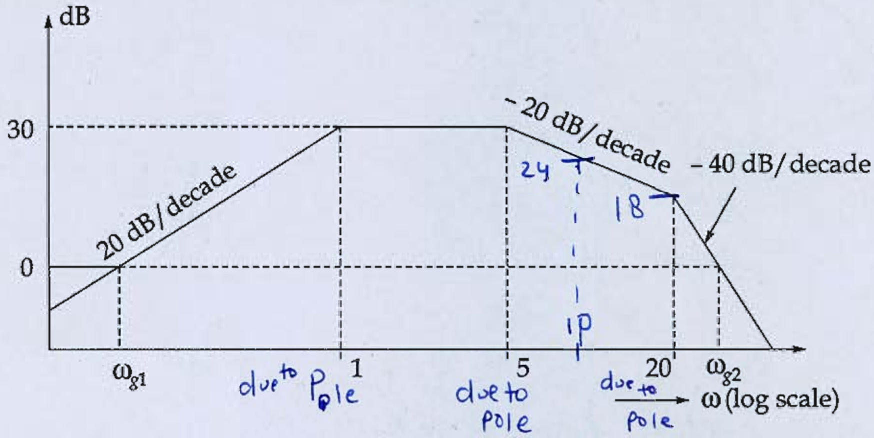
Q.5 (e) Consider a minimum phase system whose asymptotic bode magnitude plot is depicted in figure.

Determine:

(i) the transfer function $G(s)$ of the system.

(ii) the gain cross over frequencies ω_{g1} and ω_{g2}.



[6 + 6 marks]

(i) initial slope = 20 dB/decade

no. of open loop zero = 1

$\omega_1 = 1$ rad/se ; $\omega_2 = 5$ rad/sec. ; $\omega_3 = 20$ rad/sec

$$G(s) = \frac{Ks}{\left(\frac{s}{\omega_1}+1\right)\left(\frac{s}{\omega_2}+1\right)\left(\frac{s}{\omega_3}+1\right)} = \frac{Ks}{\left(\frac{s}{1}+1\right)\left(\frac{s}{5}+1\right)\left(\frac{s}{20}+1\right)}$$

6

$$\Rightarrow \boxed{G(s) = \frac{Ks}{(s+1)(0.2s+1)(0.05s+1)}}$$

Inital slope = $-20r\log\omega$

$\therefore$ gain (y) = $-20r\log\omega + 20\log K$

$\therefore$ at $\omega = 1$; gain = 30 dB

$\Rightarrow 30 = -20(1)\log(1) + 20\log K$

$$\Rightarrow \boxed{K = 31.622}$$

$\therefore$ $$G(s) = \frac{31.622\, s}{(s+1)(0.2s+1)(0.05s+1)}$$

(ii) gain cross over frequency = where gain becomes 0

for ω_{g1} :⇒ $\frac{y_2 - y_1}{\log \omega_2 - \log \omega_1}$ = slope

$\therefore$ $$\frac{30-0}{\log(1/\omega_{g1})} = 20$$

⇒ $1.5 = \log\left(\frac{1}{\omega_{g1}}\right)$

⇒ $\frac{1}{\omega_{g1}} = 10^{1.5}$

⇒ $\omega_{g1} = 0.0316$ rad/sec.

for ω_{g2} [$\omega = 5 \xrightarrow{to} 10$ (1 octave)

$\because$ -20 dB/decade $= -6$ dB/octave

$\therefore$ gain at $\omega = 10$ ⇒ $30 - 6 = 24$ dB

again $\omega = 10 \xrightarrow{to} 20$ (1 octave)

6

$\therefore$ gain at $\omega = 20$ ⇒ $24 - 6 = 18$ dB

$\therefore$ $$\frac{18-0}{\log(20/\omega_2)} = -40$$

⇒ $-\frac{18}{40} = \log\frac{20}{\omega_2}$

⇒ $\frac{18}{40} = \log\frac{\omega_2}{20}$ ⇒ $\omega_2 = 20 \times 10^{(18/40)}$

⇒ $\omega_2 = 56.367$ rad/sec

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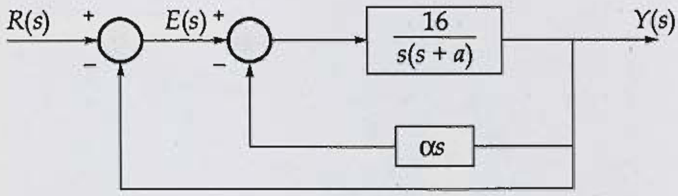
Q.6 (a) The system shown in figure is a unity-feedback. Control system with a minor rate-feedback loop.



(i) In the absence of rate feedback ($\alpha = 0$), determine the peak overshoot of the system to unit-step input and the steady-state error resulting from a unit-ramp input.

(ii) Determine the rate-feedback constant α which will decrease the peak overshoot of the system of unit-step input to 1.5%. What is steady-state error to unit-ramp input with this setting of the rate feedback constant?

(iii) Illustrate how in the system with rate feedback, the steady-state error to unit-ramp input can be reduced to the same level as in part (i), while the peak over-shoot to unit-step input is maintained at 1.5%.

[4 + 8 + 8 marks]

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Q.6 (b) Design an Armstrong indirect FM modulator to generate an FM signal with carrier frequency 97.3 MHz and Δf = 20.48 kHz. A NBFM generator with f_{c1} = 20 kHz and Δf = 10 Hz is available. Only frequency doublers can be used as multipliers. Additionally, a local oscillator (LO) with adjustable frequency between 400 kHz and 500 kHz is readily available for frequency mixing.

[20 marks]

Q.6 (c) A signal $m(t) = 3\cos(25\pi t) - 2\cos(50\pi t)$, where the unit of time is milliseconds is amplitude modulated using the carrier frequency (f_c) of 600 kHz. The AM signal is given by $s(t) = 6\cos(2\pi f_c t) + m(t)\cos(2\pi f_c t)$.

(i) Sketch the magnitude spectrum of $s(t)$. What its bandwidth?

(ii) What is the modulation index?

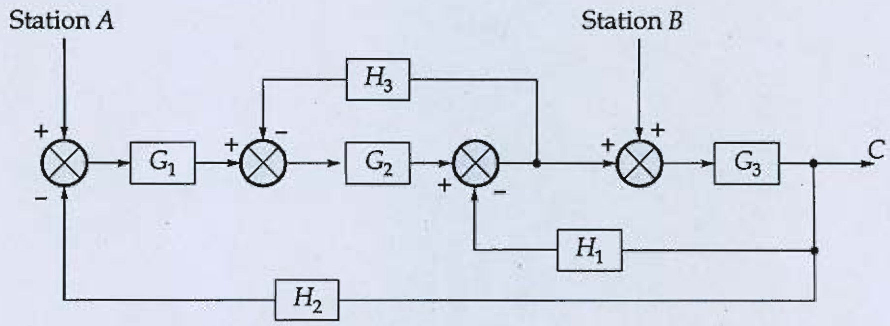
(iii) The AM signal is passed through a high-pass filter with cut-off frequency 590 kHz. (i.e., the filter passes all frequencies above 590 kHz, and cuts off all frequencies below 590 kHz). Find an explicit time-domain expression for the quadrature component of the filter output with respect to a 600 kHz frequency reference.

[8 + 2 + 10 marks]

Q.7 (a) For the system represented by the block diagram shown in figure below, evaluate the closed-loop transfer function when the input R is

(i) at station A and

(ii) at station B.



[10 + 10 marks]

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Q.7 (b) **(i)** A linear time-invariant system is characterized the homogeneous state equation.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

1. Compute the solution of the homogeneous equation assuming the initial state vector.

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

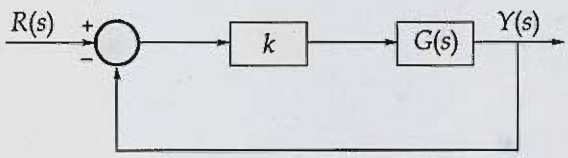
2. Consider now the system has a forcing function and is represented by the following non homogeneous state equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

where u is a unit step input.

Compute the solution of this equation assuming initial conditions of part 1.

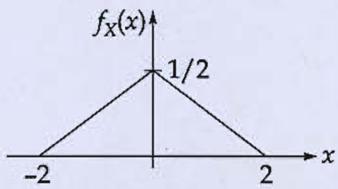
(ii) Given an open loop transfer function, discuss the stability of the closed loop system by Nyquist stability criterion.



where, $G(s) = \dfrac{8}{(s+1)(s^2+2s+2)}$.

[10 + 10 marks]

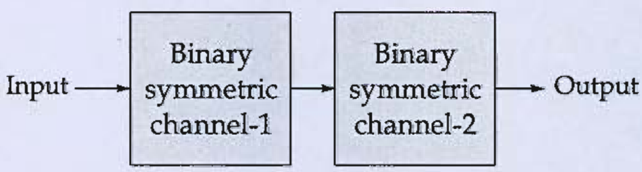
(c) (i) A signal can be modeled as a lowpass stationary process $X(t)$ whose PDF at any time t_0 is given below,



The bandwidth of this process is 5 kHz and it is desired to transmit it using a PCM system. If sampling is done at the Nyquist rate and a uniform quantizer with 32 levels is employed then determine,

1. SQNR
2. Bit Rate

(ii) Two binary symmetric channels are connected in cascade, as shown in figure. Find the overall channel capacity of the cascaded connection, assuming that both channels have the same transition probability.



Also, calculate the capacity of cascaded connection if the transition probability is 0.4.

[5 + 5 + 10 marks]

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Q.8 (a) **(i)** Derive the expression for the transfer function from the state model.

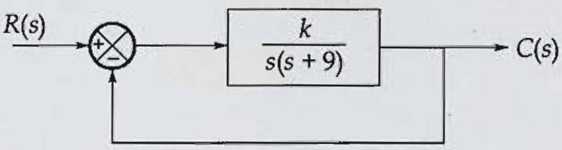
$\dot{x} = Ax + Bu$

$y = Cx + Du$

and also specify the required assumption for obtaining transfer function as

$$\frac{Y(s)}{U(s)} = T(s) = C(sI - A)^{-1}B + D$$

(ii) For the system shown below, if the resonant frequency of the system is 12 rad/sec and magnitude at resonant frequency is 1.15.



Calculate the values of

1. k **2.** a **3.** Settling time **4.** Bandwidth

[10 + 10 marks]

(i) $\dot{X} = AX + BU$

$\Rightarrow SX(s) - x(0) = AX(s) + BU(s)$

For transfer function assume initial condition $x(0) = 0$

$\therefore SX(s) = AX(s) + BU(s)$

$\Rightarrow \boxed{X(s) = (sI - A)^{-1} BU(s)}$ (1)

Do not write in this margin

$Y(s) = CX(s) + DU(s)$

$\Rightarrow Y(s) = C[sI-A]^{-1}BU(s) + DU(s)$

$$\Rightarrow \boxed{\frac{Y(s)}{U(s)} = T(s) = C[sI-A]^{-1}B + D}$$

9

(ii) $\omega_r = 12\ \text{rad/sec}$ $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.15$

$\Rightarrow \omega_r = \omega_n\sqrt{1-2\xi^2}$

$\Rightarrow \frac{1}{2\times 1.15} = 2\xi\sqrt{1-\xi^2}$

$\Rightarrow 0.189 = \xi^2 - \xi^4$

$\Rightarrow \xi^4 - \xi^2 + 0.189 = 0$

$\Rightarrow \xi^2 = 0.746\ ;\ 0.253$

$\because \left[\xi < \frac{1}{\sqrt{2}}\ (\text{for under damped}),\ \xi^2 < 0.5\right]$ $\therefore \xi^2 = 0.253$

$\boxed{\xi = 0.502}$

again $12 = \omega_n\sqrt{1-\xi^2}$

$\Rightarrow 12 = \omega_n\sqrt{1-0.253}$

$\Rightarrow \boxed{\omega_n = 13.884\ \text{rad/sec}}$

$\frac{C(s)}{R(s)} = \frac{K}{s^2+9s+K}$; $= \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

(a)

$\therefore$ (1) $\boxed{K = \omega_n^2 = 192.76}$

Sorry, there is a misprint in G(s) given.

$G(s) = \frac{k}{s(s+a)}$

where 'a' is unknown

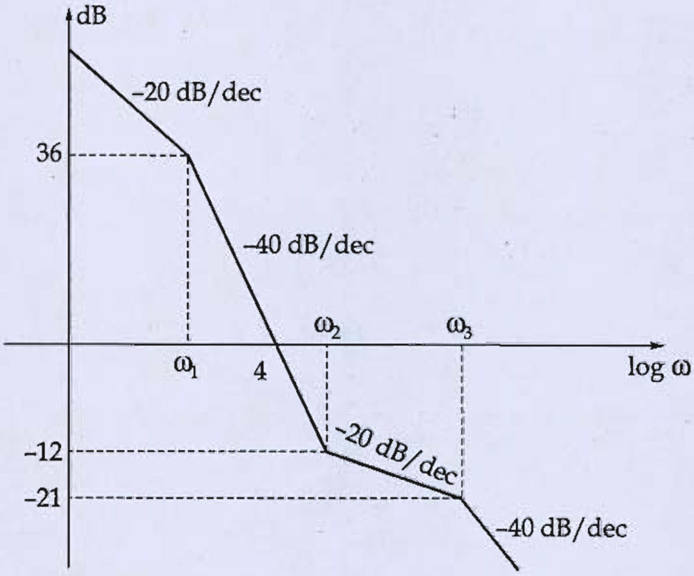
9

(2) damping factor $a = \xi \omega_n = 4.5$ ×

(3) settling $T_s = 4T = \frac{4}{\xi \omega_n} = \frac{4}{4.5}$

$T_s = 0.88$ sec ×

(b) Derive the transfer function of the system from the data given on the bode diagram shown in figure below:



[20 marks]

$$GH(s) = \frac{K\left(\frac{s}{\omega_2}+1\right)}{s\left(\frac{s}{\omega_1}+1\right)\left(\frac{s}{\omega_3}+1\right)}$$

$$\Rightarrow \quad -40 = \frac{0-36}{\log(4/\omega_1)}$$

$$\Rightarrow \quad \log\left(\frac{4}{\omega_1}\right) = \frac{36}{40}$$

$$\Rightarrow \quad \frac{4}{\omega_1} = 10^{36/40}$$

$$\Rightarrow \quad \boxed{\omega_1 = 0.503 \text{ rad/sec}}$$

again initial slop = $-20 r \log(\omega)$

$$36 = -20(1)\log\omega_1 + 20\log K$$

$$\Rightarrow \quad 20\log K = 36 + 20\log(0.503)$$

$$\boxed{K = 31.66}$$

again $-40 = \frac{-12-0}{\log\left(\frac{\omega_2}{4}\right)} = \frac{-12}{\lg\left(\frac{\omega_2}{4}\right)}$

$\Rightarrow \log \frac{\omega_2}{4} = \frac{12}{40}$

$\Rightarrow \omega_2 = 4 \times 10^{12 \div 40}$

$\Rightarrow \boxed{\omega_2 = 7.98 \text{ rad/sec}}$

again $-20 = \frac{-21-(-12)}{\log\left(\frac{\omega_3}{\omega_2}\right)} = \frac{-9}{\lg\left(\frac{\omega_3}{7.98}\right)}$

$\Rightarrow \log\left(\frac{\omega_3}{7.98}\right) = \frac{9}{20}$

$\Rightarrow \omega_3 = 7.98 \times 10^{9 \div 20}$

$\boxed{\omega_3 = 22.49 \text{ rad/sec}}$

$\therefore \boxed{GH(s) = \frac{31.66\left(\frac{s}{7.98}+1\right)}{s\left(\frac{s}{0.503}+1\right)\left(\frac{s}{22.49}+1\right)}}$

19

Good

(c) Let $s(t)$ be a digital NRZ signal ($\pm A$), which passes through the noisy channel. Channel introduces White Gaussian Noise [$\omega(t)$] having PSD of $N_0/2$. Receiver was designed using matched filter, sample and hold circuit and decision making circuit. Decision making circuit uses maximum likelihood algorithm/technique. Compute the following:

(i) Output of the sample and hold circuit when 'A' is transmitted.

(ii) Variance of the Noisy signal at the output of Sample and Hold circuit.

(iii) Compute the probability of error when 'A' is received/detected as '$-A$' and '$-A$' is interpreted as '$+A$'.

[5 + 5 + 10 marks]

OOOO

Space for Rough Work

Space for Rough Work

Space for Rough Work

$$\frac{1}{\sqrt{(1-\omega^2)^2 + (2\xi\omega)^2}}$$

$$\omega_r = \sqrt{1-2\xi^2}$$

$$\frac{1}{\sqrt{(1-1+2\xi^2)^2 + 4\xi^2(1-2\xi^2)}}$$

$$4\xi^4 + 4\xi^2 - 8\xi^4$$

$$\sqrt{4\xi^2 - 4\xi^4}$$

$$2\xi\sqrt{1-\xi^2}$$

Space for Rough Work