Detailed Solutions

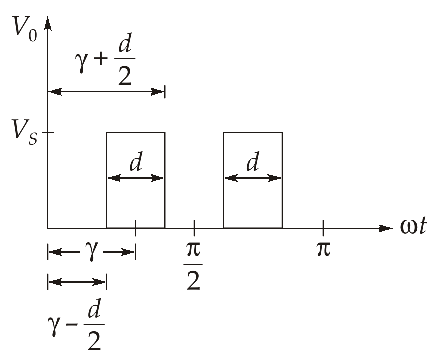
ESE-2025 Mains Test Series

Electrical Engineering Test No : 6

Section-A : Power Systems + Power Electronics & Drives + Communication Systems

Q.1 (a) Solution:

In Multiple Pulse Modulation (MPM), several equidistant pulses per half cycle are used.



The rms value of pulses in single pulse modulation and multiple pulse modulation are same.

$$V_{0r} = V_S\sqrt{\frac{2d}{\pi}}$$

(i) Positive and negative half cycle of V_0 are symmetrical about $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ respectively.

In addition these half cycles are also identical as a result coefficient $a_n = 0$.

$$b_n = \frac{2}{\pi}\int_0^{\pi} V_0 \sin n\omega t \cdot d(\omega t)$$

$$= \frac{2}{\pi} \int_{\gamma - \frac{d}{2}}^{\gamma + \frac{d}{2}} V_s \sin n\omega t \cdot d(\omega t) \times 2$$

The use of factor 2 in the above expression accounts for the two pulses from 0 to π.

$$b_n = \frac{4V_s}{n\pi} (-\cos n\omega t) \Big|_{\gamma - \frac{d}{2}}^{\gamma + \frac{d}{2}}$$

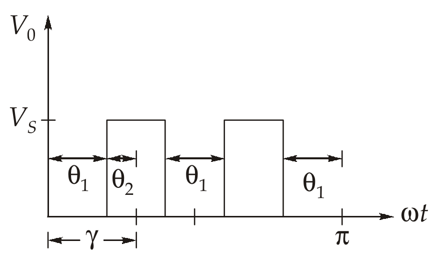
$$= \frac{4V_s}{n\pi} \left[\cos n\left(\gamma - \frac{d}{2}\right) - \cos n\left(\gamma + \frac{d}{2}\right) \right]$$

$$b_n = \frac{8V_s}{n\pi} \sin n\gamma \sin \frac{nd}{2}$$

∴ The Fourier series expansion of output voltage in MPM PWM inverter is,

$$V_0 = \sum_{n=1,3,5...}^{\infty} \frac{8V_s}{n\pi} \sin n\gamma \sin \frac{nd}{2} \sin n\omega t$$

(ii)



For N pulses per half cycle there are $(N + 1)$ intervening equidistant spaces, each of width θ_1 as shown above.

For these equidistant spaces $V_0 = 0$

Total width of these equidistant spaces,

$$(N + 1)\theta_1 = \pi - (\text{width of N pulses})$$

$$= (\pi - 2d)$$

$$\theta_1 = \frac{\pi - 2d}{N + 1}$$

$$\theta_2 = \text{half of the pulse width} = \frac{d}{N}$$

$$\gamma = \theta_1 + \theta_2$$

$$\gamma = \frac{\pi - 2d}{N+1} + \frac{d}{N}$$

Where, N = Number of pulses per half cycle

and d = pulse width

Q.1 (b) Solution:

$$P_R = 3 \times 800 \text{ kW}$$

$$\cos\phi_R = 0.8 \text{ leading}$$

$$\Rightarrow \quad \phi = \cos^{-1}(0.8) = 36.87^\circ$$

Transmission line impedance is,

$$Z = 20(0.015 + j0.02)$$

$$Z = (0.3 + j0.4)\Omega$$

$$= 0.5\angle 53.13^\circ \ \Omega$$

V_{RL} = Receiving end line voltage

V_R = Receiving end phase voltage

Now,
$$P_R = \sqrt{3} V_{RL} I_R \cos\phi$$

$$\Rightarrow \quad 3 \times 800 \times 10^3 = \sqrt{3} \times V_{RL} I_R \times 0.8$$

$$\Rightarrow \quad I_R = \frac{1732.051 \times 10^3}{V_{RL}}$$

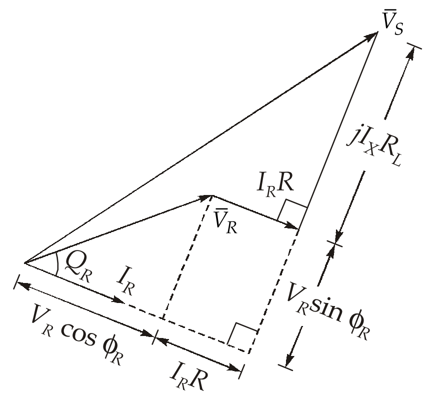
or
$$\bar{I}_R = \frac{1732.051 \times 10^3}{V_{RL}} \angle 36.87^\circ$$

$$\bar{I}_R = \frac{1732.051 \times 10^3}{\sqrt{3}\, V_R} \angle 36.87^\circ$$

$$\Rightarrow \quad \bar{I}_R = \frac{1000}{V_R} \times 10^3 \angle 36.87^\circ \quad \text{...(i)}$$

Now,
$$\bar{V}_S = \bar{V}_R + \bar{I}_R Z$$

$$= \bar{V}_R + \bar{I}_R (R + jX_L)$$



We can write,

$$V_S = \sqrt{(V_R\cos\phi_R + I_RR)^2 + (V_R\sin\phi_R + I_RX_L)^2}$$

$$\Rightarrow \quad \frac{3300}{\sqrt{3}} = \sqrt{\left(V_R \times 0.8 + \frac{1000\times10^3}{V_R}\times0.3\right)^2 + \left(V_R\times0.6 + \frac{1000\times10^3}{V_R}\times0.4\right)^2}$$

$$\frac{10890\times10^3}{3} = \left(V_R \times 0.8 + \frac{1000\times10^3}{V_R}\times0.3\right)^2\left(V_R\times0.6 + \frac{1000\times10^3}{V_R}\times0.4\right)^2$$

$$\Rightarrow \quad 3630000 = V_R^2 + 480000 + \frac{9\times10^{10}}{V_R^2} + \frac{16\times10^{10}}{V_R^2}$$

$$\Rightarrow \quad 3630000 = V_R^2 + \frac{6.25\times10^{10}}{V_R^2} + 960000$$

$$= V_R^2 + \frac{25\times10^{10}}{V_R^2} - 2670000 = 0$$

Solving this, we get

$$\Rightarrow \quad V_R = 1604.00$$

∴ Receiving and line to line voltage is

$$V_{RL} = \sqrt{3}V_R = \sqrt{3}\times1604.00 = 2778.217 \text{ V}$$

and line/phase current,

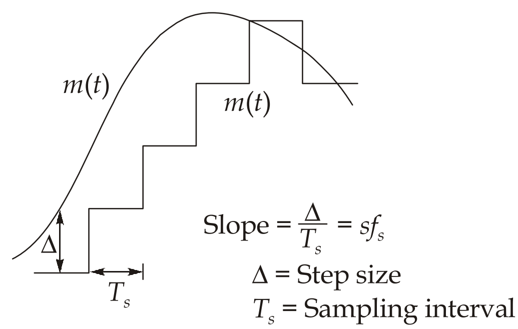
$$I_R = \frac{1000\times10^3}{V_R} = \frac{1000\times10^3}{1604.00} = 623.44 \text{ A}$$

Q.1 (c) Solution:

There are two types of error in delta modulation :

1. Slope-overload error. 2. Granular noise error

1. **Slope Overload Error :** This error occurs when the slope of staircase approximation of the modulating signal is too small as shown in figure.



Δ = Step size

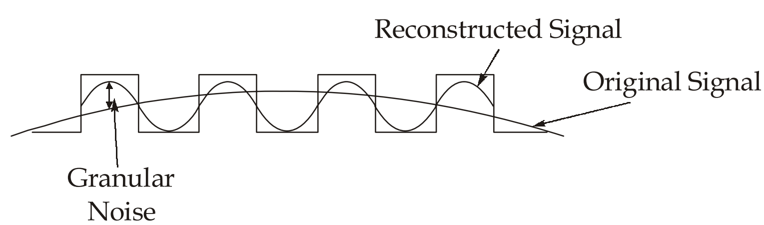
T_s = Sampling interval

To avoid the slope-overload error, the following condition must be satisfied :

$$\frac{\Delta}{T_s} \geq \left|\frac{dm(t)}{dt}\right|_{\max}$$

$$\Delta f_s \geq \left|\frac{dm(t)}{dt}\right|_{\max}$$

2. **Granular Noise :** Granular or Idle noise occurs when the step size is too large as compared to small variation in input signal. This means for a very small variation in the input signal, the staircase is changed by large amount because of large step size.

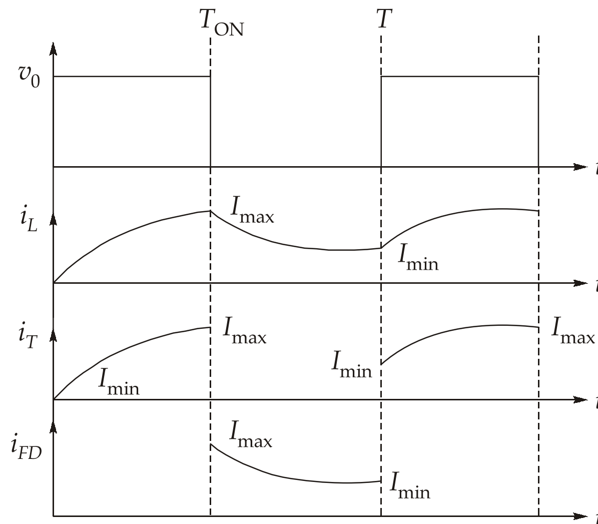


To avoid the granular noise, the step size should be made smaller so that the staircase approximation may become more close to the modulating signal.

Q.1 (d) Solution:

When the chopper is ON, freewheeling diode will be in OFF state.

Thus, by KVL, $$Vs = Ri_T + L\frac{di_T}{dt} + E \quad \ldots(i)$$



From (i), $$Ri_T dt + \frac{Ldi_T}{dt}dt = (V_S - E)dt$$

Using the waveform, the average value can be written as:

$$\frac{1}{T}\int_0^{T_{ON}} Ri_T dt + \frac{1}{T}\int_0^{T_{ON}} L\frac{di_T}{dt}dt = \frac{1}{T}\int_0^{T_{ON}} (V_S - E)dt$$

$$\Rightarrow \quad R.\frac{1}{T}\int_0^{T_{ON}} i_T dt + \frac{1}{T}\int_0^{T_{ON}} L\frac{di_T}{dt}dt = \frac{(V_S - E)}{T}.T_{ON}$$

$$R \cdot i_T \cdot \frac{T_m}{T} + \frac{L}{T}(I_{max} - I_{min}) = (V_S - E)\cdot\alpha \qquad \left[\because \frac{T_{ON}}{T} = \alpha\right]$$

$$\Rightarrow \quad I_{T\,avg} = \frac{\alpha(V_S - E)}{R} - \frac{L}{RT}(I_{max} - I_{min}) \qquad \left[\because i_T \cdot \frac{T_m}{T} = I_{Tavg}\right]$$

Hence proved.

Q.1 (e) Solution:

$$V = 15 \sin[3 \times 10^8 t + 50 \sin(2500)t]$$

Comparing this equation with equation of FM wave

$$e = A\sin(\omega_c t + m_f \sin\omega_m t)$$

(i) To find values of carrier and modulating frequencies

$$\omega_c t = 3 \times 10^8 t, \qquad \omega_c = 3 \times 10^8$$

$$2\pi f_c = 3 \times 10^8, \qquad f_c = 47.74 \text{ MHz}$$

and $$\omega_m t = 2500t$$

$\therefore$ $$2\pi f_m = 2500$$

$$f_m = 398.08 \text{ Hz}$$

(ii) To find modulating index

Comparing given equation with equation of standard FM wave

$$m_f = 50$$

(iii) To find frequency deviation

$$m_f = \frac{\text{Frequency deviation}}{\text{Maximum modulating frequency}} = \frac{\delta}{f_m}$$

$\therefore$ $$\delta = 50 \times 398.08 = 19.904 \text{ kHz}$$

(iv) To find power delivered to 75 Ω resistor

$$P = \frac{\left(\frac{A}{\sqrt{2}}\right)^2}{R} = \frac{\left(\frac{15}{\sqrt{2}}\right)^2}{75} = 1.5 \text{ W}$$

Q.2 (a) Solution:

Let, $$V_t = |V_t|\angle\alpha \text{ p.u.}$$

Then, $$P = \frac{|V_t||V|}{X}\sin\alpha$$

$$1.0 = \frac{1.0\times1.0}{(0.1+0.5 \,||\, 0.5)}\sin\alpha$$

$\Rightarrow$ $$\alpha = 20.49°$$

Now, current through the terminals

$$I = \frac{V_t - V}{X}$$

$$I = \frac{1.0\angle 20.49° - 1.0\angle 0°}{j(0.1+0.5 \,||\, 0.5)} = 1.0163\angle 10.25° \text{ p.u.}$$

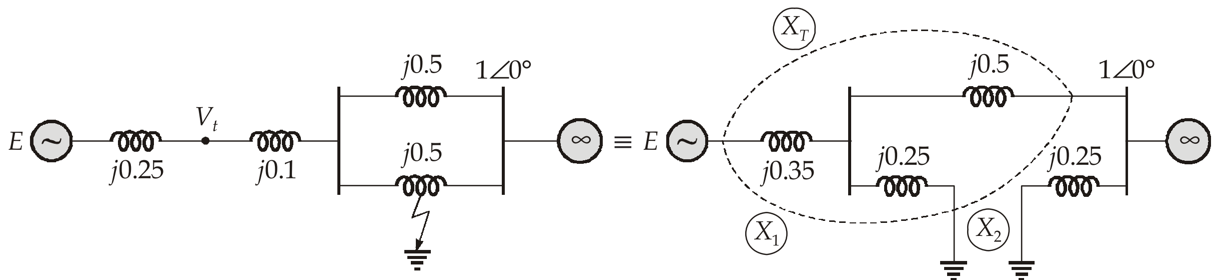
$\therefore$ Generated emf, $$E = V_t + I(jX_d)$$

$$= 1\angle 20.49° + (1.0163\angle 10.25°)(j0.25)$$

$$E = 1.0747\angle 33.94° \text{ p.u.}$$

and $$|E| = 1.0747 \text{ p.u.}$$

When one of the line is shorted in the middle



By Y-Δ conversion,

$$X_1 = 0.35 + 0.25 + \frac{0.35 \times 0.25}{0.5} = 0.775 \text{ p.u.}$$

$$X_2 = 0.25 + 0.5 + \frac{0.25 \times 0.5}{0.35} = 1.107 \text{ p.u.}$$

Transferreactance,

$$X_T = 0.35 + 0.5 + \frac{0.35 \times 0.5}{0.25} = 1.55 \text{ p.u.}$$

During the fault, the swing equation is

$$\frac{H}{180° f} \frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\Rightarrow \quad \frac{4}{180° \times 50} \frac{d^2\delta}{dt^2} = 1 - \frac{|E||V|}{X_T} \sin\delta$$

$$\Rightarrow \quad \frac{1}{2250} \frac{d^2\delta}{dt^2} = 1 - \frac{1.0747 \times 1}{1.55} \sin\delta$$

$$\Rightarrow \quad \frac{d^2\delta}{dt^2} = 2250(1 - 0.693 \sin \delta)$$

$$\frac{d^2\delta}{dt^2} = (2250 - 1560.05 \sin\delta) \quad \text{...(ii)}$$

and

$$\frac{d\delta}{dt} = (2250 - 1560.05 \sin\delta)t$$

At time $t = 0$,

$$\frac{d\delta}{dt} = 0$$ i.e., rotor speed cannot change suddenly

with initial rotor angle,

$$\left.\frac{d^2\delta}{dt^2}\right|_{t=0^+} = 2250 - 1560.5 \times \sin 33.94° = 1378.99 \text{ elec deg/sec}^2$$

Now, change in rotor angle,

$$\Delta\delta = \frac{1}{2}\times\left.\frac{d^2\delta}{dt^2}\right|_{t=0^+}\times(\Delta t)^2 = \frac{1}{2}\times 1378.99\times(0.05)^2$$

$$= 1.724 \text{ elec. degree/sec}^2$$

∴ At the end of this internal, rotor angle is

$$\delta_1 = \delta_0 + \Delta\delta$$

$$= 33.94° + 1.724°$$

$$= 35.66° \text{ elec. deg/sec}^2$$

And the angular acceleration is

$$\left.\frac{d^2\delta}{dt^2}\right|_{t=0.05} = 2250 - 1560.05 \sin \delta_1$$

$$= 2250 - 1560.05 \sin 35.66°$$

$$= 1340.53 \text{ elec. deg/sec}^2$$

Q.2 (b) Solution:

$$r = 0.125 \ \Omega/\text{km}$$

⇒ $$R = 0.125 \times 400 = 50 \ \Omega$$

$$L_1 = 1.273 \times 10^{-3} \text{ H/km}$$

⇒ $$X_L = 2\pi fL_1 l = 2\pi \times 50 \times 1.273 \times 10^{-3} \times 400$$

$$X_L = 160 \ \Omega$$

and $$Y = jyl = j2.8 \times 10^{-6} \times 400 = j1.12 \times 10^{-3} \ \mho$$

Given,

$$S_R = 48 \text{ MVA}$$

Receiving end line voltage, $$V_{RL} = 220 \text{ kV}$$

Phase voltage, $$\bar{V}_R = \frac{220}{\sqrt{3}}\times 10^3 \angle 0° \text{V}$$

$$\cos\phi_R = 0.75 \text{ leading}$$

⇒ $$\phi_R = -41.41°$$

$$\bar{I}_R = \frac{S_R}{\sqrt{3}V_{RL}}\angle -\phi_R = \frac{48\times10^6}{\sqrt{3}\times220\times10^3}\angle -(-41.41°)$$

$$\bar{I}_R = 125.967\angle 41.41° \text{ A}$$

Characteristic impedance is $Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+jX_L}{Y}}$

$$Z_C = \sqrt{\frac{50+j160}{j1.12\times10^{-3}}} = 386.872\angle -8.68°\ \Omega$$

Propagation constant of the line,

$$\gamma l = \alpha l + j\beta l = \sqrt{ZY}$$

$$= \sqrt{(50+j160)(j1.12\times10^{-3})}$$

$$= \sqrt{0.187746\angle 162.64°}$$

$$= 0.433297\angle 81.32°$$

$$\gamma l = \alpha l + j\beta l$$

Where, $\alpha l = 0.06539$ Neper

$\beta l = 0.42833$ radian

$$= 0.42833\times\frac{180°}{\pi}$$

$\Rightarrow$ $\beta l = 24.54°$

(i) Now, $A = D = \cosh\gamma l = \frac{e^{\gamma l}+e^{-\gamma l}}{2} = \frac{e^{\alpha l + j\beta l}+e^{-(\alpha l + j\beta l)}}{2}$

$$= \frac{e^{0.06539}\cdot e^{j24.54°}+e^{-0.06539}e^{-j24.54°}}{2}$$

$$= \frac{e^{0.06539}\angle 24.54° + e^{-0.06539}\angle -24.54°}{2} = 0.912\angle 1.71°$$

$$\sinh\gamma l = \frac{e^{\gamma l}-e^{-\gamma l}}{2} = \frac{e^{\alpha l}e^{j\beta l}-e^{-\alpha l}\cdot e^{j\beta l}}{2}$$

$$= \frac{e^{0.06539}-e^{-24.54°}-e^{-0.06539}\cdot e^{-j24.54°}}{2}$$

$$= \frac{e^{0.06539}\angle 24.54° - e^{-0.06534}\angle -24.54°}{2}$$

$$= 0.4205\angle 81.86°$$

$$\Rightarrow \quad B = 162.68\angle 73.18°\ \Omega$$

and
$$C = \frac{1}{Z_C}\sinh\gamma l = \frac{1}{(386.872\angle -8.68°)}(0.4205\angle 81.86°)$$

$$\Rightarrow \quad C = 1.087 \times 10^{-3}\angle 90.54°$$

(ii) We have per phase,

$$\bar{V}_S = A\bar{V}_R + B\bar{I}_R$$

$$= (0.912\angle 1.71°)\left(\frac{220}{\sqrt{3}}\times 10^3\angle 0°\right) + (162.68\angle 73.18°)(125.967\angle 41.41°)$$

$$\bar{V}_S = 109.512\angle 11.64°\text{ kV}$$

Sending end line voltage,

$$V_{SL} = \sqrt{3}\left|\bar{V}_S\right| = \sqrt{3}\times 109.512\text{ kV} = 189.68\text{ kV}$$

(iii) Sending end current,

$$\bar{I}_S = C\bar{V}_R + D\bar{I}_R$$

$$= (1.087\times 10^{-3}\angle 90.54°)\left(\frac{220\times 10^3}{\sqrt{3}}\angle 0°\right) + (0.912\angle 1.71°)(125.967\angle 41.41°)$$

$$\bar{I}_S = 231.786\angle 69.14°\text{ A}$$

∴ Sending end p.f.

$$\cos\phi_s = \cos\left[\angle\bar{V}_S - \angle\bar{I}_S\right]$$

$$= \cos[11.64° - 69.14°]$$

$$= \cos(-57.5°)$$

$$= 0.5373\text{ leading}$$

Sending end power,

$$P_S = \sqrt{3}V_{SL}I_S\cos\phi_S$$

$$= \sqrt{3}\times 189.68\times 10^3\times 231.786\times 0.5373$$

$$= 40.915\text{ MW}$$

Q.2 (c) Solution:

$$V_s = 120\sqrt{2}\sin(2\pi\times 60t)\,\text{V} = 120\sqrt{2}\sin(120\pi t)\,\text{V}$$

(i) Impedance, $Z = \sqrt{R^2 (2\pi fL)^2} = \sqrt{10^2 + (2\pi\times 60\times 20\times 10^{-3})^2}$

$$= 12.524\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{120\pi\times 20\times 10^{-3}}{10}\right) = 37.016°$$

$$\theta = 37.02° \text{ or } 0.646 \text{ radians.}$$

The general expression for load current for R.L load supplied through sinusoidal source is:

$$i_0 t = \frac{V_m}{Z}\sin(\omega t-\theta) + Ae^{-Rt/L}$$

$$= \frac{V_m}{Z}\sin(\omega t-\theta) + Ae^{-\frac{R}{\omega L}(\omega t)}$$

$$= \frac{120\sqrt{2}}{12.524}\sin(\omega - 0.646) + Ae^{-\frac{10(\omega t)}{120\pi\times 20\times 10^{-3}}}$$

$$= 13.55\sin(\omega t - 0.646) + Ae^{-1.326\,(\omega t)}$$

At $\omega t = \alpha$, $i(t) = 0$

$\therefore$ $$0 = 13.55\sin(\alpha - 0.646) + Ae^{-1.326\,(\alpha t)}$$

$$0 = 13.55\sin(1.047 - 0.646) + Ae^{-1.326\,(1.047)}$$

$\Rightarrow$ $$A = -21.2$$

$\therefore$ $$i_0 t = 13.55\sin(\omega t - 0.646) - 21.2\, e^{-1.326\,(\omega t)}$$

(ii) At extinction angle β, $i_0(t) = 0$

$\Rightarrow$ $$0 = 13.55\sin(\beta - 0.646) - 21.2\, e^{-1.326\,\beta}$$

Let $$f(\beta) = 13.55\sin(\beta - 0.646) - 21.2\, e^{-1.326\,\beta}$$

then $$f'(\beta) = 13.55\cos(\beta - 0.646) - 28.11\, e^{-1.326\,\beta}$$

By Newton-Raphson method

$$\beta_{n+1} = \beta_n - \frac{f(\beta_n)}{f'(\beta_n)}$$

$$\beta_n = \frac{13.55\sin(\beta_n - 0.646) - 212e^{-1.326\beta_n}}{13.55\cos(\beta_n - 0.646) + 28.11e^{-1.326\beta_n}}$$

Initiate guess, $\beta_0 = \pi = 3.142$

$$\beta_1 = \pi - \frac{13.55\sin(\pi - 0.646) - 21.2e^{-1.326\pi}}{13.55\cos(\pi - 0.646) + 28.11e^{-1.326\pi}} = 3.895$$

$\Rightarrow$ $$\beta_2 = \beta_1 - \frac{f(\beta_1)}{f'(\beta_1)}$$

$$= 3.895 - \frac{13.55\sin(3.895 - 0.646) - 21.2e^{-1.326\times 3.895}}{13.55\cos(3.895 - 0.646) + 28.11e^{-1.326\times 3.895}}$$

$$= 3.7768$$

& $$\beta_3 = \beta_2 - \frac{f(\beta_2)}{f'(\beta_2)}$$

$$= 3.7768 - \frac{13.55\sin(3.7768 - 0.646) - 21.2e^{-1.326\times 3.7768}}{13.55\cos(3.7768 - 0.646) + 28.11e^{-1.326\times 3.7768}}$$

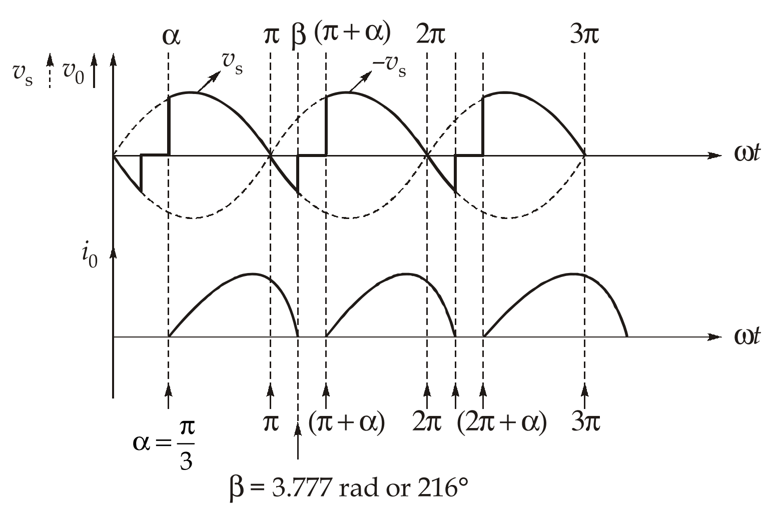
$$= 3.777$$

$\therefore$ $$\beta = 3.777 \text{ radians} = 3.777 \times \frac{180°}{\pi} \cong 216°$$

Since $\pi + \alpha = 180° + 60° = 240° > \beta$

Thus, load current will be discontinuous in nature.

(iii)



Average output voltage $= V_0 = \frac{1}{\pi}\int_{\alpha}^{\beta} V_m \sin\omega t\, d(\omega t)$

$$\frac{v_m}{\pi}[\cos\alpha - \cos\beta] = \frac{120\sqrt{2}}{\pi}[\cos 60° - \cos 216°] = 70.712 \text{ Volts}$$

$\therefore$ Average load current, $I_0 = \frac{V_0}{R} = \frac{70.712}{10}$

$\Rightarrow$ $I_0 = 7.071$ A

Q.3 (a) Solution:

The line constants are

$$R = 0.0195 \times 20 = 0.39\ \Omega$$

$$X = 2\pi fL = 2\pi \times 50 \times 0.63 \times 10^{-3} \times 20 = 3.96\ \Omega$$

(i) For short transmission line,

$$I_R = I_S = I$$

$$|I| = \frac{5000\sqrt{2}}{10} = 707.107 \text{ A}$$

We have, $|V_S| = |V_R| + |I|(R\cos\phi_R + X\sin\phi_R)$

$$= 10000 + 707.107\left(0.39 \times \frac{1}{\sqrt{2}} + 3.96 \times \frac{1}{\sqrt{2}}\right)$$

$$|V_S| = 12.175 \text{ kV}$$

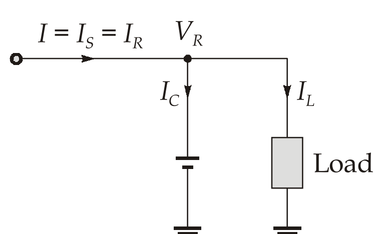
and %voltage regulation $= \frac{V_{S2} - V_{R2}}{V_{RL}} \times 100 = \frac{12.175 - 10}{10} \times 100 = 21.75\%$

(ii) Voltage regulation desired $= \frac{21.75}{2} = 10.875\%$

$\therefore$ $\frac{|V_S| - 10}{10} \times 100 = 10.875$

$\Rightarrow$ $|V_S| = 10.875$ kV

Assuming total system p.f. to be $\cos\phi_R$



$\therefore$ $|V_S| - |V_R| = |I|(R\cos\phi_R + X\sin\phi_R)$

$$\Rightarrow \quad (10.875 - 10) \times 10^3 = |I|(0.39\cos\phi_R + 3.96\sin\phi_R) \quad ...(i)$$

But

$$|I| = \frac{5000\sqrt{2}\times\left(\frac{1}{\sqrt{2}}\right)}{10\cos\phi_R} \quad (\because \text{ capacitor does not take})$$

$$= \frac{500}{\cos\phi_R} \quad ...(ii)$$

Substituting (ii) and (i),

$$1087.5 = \frac{500}{\cos\phi_R}(0.39\cos\phi_R + 3.96\sin\phi_R)$$

$$\Rightarrow \quad 1087.5 = 195 + 1980\tan\phi_R$$

$$\Rightarrow \quad \tan\phi_R = 0.4508$$

$$\phi_R = 24.26°$$

$$\Rightarrow \quad \cos\phi_R = 0.9117$$

$$\Rightarrow \quad |I| = \frac{500}{\cos\phi_R} = \frac{500}{0.9117} = 548.43 \text{ A}$$

$$\therefore \quad I_C = I_R - I$$

$$= 548.43\angle{-24.26°} - 707.107\angle{-45°}$$

$$I_C = (-0.0018 + j274.66) \text{ A}$$

Note that here real part is obtained due to approximation we took, ignore this real term

$$\therefore \quad I_C = j274.66 \text{ A}$$

$$\therefore \quad X_C = \frac{1}{2\pi fC} = \left|\frac{V_R}{I_C}\right| = \frac{10\times10^3}{274.66}$$

$$\Rightarrow \quad \frac{1}{2\pi\times50\times C} = \frac{10^4}{274.66}$$

$$\Rightarrow \quad C = 87.43 \ \mu\text{F}$$

(iii) Efficiency of transmission line

$$\%\eta = \frac{\text{Output}}{\text{Output} + \text{Loss}}\times100$$

Case (a):

$$\%\eta = \frac{5000\sqrt{2}\times\frac{1}{\sqrt{2}}}{5000\sqrt{2}\times\frac{1}{\sqrt{2}}+(707.107)^2\times 0.39\times 10^{-3}}\times 100$$

$$= 96.246\%$$

Case (b):

$$\%\eta = \frac{5000\sqrt{2}\times\frac{1}{\sqrt{2}}}{5000\sqrt{2}\times\frac{1}{\sqrt{2}}+(548.43)^2\times 0.39\times 10^{-3}}\times 100$$

$$\%\eta = 97.708\%$$

Conclusion : Placing a capacitor in parallel with the load, the receiving end power factor and the efficiency are improve while the line current reduces with reduction in voltage regulation.

Q.3 (b) Solution:

Line Voltage, $V_L = 210\ V$

Peak Line Voltage $V_{m2} = \sqrt{2}V_2 = \sqrt{2}\times 210 = 296.98\ V$

Average output voltage for field winding is

$$V_{0_f} = \frac{3VmL}{\pi}\cos\alpha_f$$

For maximum field current, α = 0°,

$$V_{0_f} = \frac{3VmL}{\pi} = \frac{3\times 296.98}{\pi} = 283.59\text{ V}$$

Maximum field current, $I_{f_{\max}} = \frac{V_{0f}}{R_f} = \frac{283.59}{130} = 2.18\text{ A}$

Now, Torque, $T_a = K_a I_f I_a$

$110 = 1.2 \times 2.181 \times I_a$

$\Rightarrow$ $I_a = 42.03\ A$

(i) Also, $E_b = K_a I_f \omega = 1.2\times 2.181\times\frac{2\pi N}{60}$

$$= 1.2\times 2.181\times 2\pi\frac{960}{60} = 263.11\text{ V}$$

By KVL to armature circuit,

$$V_t = E_b + I_a R_a$$

$$\frac{3V_{ml}}{\pi}\cos\alpha_a = E_b + I_a R_a$$

$$\frac{3\times 269.98}{\pi}\cos\alpha_\alpha = 263.11 + 42.03 \times 0.2 = 271.516$$

$\Rightarrow$ Firing angle of armature converter,

$$\alpha_a = \cos^{-1}[0.9574] = 16.78$$

(ii)

$$T_a = 110 \text{ N-m}$$

$$I_{f_{max}} = 2.181\ A$$

$$\alpha_a = 0°$$

$\Rightarrow$

$$V_t = \frac{3V_{ml}}{\pi}\cos 0° = \frac{3\times 296.98}{\pi}\times 1 = 283.6 \text{ V}$$

$$T_a = K_a I_{f_{max}} I_a$$

$$110 = 1.2 \times 2.181 \times I_a$$

$$I_a = 42.03 \text{ A}$$

Now,

$$V_{t_2} = E_{b_2} + 42.03\times 0.2$$

$$E_{b_2} = 275.194 \text{ V}$$

But,

$$E_{b_2} = K_a I_{f_{max}}\omega_2$$

$$275.194 = 1.2 \times 2.181 \times \omega_2$$

$$\omega_2 = 105.148 \text{ rads.}$$

$$\frac{2\pi N_2}{60} = 105.148$$

$\Rightarrow$

$$N_2 = 1004.0848 \text{ rpm}$$

(iii) $N_3 = 1750$ rpm

$\Rightarrow$

$$\omega_3 = \frac{2\pi N_3}{60} = \frac{2\pi\times 1750}{60} = 183.26 \text{ rad/sec.}$$

$T_a = 110$ N-m

$$V_{t_3} = V_{t_2} = 283.6 \text{ V}$$

$$E_{b_3} = V_{t_3} - I_{a_3}R_a = 283.6 - I_{a_3}\times 0.2 \quad \text{...(i)}$$

and $$E_{b_3} = K_a I_f w_3 = 1.2 I_f \times 183.26 = 219.912 I_f \quad \text{...(ii)}$$

and $$T_a = K_a I_f I_{a_3}$$

$$110 = 1.2 \times I_f \times I_{a_3}$$

$$\Rightarrow \quad I_f I_{a_3} = 91.667 \quad \text{...(iii)}$$

From equation (i) and (ii),

$$219.912 I_f = 283.6 - 0.2 I_{a_3}$$

$$\Rightarrow \quad I_{a_3} = \frac{283.6 - 219.912 I_f}{0.2} \quad \text{...(iv)}$$

Substituting equation (iv) in (iii),

$$I_f \left[\frac{283.6 - 219.912 I_f}{0.2}\right] = 91.667$$

$$\Rightarrow \quad 283.6 I_f - 219.912 I_f^2 = 18.33$$

$$\Rightarrow 219.912 I_f^2 - 283.6 I_f + 18.333 = 0$$

$$\Rightarrow \quad I_f = 1.221 \text{ A}, 0.068 \text{ A}$$

For $I_f = 1.221$ A

$$\Rightarrow \quad I_{a_3} = 75.44 \text{ A} \quad \text{(From eq}^n\text{. (iii))}$$

$$V_{0_f} = I_f R_f = 1.221 \times 130 = 158.73 \text{ V}$$

$$\frac{3V_{ml}}{\pi} \cos\alpha_f = 158.73$$

$$\Rightarrow \quad \alpha_f = \cos^{-1}(0.5597)$$

$$= 55.96°$$

(Neglected $I_f = 0.068$ A as it will result $I_{a_3} \cong 1345$ A which exceeds the rating of the machine)

Q.3 (c) Solution:

(i) Methods to improve string efficiency of an insulator:

- By using longer cross arm: The value of the string efficiency depends upon K (ratio of shunt or earth capacitance to self capacitance of the insulator). Lesser the 'K' value, the greater is the string efficiency. Hence, longer cross arm will reduce shunt/earth capacitance and maximum the string efficiency.
- By grading the insulators, such that the top most insulator unit has maximum capacitance, increasing progressively towards the bottom unit.

As, $V \propto \frac{1}{C}$

This method tends to equalize potential distribution across the units in the string.

(ii) Given, $l = 270$ m

Self weight of conductor,

$$W_S = 0.865 \text{ kg/m}$$

Diameter of conductor, $d = 2.76$ cm

Ultimate strength = 9060 kg

Radial thickness of ice, $t = 1.82$ cm

Wind pressure, $P = 3.8$ gm/cm^2

Safely factor = 2

Density of ice = 0.91 gm/cc

$$\text{Sag, } s = \frac{Wl^2}{8T}$$

Where, $$\text{Tension, } T = \frac{\text{Ultimate strength}}{\text{Safety factor}} = \frac{9060}{2} = 4530 \text{ kg}$$

$$W = \sqrt{(W_s + W_i)^2 + W_w^2}$$

$$\begin{aligned} W_i &= \text{Weight of ice} = \text{Density} \times \text{Volume of ice} \\ &= \text{Density} \times \pi t\,(d + t) \times 100 \\ &= 0.91 \times \pi \times 1.82(2.76 + 1.62) \times 100 \\ &= 2028.525 \text{ gm/m} \\ &= 2.029 \text{ kg/m} \end{aligned}$$

$$\begin{aligned} \text{Weight of wind} &= \text{Wind pressure} \times (d + 2t) \times 100 \\ &= 3.8 \times (2.76 + 2 \times 1.82) \times 100 \\ &= 1740.4 \text{ gm} = 1.7404 \text{ kg/m} \end{aligned}$$

$\therefore$ $$\begin{aligned} W &= \sqrt{(W_s + W_i)^2 + W_w^2} \\ &= \sqrt{(0.865 + 2.029)^2 + (1.7404)^2} = 3.377 \text{ kg} \end{aligned}$$

$\therefore$ $$\text{Sag, } S = \frac{Wl^2}{8T} = \frac{3.377 \times 270^2}{8 \times 4530}$$

$$S = 6.793 \text{ m}$$

Q.4 (a) Solution:

$R = 10\ \Omega$, $L = 0.03$ H, $f = 50$ Hz, $V_{dc} = 230$ V

n^{th} harmonic impedance is

$$Z_n = \sqrt{R^2 + (2\pi f n L)^2}$$

∴ Fundamental load impedance is

$$Z_1 = \sqrt{10^2 + (2\pi \times 50 \times 3 \times 0.03)^2} = 13.741\ \Omega$$

∴ 3rd harmonic load impedance is

$$Z_3 = \sqrt{10^2 + (2\pi \times 50 \times 3 \times 0.03)^2} = 29.991\ \Omega$$

(i) Square wave mode:

RMS value of n^{th} harmonic output voltage is

$$V_{on} = \frac{4V_{dc}}{n\pi\sqrt{2}}$$

∴ fundamental output voltage is

$$V_{01} = \frac{4 \times 230}{1 \times \pi\sqrt{2}}$$

and fundamental load current is

$$I_{01} = \frac{V_{01}}{Z_1} = \frac{4 \times 230}{\pi\sqrt{2}} \times \frac{1}{13.741} = 15.07 A$$

∴ 3^{rd} harmonic output voltage is

$$V_{03} = \frac{4 \times 230}{3\pi\sqrt{2}}$$

3^{rd} harmonic load current is

$$I_{03} = \frac{V_{03}}{Z_3} = \frac{4 \times 230}{3\pi\sqrt{2}} \times \frac{1}{29.991} = 2.30 A$$

RMS output/load current is

$$I_{0r} = \sqrt{I_{01}^2 + I_{03}^2} = \sqrt{15.07^2 + 2.30^2} = 15.245 A$$

Power delivered to load, $P_0 = I_{or}^2 R$

$= 15.245^2 \times 10 = 2324.1$ W

(ii) Two symmetrically spaced pulses:

The output voltage Fourier series is

$$V_0 = \sum_{n=1,3,5}^{\infty} \frac{8V_{dc}}{n\pi} \sin n\gamma \sin \frac{nd}{2} \sin \omega t$$

n^{th} harmonic RMS output voltage is

$$V_{on} = \frac{8V_{dc}}{n\pi}\sin(n\gamma)\sin\left(\frac{nd}{2}\right)$$

Where, pulse width $2d$ = (Duty Ratio) × 180°

$$= 0.5 \times 180°$$

$$d = 90°$$

$\Rightarrow$ $d = 45°$

and $$\gamma = \frac{180° - 2d}{N+1} + \frac{d}{N} \quad \text{(Where } N = \text{No. of pulses per half cycle)}$$

$$= \frac{180° - 90°}{2+1} + \frac{45°}{2} = 52.5°$$

$\therefore$ Fundamental output voltage is

$$V_{01} = \frac{8 \times 230}{1 \times \pi\sqrt{2}}\sin(1 \times 52.5°)\sin\left(\frac{45°}{2}\right) = 125.736\ V$$

Fundamental load current

$$I_{01} = \frac{V_{01}}{Z_1} = \frac{125.736}{13.741} = 9.15A$$

3^{rd} harmonic output voltage is

$$V_{03} = \frac{8 \times 230}{3 \times \pi\sqrt{2}}\sin(3 \times 52.5°)\sin\left(\frac{3 \times 45°}{2}\right) = 48.808V$$

3^{rd} harmonic load current

$$I_{03} = \frac{V_{03}}{Z_3} = \frac{48.808}{29.991} = 1.627A$$

RMS load current,

$$I_{or} = \sqrt{I_{01}^2 + I_{03}^2} = \sqrt{9.15^2 + 1.627^2}$$

$$= 9.294\ \text{A}$$

$\therefore$ Power delivered to load,

$$P_0 = I_{or}^2 R$$

$$= 9.294^2 \times 10$$

$$= 863.7841\ \text{W}$$

Q.4 (b) Solution:

(i) Fault at bus-1 with $Z_f = j0.2$ pu.

Since prefault voltages at buses are $1.0\angle 0°$ pu.

Therefore, $E_1 = E_2 = V_1 = V_2 = V_3 = 1.0\angle 0°$ pu

Now, fault current at bus-1,

$$I_f = \frac{V_1}{Z_{11} + Z_f} = \frac{1\angle 0°}{j0.0776 + j0.20} = -j3.60 \text{ pu}$$

Now, change in bus-voltages due to fault at bus (1)

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = j\begin{bmatrix} 0.0776 & 0.0448 & 0.0597 \\ 0.0448 & 0.1104 & 0.0806 \\ 0.0597 & 0.0806 & 0.2075 \end{bmatrix}\begin{bmatrix} -I_f \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = j\begin{bmatrix} 0.0776 & 0.0448 & 0.0597 \\ 0.0448 & 0.1104 & 0.0806 \\ 0.0597 & 0.0806 & 0.2075 \end{bmatrix}\begin{bmatrix} j3.60 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} -0.27936 \\ -0.16128 \\ -0.2150 \end{bmatrix}$$

Post fault voltages at buses,

$$\begin{bmatrix} V_1(F) \\ V_2(F) \\ V_3(F) \end{bmatrix} = \begin{bmatrix} V_1(P) \\ V_2(P) \\ V_3(P) \end{bmatrix} + \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1(F) \\ V_2(F) \\ V_3(F) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.27936 \\ -0.16128 \\ -0.2150 \end{bmatrix} = \begin{bmatrix} 0.72 \\ 0.838 \\ 0.785 \end{bmatrix}$$

Therefore, $I_{G1} = \dfrac{E_1 - V_1(F)}{j0.1} = \dfrac{1 - 0.72}{j0.1} = -j2.8$ pu

$$I_{G2} = \frac{E_2 - V_2(F)}{j0.2} = \frac{1 - 0.838}{j0.2} = -j0.81 \text{ pu}$$

Current between bus-2 and bus-3,

$$I_{23} = \frac{V_2 - V_3}{Z_{23}} = \frac{0.838 - 0.785}{j0.0806} = -j0.657 \text{ pu}$$

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Current between bus-3 and bus-1,

$$I_{31} = \frac{V_3 - V_1}{Z_{31}} = \frac{0.785 - 0.72}{j0.0597} = -j1.088 \text{ pu}$$

Current between bus-2 and bus-1,

$$I_{21} = \frac{V_2 - V_1}{Z_{21}} = \frac{0.838 - 0.72}{j0.0448} = -j2.64 \text{ pu}$$

Q.4 (c) Solution:

From the given figure of the power system, since there is no resistance in the transmission cable interlinking the two stations, so there will be no real power loss in the cable.

So, $$P_{G1} + P_{G2} = P_{D1} + P_{D2} = 15 + 25 = 40 \text{ p.u.}$$

For load equalization, $$P_{G1} = P_{G2} = \frac{40}{2} = 20 \text{ p.u.}$$

Equalization means that,

$$P_S = P_R = P_{G1} - P_{D1} = 20 - 15 = 5 \text{ p.u.}$$

Now, $$P_S = P_R = \frac{V_1 V_2}{X}\sin\delta_1$$

$\Rightarrow$ $$5 = \frac{1\times 1}{0.08}\sin\delta_1$$

$\Rightarrow$ $$\delta_1 = 23.58° \Rightarrow \text{torque angle}$$

Hence, $$Q_S = \frac{V_1^2}{X} - \frac{V_1 V_2}{X}\cos\delta_1$$

$$Q_S = \frac{1^2}{0.08} - \frac{1\times 1}{0.08}\cos 23.58°$$

$\Rightarrow$ $$Q_S = 1.044 \text{ p.u.} = -Q_S$$

i.e. reactive power loss in the cable is

$$Q_{\text{Loss}} = Q_S - Q_R$$
$$= 1.044 - 1.044 = 2.088 \text{ p.u.}$$

Total load on station-1 is

$$P_{G1} = P_{D1} + P_S = 15 + 5 = 20 \text{ p.u.}$$
$$Q_{G1} = Q_{D1} + Q_S = 5 + 1.044 = 6.044 \text{ p.u.}$$

$\therefore$ $$S_{G1} = P_{G1} + jQ_{C1}$$
$$= (20 + j6.044) \text{ p.u.}$$

Station-1 power factor is

$$\cos\phi_1 = \cos\left(\tan^{-1}\frac{Q_{G1}}{P_{G1}}\right) = \cos\left(\tan^{-1}\left(\frac{6.044}{20}\right)\right)$$

$$= 0.9572 \text{ p.f. lagging}$$

Section-B : Power Systems + Power Electronics & Drives + Communication Systems

Q.5 (a) Solution:

For rated dc motor conditions,

$$V_t = E_b + I_a R_a$$

$$= K_m \omega_m + I_a R_a$$

$$= K_m \times \frac{2\pi N}{60} + I_a R_a$$

$$\Rightarrow \quad 600 = K_m \times \frac{2\pi \times 1500}{60} + 70 \times 1$$

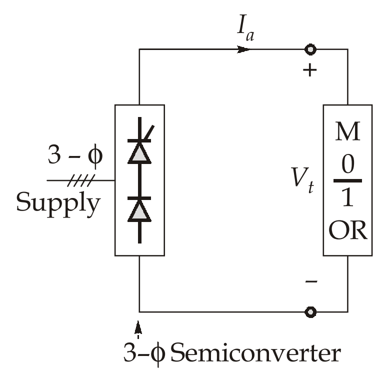
$$\Rightarrow \quad K_m = 3.374 \text{ Vs/rad or Nm/A}$$

Here, $$V_t = \frac{3V_{mL}}{2\pi}(1+\cos\alpha)$$

Where, $$V_{mL} = \sqrt{2}V_L = \sqrt{2}\times 400$$

$$\therefore \quad V_t = \frac{3\sqrt{2}\times 400}{2\pi}(1+\cos 45°)$$

$$= 461.081 \text{ Volts.}$$



At $\alpha = 45°$ and $N = 1200$ rpm.

$$V_t = E_b + I_{a_1} Ra$$

$$461.081 = K_m \times \frac{2\pi \times N_1}{60} + I_{a_1} \times 1$$

$$= 3.374 \times \frac{2\pi \times 1200}{60} + I_{a_1}$$

$$\Rightarrow \quad I_{a_1} = 37.692\ A$$

(i) RMS value of the source current,

$$I_{sr} = Ia_1\sqrt{\frac{2}{3}} = 37.092\sqrt{\frac{2}{3}} = 30.285A$$

RMS value of the thyristor current,

$$I_{sr} = Ia_1\sqrt{\frac{1}{3}} = 37.092\sqrt{\frac{1}{3}} = 21.415A$$

(ii) Average value of the thyristor current,

$$I_T = Ia_1\left[\frac{2\pi/3}{2\pi}\right] = \frac{I_{a_1}}{3} = \frac{37.092}{3} = 12.364A$$

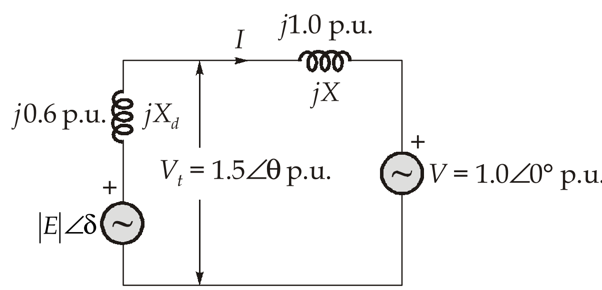
(iii) From power balance equation,

$$\sqrt{3}V_{sr}I_{sr}p.f. = V_tI_{a_1}$$

$$\sqrt{3}\times 400\times 30.285\times p.f. = 461.081 \times 37.092$$

$$p.f. = 0.8151 \text{ lagging.}$$

Q.5 (b) Solution:



For steady state maximum power limit,

$$\delta = 90°$$

Current I can be written as,

$$I = \frac{E_f\angle\delta - V_t\angle\theta}{j0.6} = \frac{V_t\angle\theta - 1\angle 0°}{j1}$$

On putting the values,

$$I = \frac{E_f\angle 90° - 1.5\angle\theta}{j0.6} = \frac{1.5\angle\theta - 1\angle 0°}{j1}$$

$$E_f\angle 90° - 1.5\angle\theta = 0.9\angle\theta - 0.6$$

$$E_f\cos 90° + jE_f\sin 90° = 2.4\angle\theta - 0.6$$

$$jE_f = 2.4\cos\theta + j2.4\sin\theta - 0.6$$

On equating the real part,

$$2.4\cos\theta - 0.6 = 0$$

$$\theta = \cos^{-1}\left(\frac{0.6}{2.4}\right) = 75.52°$$

Current I, $$I = \frac{1.5\angle\theta - 1}{j1} = \frac{(1.5\angle 75.52) - 1}{j1} = 1.581\angle 23.28 \text{ p.u.}$$

Now finding E_f,

$$E_f = (1.5\angle 75.52) + (I\,j0.6) = 2.323\angle 90°$$

Steady state power limit, $$P_{max} = \frac{|E_f|\cdot|V_t|}{X} = \frac{2.323\times 1}{1.6} = 1.451 \text{ p.u.}$$

Q.5 (c) Solution:

(i) Image frequency and its rejection ratio at 950 kHz

Given,
$$Q = 90$$
$$\text{IF} = 455 \text{ kHz},$$
$$f = 950 \text{ kHz}$$

∴ Image frequency, $f_{si} = f_s + 2\,\text{IF} = [950 + (2 \times 455)]$

∴ $f_{si} = 1860$ kHz ...(i)

Image frequency rejection ratio,

$$\alpha = \sqrt{1+Q^2\rho^2} \quad \text{...(ii)}$$

where $$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1860}{950} - \frac{950}{1860} = 1.45 \quad \text{...(iii)}$$

∴ $$\alpha = \sqrt{1+(90)^2\times(1.45)^2} = 130.5$$

(ii) Image frequency and its rejection at 10 MHz

Given,
$$Q = 90,$$
$$\text{IF} = 455 \text{ kHz},$$
$$f_s = 10 \text{ MHz}$$

∴ Image frequency, $f_{si} = 10 + (2 \times 0.455) = 10.91$ MHz ...(iv)

∴ $$\rho = \frac{10.91}{10} - \frac{10}{10.91} = 0.174$$

So, IFRR, $$\alpha = \sqrt{1+(90)^2\times(0.174)^2} = 15.72$$

So without RF amplifier the image rejection is adequate at low frequencies however it is inadequate at higher operating frequency. RF amplifier may therefore to be used at high frequencies.

Q.5 (d) Solution:

(i) The signals are band-limited to 5 kHz, 10 kHz and 5 kHz and hence Nyquist rates are 10 kHz, 20 kHz and 10 kHz respectively.

Thus, there are 10 + 20 + 10 = 40 thousand samples per second.

Number of quantization levels,

$$L = 256$$

Hence, the numbers bits N is given as

$$L = 2^N$$

$$N = \log_2 L = \log_2 256 = 8$$

Hence, output bit rate, $R_b = nf_s = 8 \times 40 = 320$ kbps

Maximum bit duration, $T_b = \frac{1}{R_b} = \frac{10^{-3}}{320} s = 3.125\ \mu\text{sec}$

(ii) Channel bandwidth required,

$$\text{B.W.} = \frac{R_b}{2} = \frac{320}{2} = 160 \text{ kHz}$$

(iii) Since there are 10000 + 20000 + 10000 = 40000 samples per seconds generated by the commutator. Hence, the samples generated in one minute is

$$60 \times 40000 = 2400000 \text{ samples/minute}$$

Commutator takes 4 samples in one rotation. Hence,

$$\text{Commutator speed} = \frac{24 \times 10^5}{4} = 6 \times 10^5 \text{ rpm}$$

(iv) For $L = 512$, n comes out to be equal to 9 (as $2^n = L$). Hence,

$$\text{Channel bandwidth} = \frac{nf_s}{2} = \frac{9 \times 40000}{2} = 180 \text{ kHz}$$

Q.5 (e) Solution:

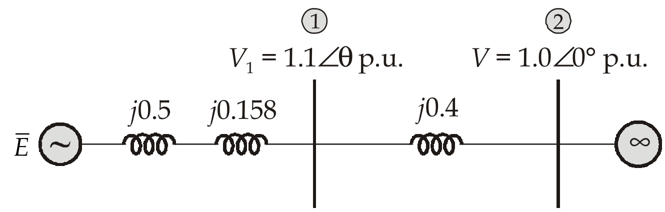
$$f = 50 \text{ Hz}$$

$$H = 5.66 \text{ MJ/MVA}$$

$$P_m = P_e = 0.95 \text{ p.u.}$$

Let, $V_1 = |V_1|\angle\theta = 1.1\angle\theta$ p.u.

and $V = 1.0\angle 0°$ p.u.



We know, $$P_e = \frac{|V_1||V|}{X}\sin\theta$$

$$\Rightarrow \quad 0.95 = \frac{1.1\times 1}{0.4}\sin\theta$$

$$\Rightarrow \quad \theta = 20.21°$$

$$\therefore \quad \bar{V}_1 = 1.1\angle 20.21° \text{ p.u.}$$

Now current through system per phase will be

$$\bar{I} = \frac{\bar{V}_1 - V}{j0.4} = \frac{1.1\angle 20.21° - 1.0\angle 0°}{j0.4} = 0.953\angle -4.85° \text{ p.u.}$$

$$\therefore \quad \bar{E} = \bar{V}_1 + (j0.5 + j0.158)\bar{I}$$

$$= 1.1\angle 20.21° + (j0.658)(0.953\angle -4.85°)$$

$$\bar{E} = 1.479\angle 42.80° \text{ p.u.}$$

(i) Excitation emf/voltage is $E = 1.479$ p.u.

And the power angle is $\delta = 42.80°$

(ii) Now, swing equation (in per unit) is given by

$$\frac{H}{180° f}\frac{d^2\delta}{dt^2} = P_m - P_{max}\sin\delta$$

$$\frac{5.66}{180\times 60}\frac{d^2\delta}{dt^2} = 0.95 - \frac{E_x V}{X_T}\sin\delta$$

$$\Rightarrow \quad \frac{5.666}{180\times 60}\frac{d^2\delta}{dt^2} = 0.95 - \frac{1.479\times 1}{1.058}\sin\delta$$

Swing equation:

$$\frac{d^2\delta}{dt^2} = (1812.721 - 2667.41\sin\delta) \text{ elec. degree/sec}^2$$

Q.6 (a) Solution:

(i) Fourier series of output.

$$V_0(t) = V_{dc} + \sum_{n=1,2,3,...}^{\infty} [a_n \cos\omega t + b_n \sin n\omega t]$$

$$V_{dc} = \frac{1}{2\pi}\int_0^{2\pi} V_0(t)\,d(\omega t) = 0$$

[∵ Symmetrical positive and negative pulses.]

$$a_n = \frac{1}{\pi}\int_0^{2\pi} V_0(t)\cos(n\omega t)\,d(\omega t) = 0$$

[∵ $V_O(t)$ has half wave odd symmetruity.]

$$b_n = \frac{1}{\pi}\int_0^{2\pi} V_0(t)\sin(n\omega t)\,d(\omega t)$$

$$= \frac{4}{\pi}\int_0^{\pi/2} V_0(t)\sin(n\omega t)\,d(\omega t) \quad \text{[Using symmetry]}$$

$$= \frac{4}{\pi}\left\{\int_0^{\alpha_1} \sin(n\omega t)\,d(\omega t) + \int_{\alpha_1}^{\alpha_2}(-V_s)\sin(n\omega t)\,d(\omega t) + \int_{\alpha_2}^{\pi/2} V_s \sin(n\omega t)\,d(\omega t)\right\}$$

$$= \frac{4V_s}{\pi}\left\{\frac{1-\cos n\alpha_1}{n} - \left(\frac{\cos n\alpha_1 - \cos n\alpha_2}{n}\right) + \frac{\cos n\alpha_2}{n}\right\}$$

$$= \frac{4V_s}{\pi}[1 - 2\cos n\alpha_1 + 2\cos n\alpha_2] \text{ for n = 1, 3, 5, ...}$$

Hence, $$V_0(t) = \frac{4V_s}{\pi}\sum_{n=1,3,5,...}^{\infty}\left[\frac{1-2\cos n\alpha_1 + 2\cos n\alpha_2}{n}\right]\sin(n\omega t)$$

(ii) To eliminate 3rd and 5th harmonic.

$$V_{03} = 0$$

$$\Rightarrow \quad 1 - 2\cos 3\alpha_1 + 2\cos 3\alpha_2 = 0 \quad \text{...(i)}$$

$$\text{and } 1 - 2\cos 5\alpha_1 + 2\cos 5\alpha_2 = 0 \quad \text{...(ii)}$$

Solving the pair of two non-linear equations using iterations method with initial guesses,

$$\alpha_2 = 33.33°$$
$$\alpha_1 = 23.65°$$

Q.6 (b) Solution:

(i)

$$Y_{23} = Y_{32} = -y_{23} = -(16 - j32)$$
$$= 35.77\angle 116.6° \text{ pu}$$

$\because$ $$P_i = P_{Gi} - P_{Li}$$
$$P_2 = P_{G2} - P_{L2}$$
$$= 50 - 305.6 = -255.6 \text{ MW} = -2.556 \text{ pu}$$

Similarly, $$P_3 = P_{G3} - P_{L3}$$
$$= -138.6 \text{ MW} = -1.386 \text{ pu}$$

Also, $$Q_2 = Q_{G2} - Q_{L2}$$
$$= 30 - 140.2 = -110.2 \text{ MVAR}$$
$$= -1.102 \text{ pu}$$
$$Y_{22} = y_{12} + y_{22}$$
$$= \frac{1}{0.02 + j0.04} + \frac{1}{0.0125 + j0.025}$$
$$= 58.13\angle -63.4° \text{ pu}$$

Iterative computation:

$$\Rightarrow \quad V_2^{(p+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^{(p)})^*} - Y_{21}V_1 - Y_{23}V_3^{(p)}\right]$$

$$\Rightarrow \quad V_3^{(p+1)} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^{(p)})^*} - Y_{31}V_1 - Y_{32}V_2^{(p+1)}\right]$$

Slack bus voltage, $V_1 = (1.05 + j0.0)$

Starting voltage, $V_2^{(0)} = (1 + j0)$ pu; $V_3^{(0)} = (1 + j0)$ pu

Now, $$\frac{P_2 - jQ_2}{Y_{22}} = \frac{(-2.556 + j1.102)}{58.13\angle -63.4°} = 0.0478\angle 220.1°$$

$$\frac{Y_{21}}{Y_{22}} = -0.3846$$

$$\frac{Y_{23}}{Y_{22}} = -0.6153$$

$\Rightarrow$
$$V_2^{(p+1)} = \left[\frac{0.0478\angle 220.1^\circ}{[V_0^{(p)}]^*} + 0.3846\,V_1 + 0.6153\,V_3^p\right]$$

Now,
$$\frac{P_3 - jQ_3}{Y_{33}} = 0.0217\angle 229.2^\circ$$

$$\frac{Y_{31}}{Y_{33}} = 0.47\angle 175.6^\circ;$$

$$\frac{Y_{32}}{Y_{33}} = 0.532\angle 183.8^\circ$$

$\Rightarrow$
$$V_3^{(p+1)} = \left[\frac{0.0217\angle 229.2}{(V_3^{(p)})^*} - 0.47\angle 175.6^\circ\,V_1 - 0.532\angle 183.8^\circ\,V_2^{(p+1)}\right]$$

Now solving equations:

For first iteration assuming,

$$p = 0$$

$\therefore$
$$V_2' = \frac{0.0478\angle 220.1^\circ}{(V_2^{(0)})^*} + 0.3846\,V_1 + 0.6153\,V_3^{(0)}$$

$$= 0.98305\angle -1.8^\circ \text{ pu}$$

$$V_3' = 1.0011\angle -2.06^\circ \text{ pu}$$

For second iteration:

Let, $p = 1$

$\Rightarrow$
$$V_2^{(2)} = \frac{0.0478\angle 220.1^\circ}{(0.98305\angle -1.8^\circ)^*} + 0.3846\times(1.05 + j0.0) + 0.6153\times 1.0011\angle -2.06^\circ$$

$$= 0.9847\angle -3.18^\circ \text{ pu}$$

$$V_3^{(2)} = 1.0036\angle -2.83^\circ \text{ pu}$$

(ii) Computation of slack bus power:

After 2nd iteration slack bus power is computed,

$$P_1 = \sum_{k=1}^{3} |V_1||V_k||Y_{1k}|\cos(\theta_{1k} - \delta_1 + \delta_k)$$

$\therefore$
$$P_1 = |V_1|^2|Y_{11}|\cos\theta_{11} + |V_1||V_2||Y_{12}|\cos(\theta_{12} - \delta_1 + \delta_2) + |V_1||V_3||Y_{13}|\cos(\theta_{13} + \delta_3 - \delta_1)$$

$\therefore$
$$P_1 = (1.05)^2 \times 53.85\cos(-68.2^\circ) + 1.05 \times 0.9847 \times 22.36\cos(116.56^\circ - 0 - 3.18^\circ) + 1.05 \times 1.0036 \times 31.62\cos(108.4^\circ - 0 - 2.83^\circ)$$

$$= 3.93 \text{ pu} = 393 \text{ MW}$$

$$Q_1 = -\sum_{k=1}^{3} |V_1||V_k||Y_{1k}|\sin(\theta_{1k} - \delta_1 + \delta_k)$$

$$Q_1 = 1.8055 \text{ pu} = 180.55 \text{ MVAR}$$

Q.6 (c) Solution:

$$f = 50 \text{ Hz}$$

$$\text{Poles, } P = 4$$

$$G = 200 \text{ MVA}$$

$$J = 81000 \text{ kg-m}^2$$

$$P_{e1} = P_{m1} = 40 \text{ MW}$$

$$P_{m2} = 60 \text{ MW}$$

(i) Kenetic energy, $\text{KE} = \frac{1}{2}J\omega_s^2$

Where, $\omega_s = \frac{2\pi}{50}\times\frac{120f}{P} = \frac{2\pi}{60}\times\frac{120\times 50}{4} = 157.08 \text{ rad/sec}$

Now, $\text{K.E.} = \frac{1}{2}\times 81000\times(157.08)^2 = 1000 \text{ MJ}$

Inertia constant, $H = \frac{K.E.}{G} = \frac{1000\,\text{MJ}}{200\text{ MVA}} = 5 \text{ mJ/MVA}$

(ii) Swing equation:

$$(P_m - P_e) = P_a = \frac{GH}{180° f}\frac{d^2\delta}{dt^2}$$

$$(60 - 40) = \frac{200\times 5}{180°\times 50}\frac{d^2\delta}{dt^2}$$

$$\Rightarrow \quad \frac{d^2\delta}{dt^2} = \alpha = 180 \text{ elec. degree/sec}^2$$

$$\alpha = 180\times\frac{2}{P} \text{ mech. degree/sec}^2$$

$$= 180\times\frac{2}{4} \text{ mech. degree/sec}^2$$

$$= 90 \text{ mech. degree/sec}^2$$

$$\alpha = 90 \times \left(\frac{60°}{360°}\right) \text{ rpm/sec}$$

$$\alpha = 15 \text{ rpm/sec}$$

(iii) $t = 15 \text{ cycle} = \frac{15}{50} = 0.3 \text{ sec}$

Now, $\frac{d^2\delta}{dt} = 180 \text{ elec. degree/sec}^2$

$\Rightarrow$ $\frac{d\delta}{dt} = 180t + K_1$

At time $t = 0^+$, $\frac{d\delta}{dt} = \omega_s - \omega_s = 0$

$\Rightarrow$ $K_1 = 0$

$\Rightarrow$ $\frac{d\delta}{dt} = 180t$ ...(i)

$$\delta(t) = 90t^2 + K_2$$

At time, $t = 0$

$$\delta(t = 0) = \delta_0 = K_2$$

$\Rightarrow$ $\delta(t) = 90t^2 + \delta_0$

Change in rotor angle,

$$\Delta\delta(t) = \delta(t) - \delta_0 = 90t^2$$

$\Rightarrow$ $\Delta\delta(t) = \delta(0.3) - \delta_0 = 90 \times (0.3)^2 = 8.1 \text{ elec. degree}$

Now from equation (i),

$$\frac{d\delta}{dt} = 180t \text{ elec. degree} = \omega$$

At the end of $t = 0.3$ sec i.e. 15 cycles

$$\omega = \frac{d\delta}{dt} = 180 \times 0.3 = 54 \text{ elec. degree/sec}$$

$$\omega_n = \frac{54}{(P/2)} \text{mech. degree/sec}$$

$$= \frac{54}{(4/2)} \text{mech. degree/sec}$$

$$= 27 \text{ mech. degree/sec}$$

$$\omega_n = 27 \times \frac{\pi}{180°} \text{mech rad/sec} = 0.4712 \text{ mech rad/sec}$$

⇒ Change in speed from synchronous speed,

$$N = \frac{60 \times \omega_n}{2\pi} = \frac{60 \times 0.4712}{2\pi} \text{rpm} = 4.4996 \text{ rpm}$$

Synchronous speed, $$N_S = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

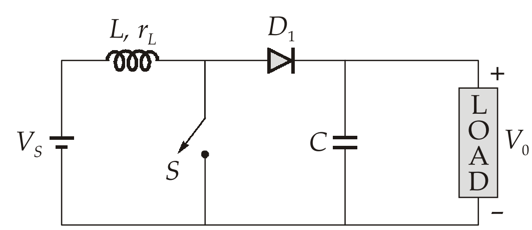
∴ Rotor speed at the end of 15 cycles will be

$$N_r = N_S + N = 1500 + 4.4996 \text{ rpm}$$

⇒ $$N_r \approx 1504.5 \text{ rpm}$$

Q.7 (a) Solution:

(i)



$$P_S = P_0 + P_L$$

$$V_S I_L = V_0 I_D + I_L^2 r_L$$

$$V_S I_L = V_0 I_L (1 - D) + I_L^2 r_L$$

$$V_S = V_0(1 - D) + I_L r_L$$

$$I_D = I_L(1 - D)$$

$$I_L = \frac{V_0}{R(1-D)}$$

$$V_S = \frac{V_0 r_L}{R(1-D)} + V_0(1-D)$$

$$V_0 = \frac{V_S}{(1-D)}\left[\frac{1}{1+\left[r_L / R(1-D)^2\right]}\right]$$

$$\eta = \frac{P_0}{P_0 + P_{\text{Loss}}} = \frac{\frac{V_0^2}{R}}{\frac{V_0^2}{R_L} + I_L^2 r_L}$$

$$\eta = \frac{\frac{V_0^2}{R}}{\frac{V_0^2}{R} + \left[\frac{V_0}{R(1-D)}\right]^2 r_L} = \frac{1}{\left[1 + \frac{r_L}{R(1-D)^2}\right]}$$

(ii)

$$D = 1 - \frac{V_S}{V_0} = 1 - \frac{5}{10} = 0.5$$

$$\eta = \frac{1}{1 + \frac{0.048}{10(1-0.5)^2}} \times 100 = 98.12\%$$

Q.7 (b) Solution:

Universal Torque Equation :

$$T_d = K_1 I^2 + K_2 V^2 + K_3 VI\cos(\theta - \tau) - K_4$$

where,

T_d = Operating/driving torque of relay

$K_1 I^2$ = Over-current term

$K_2 V^2$ = Over-voltage term

$K_3\ VI\cos(\theta - \tau)$ = Directional term

K_4 = Spring restraining torque

(i) Impedance Relay (Z) :

- Also known as Voltage Restraint Overcurrent Relay.
- Assuming $K_3 = K_4 = 0$; K_2 = negative in UTE considering it restraining torque.

$$T_d = K_1 I^2 - K_2 V^2$$

For tripping condition,

$$K_1 I^2 > K_2 V^2$$

$$\sqrt{\frac{K_1}{K_2}} > \frac{V}{I}$$

$$Z_l = \sqrt{\frac{K_1}{K_2}} = \%\ \text{PSB setting}$$

Z_f = Fault impedance

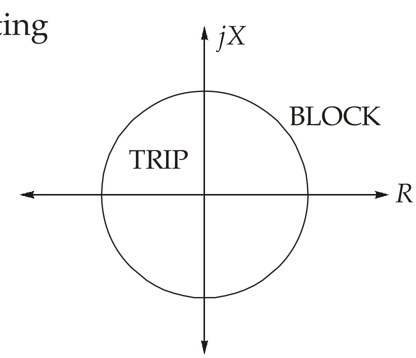
For tripping, $Z_f < Z_l$

For blocking, $Z_f > Z_l$

Locus of impedance relay,

$$Z_l = R_l + jX_l$$

$$|Z_l| = \sqrt{R_l^2 + X_l^2}$$



$$|Z_l|^2 = R_l^2 + X_l^2 \quad \text{...(equation of circle)}$$

$\therefore$ Locus of impedance relay is a circle with radius $r = Z_l$.

(ii) **Admittance Relay/MHO Relay :**

- Also known as Voltage Restraint Directional Relay.
- Assuming $K_1 = K_4 = 0$; K_2 = Negative in UTE.

$$T_d = K_3 VI\cos(\theta - \tau) - K_2 V^2$$

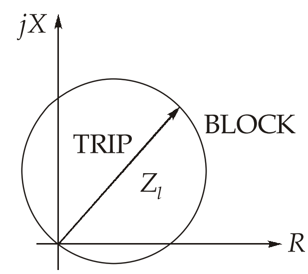
For tripping condition :

$$K_3 VI\cos(\theta - \tau) > K_2 V^2$$

$$\frac{K_3}{K_2} > \frac{V}{I\cos(\theta - \tau)}$$

$$Y\cos(\theta - \tau) > \left(\frac{K_3}{K_2} = \frac{1}{Z_l}\right)$$

Locus of MHO relay is a circle passing through origin, with diameter = Z_l.



(iii) **Reactance Relay :**

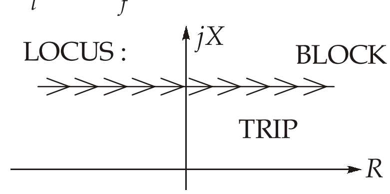
- Also known as Directional Restraint OC Relay.
- Assuming $K_2 = K_4 = 0$; K_3 = Negative in UTE.

$$T_d = K_1 I^2 - K_3\, VI\cos(\theta - \tau)$$

For tripping : $K_1 I^2 > K_3\, VI\cos(\theta - \tau)$

$$\frac{K_1}{K_3} > Z\cos(\theta - \tau)$$

$$X_l > X_f$$



Q.7 (c) Solution:

Given, $V_L = 220$ kV

$$V_m = \sqrt{2}V_{ph} = \sqrt{2} \times \frac{1}{\sqrt{3}} V_L = \sqrt{\frac{2}{3}} \times 220\,\text{kV}$$

Power factor, $\cos\phi = 0.5$ lag

$$\phi = \cos^{-1}(0.5)$$

$$\sin\phi = \sin(\cos^{-1}(0.5))$$

$$K_3 = \sin\phi = \frac{\sqrt{3}}{2}$$

K_1 = First pole to clear factor

K_2 = Armature reaction component = 0.9

(i)

$$(\text{RRRV})_{max} = K_1K_2K_3\ \omega_0 V_m$$

$$= K_1K_2K_3 \times 2\pi f_0\ V_m$$

- For grounded fault, $K_1 = 1$

$$\therefore \quad (\text{RRRV})_{max} = 1\times 0.9\times\frac{\sqrt{3}}{2}\times 2\pi\times 20\times 10^3\times\sqrt{\frac{2}{3}}\times 220\times 10^3$$

$$= 1.75938 \times 10^{10} \text{ V/sec}$$

$$= 17.594 \text{ kV/}\mu\text{sec}$$

- For ungrounded fault,

$$K_1 = 1.1$$

$$\therefore \quad (\text{RRRV})_{max} = 1.1\times 0.9\times\frac{\sqrt{3}}{2}\times 2\pi\times 20\times 10^3\times\sqrt{\frac{2}{3}}\times 220\times 10^3$$

(ii)

$$\text{Average RRRV} = \frac{(TRV)_{max}}{\text{Time at which it occurs}}$$

$$= \frac{1.632\times K_1K_2K_3V_L}{\pi\sqrt{LC}} = \frac{1.632\times K_1K_2K_3V_L}{\pi/2\pi f_0}$$

$$= 1.632 \times 2f_0K_1K_2K_3\ V_L$$

- For grounded fault, $K_1 = 1$

$$\text{Averge RRRV} = 1.632\times 2\times 20\times 10^3\times 1\times 0.9\times\frac{\sqrt{3}}{2}\times 220\times 10^3$$

$$= 1.1194 \times 10^{10} \text{ V/sec}$$

$$= 11.194 \text{ kV/}\mu\text{sec}$$

- Ungrounded fault, $K_1 = 1.1$

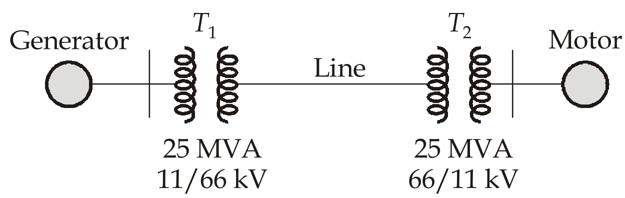
$$\text{Average RRRV} = 1.632\times 2\times 20\times 10^3\times 1.1\times 0.9\times\frac{\sqrt{3}}{2}\times 220\times 10^3$$

$$= 12.313 \text{ kV/}\mu\text{sec}$$

(iii) Time for maximum TRV is

$$t = \pi\sqrt{LC} = \frac{\pi}{2\pi f_0} = \frac{1}{2f_0} = \frac{1}{2\times 20\times 10^3} = 25\ \mu\text{sec}$$

Q.8 (a) Solution:



Given :Gen, Motor : 25 MVA, 11 kV,

$$X'' = 15\%$$

T/F : $X = 10\%$

Line : 25 MVA, 66 kV, $X = 10\%$

$$P_{3\phi} = 15\text{ MW, pf} = 0.8\text{ lead,}$$

$$P_{pv} = \frac{15\text{ MW}}{25\text{ MVA}} = 0.6\text{ p.u.}$$

Terminal voltage of motor = 10.6 kV

Let base voltage at generator side = 11 kV

Base MVA = 25 MVA

Prefault Conditions :

Terminal voltage of the motor in p.u. = $\frac{10.6}{11}$

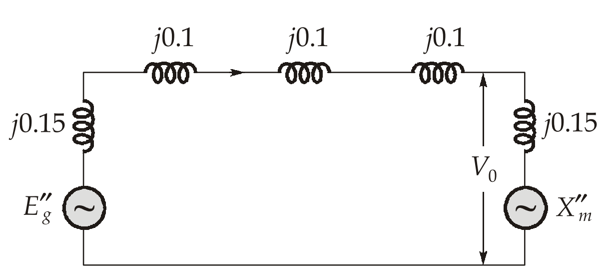
$$V_0 = 0.963\text{ p.u.}$$

$$(P_{\text{p.u.}}) = V_{\text{p.u.}} \times I_{\text{p.u.}} \times \cos\phi$$

$$\Rightarrow \quad \frac{15}{25} = 0.963 \times I_{pu} \times 0.8$$

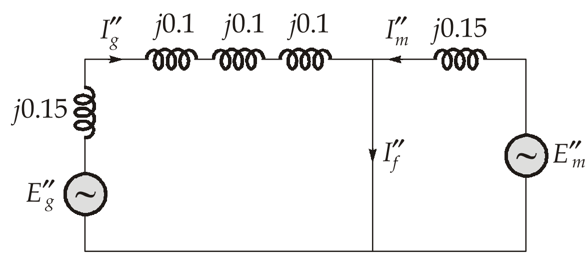
$$I_{pu} = 0.778\angle 36.86$$

Equivalent circuit :



$$E''_g = V_0 + I_{pu}(X_{T1} + V_{T2} + X_L + X''_g)$$
$$= 0.963\angle 0° + 0.778\angle 36.86\ (j0.1 + j0.1 + j0.1 + j0.15)$$
$$= 0.8034\angle 20.41°$$
$$E''_M = V_0 - I_{pu}(X''_M) = 0.963\angle 0° - 0.778\angle 36.86\ (j0.15)$$
$$= 1.03\angle -5.16°$$

At fault :



$$I''_0 = \frac{E''_g}{j0.15 + j0.1 + j0.1 + j0.1}$$
$$= \frac{0.8034\angle 20.41°}{0.45\angle 90°} = 1.785\angle -69.59°\ \text{p.u.}$$
$$I''_{g\ (\text{actual})} = 1.785\angle -69.58 \times \frac{25}{\sqrt{3} \times 11}\ \text{kA}$$
$$I''_{g\ (\text{actual})} = 2.34\angle -69.58\ \text{kA}$$
$$I''_m = \frac{E''_m}{j0.15} = \frac{1.03\angle -5.16}{j0.15} = 6.87\angle -85.6°\ \text{pu}$$
$$I''_{m\ (\text{actual})} = 6.87\angle -95.16 \times \frac{25}{\sqrt{3} \times 11}\ \text{kA}$$
$$I''_{m\ (\text{actual})} = 9.01\angle -95.16\ \text{kA}$$
$$I''_f = I''_g + I''_m$$
$$= 1.785\angle -69.59° + 6.87\angle -95.16°\ \text{pu} = 8.51\angle -89.96°$$
$$I''_{f\ (\text{actual})} = 8.51\angle -89.96 \times \frac{25}{\sqrt{3} \times 11}\ \text{kA}$$
$$I''_{f\ (\text{actual})} = 11.17\angle -89.96\ \text{kA}$$

Subtransient current in generator = 2.34 kA

Subtransient current in motor = 9.01 kA

Subtransient fault current = 11.17 kA

Q.8 (b) (i) Solution:

Given, modulation index, $\mu = 0.3$

Transmission efficiency (η) :

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{(0.3)^2}{2+(0.3)^2}$$

$$\therefore \quad \eta = 0.043 \text{ or } 4.3\%$$

% power saving for $m = 0.3$

Power saving in DSB-SC

$$P_t = P_c\left[1+\frac{m^2}{2}\right]$$

For $m = 0.3$

$$P_t = 1.045P_c$$

$$\% \text{ power saving} = \frac{P_c}{1.045P_c} = 95.69\%$$

Power saving in SSB

$$\% \text{ power saving} = \frac{P_c\left[1+\frac{m^2}{4}\right]}{P_c\left[1+\frac{m^2}{2}\right]} = \frac{\left[1+\left(\frac{m^2}{4}\right)\right]}{\left[1+\left(\frac{m^2}{2}\right)\right]}$$

$$\% \text{ power saving} = \frac{1.0225}{1.045} = 97.84\%$$

Q.8 (b) (ii) Solution:

Given, $m(t) = 2\cos 100t + 18\cos 2000\pi t$

$A = 10,\ \omega_c = 10^6,\ k_f = 1000\pi$ and $k_p = 1$

1. **Expression for FM and PM waves**

Expression for PM :

$$\phi_{PM}(t) = A\cos[\omega_c t + k_p\, m(t)]$$
$$= 10\cos[10^6 t + (2\cos 100t + 18\cos 2000\pi t)]$$

Expression for FM:

$$\phi_{FM}(t) = A\cos\left[\omega_c t + K_f \int m(t)dt\right]$$

$$= 10\cos\left[10^6 t + 1000\pi\int(2\cos 100t + 18\cos 2000\pi t)dt\right]$$

$$= 10\cos\left[10^6 t + \frac{2000\pi \sin 100t}{100} + \frac{18\times 10^3\pi}{2000\pi}\sin 2000\pi t\right]$$

$$\therefore \quad \phi_{FM}(t) = 10\cos[10^6 t + 20\pi \sin 100t + 9\sin 2000\pi t]$$

2. Bandwidth

For FM:

Considering the expression of $\phi_{FM}(t)$, we can obtain the maximum frequency deviation as follows:

$$\delta_{max} = K_f |m(t)|_{max}$$

$$= 500 \text{ Hz} \times 20 \text{ Volt}$$

$$= 10 \text{ kHz}$$

$$\therefore \quad BW = 2(\delta_{max} + \phi_m)$$

$$= 2(10 \text{ kHz} + 1 \text{ kHz}) = 22 \text{ kHz}$$

For PM:

$$m(t) = (2\cos 100t + 18\cos 2000\pi t)$$

$$\therefore \quad m'(t) = \frac{d}{dt}m(t) = 200\sin 100t - 36\times 10^3\pi\sin 2000\pi t$$

$$\therefore \quad [m'(t)]_{max} = 200 + 36\times 10^3\pi \approx 36\times 10^3\pi$$

$$\therefore \quad \Delta f = \frac{k_p \times [m'(t)]_{max}}{2\pi} = \frac{1\times 36\times 10^3\pi}{2\pi} = 18 \text{ kHz}$$

$$\therefore \quad \text{BW of PM} = 2(\Delta f + f_m) = 2(18+1)\text{kHz} = 38 \text{ kHz}$$

Q.8 (c) Solution:

Line length, $l = 150$ km

Frequency, $f = 60$ Hz

Total resistance of the line, $R = \frac{0.2}{0.5}\times 150 = 60\ \Omega$

Total series reactance of the line,

$$X = \frac{0.4}{0.5}\times 150 = 120\ \Omega$$

($\because$ resistance and reactance given are for per 0.5 km)

Series impedance of the line per phase,

$$Z = R + jX = (60 + j120)\Omega$$
$$= 134.16\angle 63.43°\ \Omega$$

Capacitance shunt admittance of the line per phase

$$Y = (j2.5 \times 10^{-6}) \times 150$$
$$\Rightarrow \quad Y = 3.75 \times 10^{-4}\angle 90°\ S$$

% receiving end load power, $P_R = 40$ MW

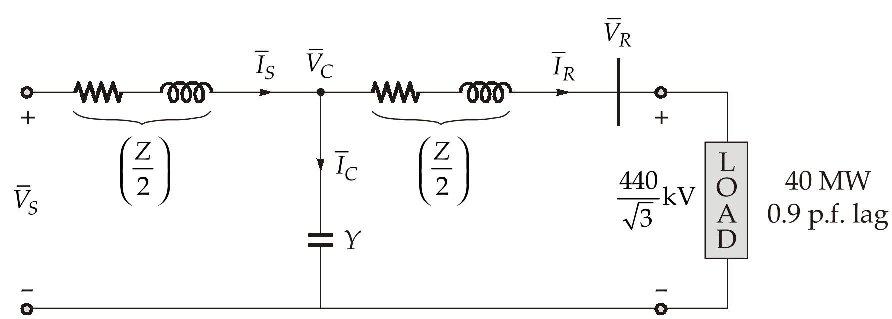
P.f. $\cos\phi_R = 0.9$ lagging

$$\Rightarrow \quad \phi_R = 25.84°$$

Receiving end current, $\bar{I}_R = \dfrac{P_R}{\sqrt{3}V_{RL}\cos\phi_R}\angle -\phi_R$

$$\bar{I}_R = \frac{40\times10^6}{\sqrt{3}\times220\times10^3\times0.9}\angle -25.84°\ A$$
$$= 116.62\angle -25.84°\ A$$

T-model



Node voltage, $\bar{V}_C = \bar{V}_R + \bar{I}_R\left(\dfrac{Z}{2}\right)$

$$= 127\times10^3\angle 0° + (116.62\angle -25.84°)\left(\frac{134.16}{2}\angle 63.43°\right)$$
$$\Rightarrow \quad \bar{V}_C = 133.28\angle 0.05°\ kV$$

Charging current, $\bar{I}_C = \bar{V}_C Y$

$$= (133.28\angle 0.05°)(3.75\times10^{-4}\angle 90°)$$
$$= 49.98\angle 92.05°\ A$$

(i) Sending end current, $\bar{I}_S = \bar{I}_C + \bar{I}_R$

$$= 49.98\angle 92.05° + 116.62\angle -25.84° = 103.17\angle -0.48°\ A$$

Sending and voltage per phase,

$$\bar{V}_S = \bar{V}_C + \bar{I}_S\left(\frac{Z}{2}\right)$$

$$= (133.28\angle 0.205°) + (0.103172 - 0.48)\times\left(\frac{134.16\angle 63.43°}{2}\right)$$

$$\bar{V}_S = 136.78\angle 9.58° \text{ kV}$$

Sending end line voltage,

$$V_{SL} = |\bar{V}_S|\sqrt{3}$$

$$= 136.78 \text{ kV}$$

$$= 236.91 \text{ kV}$$

(ii) Power loss in the line,

$$P_L = 3\left(|\bar{I}_S|^2 + |\bar{I}_R|^2\right)\frac{R}{2}$$

$$= 3\times\left((103.17)^2 + (116.62)^2\right)\times 30 = 2.182 \text{ MW}$$

$$\text{\% efficiency, } \%\eta = \frac{P_R}{P_R + P_L}\times 100 = \frac{40}{40 + 2.182}\times 100 = 94.83\%$$

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