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ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Mechanical Engineering

Test-4 : Theory of Machines + Industrial and Maintenance Engineering

Name :

Roll No :

Test Centres	Student's Signature
Delhi <input checked="" type="checkbox"/> Bhopal <input type="checkbox"/> Jaipur <input type="checkbox"/> Pune <input type="checkbox"/> Kolkata <input type="checkbox"/> Hyderabad <input type="checkbox"/>	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	18
Q.2	12
Q.3	—
Q.4	—
Section-B	
Q.5	30
Q.6	54
Q.7	—
Q.8	23
Total Marks Obtained	137

Signature of Evaluator

Cross Checked by

Caan Shan

Well done! Keep it up.

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section B : Theory of Machines + Industrial and Maintenance Engineering

- (a) A rail car has a total weight of 1500 N. The moment of inertia of each wheel together with its gearing is 25 kg.m^2 . The centre distance between the two on an axle is 1.6 m and each wheel is 420 mm radius. Each axle is driven by a motor the speed ratio between the two being is 1 : 3. Each axle is driven by a motor the speed ratio between the two being is 1 : 3. Each motor with its gear has a moment of inertia of 18 kgm^2 and runs in the direction opposite to that of its axle. The centre of gravity of the car is 1 m above the rails. Determine the limiting speed for the car when moving on a curve of 250 m radius such that no wheel leaves the rails.

[12 marks]

Given : weight = 1500 N

$$M. I. = 25 \text{ kg.m}^2$$

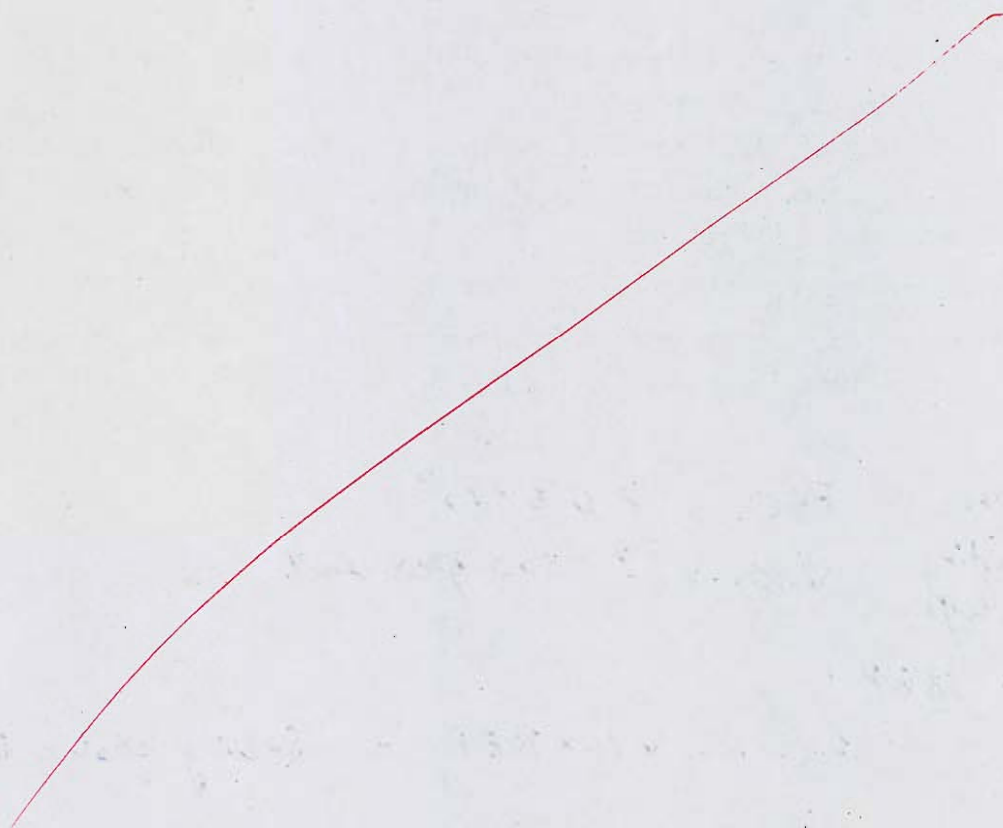
$$C = 1.6 \text{ m}$$

$$r_w = 0.42 \text{ m}$$

Y @

- (b) Write the basic principle of maintenance planning. Also write the objective of planned maintenance activity.

[12 marks]



Q.1 (c) A plant is manufacturing 5000 heavy duty lathes per year and is operating at 80% of its capacity. The annual sales return is ₹3.0 × 10⁷. The fixed cost of the plant is ₹0.5 × 10⁷ and variable cost of ₹4500 per unit. There is a proposal to utilise spare capacity by manufacturing precision lathes which would increase the fixed cost by ₹800000 but reduce the variable cost by ₹800 per unit.

- (i) Is the proposal economical? Give reasons for your answer.
- (ii) If a reduction in selling price by ₹500 per unit requires the plant to be run at 90% of its capacity to break even, would this be a better proposal than the earlier one?

[12 marks]

Given: F.C. = ₹0.5 × 10⁷
Existing setup V.C. = ₹4500 per unit

At BEP:

$$F.C. + V.C. \times BEP = \text{Selling cost} \times BEP$$

$$\Rightarrow 0.5 \times 10^7 + 4500 \times x = 3 \times 10^7$$

$$\Rightarrow x = \frac{2.5 \times 10^7}{4500} = 5.555 \times 10^3 \text{ units}$$

$$\begin{aligned}\text{Capacity of the plant} &= \frac{5000}{0.8} \\ &= 6250 \text{ units}\end{aligned}$$

7

- Q.1 (d) The crank and the connecting rod of a vertical single cylinder gas engine running at 2000 rpm are 80 mm and 300 mm respectively. The diameter of the piston is 90 mm and the mass of the reciprocating parts is 2 kg. At a point during the power stroke when the piston has moved 20 mm from the stop dead centre position, the pressure on the piston is 1000 kN/m^2 . Determine the
- net force on the piston.
 - thrust in the connecting rod.
 - thrust on the sides of cylinder walls.
 - engine speed at which the above values are zero.

[12 marks]

Given : $r = 0.08 \text{ m}$

$l = 0.3 \text{ m}$

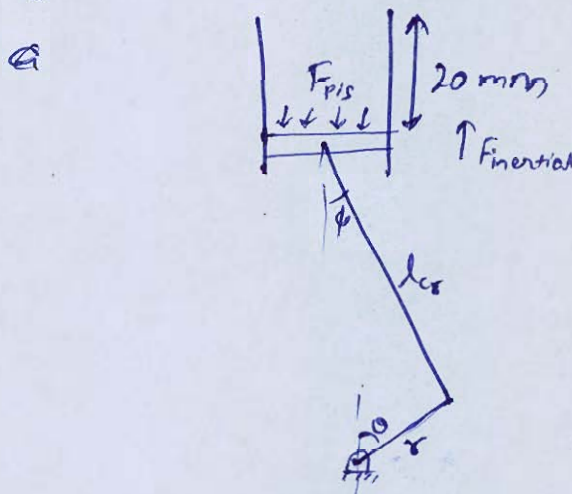
$n = \frac{1}{r} = \frac{15}{4}$

$d_p = 0.09 \text{ m}$

$m_{rec} = 2 \text{ kg}$

$N = 2000 \text{ rpm} \Rightarrow \omega = 2\pi \times \frac{2000}{60} = \frac{200\pi}{3}$

(i) Net force on the piston = F_{net}



$x = (l + r) - (l \cos \phi + r \cos \theta)$

$20 = (300 + 80) - (300 \cos \phi + 80 \cos \theta)$

$\Rightarrow 300 \cos \phi + 80 \cos \theta = 360$

$\Rightarrow 15 \cos \phi + 4 \cos \theta = 18 \quad \text{--- (1)}$

$r \sin \theta = l \sin \phi$

$\Rightarrow 80 \sin \theta = 300 \sin \phi$

$$\Rightarrow \sin \theta = \frac{15}{4} \sin \phi$$

$$\Rightarrow 1 - \cos^2 \phi = \frac{16}{225} (1 - \cos^2 \theta)$$

$$\Rightarrow \cancel{1 - \cos^2 \phi} \cos^2 \phi = 1 - \frac{16}{225} + \frac{16}{225} \cos^2 \theta$$

$$\cos^2 \phi = \frac{209}{225} + \frac{16}{225} \cos^2 \theta \quad \text{--- (ii)}$$

$$\text{From (i)} : \cos^2 \phi = \left(\frac{18 - 4 \cos \theta}{15} \right)^2$$

$$\Rightarrow \frac{209}{225} + \frac{16}{225} \cos^2 \theta = \frac{324 + 16 \cos^2 \theta - 144 \cos \theta}{225}$$

$$\Rightarrow 144 \cos \theta = 115$$

$$\Rightarrow \theta = 37^\circ ; \phi = 9.235^\circ$$

$$\begin{aligned} a_{\text{piston}} &= \omega^2 r \left(\cos \theta + \frac{\cos 2\theta}{n} \right) \\ &= \left(\frac{200\pi}{3} \right)^2 \times 0.08 (0.798 + 0.0735) \\ &= \left(\frac{200\pi}{3} \right)^2 \times 0.07 \\ &= 3070.5 \text{ m/s}^2 \end{aligned}$$

$$F_{\text{net}} = F_{\text{pis}} - F_{\text{inertia}} + \cancel{F_{\text{spring}}} 209.61$$

$$\begin{aligned} \textcircled{12} &= \left(1000 \times 10^3 \times \frac{\pi}{4} \times (0.09)^2 \right) - 2 \times 3070.5 + 19.62 \\ &= 240.345 \text{ N} \quad \underline{\text{Ans}} \end{aligned}$$

$$\text{(ii)} \quad F_Q = \frac{F_{\text{net}}}{\cos \phi} = 243.5 \text{ N} \quad \underline{\text{Ans}}$$

$$\text{(iii)} \quad F_{\text{thrust on side walls}} = F_{\text{net}} \times \tan \phi = 39 \text{ N} \quad \underline{\text{Ans}}$$

$$\text{(iv)} \quad F_{\text{net}} = 0 \Rightarrow 1000 \times 10^3 \times \frac{\pi}{4} \times (0.09)^2 + 19.62 = 2\omega^2 \times 0.07$$

$$\Rightarrow \omega = 213.5 \text{ rad/s}$$

$$\Rightarrow N = \frac{\omega \times 60}{2\pi} = \cancel{2038.7} \text{ rpm} \quad \underline{\text{Ans}}$$

- Q.1 (e) The centre distance between two spur gears in a mesh is to be approximately 280 mm. The gear ratio is 9 to 1. The pinion transmit 400 kW at 2000 rpm. The pressure angle of the involute teeth is 20° and the addendum is equal to one module. The limiting value of normal tooth pressure is 1 kN/mm of width. Determine the
- nearest standard module so that interference does not occur and number of teeth on each gear wheel.
 - width of pinion.

[12 marks]

Given : $C = 0.28 \text{ m}$

G.R. = $\frac{9}{1}$

$P = 400 \text{ kW}$ @ 2000 rpm

$\phi = 20^\circ$

$\Rightarrow \omega = \frac{2\pi \times 2000}{60}$
 $= \frac{200\pi}{3}$

$\frac{T_g}{T_p} = \frac{d_g}{d_p} = \frac{r_g}{r_p} = 9$

$\therefore r_g + r_p = 0.28$

$\Rightarrow r_p = 0.028 \text{ m}$

$\therefore r_g = 0.252 \text{ m}$

$P = T \times \omega$

$\Rightarrow T = \frac{400 \times 10^3}{\left(\frac{2000}{60}\right)} = \frac{3}{2\pi} \times 10^3$
 $= 477.46 \text{ N-m}$

For no interference :

$T_g = \frac{2}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1}$
 $= \frac{2}{\sqrt{1 + \frac{1}{9} \left(\frac{1}{9} + 2 \right) \sin^2 20^\circ} - 1}$

$= 146.72$

$\approx 148 \text{ or } 150 \text{ teeth}$

$$T_r = \frac{150}{9} = 16.67$$

$$\approx 17$$

$$\Rightarrow T_g = 17 \times 9 = \underline{153} \text{ teeth}$$

$$m = \frac{28}{17} \quad \text{or} \quad \frac{252}{153}$$

$$= \underline{1.647} \quad = \underline{1.647} \quad \underline{\underline{\text{Ans}}}$$

(ii)

$$T = F \times r_p$$

$$\text{Torque, } F = 1000 \text{ N} \times w$$

$$\Rightarrow 477.46 = (1000 w) \times 0.028$$

$$\Rightarrow \underline{w = 17.05 \text{ mm}} \quad \underline{\underline{\text{Ans}}}$$

Q.2 (a) Five spare parts are made, each of which must go through machines M1, M2, M3 in order M1, M3, M2. Processing times (in hours) are given below :

1. Determine the optimal sequence and total elapsed time of each machine.
2. If it can be processed by the sub-contract services by three outside parties P1, P2 and P3 to process on M1, M2 and M3 respectively, schedule the parties optimally under the condition that the parties may be called on any day but the contract once started should be continued till the last job of the respective party is completed and the payment should be made for the process delay also.
3. What will be the amount paid to each party if it costs Rs. 10/- per working hour and Rs. 5/- per waiting hour?

Spare Part	1	2	3	4	5
M1	8	5	4	6	5
M2	10	13	11	10	12
M3	6	2	9	7	4

[20 marks]

For Johnson rule to be applicable :

Order $M_1 \rightarrow M_3 \rightarrow M_2$

~~Max. of $M_1 \geq$ Min. of M_3~~

Min. of $M_1 \geq$ Max. of M_3

OR Min. of $M_2 \geq$ Max. of M_3
(10) \geq (9) \rightarrow satisfied.

Let new machines : $W = M_1 + M_3$

$X = M_3 + M_2$

	$W = M_1 + M_3$	$X = M_3 + M_2$
1	14	16
2	7	15
3	13	20
4	13	17
5	9	16

Order sequence : $\underline{2}, \underline{5}, \underline{3}, \underline{4}, \underline{1}$

OR $\underline{2}, \underline{5}, \underline{4}, \underline{3}, \underline{1}$

Ans

Total elapsed time of each machine :

	M1		M3		M2	
	In	Out	In	Out	In	Out
2	0	5	5	7	7	20
5	5	10	10	14	20	32
3	10	10 14	10 14	20 23	32	43
4	14	20	23	30	43	53
1	20	28	30	36	53	63

$$M1 : 63 - (8 + 5 + 4 + 6 + 5) = \underline{35 \text{ hours}}$$

$$M3 : 63 - (6 + 2 + 9 + 7 + 4) = \underline{35 \text{ hours}}$$

$$M2 : 63 - (10 + 13 + 11 + 10 + 12) = \underline{7 \text{ hours}}$$

2-3: P1 : Amount = $10 \times 28 + 5 \times 35 = \underline{£455}$ Ans

P3 : Amount = $10 \times 28 + 5 \times 35 = \underline{£455}$ Ans

P2 : Amount = $10 \times 56 + 7 \times 5 = \underline{£595}$ Ans

Q.2 (b) A symmetrical tangent cam operating a roller follower has the following particulars:
Radius of base circle of cam = 45 mm; Roller radius = 25 mm; Angle of ascent = 75°
Total lift = 25 mm; Speed of cam shaft = 360 rpm
Determine

- (i) The principal dimension of the cam.
- (ii) The equation of the displacement curve, when the follower is in contact with the straight flank.
- (iii) The acceleration of the follower, when it is in contact with the straight flank where it merges into the circular nose.

[20 marks]

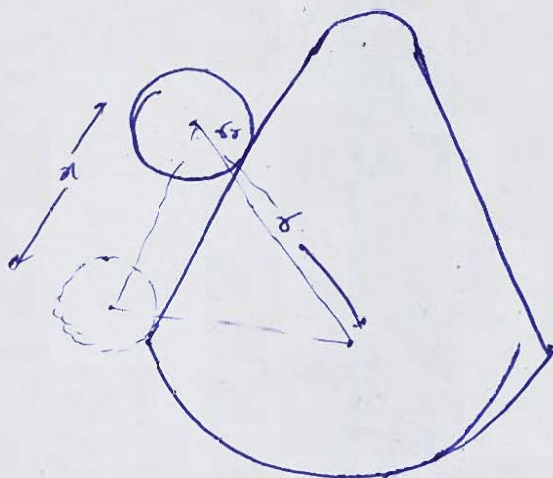
Given :

$$r_{b,c} = 45 \text{ mm}$$
$$r_f = 25 \text{ mm}$$
$$h = 25 \text{ mm}$$
$$\alpha = 75^\circ$$

$$(v) \quad (r_e - r_b) = h$$

$$\Rightarrow r_e = 25 + 45 = 70 \text{ mm}$$

(vi)



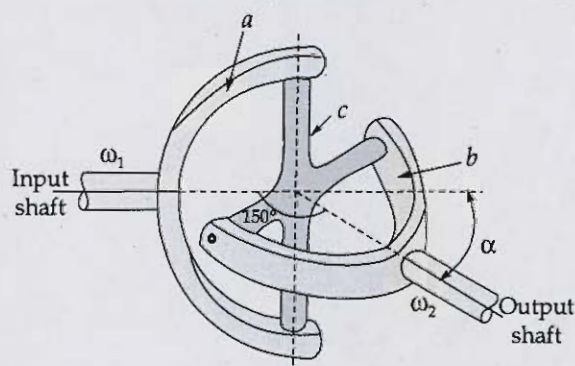
$$\vec{r} = (\vec{r} + \vec{r}_r) - (\vec{r}_b + \vec{r}_b)$$



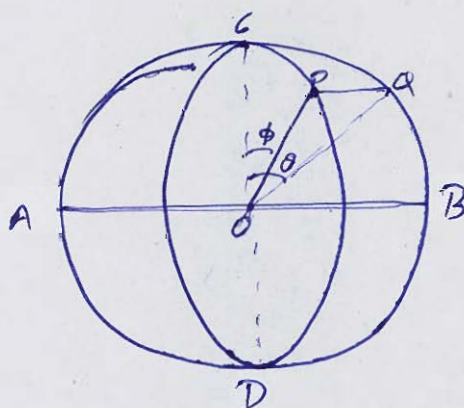
(oo)



- 2 (c) A Hooke's joint is to connect two shafts whose axes intersect at 150° . The driving shaft rotates uniformly at 120 rpm. Deduce a general expression for the angular velocity of the driven shaft. The driven shaft operates against a steady torque of 135 Nm and carries a flywheel whose weight is 45 kg and radius of gyration 0.15 m. What is the maximum value of the torque which must be exerted by the driving shaft?

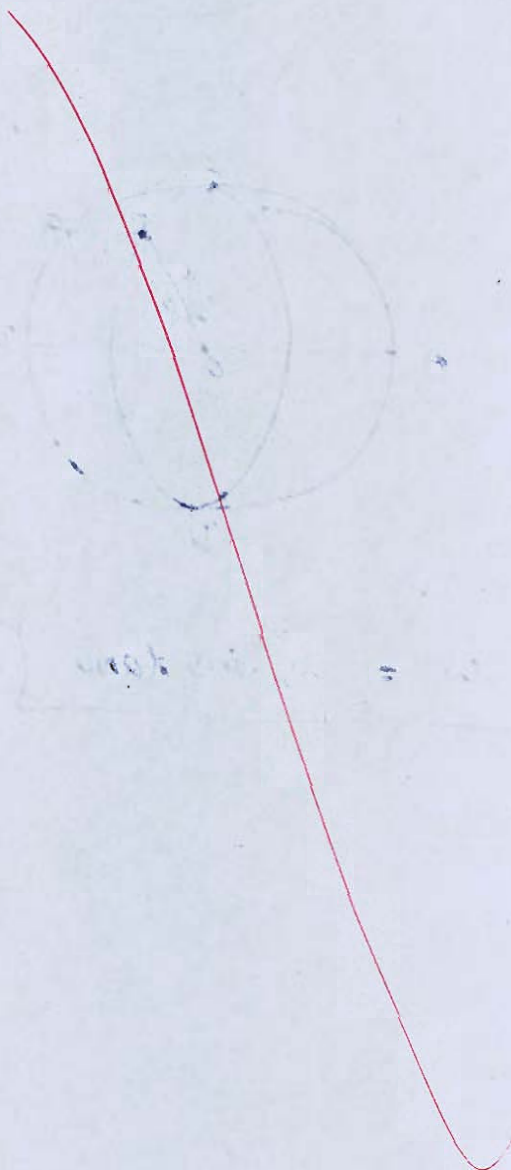


[20 marks]



$$\omega_2 = \omega_1 \sin \alpha \cos \theta$$







3 (a) The following data refer to a Proell governor :

Mass of each ball = 6 kg

Mass of sleeve = 60 kg

Length of each arm = 300 mm

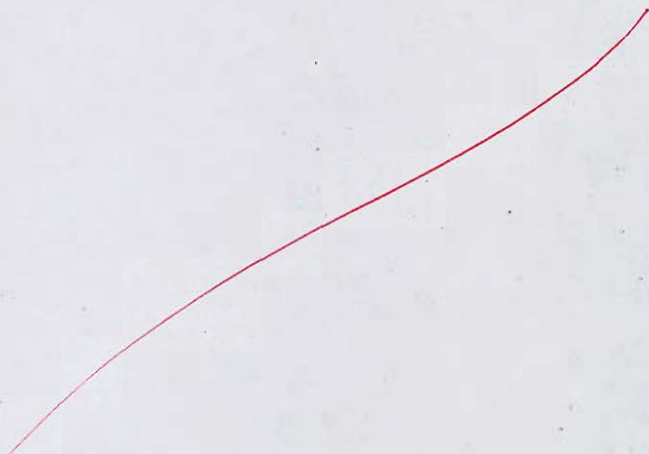
Distance of pivots of lower arms from axis of rotation = 30 mm

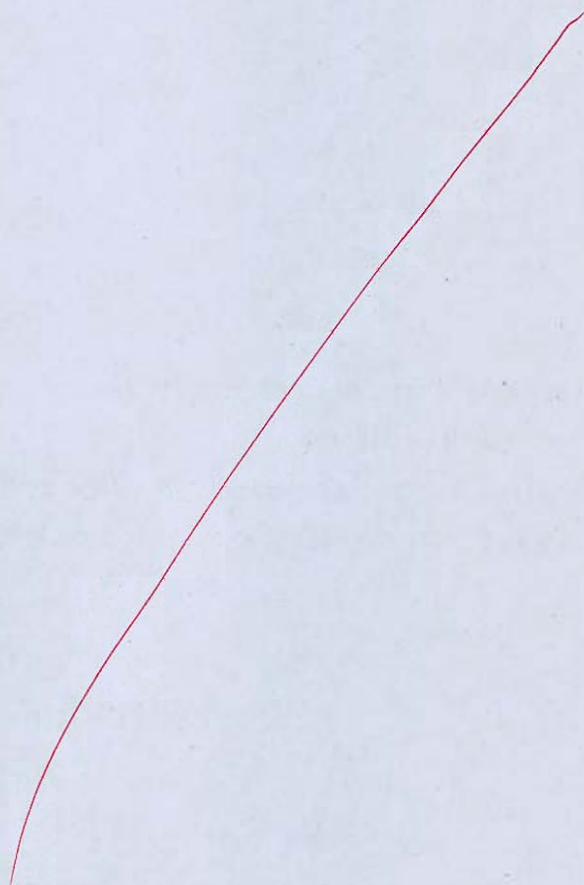
Length of extensions of lower arms = 100 mm

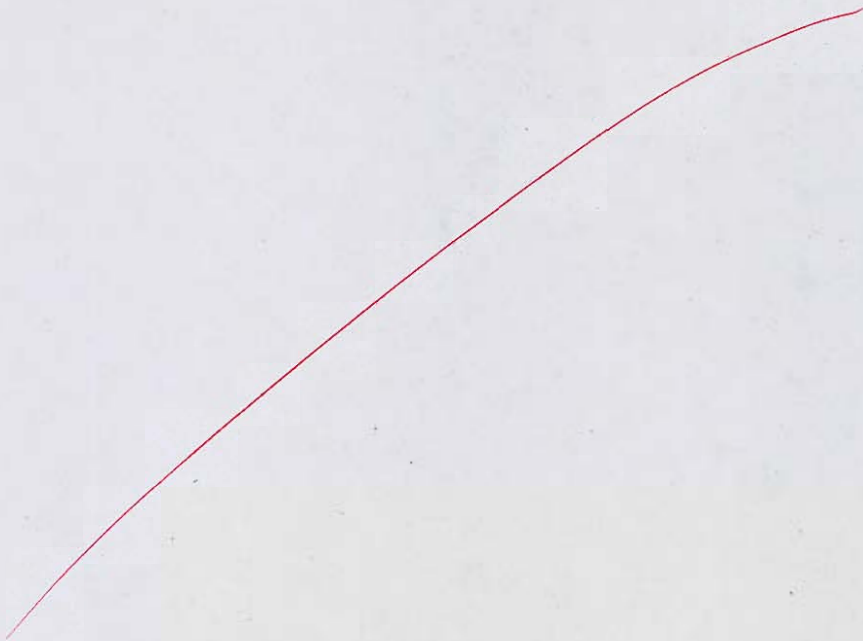
The extensions arms are parallel to the axis of the governor at the minimum radius.

Determine the equilibrium speeds corresponding to extreme radii are 150 mm and 200 mm.

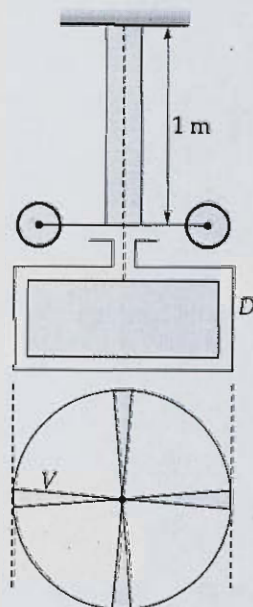
[20 marks]







- Q.3 (b) A flywheel of moment of inertia 25 kg.m^2 is fixed to one end of a vertical shaft diameter 2.54 cm and the length 1 m . The other end of the shaft is fixed. The torsional oscillations of the flywheel are damped by means of a vane as shown in figure, which moves in a dashpot D filled with oil. The amplitude of oscillations is found by experiment to diminish to $\left(\frac{1}{20}\right)^{\text{th}}$ of its initial value in three complete oscillations. Assuming the damping torque to be directly proportional to the angular velocity, find its magnitude at a speed of 1 rad/s . The modulus of rigidity of the shaft material is 85 GPa and compare later with the frequency of the free vibrations.

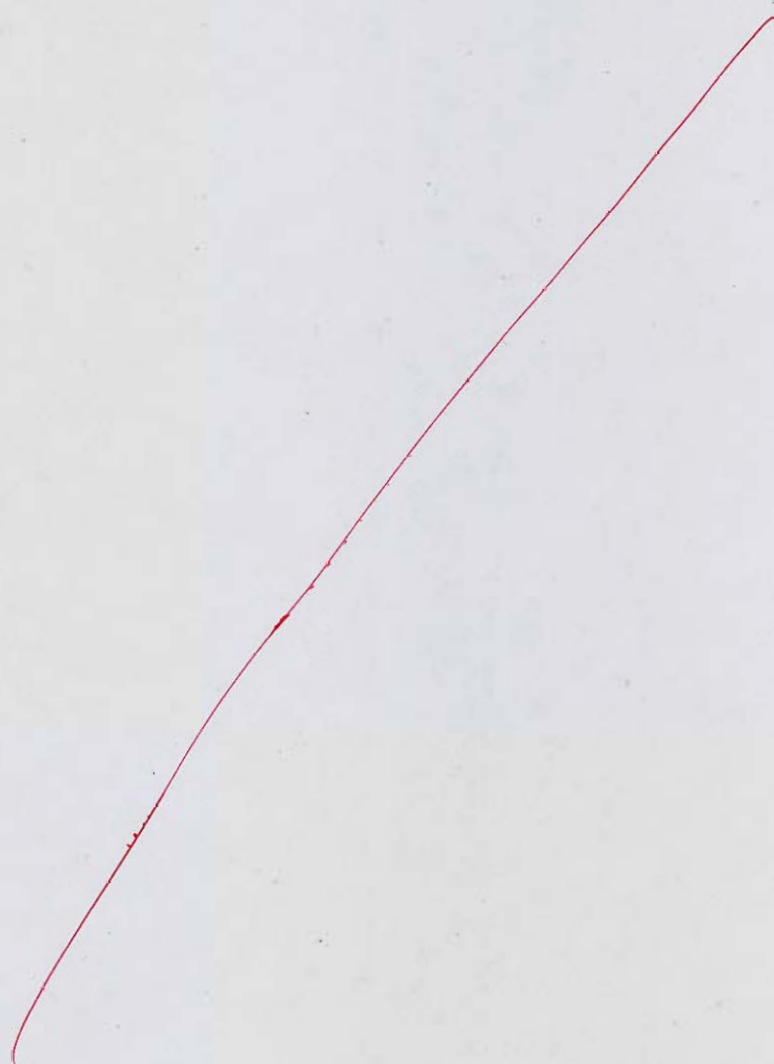


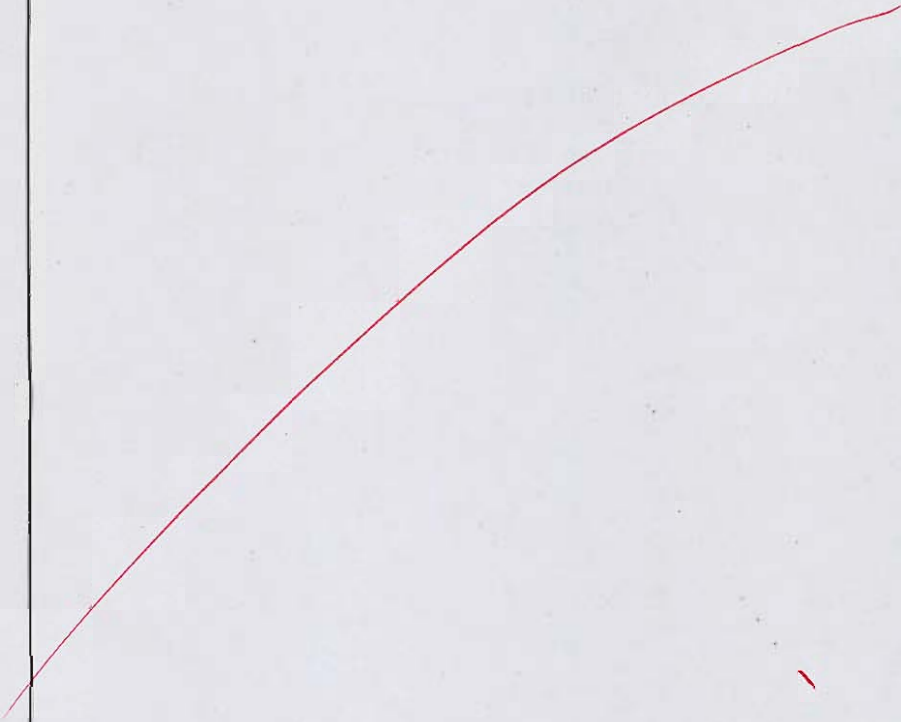
[20 marks]



Q.3 (c) Why is wear debris analysis done? What are wear debris characteristics? Name the different wear mechanisms and wear modes. What are different wear debris methods?

[20 marks]





Q.4 (a) The following data refer to a two cylinder uncoupled locomotive:

Rotating mass per cylinder = 350 kg; Reciprocating mass per cylinder = 400 kg

Distance between the wheels = 1.5 m; Distance between the cylinder centres = 0.6 m

Diameter of threads of the driving wheels = 1.8 m; Crank radius = 0.4 m

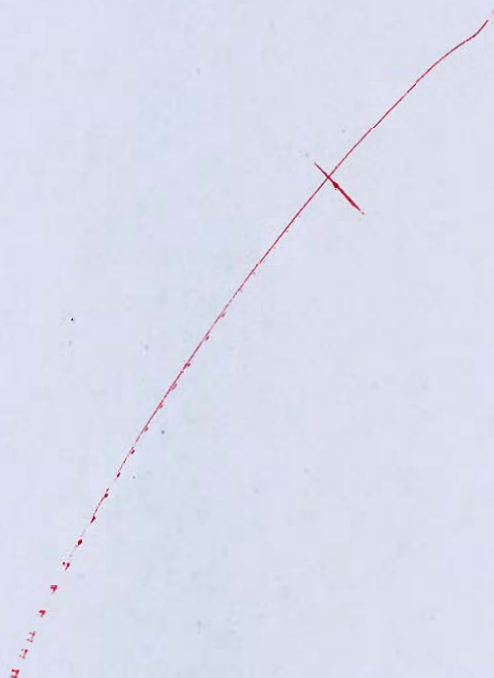
Radius of centre of the balance mass = 0.6 m; Speed of the locomotive = 54 km/h

Angle between the cylinder cranks = 90° ; Dead load on each wheel = 40 kN

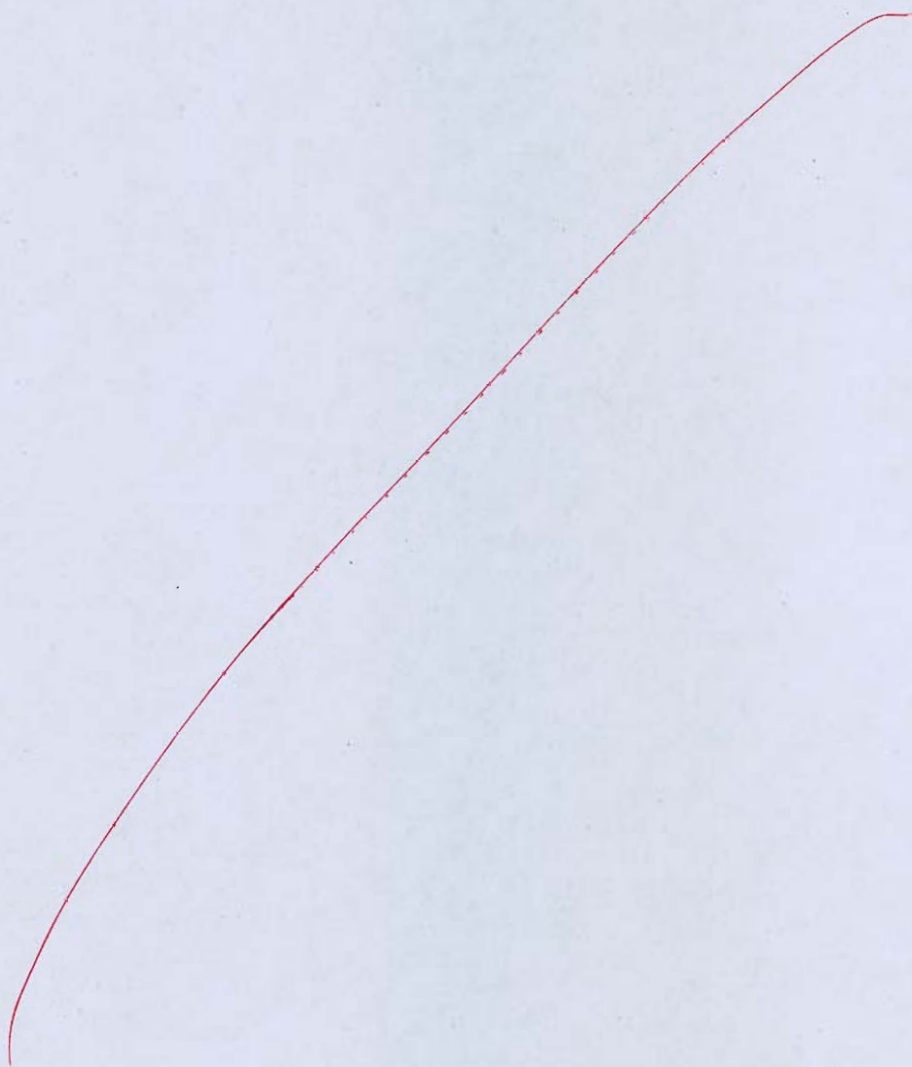
Determine:

- (i) the balancing mass required in the planes of driving wheels if the complete revolving and $1/2^{\text{nd}}$ of the reciprocating masses are to be balanced.
- (ii) Swaying couple.
- (iii) Variation in tractive effort.
- (iv) Maximum ~~and~~ minimum pressure on rails.
- (v) Maximum speed of locomotive without lifting the wheels from the rails.

[20 marks]



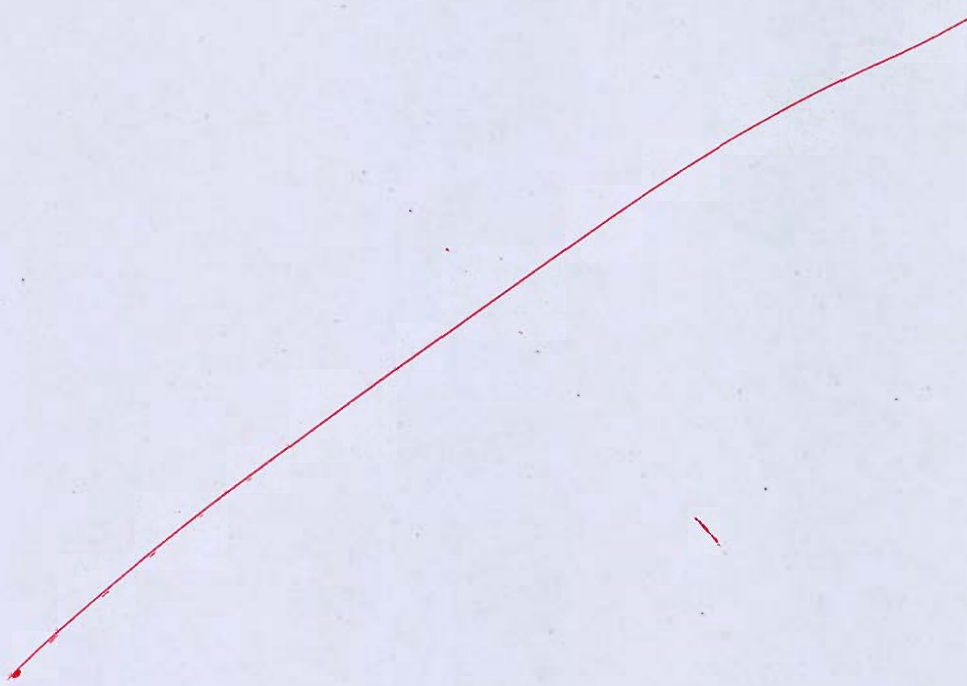


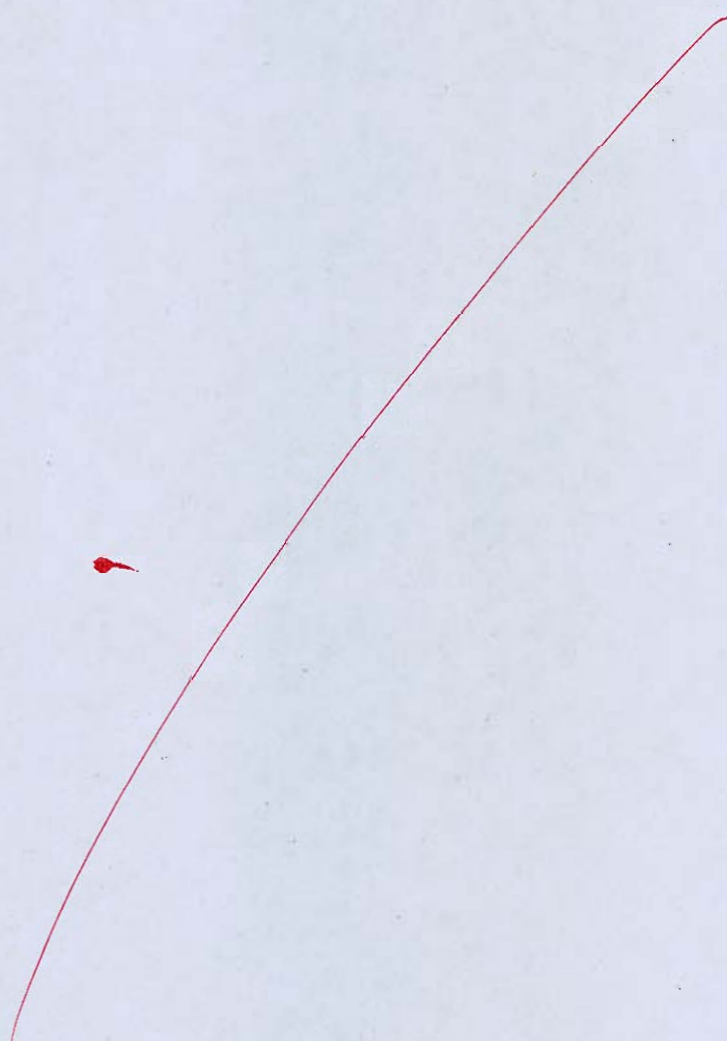


2.4 (b) Explain how the following NDT methods are used for condition monitoring and provide an application for each:

- (i) Ultrasonics
- (ii) Acoustic emissions
- (iii) Eddy current testing
- (iv) Liquid color penetrant testing
- (v) Radiography

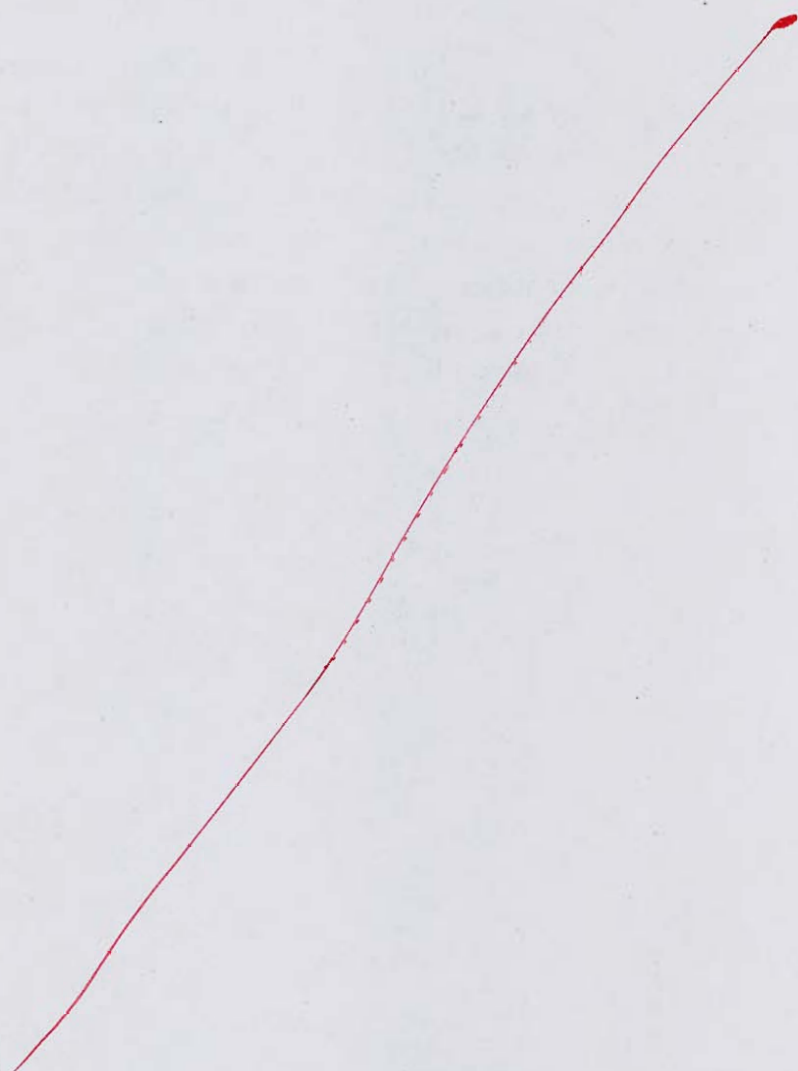
[20 marks]





- 2.4 (c) The cylinder of a twin V-engine are set at 60° angle with both piston connected to a single crank through their respective connecting rods. Each connecting rod is 540 mm long and the crank radius 120 mm. The total rotating mass is equivalent to 2.2 kg at the crank radius and the reciprocating mass is 1.1 kg per piston. A balance mass is also fitted opposite to the crank equivalent to 2.3 kg at a radius of 140 mm. Determine the maximum and minimum values of the primary and secondary forces due to inertia of the reciprocating and the rotating masses if the engine speed is 900 rpm.

[20 marks]



Section B : Theory of Machines + Industrial and Maintenance Engineering

- Q.5 (a) An activity has four elements. The activity is work-measured and the times (r) are recorded on continuous basis over five cycles. The observations are given in table :

Work Element	Observations					Rating factor
	Cycle → 1	2	3	4	5	
1	0.50	3.30	5.70	8.20	10.85	1.1
2	0.70	3.45	5.95	8.55	11.10	1.2
3	1.45	4.05	6.50	9.25	11.75	1.2
4	2.75	5.25	7.60	10.35	13.05	0.9

The time r is recorded at the end of work-element.

- Calculate the standard time for the activity, if the allowance is 20%.
- What is the appropriate sample size for estimating the time for element 2 within $\pm 10\%$ of the true mean with 95% confidence; the standard deviation for the time for element 2 is 0.0742?

[12 marks]

Work Element	Average time $[(1)+(2)+(3)+(4)+(5)] \div 5$	Rating Factor	N.T.
1	5.71	1.1	6.281
2	5.95	1.2	7.14
3	6.6	1.2	7.92
4	7.8	0.9	7.02

$$(S.T.)_1 = N.T. \left(1 + \frac{\text{Allowance}}{100} \right)$$

$$= 6.9091$$

$$(S.T.)_2 = 8.568$$

$$(S.T.)_3 = 9.504$$

$$(S.T.)_4 = 8.424$$

$$(i) \text{ S.T. of activity} = 33.4051$$

Ans

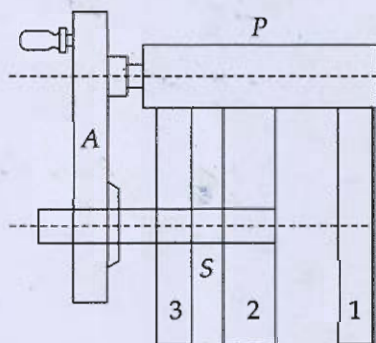
$$(ii) \quad Z = \frac{t - \mu}{\sigma}$$

$$P(Z = 1.95) = 0.95$$

$$\Rightarrow Z = 1.95$$

$$\text{Given: } |t - \mu| = 0.1 \mu$$

- 5 (b) The Ferguson's paradox epicyclic gear train is shown in figure. Gear 1 is ^{fixed} to the frame. The arm A and gear 2 and 3 are free to rotate on the shaft S. Gear 1, 2 and 3 have 100, 120 and 80 teeth respectively. The planet gear has 24 teeth. The pitch circle diameter of all the gears is the same so that the planet gear P meshes with all of them. The revolutions of gears 2 and 3 for one revolution of the arm A.



Ferguson's paradox epicyclic gear train

[12 marks]

Arm	P	3	2	1
Arm fixed	x	$x \cdot \frac{T_3}{T_P}$	$x \cdot \frac{T_2}{T_P}$	$x \cdot \frac{T_1}{T_P}$
Arm rotates (+y)	$x+y$	$x \cdot \frac{T_3}{T_P} + y$	$x \cdot \frac{T_2}{T_P} + y$	$x \cdot \frac{T_1}{T_P} + y$

Now, gear 1 is fixed:

$$x \cdot \frac{T_1}{T_P} + y = 0$$

$$\Rightarrow x \cdot \frac{100}{24} + y = 0$$

$$\Rightarrow y = -x \times \frac{25}{6}$$

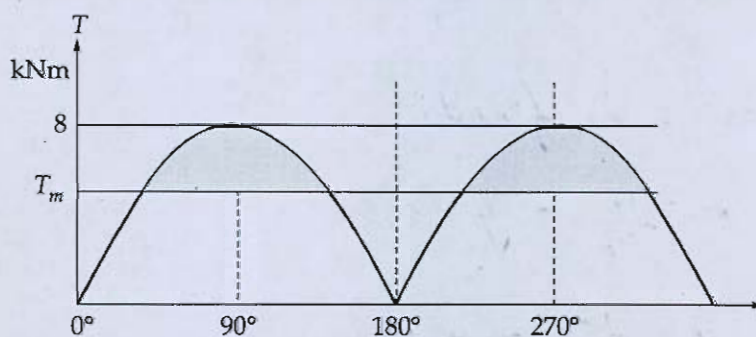
For $y=1$ $\therefore x = \frac{-6}{25}$ revolutions.

gear 2 : revolutions = $\left(\frac{-6}{25}\right) \times \left(\frac{120}{24}\right) + 1$

= $\frac{-1}{5}$ revolutions Ans

$$\begin{aligned}
 \text{Gear 3 : revolutions} &= x \cdot \frac{T_3}{T_P} + y \\
 &= \left(\frac{-6}{25} \right) \times \frac{80}{243} + 1 \\
 &= \frac{-4}{5} + 1 \\
 &= \frac{1}{5} \text{ revolutions} \quad \underline{\text{Ans}}
 \end{aligned}$$

- Q.5 (c) A single-cylinder double acting pump is driven through gearing at 60 rpm. The resisting torque of pump shaft may be assumed to follow a sine curve in half revolution with a maximum value of 8 kNm at 90° and 270° . Find the weight of the flywheel required to be mounted on a pump shaft to keep the speed within 1.5% of mean speed, if the radius of gyration of the flywheel is 1.5 m. The effect of motor armature and gear wheel is equivalent to a flywheel of 5 kN with a radius of gyration of 1 m on the pump shaft.



Assuming equation of curve b/w 0° to 180° as [12 marks]

$$\Rightarrow y = a \sin \theta + b \cos \theta$$

$$\text{At } \theta = 0^\circ, y = 0 \Rightarrow b = 0$$

$$\text{At } \theta = 90^\circ, y = 8 \Rightarrow 8 = a \times 1 \Rightarrow a = 8$$

$$\Rightarrow y = 8 \sin \theta$$

$$\Rightarrow T = 8 \sin \theta$$

$$T_{\text{mean}} = \int_{\theta=0}^{180} 8 \sin \theta \, d\theta / \pi$$

$$= \frac{8 (-\cos \theta)_0^{180}}{\pi} = \frac{16}{\pi}$$

At $\theta = 0$, $T_{\text{mean}} = 8 \sin \theta$

$$\Rightarrow \sin \theta = \frac{2}{\pi}$$

$$\Rightarrow \theta = 39.54^\circ, 140.46^\circ$$

$$\Delta E = \int_{\theta=0}^{\theta_2} T \, d\theta$$

$$= 8 (-\cos \theta)_{39.54}^{140.46}$$

$$= 12.34 \text{ KJ}$$

5

$$I = m_{\text{fly}} \times (1.5)^2 + \frac{5000 \times 1^2}{8}$$

Since, $\Delta E = I \omega^2 C_s$

$$\omega = \frac{2\pi \times 60}{60} = 2\pi$$

$$\Rightarrow 12340 = I \times (2\pi)^2 \times 0.015$$

$$\Rightarrow I = 20836.5$$

$$\Rightarrow m_{\text{fly}} \times 2.25 + 509.68 = 20836.5$$

$$\Rightarrow m_{\text{fly}} = 9034.1 \text{ kg}$$

$$\Rightarrow \text{Weight} = 88.625 \text{ kN} \quad \underline{\text{Ans}}$$

Q.5 (d) A manufacturing company needs 3000 units of a particular component every year. The company buys it at the rate of ₹35 per unit. The order processing cost for this part is estimated as ₹15 per order and the cost of carrying a part in stock comes to about ₹3 per year.

The company can manufacture this part internally. In that case, it saves 20% of the price of the product. However, it estimates a set-up cost of ₹240 per production run. The annual production rate would be 4500 units. However, the inventory holding cost remain unchanged.

1. Determine the EOQ and the optimal number of orders placed in a year.
2. Determine the optimum production lot size and the average duration of the production run.
3. Should the company manufacture the component internally or continue to purchase it from the supplier?

[12 marks]

Given : $D = 3000 \text{ units/year}$

$\text{Cost} = ₹ 35 / \text{unit}$

$A = ₹ 15 / \text{order}$

$C_c = ₹ 3 / \text{year}$

$$4) \text{ EOQ} = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2 \times 15 \times 3000}{3}} = 173.2$$

$$\text{Optimal no. of orders} = \frac{3000}{173.2} = 17.32 \text{ orders}$$

Ans

$$(i) \text{ New price on manufacturing} = 35 \times 0.8 \\ = \text{£}28$$

$$EOQ = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2 \times 240 \times 3000}{3}} \\ = \underline{692.8} \quad \underline{\text{Ans}}$$

4500 units \rightarrow in a year

$$\therefore \text{per day production} = \frac{4500}{365} \text{ units}$$

Average duration of production

$$\text{run} = \frac{692.8}{\frac{4500}{365}} \text{ days}$$

$$= \underline{56.2 \text{ days}} \quad \underline{\text{Ans}}$$

$$(iii) \text{ Cost of buying} = 3000 \times 35 + 15 \times (17.32) \\ + \sqrt{2 \times 15 \times 3000 \times 3}$$

$$= \underline{\text{£}105519.6}$$

$$\text{Cost of manufacturing} = 3000 \times 28 + \sqrt{2 \times 240 \times 3000 \times 3}$$

$$= \underline{\text{£}86078.46}$$

10

So company should manufacture it.

- Q.5 (e) The following information is known about a group of items. Classify the items as A, B and C.

Item no.	Quantity consumed in a year	Cost per unit (₹)	Quantity x Cost
1	3	45	135
✓ 2	200	6	1200
✓ 3	40	1200	48000
✓ 4	30	25	750
5	5	20	100
✓ 6	17	2100	35700
✓ 7	25	50	1250
✓ 8	6	40	240
✓ 9	100	7	700
✓ 10	100	8	800
✓ 11	250	4	1000
✓ 12	120	8	960
✓ 13	140	7	980
14	9	10	90
✓ 15	20	10	200

[12 marks]

Arranging the items based on value in
descending order:

Item	Value	Cumulative value	%	
3	48000	48000	52.11	A
6	35700	83700	90.87	
15 7	1250 1250	84950	92.23	B
11 2	1200 1200	86150	93.53	
11	1000	87150	94.62	
13	980	88130	95.68	
12	960	89090	96.726	
10	800	89890	97.6	C
4	750	90640	98.41	
9	700	91340	99.17	
8	240	91580	99.43	
15	200	91780	99.647	
1	135	91915	99.8	
5	100	92015	99.9	
14	90	92105	100	

A includes 90.87 % worth of item 3 and 6

B includes 5.856 % worth of item 7, 2, 11,
13 and 12

Remaining item in C.

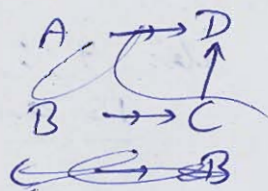
10

- Q.6 (a) A, B, C and D are the activities. Their normal and crash durations and associated costs are given in the table below:

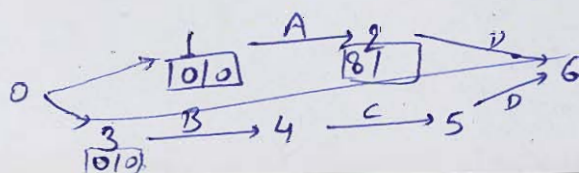
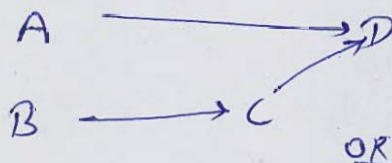
Activity	Duration in days		Direct cost in ₹	
	Normal	Crash	Normal	Crash
A	8	4	6000	12000
B	4	2	2000	14000
C	10	4	4000	8000
D	6	4	4000	8000

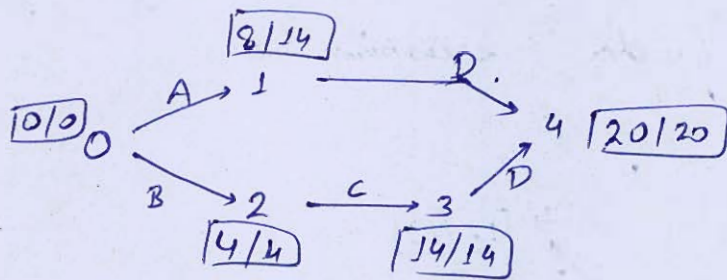
For the entire project the indirect cost is ₹ 1000 per day. A and B are starting activities; C follows B; D follows A and C; D is the finishing activity. Draw CPM Network. Calculate points for PTC graph and plot the same. Determine the optimum cost and optimum duration for the project. PTC is project-Time-Cost-Trade-Off graph.

[20 marks]



CPM Network :-





Critical Path : $B \rightarrow C \rightarrow D$: Time = 20 days

$$\begin{aligned} \text{Cost} &= 6000 + 2000 + 4000 + 4000 + (1000 \times 20) \\ &= ₹36000 \end{aligned}$$

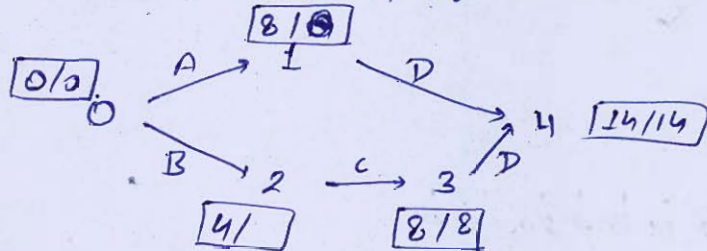
Crashing : $(\text{Slope})_B = \frac{14000 - 2000}{4 - 2} = 6000$

$$(\text{Slope})_C = \frac{8000 - 4000}{10 - 4} = \frac{2000}{3}$$

So, crashing B & C :

$$\text{Total time} = \cancel{8} + 2 + 10 + 6 = 18 \text{ days}$$

$$\text{Cost} = \cancel{6000} + 20$$



$$\text{Total time} = 14 \text{ days}$$

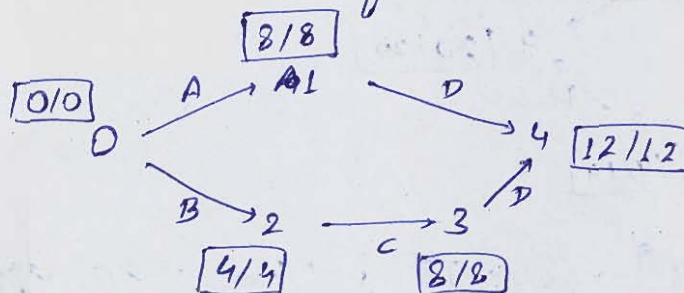
$$\begin{aligned} \text{Cost} &= 6000 + 2000 + 8000 + 4000 + (1000 \times 14) \\ &= ₹34000 \end{aligned}$$

Further crashing : $(\text{Slope})_A = \frac{6000}{4} = \frac{3000}{2} = 1500$

This is greater than the indirect cost.

$$\rightarrow \text{Cost} = 12000 + 2000 + 8000 +$$

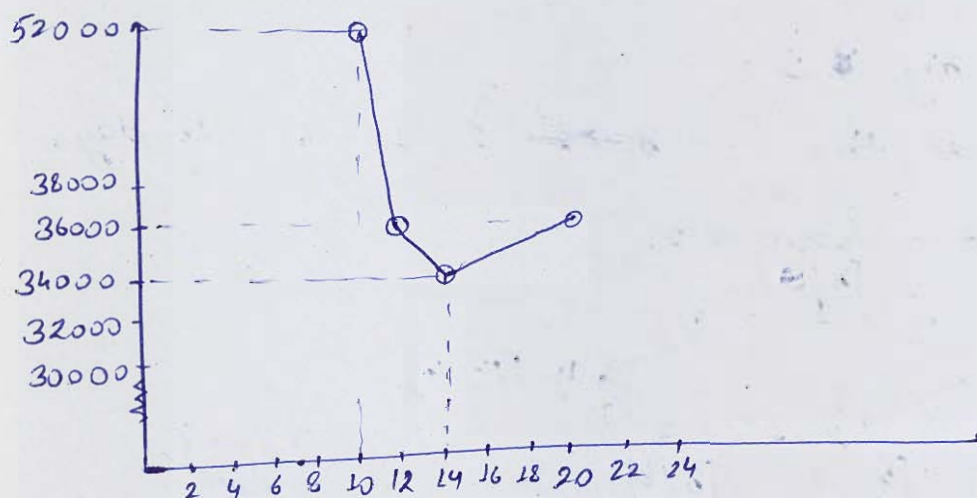
To reduce time further, crashing D.



Total time = 12 days

$$\begin{aligned}
 \text{Cost} &= 6000 + 2000 + 8000 + 8000 \\
 &\quad + 12 \times 1000 \\
 &= ₹36000
 \end{aligned}$$

PTC graph



Further crashing A and B:

Total time = 10 days

$$\begin{aligned}
 \text{Cost} &= 12000 + 14000 + 8000 + 8000 + 10 \times 1000 \\
 &= ₹52000
 \end{aligned}$$

Optimum cost = ₹ 34,000 Ans

Optimum duration = 14 days Ans

(20)

- Q.6 (b) A travel agent is planning a charter trip Thailand. The eight day/seven night package includes the fare for round-trip travel, surface transportation, board and lodging and selected tour options. The charter trip is restricted to 200 persons and past experience indicates that there will not be any problem for getting 200 persons. The problem for the travel agent is to determine the number of Deluxe, Standard and Economy tour packages to offer for this charter. These three plans differ according to seating and service for the flight, quality of accommodations, meal plans and tour options. The following table summarizes the estimated prices for the three packages and the corresponding expenses for the travel agent. The travel agent has hired an aircraft for the flat fee of Rs. 2,15,000 for the entire trip.

Tour Plan	Prices and costs for tour packages per person		
	Price (Rs.)	Hotel costs (Rs.)	Meals & other expenses (Rs.)
Deluxe	12000	4000	5500
Standard	8000	2600	2750
Economy	6750	1950	2350

In planning the trip, the following considerations must be taken into account:

- At least 10 per cent of the packages must be deluxe type.
- At least 35 per cent but not more than 70 per cent must be of the standard type.
- At least 30 per cent must be of the economy type.
- The maximum number of deluxe packages available in any aircraft is restricted to 60.
- The hotel desires that at least 120 of the tourists should be on the deluxe and standard packages together.

The travel agent wishes to determine the number of packages to offer in each type so as to maximize the total profit.

- Formulate the above as a linear programming problem.
- Restate the above linear programming problem in terms of two decision variables, taking advantage of the fact that 200 packages will be sold.
- Find the optimum solution using graphical method for the restated linear programming problem and interpret your results.

[20 marks]

Let deluxe packages be d , standard packages be s and economy packages be e .
Total packages be P .

Given conditions:

$$d \geq 0.1P$$

$$0.35P \leq s \leq 0.7P$$

$$e \geq 0.3P$$

$$d \leq 60$$

$$d + s \geq 120$$

$$P \leq 200$$

$$\text{Total deluxe cost} = 21500$$

$$\text{Standard cost} = 13350$$

$$\text{Economy cost} = 11050$$

(i) Cost
~~Profit~~ equation,
$$Z = 21500d + 13350s + 11050e + 2,15,000$$

Ans

(ii) given $P = 200 \Rightarrow e = 200 - d - s$

$$Z = 10450d + 2300s + 2425000$$

Conditions:

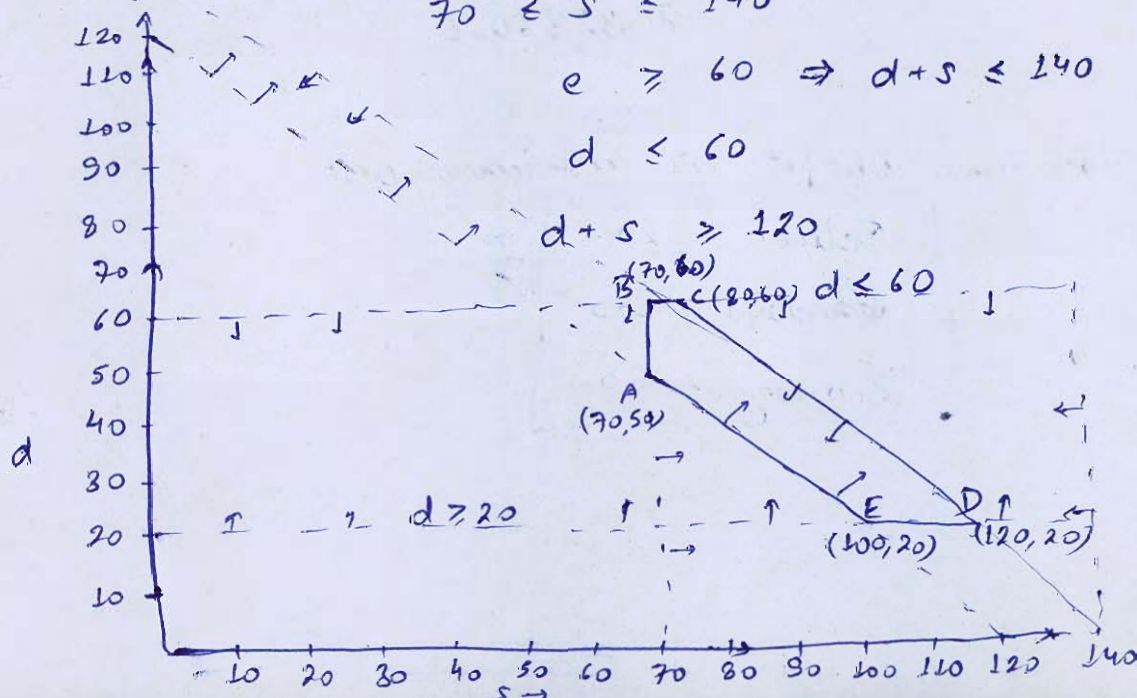
$$d \geq 0.1P \Rightarrow d \geq 20$$

$$70 \leq s \leq 140$$

$$e \geq 60 \Rightarrow d + s \leq 140$$

$$d \leq 60$$

$$d + s \geq 120$$



Corner points : A, B, C, D, E

~~For cost minimum : E (100, 20)~~

$$\begin{aligned} \text{(Cost)}_E &= 10450 \times 100 + 2300 \times 20 + 2425000 \\ &= \text{₹ } 3516000 \end{aligned}$$

~~For cost maximum : C (80, 60)~~

$$\begin{aligned} \text{(Cost)}_C &= 10450 \times 80 + 2300 \times 60 + 2425000 \\ &= \text{₹ } 3399 \end{aligned}$$

(c)

For cost minimum : E (100, 20)

$$\begin{aligned} \text{(Cost)}_E &= 10450 \times 20 + 2300 \times 100 + 2425000 \\ &= \text{₹ } 2864000 \end{aligned}$$

For cost maximum : C (80, 60)

$$\begin{aligned} \text{(Cost)}_C &= 10450 \times 60 + 2300 \times 80 + 2425000 \\ &= \text{₹ } 3236000 \end{aligned}$$

For max. profit or minimum cost :

Delux	= 20
Standard	= 100
Economy	= 80

- 6 (c) In a spring loaded governor of the Hartnell type, the mass of each ball is 1 kg, length of vertical arm of the bell crank lever is 100 mm and that of the horizontal arm is 50 mm. The distance of fulcrum of each bell crank lever is 80 mm from the axis of rotation of the governor. The extreme radii of rotation of the balls are 75 mm and 112.5 mm. The maximum equilibrium speed is 5 per cent greater than the minimum equilibrium speed which is 360 rpm. Find, neglecting obliquity of arms, initial compression of the spring and equilibrium speed corresponding to the radius of rotation of 100 mm.

[20 marks]

$$m_{\text{ball}} = 1 \text{ kg}$$

$$y = 0.1 \text{ m}$$

$$x = 0.05 \text{ m}$$

$$r = 0.08 \text{ m}$$

$$r_1 = 0.075 \text{ m} \quad \text{and} \quad r_2 = 0.1125 \text{ m}$$

$$N_1 = 360 \text{ rpm}, \text{ so } N_2 = 1.05 \times 360$$

$$= 378 \text{ rpm}$$

$$\omega_1 = 37.7 \text{ rad/s}$$

$$\omega_2 = 39.58 \text{ rad/s}$$

$$F_{c1} = 1 \times (37.7)^2 \times 0.075$$

$$= 106.6 \text{ N}$$

$$F_{c2} = 176.24 \text{ N}$$

$$\begin{aligned}
 k &= 2 \left(\frac{1}{y} \right)^2 \frac{F_2 - F_1}{x_2 - x_1} \\
 &= 2 \left(\frac{0.05}{0.1} \right)^2 \times \frac{176.24 - 106.6}{0.0375} \\
 &= 928.53 \text{ N/m} \quad \text{A}
 \end{aligned}$$

For initial compression of spring:

$$F_c \times y = \frac{(k \cdot x_0 + mg) \times}{2}$$

$$\Rightarrow \frac{21.32}{0.05} - 9.81 = 928.53 x_0$$

$$\Rightarrow x_0 = 0.4486 \text{ m} \quad \underline{\text{Ans}}$$

Equilibrium speed @ $x = 0.1 \text{ m}$

$$\frac{h_0 - h_1}{x} = \frac{x - x_1}{y}$$

$$\begin{aligned}
 \Rightarrow h &= h_1 + \frac{x}{y} (x - x_1) \\
 &= 0.4486 + \frac{1}{2} (0.1 - 0.075) \\
 &= 0.4611
 \end{aligned}$$

$$h = 2 \left(\frac{x}{y} \right)^2$$

$$\frac{F_c - F_1}{x - x_1} = \frac{F_2 - F_1}{x_2 - x_1}$$

$$\Rightarrow \frac{F_c - 106.6}{0.1 - 0.075} = \frac{176.24 - 106.6}{0.0375}$$

$$\Rightarrow F_c = 153 \text{ N}$$

$$1(\omega^2) \times 0.1 = 153$$

$$\Rightarrow \omega = 39.112$$

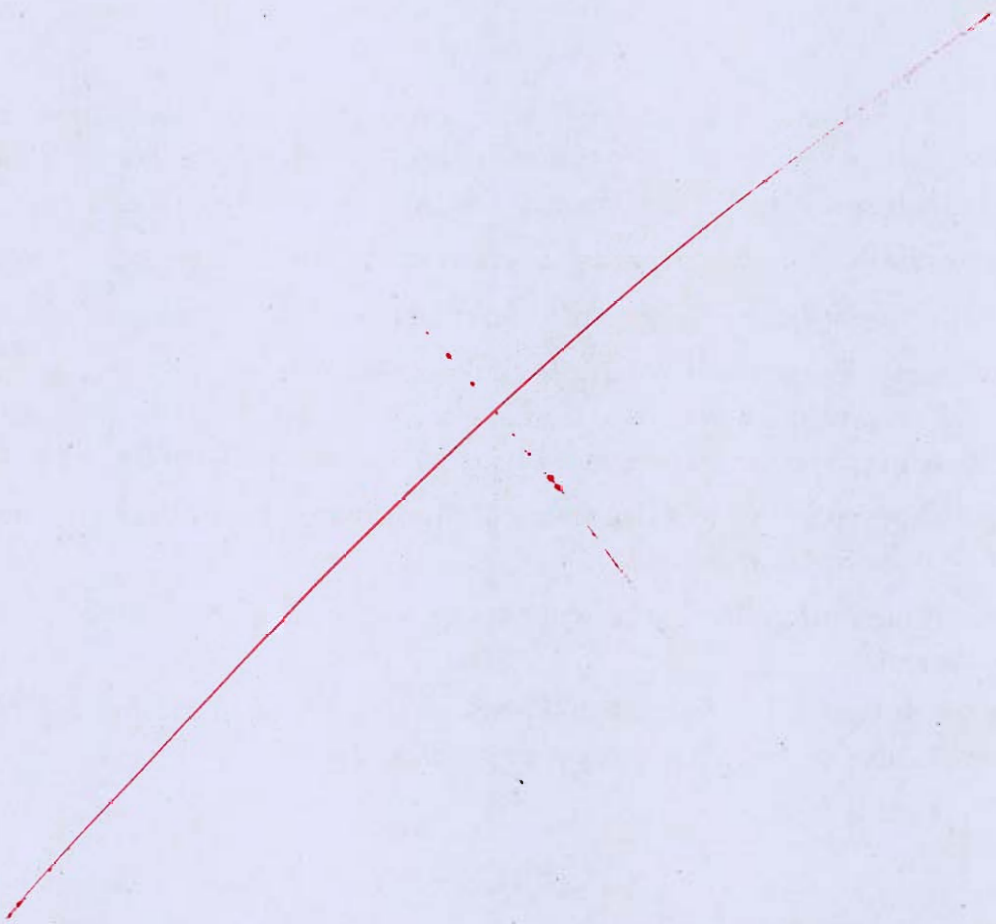
$$N = 373.55 \text{ rpm}$$

Ans
20

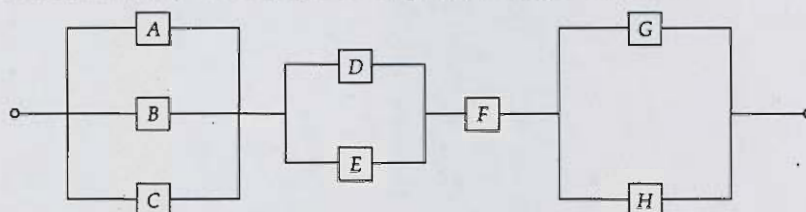
- Q.7 (a) A company has 5 jobs to be done. The following matrix shows the return in rupees of assigning i^{th} machine ($i = 1, 2, \dots, 5$) to the j^{th} job ($j = 1, 2, \dots, 5$). Assign the five jobs to the five machines so as to maximize the total expected profit.

		Job				
		1	2	3	4	5
Machine	1	5	11	10	12	4
	2	2	4	6	3	5
	3	3	12	5	14	6
	4	6	14	4	11	7
	5	7	9	8	12	5

[20 marks]

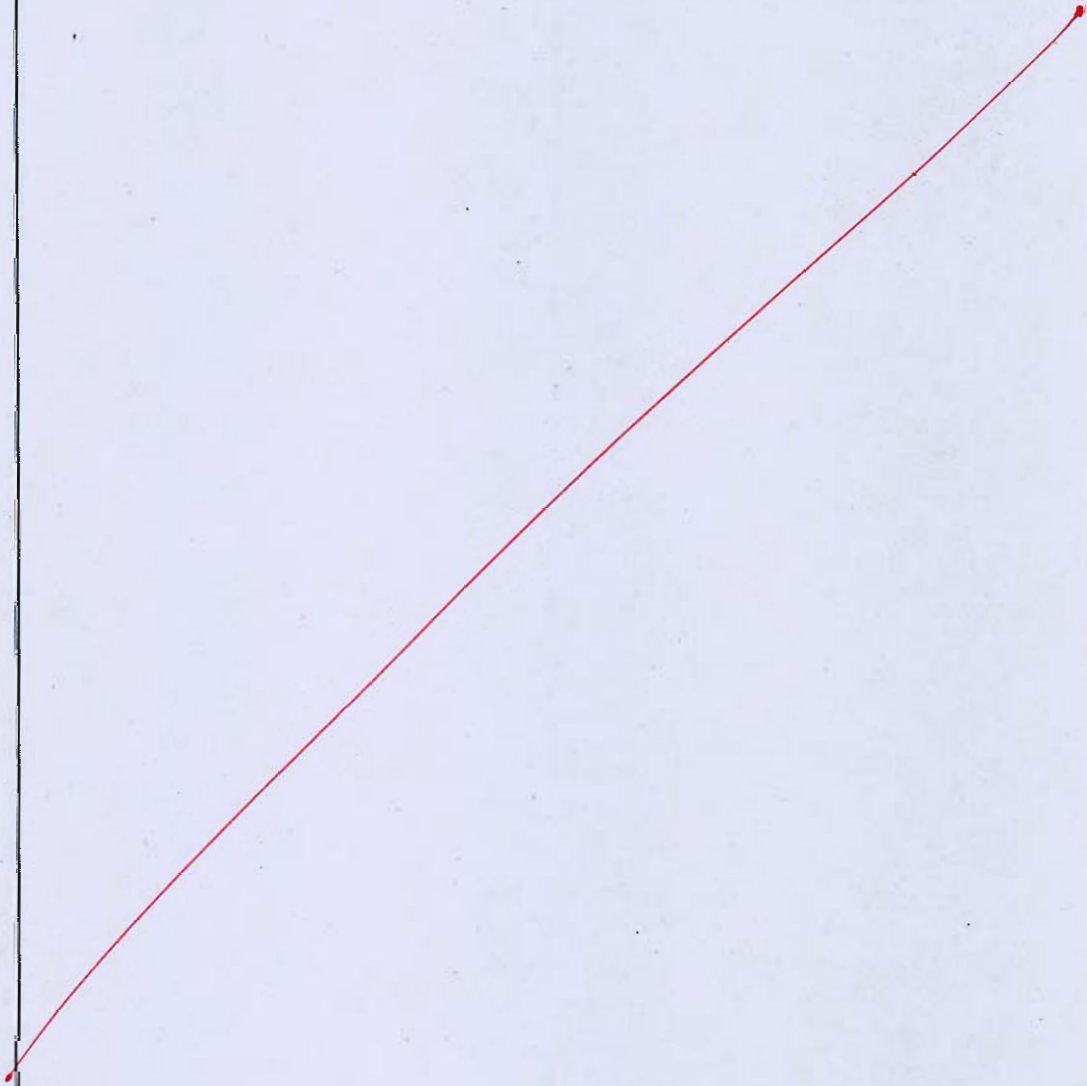


- Q.7 (b) (i) Arrival rate of customers at a milk booth are according to Poisson distribution with an average time of 9 minutes between two consecutive arrivals. The length of milk distribution is assumed to be exponentially distributed with mean 3 minutes.
1. Determine the probability that a person arriving at the booth will have to wait.
 2. Find the average queue length that is formed from time to time.
 3. Milk plant company will install a second milk booth when convinced that an arrival would expect to have to wait at least 4 minutes for the milk. Find the increase in flow of arrivals which will justify a second milk booth.
 4. What is the probability that an arrival will have to wait for more than 10 minutes before the milk booth is free?
 5. What is the probability that he will have to wait for more than 10 minutes in the system?
- (ii) What is the difference between reliability and availability of an equipment? Also find the reliability of the following system given below.

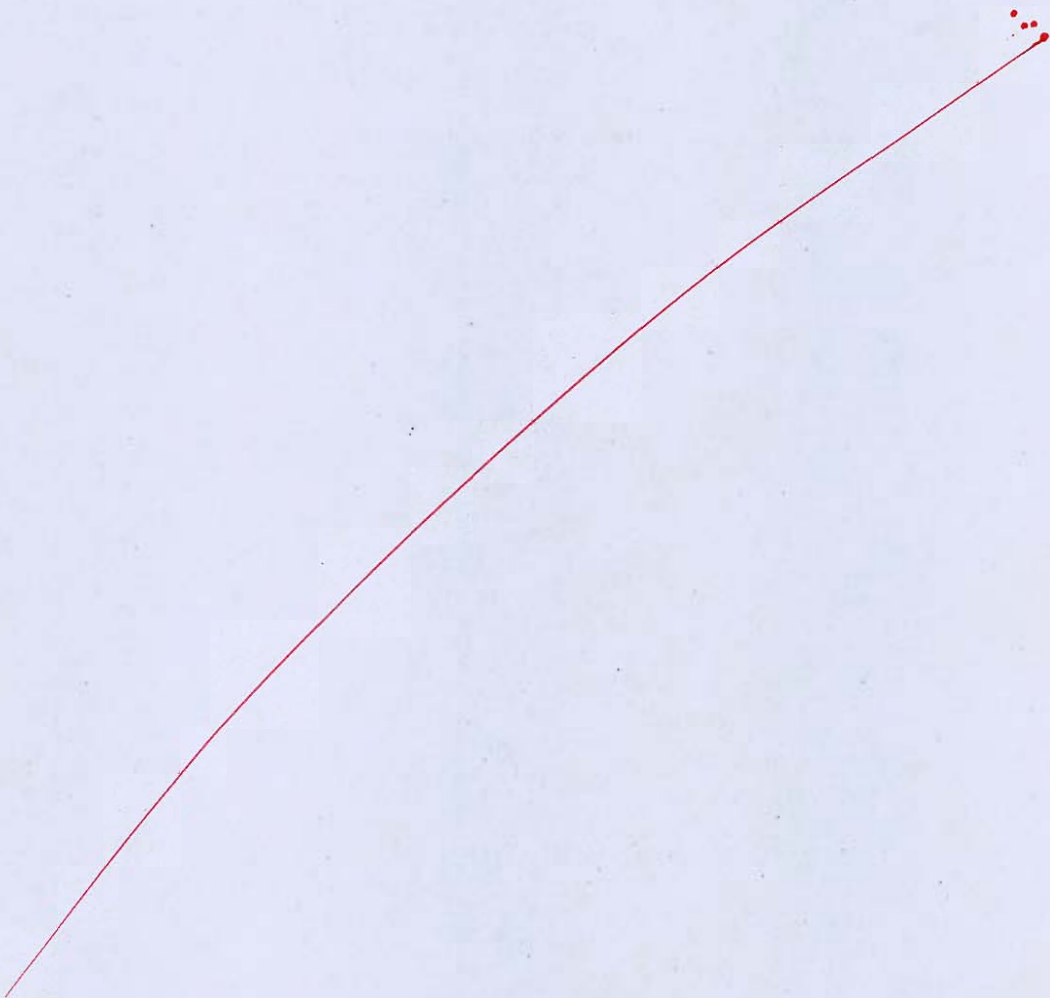


If reliability of each unit is 0.35.

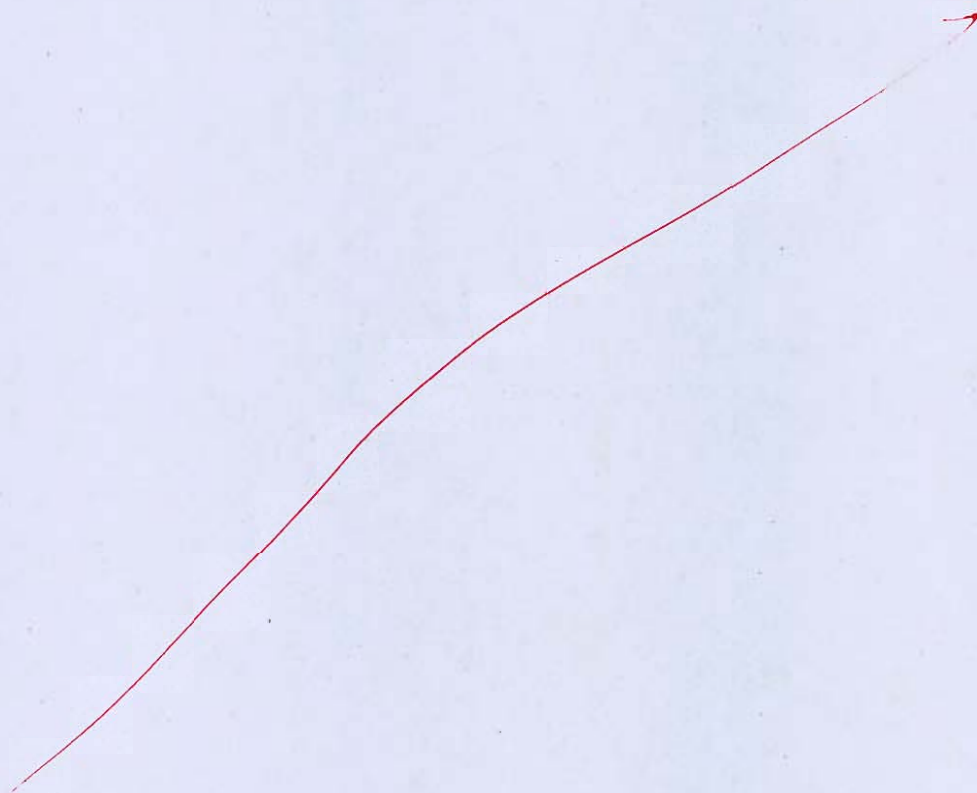
[10 + 10 marks]

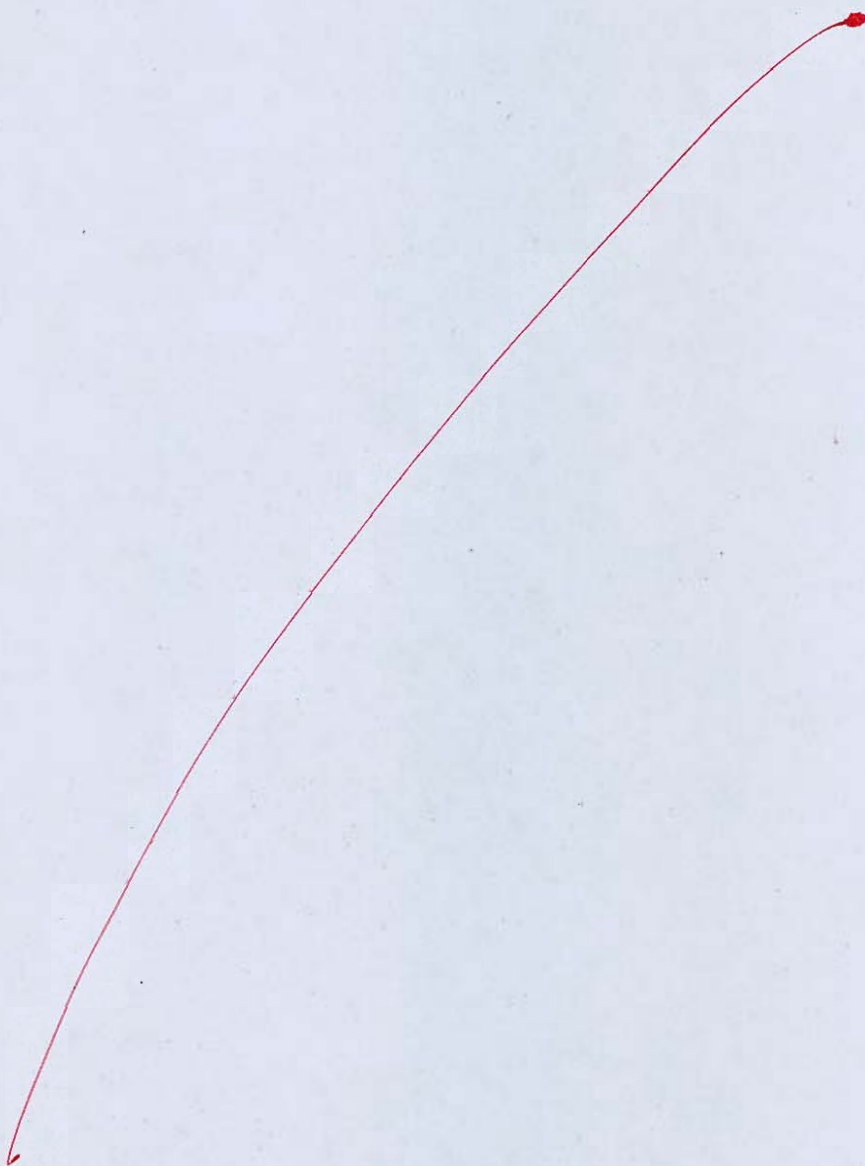






[20 marks]





- (a) A disc of mass 5 kg is mounted midway between bearing which may be assumed to be simple supports. The bearing span is 60 cm. The steel shaft is 20 mm diameter and is horizontal. The centre of gravity of the disc is displaced 2 mm from the geometric centre. The equivalent viscous damping at the centre of the disc-shaft may be assumed as 60 N-sec/m. If the shaft rotates at 360 rpm. Take $E = 2 \times 10^{11} \text{ N/m}^2$.
- (i) Determine the maximum stress in shaft.
- (ii) The power required to drive the shaft at the speed 360 rpm.

[20 marks]

$$m_{\text{disc}} = 5 \text{ kg}$$

$$l = 0.6 \text{ m}$$

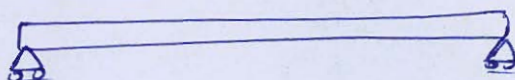
$$d_s = 0.02 \text{ m}$$

$$c = 60 \text{ Nsec/m}$$

$$N = 360 \text{ rpm} \Rightarrow \omega = 12\pi$$

$$E = 2 \times 10^{11} \text{ N/m}^2$$

$$e = 2 \text{ mm}$$



(i) Force due to eccentricity = $m\omega^2 e$

$$= 5 \times (12\pi)^2 \times (0.002)$$

$$= 14.212 \text{ N}$$

①

- 3 (b) The demand for six consecutive periods of a product is as follows: 95, 98, 102, 106, 110, 120. Establish a linear forecaster. Determine demand in 12th period. Also calculate the coefficient of determination and mean square error for the line of best fit.

[20 marks]

x (Period)	y (D)	xy	x^2
1	95	95	1
2	98	196	4
3	102	306	9
4	106	424	16
5	110	550	25
6	120	720	36
Σ	21	631	91

$$\text{Let } y = ax + b$$

$$\Sigma y = a \Sigma x + b.(n)$$

$$\Rightarrow 631 = 21a + 6b \quad \text{--- (i)}$$

$$\sum xy = a \sum x^2 + b \sum x$$

$$\Rightarrow 2291 = 91a + 21b \quad \text{--- (ii)}$$

From (i) and (ii) :

$$a = \frac{33}{7} \quad b = \frac{266}{3}$$

$$D_{12} = ax + b$$

$$= \frac{33}{7} \times 12 + \frac{266}{3}$$

$$= \underline{145.24 \text{ units}} \quad \underline{\text{Ans}}$$

For mean square error :

$$F_1 = \frac{33}{7} \times 1 + \frac{266}{3} = 93.38$$

$$F_2 = 98.09$$

$$F_3 = 102.81$$

$$F_4 = 107.5$$

$$F_5 = 112.24$$

$$F_6 = 116.9$$

$$M.S.E. = \frac{\sum (D - F)^2}{6}$$

$$= \frac{19.844}{6} = \underline{3.307} \quad \underline{\text{Ans}}$$

14

$$\text{Correlation (r)} = \frac{\sum [(x - x_m)(y - y_m)]}{\sqrt{\sum (x - x_m)^2 \sum (y - y_m)^2}}$$

$$\text{C.O.D.} = r^2$$

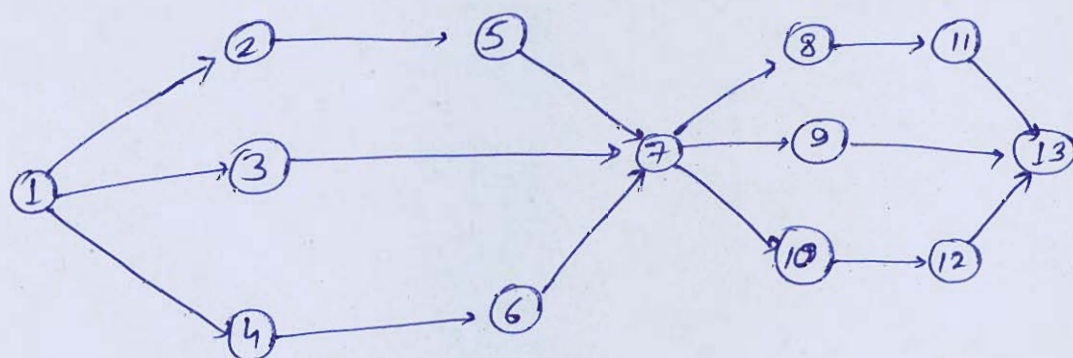
- Q.8 (c) A manufacturing line consists of 13 work elements. The precedence relationships among them are shown in the precedence diagram in figure. The time required for each work element and their immediate precedence are tabulated below:

Work element	Duration (min)	Immediate precedence
1	9	—
2	4	1
3	4	1
4	4	1
5	6	2
6	8	4
7	6	3, 5, 6
8	4	7
9	3	7
10	6	7
11	8	8
12	6	10
13	11	9, 11, 12

- Draw the precedence diagram for the work elements based on given table.
- Using the largest candidate rule, perform line balancing for the given data. Assign tasks to workstations, if station time is given 17 min.
- Compute the following performance measures.
 - Number of workstations required.
 - Line efficiency
 - Balance delay
 - Smoothness index

[20 marks]

i) Precedence diagram :



$$T_{wc} = 79 \text{ min}$$

(ii) Line balancing

Workstation	Elements	Duration
<u>I</u>	1	9
	2	4
	4	4
<u>II</u>	6	8
	5	6
<u>III</u>	3	4
	7	6
	10	6
<u>IV</u>	8	4
	11	8
	9	3
<u>V</u>	12	6
	13	11

(iii)

1. No. of Workstations required, $n = \frac{T_{wc}}{T_{si}}$

$$= \frac{79}{17}$$

$$= 4.65$$

$$\approx \underline{5} \quad \underline{\text{Ans}}$$

$$2. \quad \eta_{line} = \frac{79}{17 \times 5} = \underline{92.94\%} \quad \underline{\text{Ans}}$$

$$3. \quad \text{Balance delay} = 1 - \eta_{line}$$

$$= \underline{7.06\%} \quad \underline{\text{Ans}}$$

or 6 days min

$$\begin{aligned} 4. \quad S.I. &= \frac{\sqrt{(17-17)^2 + (17-14)^2 + (17-16)^2 + (17-15)^2 + (17-17)^2}}{5} \\ &= \frac{\sqrt{9 + 1 + 4}}{5} = \underline{\underline{0.74833}} \quad \text{Ans} \end{aligned}$$

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

$$\underline{C_1 + C_2 = 0.28}$$

$$\underline{r_1, r_2 \checkmark}$$

$$\underline{N_1 T_1 = N_2 T_2}$$

$$n \times T_p$$

$$d_p =$$