MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Mechanical Engineering Test No : 6

Section A : Production Engineering and Material Science + Mechatronics and Robotics

1. (a)

Given: $\tau_s = 400\ \text{N/mm}^2$, $b = 2$ mm, $t = 0.2$ mm, $\mu = 0.5$

(i) Merchant's theory

$$2\phi + \beta - \alpha = 90^\circ$$

$$\beta = \tan^{-1}(\mu)$$

$$\beta = \tan^{-1}(0.5)$$

$$\beta = 26.565^\circ$$

and $\alpha = 10^\circ$ (Given)

$$\phi = \frac{(90+10-26.565)}{2} = 36.717^\circ$$

$$\phi = 36.717^\circ$$

We know that, $\tau_s = \dfrac{F_s \sin\phi}{bt}$

$$F_s = \frac{\tau_s \times bt}{\sin\phi} = \frac{400 \times 2 \times 0.2}{\sin(36.717)^\circ} = 267.62\ \text{N}$$

Resultant machining force, $R = \dfrac{F_s}{\cos(\phi + \beta - \alpha)}$

$$= \frac{267.62}{\cos(36.717 + 26.565 - 10)^\circ} = 447.617\ \text{N}$$

$\therefore$ $F_c = R\cos(\beta - \alpha) = 447.617 \times \cos(26.565 - 10)^\circ = 429.04\ \text{N}$

$\therefore \quad F_T = R\sin(\beta - \alpha) = 447.617 \times \sin(26.565 - 10)° = 127.617$ N

$\therefore \quad F_c = 429.04$ N and $F_T = 127.617$ N

(ii) Lee and Shaffer relation,

$$\phi + \beta - \alpha = 45°$$

$$\therefore \quad \phi = 45 + 10 - 26.565 = 28.435°$$

$$\phi = 28.435°$$

$$F_s = \frac{\tau_s \times bt}{\sin\phi} = \frac{400 \times 2 \times 0.2}{\sin(28.435)°} = 336.02 \text{ N}$$

and $\quad$ Resultant, $R = \dfrac{F_s}{\cos(\phi + \beta - \alpha)} = \dfrac{336.02}{\cos(28.435 + 26.565 - 10)°}$

$$= 475.2 \text{ N}$$

Cutting and thrust forces are:

$$F_c = R\cos(\beta - \alpha) = 475.2 \times \cos(26.565 - 10)°$$

$$= 455.5 \text{ N}$$

$$F_T = R\sin(\beta - \alpha) = 475.2 \times \sin(26.565 - 10)°$$

$$= 135.5 \text{ N}$$

$$F_c = 455.5 \text{ N and } F_T = 135.5 \text{ N}$$

It is found that forces obtained using Lee and Shaffer relation is greater than Merchant's theory.

1. (b)

Frame B in homogeneous transformation matrix can be written as

$${}^{A}T_B = \left[\begin{array}{c|c} {}^{A}R_B & {}^{A}P_{BORG} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$${}^{A}T_B = \begin{bmatrix} \cos 30° & -\sin 30° & 0 & 10 \\ \sin 30° & \cos 30° & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

As given $\quad {}^{B}P = \begin{bmatrix} 3.0 \\ 7.0 \\ 0.0 \end{bmatrix}$

General transformation vector can be written as

$$A_P = {}^A T_B {}^B P$$

$$A_P = \begin{bmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 \times 3 - 7 \times 0.5 + 10 \times 1 \\ 0.5 \times 3 + 7 \times 0.866 \times 7 + 5 \times 1 \\ 0 \times 3 + 0 \times 7 + 0 + 1 \times 1 \\ 0 \times 3 + 0 \times 7 + 0 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 9.098 \\ 12.562 \\ 0 \\ 1 \end{bmatrix}$$

So, $A_P = [9.098, 12.562, 0]$

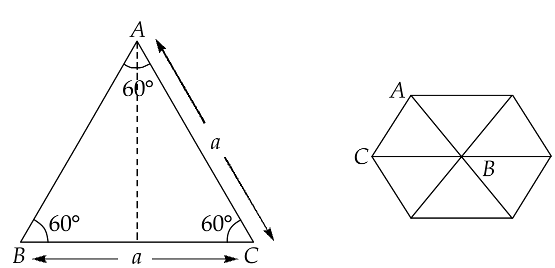
1. (c)

Titanium has HCP unit cell.

Ratio of lattice parameter $(c/a) = 1.58$

Radius of the Ti-atom, $R = 0.1445 \text{ nm} = 0.1445 \times 10^{-9}$ m

Volume of unit cell



$$\text{Area} = \frac{1}{2} \times a \times a \sin 60° = \frac{\sqrt{3}}{2} \cdot \frac{a^2}{2} = \frac{\sqrt{3} \cdot a^2}{4}$$

$$\text{Total base area} = 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3} a^2 \times 3}{2}$$

$$V_C = \frac{3\sqrt{3} a^2}{2} \times c$$

$\because$ $a = 2R$

$$V_C = \frac{3\sqrt{3}(2R)^2}{2} \times c = 6\sqrt{3} R^2 \times c$$

$$\text{Volume of cell} = 6\sqrt{3} R^2 \times 1.58a$$

$$= 6\sqrt{3} R^2 \times 1.58 \times 2R$$

$$\text{Volume of cell} = 32.8396R^3$$
$$= 32.8396 \times (0.1445 \times 10^{-9})^3$$
$$\text{Volume of unit cell} = 9.908 \times 10^{-29}\ \text{m}^3/\text{unit cell}$$

Theoretical density of Ti, $\rho = \dfrac{n \times A_{Ti}}{V_C \times N_A}$

$$= \frac{(6\ \text{atoms/unit cell}) \times (47.87\ \text{gm/mol})}{(9.908 \times 10^{-29} \times 10^6\ \text{cm}^3/\text{unit cell}) \times (6.022 \times 10^{23}\ \text{atoms/mol})}$$

$$\text{Density, } \rho = 4.8138\ \text{gram/cm}^3$$

$$\%\ \rho = \frac{4.51 - 4.8138}{4.8138}$$

$$\%\ \rho = -6.311\%$$

Actual density is 6.311% less than the theoretical density.

1. (d)

(i) Gibbs phase rule and level rule: Gibbs phase rule applies to non-reactive multi components heterogeneous systems in equilibrium and is given by the equality (F = C – P + 2) Where,

F is the number of degrees of freedom

C is the number of components

P is the number of phases in equilibrium

The lever rule is a tool used to determine weight percentages of each phase of a binary equilibrium phase diagram. In an alloy with two phases, α and β, which themselves contain two elements, A and B, the lever rule states that the weight percentages of the α phase is

$$X_\alpha = \left(\frac{C - b}{a - b}\right)$$

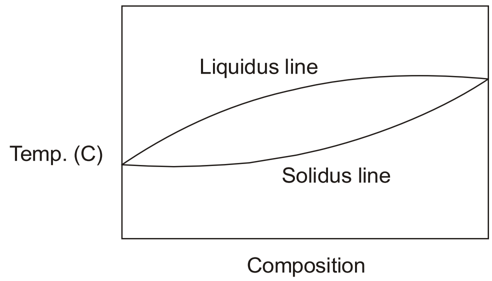
Where,

a is wt% of element B in α-phase

b is wt% of element B in β-phase

C is wt% of element B in entire alloy

(ii) Isomorphous System: Isomorphous system is one, where complete liquid and solid solubility occurs. The example is the Cu-Ni alloy where complete solubility occurs because Cu and Ni have the same crystal structure, electronegativity and valence.



Isomorphous system phase diagram

(iii) Peritection reaction in steel: In Peridotic reaction, a liquid and solid phase of fixed proportions react at a fixed temperature to yield a single solid phase.

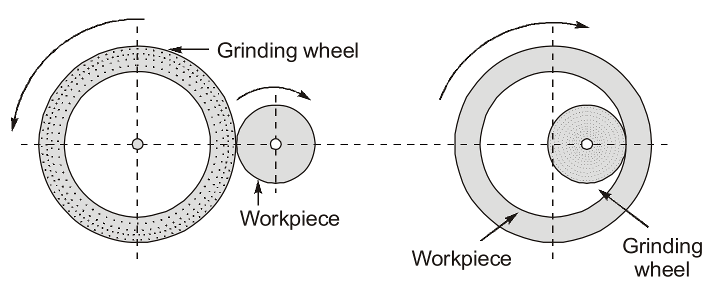
$$\underset{\text{(Molten liquid)}}{L} + \underset{\text{(Delta ferrite)}}{\delta} \underset{(1495°C)}{\overset{(0.18\%\ \text{Carbon})}{\rightleftharpoons}} \underset{\text{(Gamma–Austenite)}}{\gamma}$$

1. (e)

Grinding operations or machines can be classified as:

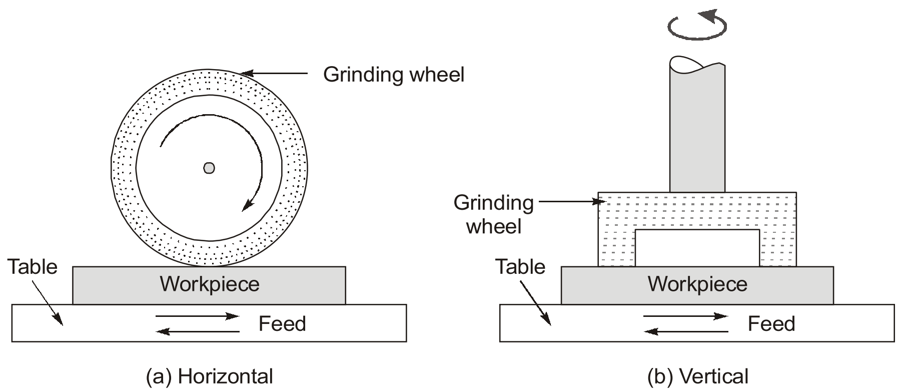
(a) cylindrical grinding machines
(b) surface grinding machines
(c) centreless grinding machines

Cylindrical grinding is used to produce cylindrical surface as shown in figure. Both internal and external surfaces can be ground. The workpiece is held between the centre on the machine similar to lathe machine and it is rotated at much lower speed and opposite direction to that of grinding wheel.



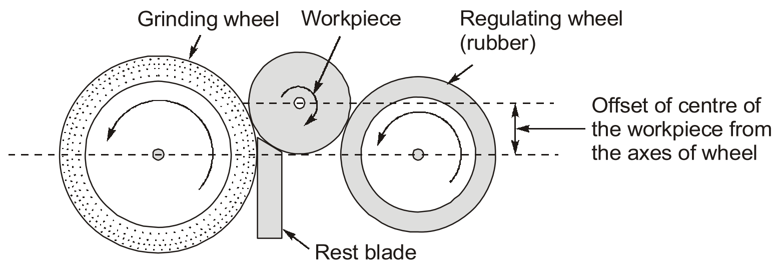
Cylindrical grinding

Surface grinding is used to produce flat surfaces as shown in figure. The surface grinders can be three types based on the position of the axis of spindle which can be (i) horizontal, (ii) vertical and (iii) universal. The workpiece is clamped on the table and grinding wheel is made to rotate for carrying out grinding. The table can be moved in transverse and lateral directions for finishing the workpiece lengthwise and widthwise. The table is given reciprocating motion from the saddle.



Surface grinding

Centreless grinding is used for axi-symmetric shaped as jobs shown in figure. It is possible to grind cylindrical jobs without fixing them either in centre or in the chuck of the machine. As job is not fixed on the machine, hence it is not required to be provided with any rotary motion by the machine. The job is supported with a work rest blade in between a large grinding wheel and a small regulating wheel made of rubber. The regulating wheel holds the job against grinding wheel and the job is rotated at the same surface speed by the regulating wheel. The centre of the job is kept slightly above the centres of the wheels.



Centreless grinding

2. (a)

Side riser:

$$\text{For side riser, volume, } V = \pi r^2 h$$

$$\text{Surface area, } A = 2\pi r^2 + 2\pi rh$$

$$A = 2\pi r^2 + 2\pi r \times \left(\frac{V}{\pi r^2}\right)$$

$$A = 2\pi r^2 + \frac{2V}{r}$$

$$\frac{dA}{dr} = 4\pi r - \frac{2V}{r^2}$$

For minimum surface area of riser,

$$\frac{dA}{dr} = 0$$

$$4\pi r - \frac{2V}{r^2} = 0$$

$$\frac{4\pi r^3}{2} = V$$

$$\text{Volume, } V = 2\pi r^3 = \pi r^2 h$$

$$\text{On comparing, } h = 2r$$

$$\text{Volume of casting, } V_c = 30 \text{ cm} \times 18 \text{ cm} \times 12 \text{ cm} = 6480 \text{ cm}^3$$

$$\text{Surface area of casting, } A_c = 2[30 \times 18 + 18 \times 12 + 12 \times 30] = 2232 \text{ cm}^2$$

$$\text{Volume of riser, } V_r = 3 \times (\text{\% of shrinkage volume of casting})$$

$$= 3 \times [0.07 \times 30 \times 18 \times 12]$$

$$V_r = 1360.8 \text{ cm}^3$$

For side riser, $$\left(\frac{V}{A}\right)_{\text{riser}} = \frac{\pi r^2 h}{2\pi r^2 + 2\pi rh} = \frac{2\pi r^3}{2\pi r^2 + 4\pi r^2} = \frac{2\pi r^3}{6\pi r^2} = \frac{r}{3}$$

Now, we know that Riser volume= $\pi r^2 h$

$$1360.8 = \pi r^2 \times 2r$$

$$r^3 = 216.578$$

$$r = 6.00 \text{ cm}$$

$$\left(\frac{V}{A}\right)_{\text{riser}} = \frac{r}{3} = \frac{6}{3} = 2$$

$$\left(\frac{V}{A}\right)_{\text{Casting}} = \frac{6480}{2232} = 2.903$$

Now, as we can see that,

$$\left(\frac{V}{A}\right)_{\text{Casting}} > \left(\frac{V}{A}\right)_{\text{riser}}$$

So riser will solidify prior to casting, so these dimensions are not correct since at least riser should solidify along with the casting.

So, $$\left(\frac{V}{A}\right)_{riser} \geq \left(\frac{V}{A}\right)_{casting}$$

$$\frac{r}{3} \geq 2.903$$

$$r \geq 8.709 \text{ cm}$$

Radius of side riser, $r \geq 8.71$ cm

Height of side riser, $h = 2r = 2 \times 8.71 = 17.42$ cm

Side riser dimensions:

Radius of side riser, $r = 8.71$ cm

Height of side riser, $h = 17.42$ cm

Top riser:

For top riser, volume, $V = \pi r^2 h$

Surface area, $A = \pi r^2 + 2\pi rh = \pi r^2 + 2\pi r \times \frac{V}{\pi r^2}$

$$A = \pi r^2 + \frac{2V}{r}$$

$$\frac{dA}{dr} = 2\pi r - \frac{2V}{r^2}$$

For minimum surface area of riser, $\frac{dA}{dr} = 0$

$$2\pi r - \frac{2V}{r^2} = 0$$

Volume, $V = \pi r^3 = \pi r^2 h$

On comparing, height of riser, $h = r$

Volume of riser, $V_r = 1360.8 \text{ cm}^3$

$$\pi r^3 = 1360.8$$

For top riser,

Radius of riser, $r = 7.566$ cm

Height of riser, $h = r = 7.566$ cm

Now, $$\left(\frac{V}{A}\right)_{riser} = \frac{\pi r^2 h}{\pi r^2 + 2\pi rh} = \frac{\pi r^3}{\pi r^2 + 2\pi r^2} \qquad \{\text{As, } r = h\}$$

$$\left(\frac{V}{A}\right)_{riser} = \frac{r}{3}$$

$$\left(\frac{V}{A}\right)_{riser} = \frac{7.566}{3} = 2.522$$

$$\text{Volume of casting, } V_c = 30 \times 18 \times 12 = 6480 \text{ cm}^3$$

$$\text{Surface area of casting, } A_c = 2[30 \times 18 + 18 \times 12 + 12 \times 30] - \pi r^2$$

$$A_c = 2232 - \pi r^2$$

$\left(\frac{V}{A}\right)_{casting}$ for top riser will be more than $\left(\frac{V}{A}\right)_{casting}$ of side riser.

So, $$\left(\frac{V}{A}\right)_{casting} > \left(\frac{V}{A}\right)_{riser}$$

So riser will solidify prior to casting, so these dimensions are not correct since atleast riser should solidify along with the casting.

So, $$\left(\frac{V}{A}\right)_{riser} \geq \left(\frac{V}{A}\right)_{casting}$$

$$\frac{r}{3} \geq \left(\frac{6480}{2232 - \pi r^2}\right)$$

$$2232r - \pi r^3 \geq 19440$$

$$\pi r^3 - 2232r + 19440 \leq 0$$

So,for solution of equation, $\pi r^3 - 2232r + 19440 = 0$

Now, $r = -30.25;\ 20.0435,\ 10.206$

Hence, the most optimum result is, $r = 10.206$ cm

Hence, radius of top riser, $r = 10.206$ cm

height of top riser, $h = r = 10.206$ cm

2. (b)

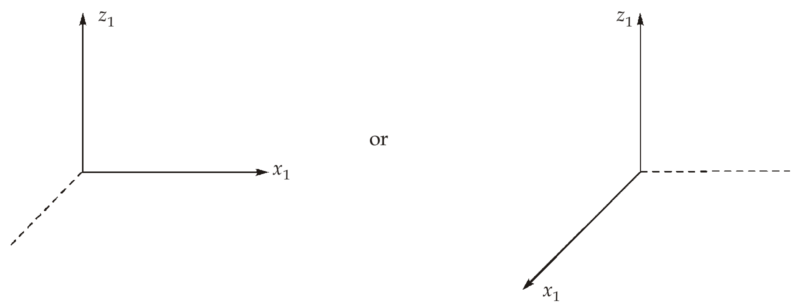
In cylindrical manipular, it has 3 joints, one revolute and two prismatic.

Algorithm for assigning frames and axis.

Step 0: The three joints are numbered as 1 2 and 3 starting with immobile base as 0.

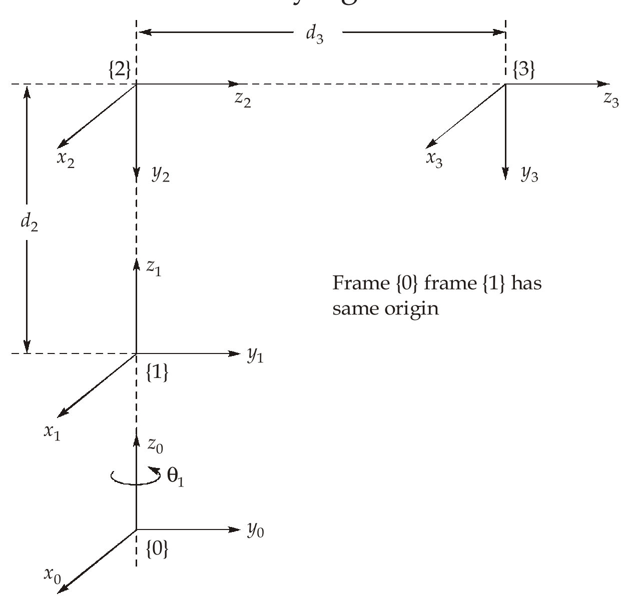
Step 1: We know frame {0} location is arbitrary, its choice is made based on simplification of the model and some convenient reference in workspace. Its axis is made in such a way that it should not add unnecessary variables (θ_i, d_i). Assign all joint z axis. Start with frame {1} to last frame. Frame {0} can be made in last and it will be same as joint 1 axis.

Step 2: The x_1- axis is set in the direction of perpendicular to plane containing z_1 and z_o axis. As z_o and z_1 coincide it can be placed in 2 ways:



To remove this ambiguity, start with x_2. Similarly x_2 will be perpendicular to plane assigned. x_1 should be parallel to x_2 because it should not add extra joint angle (θ_i) (angle between x_{i-1} and x_i axis). x_3 can be made similar to x_2 because frame {3} is at the face plate for attaching wrist i.e. can be chosen arbitrary. Frame {0} and frame {n} containing x_2 and z_2. So, x_2 can be asigned. (last frame) is assigned in same way.

Step 3: The y-axis for all frame is fixed by right hand rule.



Joint-Link parameter table:

Link(i)	a_i	α_i	d_i	θ_i	q_i	$C\theta_i$	$S\theta_i$	$C\alpha_i$	$S\alpha_i$
1	0	0	0	θ_1	θ_1	C_1	S_1	1	0
2	0	–90	d_2	0	d_2	1	0	0	–1
3	0	0	d_3	0	d_3	1	0	1	0

Transformation matrices:
We know that

$$^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\alpha_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_3 = {}^0T_1 {}^1T_2 {}^2T_3 = \begin{bmatrix} C_1 & 0 & -S_1 & -d_3S_1 \\ S_1 & 0 & C_1 & d_3C_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. (c)

1. **Process Annealing :** Process annealing or sub-critical annealing is done on cold worked low carbon steel sheet, wire or tubing to relieve internal stresses and to soften the material. The process is as follows :
   (i) The steel is heated to 550-650°C, which is just below the lower critical temperature on iron-carbon diagram for steel.
   (ii) Stresses throughout the metal are relieved and recrystallization causes new grains to form and grow.

Heating period is followed by slow cooling. Prolonged annealing causes the cementite in the pearlite to "ball up" or spheroidise. Ferrite grain growth also occurs. Obviously, annealing time and temperature control is very essential for proper process annealing.

2. **Full Annealing :** This operation removes all structural imperfections by complete recrystallization. This operation is often utilized in low and medium carbon steels that will be machined or will experience extensive plastic deformation during a forming operation.

   This operation consists of :

   (i) Heating the hypoeutectoid steel to about 50-70°C above the upper critical temperature (for hypoeutectoid steels) and by the same temperature above the lower critical temperature for hypereutectoid steels until equilibrium is achieved. This ensures that the metal is heated thoroughly and phase transformation has taken place throughout the volume.

   (ii) The alloy is then furnace cooled; i.e., the heat-treating furnace is turned off and both furnace and steel cool to room temperature at the same rate, which takes several hours. The microstructural product of full anneal is coarse pearlite (in addition to any proeutectoid phase) that is relatively soft and ductile. The full-annealing cooling procedure is time consuming; however, a microstructure having small grains and a uniform grain structure results.

3. **Normalizing :** For this process heating and soaking is same as in the full annealing but part is allowed to cool in air so that cooling rate is much faster. An annealing heat treatment called normalizing is used to refine the grains (i.e., to decrease the average grain size) and produce a more uniform and desirable size distribution. Fine grained pearlite steels are tougher than coarse-grained ones. The fine grain structure increases the yield and ultimate strengths, hardness and impact strength. Normalizing is accomplished by heating at approximately 55 to 85°C above the upper critical temperature, which is, of course, dependent on composition. Normalizing often applied to castings and forgings is stress relieving process. To some extent, it increases strength of medium carbon steel. It improves machinability, when applied to low carbon steel. Alloy steels in which the austenite a procedure termed austenizing is very stable can be normalized to produce hard martensite structure. Cooling in air produces high rate of cooling which can decompose the austenitic structures in such steels and martensite is produced. This increases the hardness to great extent.

4. **Spherodizing :** Tool steels for cutting tools, measuring instrument and cold forming dies have a high concentration of carbon (0.7-2%), which provides high hardness and makes these steels poorly machinable. The hardness of tool steels can be reduced by annealing. Besides, spheroidizing annealing of hypereutectoid steels prepare their structure to hardening. This type of heat treatment produces carbide in the form of round or globular (spheroids) instead of plates as in pearlite.

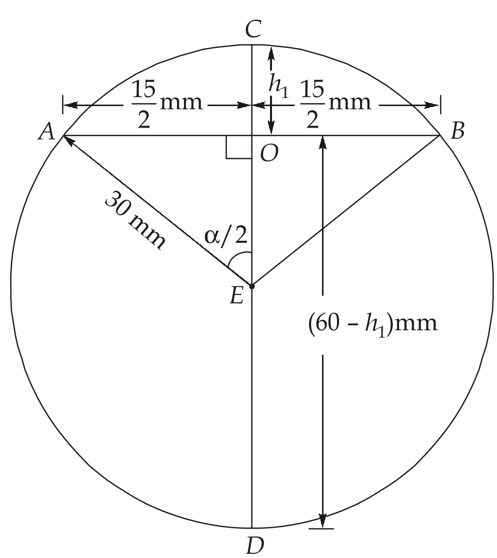
   This structure gives: good machinability, high ductility and improvement in formability. The hardness of steel is the lowest when the steel structure consists of granular pearlite with inclusions of rounded-off (spherical) cementite grains. From this the name 'spheroidizing' annealing transpires. The spheroidizing heat treatment consists of heating the alloy at a temperature just below the eutectoid phase diagram at about 700°C in the α + Fe_3C region of the phase diagram. If the precursor microstructure contains pearlite, spheroidizing times will ordinarily range between 15 to 25 hours. During this heat treatment there is coalescence of the Fe_3C to form the spheroid particles.

3. (a)

Reinforcement is a part of circle of radius 30 mm.

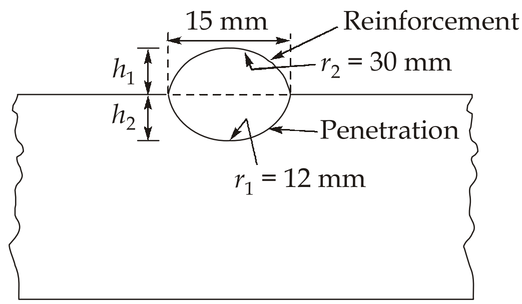
By circle property, perpendicular drawn from the centre on the chord bisect the chord.

By circle property,



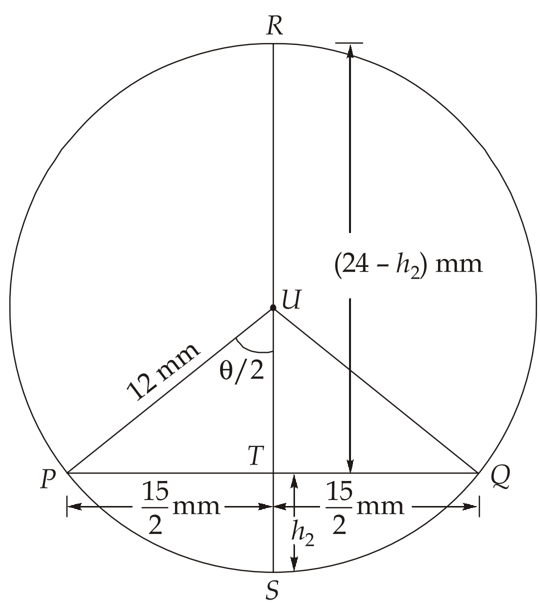
OE can be directly solved,

$$OE = \sqrt{30^2 - 7.5^2} = 29.0474 \text{ mm}$$



Penetration is also a part of the circle of radius 12 mm.

From circle property,



$$UT = \sqrt{12^2 - 7.5^2} = 9.3675 \text{ mm}$$

We know that,

$$\% \text{ dilution} = \left(\frac{A_p}{A_p + A_R}\right) \times 100$$

where, A_p = Penetration area, A_R = Reinforcement area

Now for penetration area, which is part of 12 mm radius circle.

$$\sin\frac{\theta}{2} = \left(\frac{\frac{15}{2}}{12}\right) = \frac{15}{24} = \frac{5}{8}$$

$$\frac{\theta}{2} = 38.682°$$

$$\theta = 77.364°$$

$$\text{Area of sector of circle, } A_1 = \frac{\theta}{360}\times\pi r_1^2 = \frac{77.364}{360}\times\pi\times12^2$$

$$A_1 = 97.218 \text{ mm}^2$$

$$\text{Area of } \Delta PQU, A_2 = \frac{1}{2}\times PQ\times TU$$

$$= \frac{1}{2}\times15\times(12-2.633) = 70.2525 \text{ mm}^2$$

Now, $$\text{Area of penetration, } A_p = A_1 - A_2 = 97.218 - 70.2525$$

$$A_p = 26.9655 \text{ mm}^2$$

Reinforcement is part of circle of radius 30 mm.

In ΔAOE, $$\sin\frac{\alpha}{2} = \frac{OA}{AE} = \frac{\left(\frac{15}{2}\right)}{30} = \frac{1}{4}$$

$$\frac{\alpha}{2} = 14.4775°$$

$$\alpha = 28.955°$$

$$\text{Area of sector of circle, } A_3 = \frac{\alpha}{360}\times\pi r_2^2 = \frac{28.955}{360}\times\pi\times30^2$$

$$A_3 = 227.412 \text{ mm}^2$$

$$\text{Area of } \Delta ABE, A_4 = \frac{1}{2}\times AB\times OE = \frac{1}{2}\times15\times(29.0474) = 217.8525 \text{ mm}^2$$

$$\text{Area of reinforcement, } A_R = A_3 - A_4 = 227.412 - 217.8525$$

$$A_R = 9.5595 \text{ mm}^2$$

$$\%\text{Dilution} = \left(\frac{A_P}{A_p + A_R}\right)\times100 = \left(\frac{26.9655}{26.9655+9.5595}\right)\times100$$

$$\%\text{Dilution} = 73.8275\%$$

3. (b)

The unintentional deterioration of a material by an electrochemical process is known as corrosion. Corrosion is the reverse process of extracting metal from ores, because pure metals tend to revert to their original state of oxides, sulphite and so on. Except noble metals, gold and platinum, all other metal exist in nature in the form of oxides, sulphides, carbonates and silicates. Lots of energy is consumed in extracting pure metal from its ores. So pure metal are of high-energy state, which is unstable, therefore when these metals comes in contact with environment, gases and liquids, they tend to revert to their original form or they want to be oxidized.

When corrosion takes place in the presence of an electrolyte, an aqueous solution of acid, salt or alkali, then it is known as wet corrosion. If corrosion takes place without the presence of an electrolyte, as oxidation of metal in furnace, it is known as 'dry corrosion'.

Corrosion is a localized surface phenomenon, it reduces the mechanical strength of the metal sometimes, because of severe pitting on the metal, holes and cracks occurs in the metal. Generally, the effects of corrosion are more pronounced on metals. In case of ceramics, there is degradation of properties at high temperature or in extreme environments, this is also known as corrosion. For polymers, mechanisms and consequences differ from those of metals and ceramics, and the term, 'degradation' is most commonly used.

Electrochemical reaction: Corrosion of metal is an example of electrochemical reaction. Considering the corrosion of steel in a solution of hydrochloride acid (HCl). Ions of hydrogen (H^+) and chlorine (Cl^-) are dissolved in liquid, and the solution of dissolved ions is known as electrolyte. For a metal to dissolve in a liquid, it must transform from a solid atom to an ion as follows

$$Fe \rightarrow Fe^{2+} + 2e^- \qquad \text{... (At anode)}$$

Because of liberation of electrons the metal becomes charged (or polarized). If a sample of iron is placed in a solution of HCl, a corrosion reaction occurs. Iron goes into the solution and chloride ion does not really go into the chemical reaction.

$$Fe + 2H^+ \rightarrow Fe^{2+} + H_2 \qquad \text{... (At anode)}$$

Above equation shows that when elemental iron reacts with the hydrogen ions in the acid it produces dissolved iron (as ions) and hydrogen gas.

Partial cathodic reaction,

$$2H^+ + 2e^- \rightarrow H_2$$

When iron is converted into its elemental form Fe^{2+}, it is said to be oxidized. Hydrogen ion after accepting the electron become elemental hydrogen or convert in a most stable state as H_2 molecule. In this reaction, hydrogen is said to be reduced. Chemical reaction between, ions and electrons is known as 'electro chemistry'.

Corrosion process requires simultaneous occurrence of both oxidation and reduction process. Corrosion process will stop if any of these two processes is stopped. If either half of the process is accelerated, corrosion rate also increases.

For a metallic corrosion to occur, an environment or a system is required, which accepts the electrons liberated during metal oxidation. When a metal corrodes, the surface of the metal contains many microscopic anode and cathode sites. Defects, impurities, inclusion and grain boundaries act as anodic or cathodic sites.

Electrochemical process: Corrosion is an electrochemical process, in which a metal is corroded and metal ions are removed at anode from the anode,

$$M \rightarrow M^{n+} + ne^- \qquad \text{[Oxidation reaction]}$$

A metal is oxidized to metal in M^{n+} and electron ne^-, where n is the number of valency electrons. This process of oxidation occurs at anode.

Examples:

$$Fe \rightarrow Fe^{2+} + 2e^-$$
$$Al \rightarrow Al^{3+} + 3e^-$$
$$Cu \rightarrow Cu^{2+} + 2e^-$$

(i) Acid solution contains high concentration of hydrogen ions H^+.

$$2H^+ + 2e^- \rightarrow H_2 \qquad \text{[Reduction process]}$$

(ii) If acid solution contain dissolved oxygen and hydrogen ions H^+, then the reduction reaction will be,

$$O_2 + 4H^+ + 4e^- \rightarrow 2H_2O \qquad \text{[Reduction process]}$$

(iii) If water contains dissolved oxygen,

$$2H_2O + O_2 + 4e^- \rightarrow 4(OH)^- \qquad \text{[Reduction process]}$$

(iv) $$M^{n+} + ne^- \rightarrow M \qquad \text{[Total reduction]}$$

Site at which reduction takes place is called a 'cathode' overall electrochemical reaction must consist of an oxidation and a reduction process. Individual oxidation reaction or reduction reaction called 'half reaction'.

3. (c)

1. System type

$$G(s) = \frac{K_i}{s},\ G_p(s) = \frac{1}{s(s+4)}$$

Therefore $$\text{OLTF} = G(s)\,G_p(s) = \frac{K_i}{s^2(s+4)}$$

As the order of s in denominator is two, this a type "2" system.

2. Steady state errors.

For step input $$R(s) = \frac{1}{s}$$

Therefore, $$e_{ss} = \lim_{s\to 0}\left[\frac{sR(s)}{1+G(s)}\right]$$

$$= \lim_{s \to 0} \left[s \frac{1}{1+[K_i / s^2(s+4)]} \frac{1}{s} \right]$$

$$= \lim_{s \to 0} \left[\frac{1}{1+\frac{K_i}{s^2(s+4)}} \right] = \frac{1}{\infty} = 0$$

For a ramp input $R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s \to 0} \left[s \frac{1}{1+\frac{K_i}{s^2(s+4)}} \frac{1}{s^2} \right] = \lim_{s \to 0} \left[\frac{1}{s+\frac{K_i}{s(s+4)}} \right] = \frac{1}{\infty} = 0$$

3. Stability analysis with a proportional controller: for a proportional controller, the controller transfer function is K_p. Therefore,

$$G(s) = K_p \frac{1}{s(s+4)}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K_p / s(s+4)}{1+\frac{K_p}{s(s+4)}} = \frac{K_p}{s(s+4)+K_p}$$

$$\frac{C(s)}{R(s)} = \frac{K_p}{s^2+4s+K_p}$$

Arranging the denominator of the above equation in Routh's array, we have

s^2	1	K_p
s^1	4	0
s^0	K_p	

For stability all elements in the first column of Routh's array should be positive. Therefore the system is stable for $K_p > 0$

4. Stability analysis with an integral controller

$$\frac{C(s)}{R(s)} = \frac{K_i / s^2(s+4)}{1+[K_i / s^2(s+4)]} = \frac{K_i}{s^3+4s^2+K_i}$$

Characteristic equation is $s^3 + 4s^2 + K_i = 0$, computing Routh's array, We get

s^3	1	0
s^2	4	K_i
s^1	$\frac{4\times 0 - K_i}{4} = \frac{-K_i}{4}$	0
s^0	$\frac{-\frac{K_i}{4}\times K_i - 0}{\frac{-K_i}{4}} = K_i$	0

The system is unstable for all the values of K_i as there are sign changes in the first column of Routh's table.

4. (a)

As per given information,

Cross section area of composite, $A_c = 320\ \text{mm}^2$

Longitudinal load, $F_c = 44500$ N

$V_f = 0.3,\ V_m = 0.7,\ E_f = 131$ GPa, $E_m = 2.4$ GPa

1. Fibre matrix load ratio $\left(\frac{F_f}{F_m}\right)$

$$\frac{F_f}{F_m} = \frac{E_f V_f}{E_m V_m} = \frac{131\times 0.30}{2.4\times 0.70} = 23.4$$

$$\text{Load ratio} = \frac{F_f}{F_m} = 23.4$$

2. As

$$\frac{F_f}{F_m} = 23.4$$

$$F_f = 23.4\ F_m$$

$$\text{Total load} = F_f + F_m$$

$$F_f + F_m = 44500\ \text{N}$$

$$23.4\ F_m + F_m = 44500\ \text{N}$$

$$24.4 F_m = 44500\ \text{N}$$

$$F_m = 1823.77\ \text{N}$$

$$F_f = 44500 - 1823.77 = 42676.23\ \text{N}$$

3. Cross sectional area of fibre,

$$A_f = V_f A_c = 0.30 \times 320 = 96\ \text{mm}^2$$

Cross sectional area of matrix,

$$A_m = V_m A_c = 0.70 \times 320 = 224\ \text{mm}^2$$

Stress induced in fibre phase,

$$\sigma_f = \frac{F_f}{A_f} = \frac{42676.23\text{ N}}{96} = 444.544\text{ MPa}$$

Stress induced in matrix phase,

$$\sigma_m = \frac{F_m}{A_m} = \frac{1823.77}{224} = 8.1418\text{ MPa}$$

4. Strain in the composite is the same as the strain on each of the matrix and fibre phases:

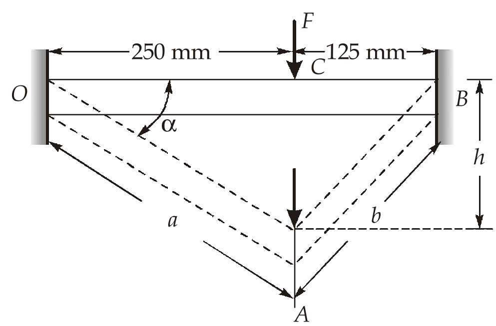
$$\varepsilon_m = \frac{\sigma_m}{E_m} = \frac{8.1418}{2.4 \times 10^3} = 3.3924 \times 10^{-3}$$

$$\varepsilon_f = \frac{\sigma_f}{\varepsilon_f} = \frac{444.544}{131 \times 10^3} = 3.393 \times 10^{-3}$$

$\Rightarrow$ $$\varepsilon_{composite} = \varepsilon_m = \varepsilon_f = 3.393 \times 10^{-3}$$

4. (b)

Because the cross-section is very small compared to the length of the part, this operation is equivalent to stretching a piece of metal from 375 mm to a length of $(a + b)$.



For $\alpha = 20°$, $$\cos\alpha = \frac{250}{a}$$

$$a = \frac{250}{\cos 20°} = 266.044\text{ mm}$$

$$\sin\alpha = \frac{h}{266.044}$$

$$h = 266.044\sin 20° = 90.9924\text{ mm}$$

Now, In ΔABC, $$b^2 = h^2 + 125^2$$

$$b^2 = (90.9924)^2 + 125^2$$

$$b = 154.611\text{ mm}$$

Now, Total final length $= a + b$

$= 266.044 + 154.611 = 420.655$ mm

$$\text{True strain, } \in = \ln\left(\frac{L_f}{L_0}\right) = \ln\left(\frac{420.655}{375}\right) = 0.11488$$

Work done per unit volume is given by,

$$u = \int_0^{0.11488} \sigma d\in = \int_0^{0.11488} (700\times10^6)\in^{0.3} d\in$$

$$= 700\times10^6\left[\frac{\in^{1.3}}{1.3}\right]_0^{0.11488}$$

$$= 700\times10^6\left[\frac{(0.11488)^{1.3}}{1.3}\right] = 32.32\times10^6 \text{ N/m}^2$$

$$u = 32.32 \text{ MN-m/m}^3 \text{ or } 32.32 \text{ MJ per unit volume}$$

Volume of the workpiece is,

$$V = 0.375 \times 5 \times 10^{-4} = 1.875 \times 10^{-4} \text{ m}^3$$

$$\text{Total work done, } W = (u) \times (V)$$

$$= 32.32 \times 10^6 \times 1.875 \times 10^{-4} \text{ Nm} = 60.60 \times 10^2 \text{ Nm}$$

$$\text{Total work done, } W = 6060 \text{ Nm}$$

(b) The necking limit for uniaxial tension. For maximum strain $\in = n$.

$$\text{We know that, True strain, } \in = \ln\left(\frac{L_f}{L_0}\right)$$

$$\frac{L_f}{L_0} = e^{\in} \qquad \text{(For maximum limit, } \in = n)$$

$$L_f = L_0 \times (e)^n$$

$$L_f = 0.375 \times (e)^{0.3} = 0.5062 \text{ m} = 506.2 \text{ mm}$$

For maximum strain condition,

$$(a + b) = L_f = 506.2 \text{ mm} \qquad ...(1)$$

For both ΔAOC and ΔABC,

Height 'h' will be same for both triangles.

$$h^2 = a^2 - 250^2 \qquad ...(2)$$

$$h^2 = b^2 - 125^2 \qquad ...(3)$$

Now, from equation (2) and (3),

$$a^2 - 250^2 = b^2 - 125^2$$
$$a^2 - b^2 = 250^2 - 125^2$$
$$a^2 - b^2 = 46875$$
$$(a + b)(a - b) = 46875$$
$$(a - b) = \frac{46875}{506.2} = 92.602$$

$\therefore$ $$a - b = 92.602 \quad \text{...(4)}$$

Adding equation (1) and (4), $$2a = 598.802$$
$$a = 299.401 \text{ mm}$$

Putting value of 'a' in equation (1),

$$b = 506.2 - 299.401 = 206.799 \text{ mm}$$

Now, $$\cos\alpha_{max} = \frac{250}{a_{max}} = \frac{250}{299.401} = 0.835$$
$$\alpha_{max} = 33.384°$$

4. (c)

1. Given : $h_o = 10$ mm, $R = 150$ mm, $K = 350$ MPa, $w = 200$ mm, $h_f = 6$ mm, $\mu = 0.1$, $n = 0.26$

Flow stress, $$\sigma_f = K\varepsilon^n$$

where, $$\varepsilon = \text{True strain} = \ln\left(\frac{h_o}{h_f}\right) = \ln\left(\frac{10}{6}\right) = 0.5108$$

$\therefore$ $$\sigma_f = 350 \times (0.5108)^{0.26} = 293.9131 \text{ MPa}$$

Roll pressure at entry and exit,

$$\sigma'_f = \frac{2}{\sqrt{3}}\sigma_f = \frac{2}{\sqrt{3}} \times 293.9131$$
$$\sigma'_f = 339.3816 \text{ MPa}$$
$$\Delta h = 2R(1 - \cos\alpha)$$
$$(10 - 6) = 2 \times 150(1 - \cos\alpha)$$

On solving, we get $$\alpha = 9.366° = \left(9.366 \times \frac{\pi}{180}\right) \text{ radian}$$

$$H = 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\,\phi\right)$$

At entry, $\phi = \alpha$, hence $H = H_o$

$$\therefore \quad H_o = 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\alpha\right)$$

$$= 2\sqrt{\frac{150}{6}}\tan^{-1}\left(\sqrt{\frac{150}{6}}\times 9.366\times\frac{\pi}{180}\right)$$

H_o = 6.8522 (use calculator in radian mode)

$$H_n = \frac{1}{2}\left(H_o - \frac{1}{\mu}\ln\frac{h_o}{h_f}\right) = \frac{1}{2}\left(6.8522 - \frac{1}{0.1}\ln\left(\frac{10}{6}\right)\right)$$

$H_n = 0.872$

$$\phi_n = \sqrt{\frac{h_f}{R}}\tan\left(\frac{H_n}{2}\sqrt{\frac{h_f}{R}}\right) = \sqrt{\frac{6}{150}}\tan\left(\frac{0.872}{2}\sqrt{\frac{6}{150}}\right)$$

ϕ_n = 0.017484 radian

$$\therefore \quad h_n = h_f + 2R(1 - \cos\phi_n)$$

$$= 6 + 2 \times 150(1 - \cos(0.017484)) = 6.0458 \text{ mm}$$

$$P - \sigma_x = \sigma'_f$$

At entry and exit if no back tension, no front tension is there then $\sigma_x = 0$.

$$P = \sigma'_f$$

P = 339.816 MPa

Now, pressure at neutral point,

$$P_n = \sigma'_f \cdot \frac{h_n}{h_f}e^{\mu H_n}$$

$$= 339.3816 \times \frac{6.0458}{6}e^{0.1\times 0.872}$$

P_n = 373.131 MPa

2. Slipping of the rolls means that the neutral point has moved all the way to the exit in the roll gap. Thus, the whole contact area becomes the entry zone.

So, when $\phi = 0$, $H = 0$; Pressure at exit,

$$P_{\phi=0} = \left(\bar{\sigma}'_f - P_b\right)\frac{h_f}{h_o}e^{\mu H_o}$$

However, the pressure at the exit is equal to $\bar{\sigma}'_f$.

$$\therefore \qquad \left(\bar{\sigma}'_f - P_b\right)\frac{h_f}{h_o}e^{\mu H_o} = \bar{\sigma}'_f$$

Rearranging we get,

$$P_b = \sigma'_f\left(1-\left(\frac{h_o}{h_f}\right)e^{-\mu H_o}\right)$$

$$P_b = 339.3816\left(1-\left(\frac{10}{6}\right)e^{-0.1\times 6.8522}\right)$$

$$P_b = 54.3127 \text{ MPa}$$

Section B : Production Engineering and Material Science + Mechatronics and Robotics

5. (a)

Given : $\alpha_{Ni} = 0.70,\ \alpha_{Cr} = 0.20,\ \alpha_{Fe} = 0.05$

$\alpha_{Ti} = 0.05$, Area $(A) = 1600 \text{ mm}^2$

We know that, Current $(I) = 1500$ A

$$\frac{1}{\rho_{alloy}} = \frac{\alpha_{Ni}}{\rho_{Ni}}+\frac{\alpha_{Cr}}{\rho_{Cr}}+\frac{\alpha_{Fe}}{\rho_{Fe}}+\frac{\alpha_{Ti}}{\rho_{Ti}}$$

$$= \frac{0.70}{8.9}+\frac{0.20}{7.19}+\frac{0.05}{7.86}+\frac{0.05}{4.51}$$

$$\rho_{alloy} = 8.07 \text{ gm/cc}$$

$$\left(\frac{Z}{A}\right)_{alloy} = \frac{\alpha_{Ni}Z_{Ni}}{A_{Ni}}+\frac{\alpha_{Cr}Z_{Cr}}{A_{Cr}}+\frac{\alpha_{Fe}Z_{Fe}}{A_{Fe}}+\frac{\alpha_{Ti}Z_{Ti}}{A_{Ti}}$$

$$= \frac{0.70\times 2}{58.71}+\frac{0.20\times 2}{51.99}+\frac{0.05\times 2}{55.85}+\frac{0.05\times 3}{47.9}$$

$$= 0.03646$$

$$\text{MRR} = \left(\frac{AI}{\rho ZF}\right)_{Alloy} = \frac{I}{\rho F\left(\frac{Z}{A}\right)_{Alloy}} = \frac{1500}{8.07\times 96500\times(0.03646)}$$

$$\text{MRR} = 0.05283 \text{ cm}^3/\text{s} = 52.83 \text{ mm}^3/\text{s}$$

$$\text{MRR} = 3169.75 \text{ mm}^3/\text{min}$$

$$\text{Rate of dissolution} = \frac{\text{MRR}}{\text{Area}} = \frac{3169.75}{1600} = 1.981 \text{ mm/min}$$

5. (b)

AWJM is a process that uses a very high speed water jet (supersonic Mach number $\simeq 2.5$), which is mixed with abrasives to cut any material without affecting work material and environment.

Greatest advantage of AWJM is that it cuts material without any heating effect, so the work material is free from thermal and mechanical distortions.

Equipments:

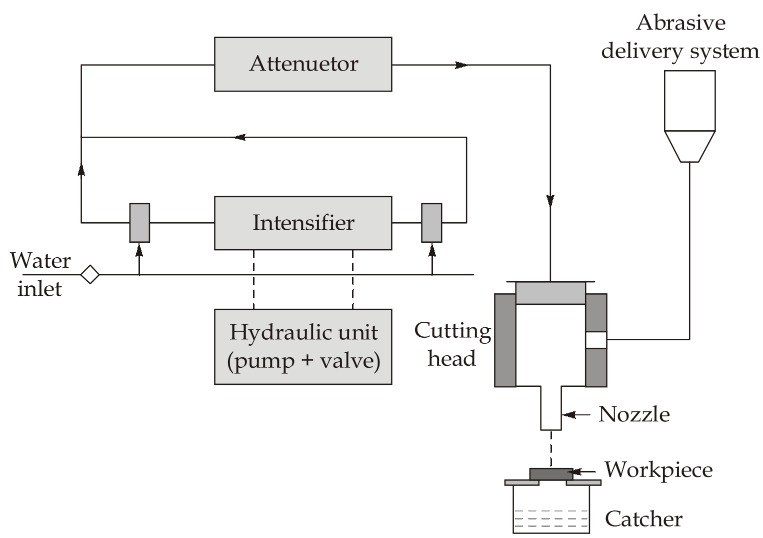
1.An intensifier pump to provide high pressure water.

2.The abrasive delivery system.

3.A cutting head for producing abrasive water jet.

4.A computer controlled manipulator to provide desired motion to cutting head.

5.A catcher that dissipates the remaining jet energy.



Pressure at Nozzle = 1300 – 4000 bar

Nozzle dia = 0.18 – 0.40 mm

Process parameters:

1. Jet velocity
2. Feed rate
3. Abrasive size
4. Work material and its thickenss.

Advantages:

1. Cuts any material irrespective of hardness.
2. Low cost than other machining processes.
3. No heat affected zone.
4. Scrap material left can be reused.
5. Water jet cuts with very little force so it is possible to cut thin walled parts.

Applications: The main applications of the process is, in machining inaccessible areas like inside surface of bottle, cleaning metallic moulds, producing very fine holes in the tool for the purpose of lubrication etc.

5. (c)

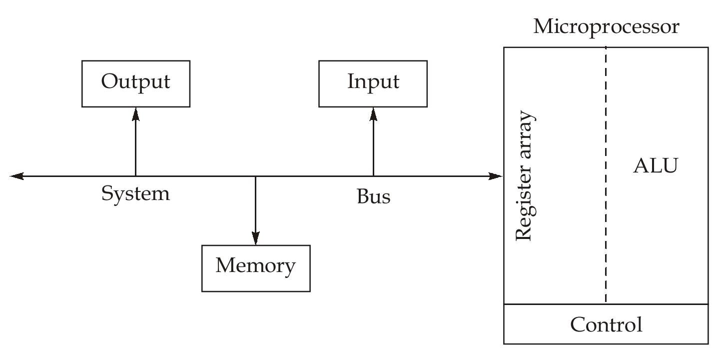


Figure: Block diagram of a microcomputer

The microprocessor consists of the following three segments:

1. Arithmetic/logic unit (ALU): In this area of microprocessor, computing functions are performed on data. The ALU performs airthmetic operations such as addition and subtraction, and logic operations such as AND, OR and exclusive OR. Results are stored either in registers or in memory or sent to output devices.
2. Register unit: This area of the microprocessor consists of various registers. The registers are used primarily to store data temporarily during the execution of a program. Some of the registers are accessible to the user through instructions.
3. Control unit: The control unit provides the necessary timing and control signals to all the operations in the microcomputer. It controls the flow of data between the microprocessor and peripherals including memory.

Uses/Application areas of microprocessor

- Instrumentation application
- Process control

- Instrumentation
- Monitoring and control
- Data acquisition

– Medical electronics:
  - Patient monitoring in ICU
  - Pathological analysis
  - Measurement of parameters like blood pressure and temperature

– High level language computers

– Home entertainment and games

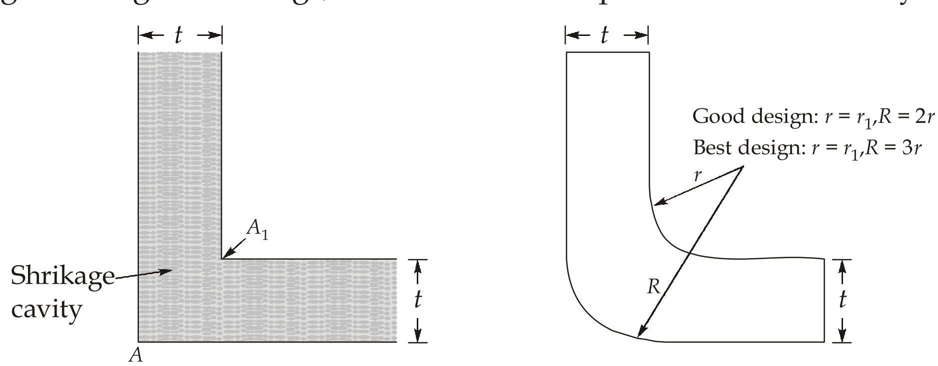
– Computer peripheral control

– Control of automation and continuous processes

– Inventory control system, pay roll banking etc.

5. (d)

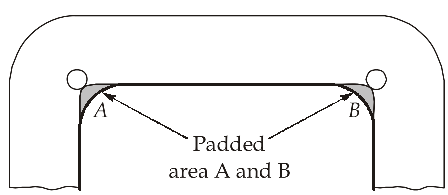
Hot spots are the last portions of the casting to solidify and they usually occur at points where one section joins another, or where a section is heavier than that adjoining it, for example, a corner of a right angle figure. Hot spots are weak and metal may tear where they occur. Wherein for casting steel and other high-temperature metals, the hot spot must be fed by molten metal (through proper feeder or risers) to avoid formation of cavities at the hot spot. Shape of the casting should avoid abrupt changes from heavy to thin section and should be such so as to make directional solidification possible. Besides improving the design of castings, occurrence of hot spots can be avoided by using chills.



(i) Bad design (ii) Good design

Hot tears are cracks at various points in a casting brought about by internal stresses resulting from restricted contraction. The strength of metal near solidification or freezing point is very low. Stresses imposed at this temperature may lead to incipient flaws which later develop into well defined cracks or hot tears upon cooling. Precautions must be taken to avoid stress concentrations caused by shrinkage stresses at weak points. Sharp angles and abrupt changes in cross-section contribute to large temperature differences within a casting, which may result in hot tear. Hot tears and excessive residual stresses in castings occur because sections of a casting are restrained from shrinking by the presence of either massive coarse or hard rammed molds.

Care should be taken to avoid stress concentrations caused by shrinkage stresses at weak points. Take an example of a product, a high quality alloy steel casting shown in figure. The design is one that is conductive to hot tearing at the natural hot spots in the inside corners. Cracks may occur in this radius area as this is hottest, weakest and subjected to highest stress concentration. Use of exothermic pads can solve the problem. The pads are made of exothermic mixture which comprises of mill scales, aluminium powder, charcoal burn and the heat evolved due to exothermic reaction (during casting) keeps the metal in molten state long enough so that excessive strains are not built up during final stage of solidification and shrinkage, thus avoiding the possibility of hot tears.



Hot tearing is rupture of skin by thermal contraction and is observed where a thin section joins a thick section. The thin section solidifies first, whereas the thicker section still contains liquid metal. The thinner section which begins to solidify first contracts thermally pulling itself away from the large section which is still partly in molten state and the fracture results at the corner of the thicker section.

5. (e)

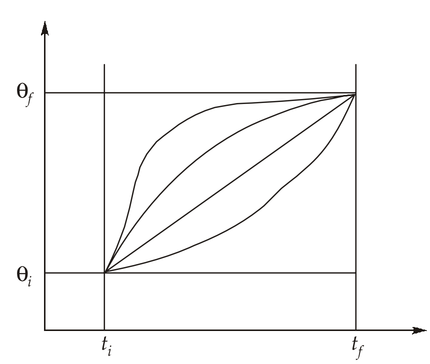
A cubic polynomial has the form

$$\theta(t) = a_1 + a_2 t + a_3 t^2 + a_4 t^3$$

The velocity and acceleration can be expressed by

$$\dot{\theta}(t_f) = a_2 + 2a_3 t + 3a_4 t^2$$

$$\ddot{\theta}(t) = 2a_3 + 6a_4 t$$



Curves connecting initial and final point

When a link moves, it starts from rest and stops at rest, hence the initial and final velocities have to be zero at initial time t_i and final time t_f.

$$\dot{\theta}(0) = 0$$

$$\dot{\theta}(t_f) = 0$$

Hence combining the two velocity constraints and two position constraints (start position and end position) and using equations.

$$\theta_i = a_1$$
$$\theta_f = a_1 + a_2 t_f + a_3 t_f^2 + a_4 t_f^3$$
$$0 = a_2$$
$$0 = a_2 + 2a_3 t_f + 3a_4 t_f^2$$

Solving above four equations, we get

$$a_1 = \theta_i$$
$$a_2 = 0$$
$$a_3 = \frac{3}{t_f^2}(\theta_f - \theta_i)$$

In the given problem, $\theta_i = 0°$, $\theta_f = 60°$, time, $t = 5$ seconds

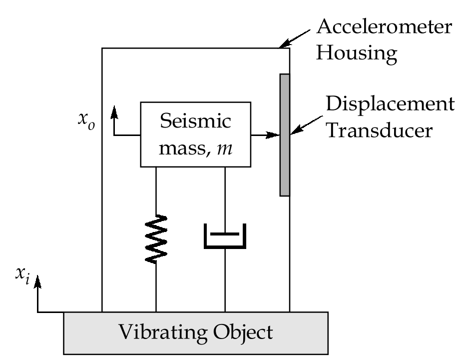
$$a_1 = 0$$
$$a_2 = 0$$
$$a_3 = \frac{3}{t^2}(\theta_f - \theta_i) = \frac{3}{25} \times (60 - 0) = 7.2$$
$$a_4 = -\frac{2}{t^3}(\theta_f - \theta_i) = -\frac{2}{125} \times (60 - 0) = -0.96$$

The required cubic polynomial is given by

$$\theta(t) = 7.2t^2 - 0.96\, t^3$$ Answer

6. (a)

As per given information,



$$m = 50 \text{ gram} = 0.05 \text{ kg}$$

Spring constant, $K = 5000$ N/m

Damping constant, $C = 30$ N.s/m

General equation of motion,

$$\ddot{x}_r + 2\zeta\omega_n \dot{x}_r + \omega_n^2 x_r = -\ddot{x}_i$$

where $x_r = x_o - x_i$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{5000}{0.05}} = 316.227 \text{ rad/sec}$$

$$\zeta = \frac{C}{2\sqrt{km}} = \frac{30}{2\sqrt{5000 \times 0.05}} = 0.9486$$

(i) Actual acceleration magnitude

$$(\ddot{x}_i) = x_{in}\omega^2$$

$$= 5 \times (100)^2 = 5 \times 10^4 \text{ mm/sec}^2$$

$$\ddot{x}_i = 50 \text{ m/sec}^2$$

(ii) Amplitude of steady state relative displacement

$$X_r = \frac{1}{(\omega_n)^2} H_a(\omega)(X_i\omega^2)$$

$$H_a(\omega) = \frac{X_r(\omega_n^2)}{X_i(\omega^2)} = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

$$X_r = \frac{X_i\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

$$X_r = \frac{5 \times \left(\frac{100}{316.227}\right)^2}{\sqrt{\left(1-\left(\frac{100}{316.227}\right)^2\right)^2 + \left(2 \times 0.9486 \times \frac{100}{316.227}\right)^2}}$$

$$X_r = \frac{0.5000}{\sqrt{0.8099 + 0.3599}} = 0.4623 \text{ mm}$$

$$X_r = 0.4623 \text{ mm}$$

(iii) The acceleration amplitude, measured by accelerometer

$$\left|\left(\ddot{X}_{in}\right)_{measured}\right| = X_r\omega_n^2$$

$$= 0.4623 \times 100^2 \text{ mm/sec}^2 = 4.623 \times 10^3 \text{ mm/sec}^2$$

$$\left(\ddot{X}_{in}\right)_{measured} = 4.623 \text{ m/sec}^2$$

(iv) $x_r(t) = X_r \sin(\omega t + \phi)$

$$\phi = \tan^{-1}\left[\frac{2\zeta\frac{\omega}{\omega_n}}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}\right]$$

$$\phi = \tan^{-1}\left[\frac{2\times 0.9486\times\frac{100}{316.227}}{\left(\frac{100}{316.227}\right)^2 - 1}\right]$$

$$\phi = -33.687° = -0.5879 \text{ rad}$$

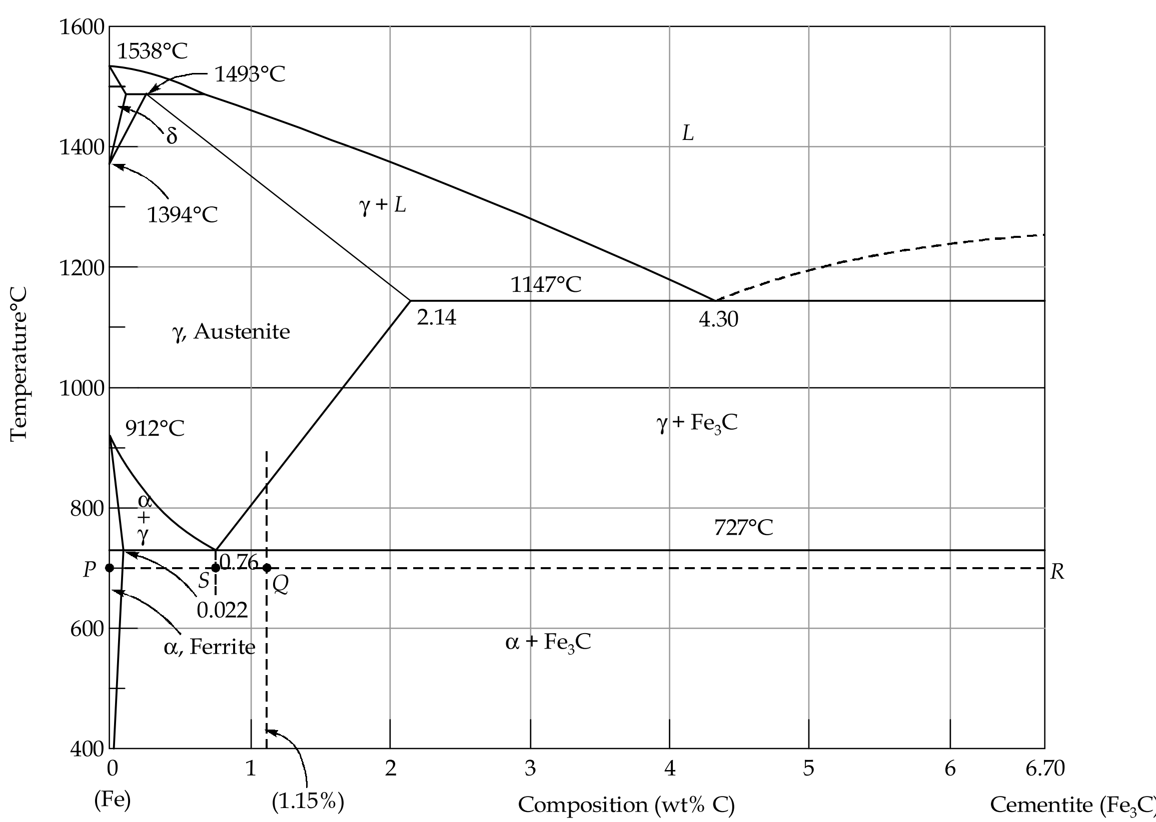
$$X_r = 0.4623 \text{ mm}$$

$$x_r(t) = 0.4623 \sin(100t - 33.687°)$$

$$x_r(t) = 0.4623 \sin(100t - 0.5879) \text{ mm}$$

6. (b)

As per given information, by considering 1.0 kg of austenite containing 1.15 wt.% C, cooled to below 727°C.



(i) The proeutectoid phase will be Fe_3C since 1.15 wt% C is greater than the eutectoid composition (0.76 wt% *C*)

(ii) Objective is to determine how much total ferrite and cementite is formed. By applying the lever rule in line *PQR*

Total ferrite, $$W_\alpha = \frac{C_{Fe_3C} - C_Q}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - 1.15}{6.70 - 0.022}$$

$$W_\alpha = 0.8310$$

This when multiplied by the total mass of the alloy (1.0 kg) gives 0.831 kg of total ferrite.

Similarly, for total cementite, by applying lever rule in line *PQR*

$$W_{Fe_3C} = \frac{C_Q - C_\alpha}{C_{Fe_3C} - C_\alpha}$$

$$W_{Fe_3C} = \frac{1.15 - 0.022}{6.70 - 0.022} = 0.169$$

$$W_{Fe_3C} = 0.169 \text{ kg}$$

and the mass of total cementite that is formed

$$W_{Fe_3C} = 0.169 \times 1.0 \text{ kg} = 0.169 \text{ kg}$$

(iii) Objective is to find each of pearlite and the proeutectoid phase formed in kilogram.

By applying lever rule in *SQR*

$$W_{\text{pearlite}} = \frac{C_{Fe_3C} - C_Q}{C_{Fe_3C} - C_{\text{eutectoid}}}$$

$$W_{\text{pearlite}} = \frac{6.70 - 1.15}{6.70 - 0.76} = 0.9343$$

Therefore total mass of pearlite

$$= 0.9343 \times 1 \text{ kg} = 0.9343 \text{ kg}$$

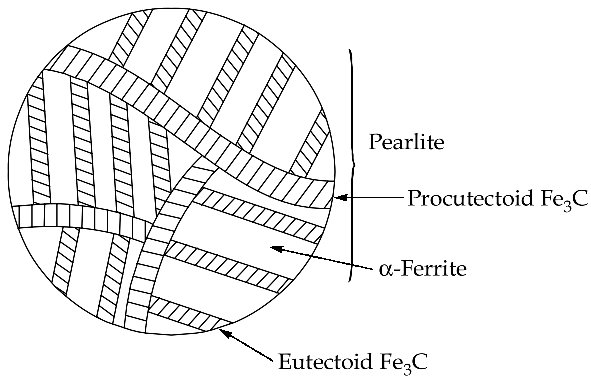
$$W_{\text{cementite}} = \frac{C_Q - C_{\text{eutectoid}}}{C_{Fe_3C} - C_{\text{eutectoid}}}$$

$$= \frac{1.15 - 0.76}{6.70 - 0.76} = 0.06565$$

Therefore, total mass of cementite that is formed

$$= 0.6565 \times 1 \text{ kg} = 0.6565 \text{ kg}$$

(iv) The resulting microstructure.



6. (c) (i)

We know that for hole 'H', fundamental deviation of hole is zero.

Geometric mean dimension, $D = \sqrt{D_1 D_2} = \sqrt{50 \times 80} = 63.245$ mm

Tolerance unit, $i = 0.45D^{1/3} + 0.001D = 0.45\,(63.245)^{1/3} + 0.001 \times 63.245$

$i = 1.86\ \mu m$

For hole H_8, tolerance $= 25i = 25 \times 1.86 = 46.5\ \mu m$

For shaft f_7, tolerance $= 16i = 16 \times 1.86 = 29.76\ \mu m$

Fundamental deviation of shaft $= -5.5D^{0.41} = -5.5(63.245)^{0.41}$

$= -30.11\ \mu m$

Negative sign indicates the fundamental deviation is upper deviation,

For hole basis system, LL of hole = Basic size = 65 mm

and HL of hole = LL + tolerance

= 65 + 0.0465 = 65.0465 mm

Hole size = $65^{+0.046}_{+0.000}$

For shaft, HL of shaft = Basic size – Fundamental deviation

= 65 – 0.030 = 64.97 mm

LL of shaft = HL – Tolerance = 64.97 – 0.0297 = 64.94 mm

Shaft size = $65^{-0.03}_{-0.06}$

Where, HL = Higher limit, LL = Lower limit

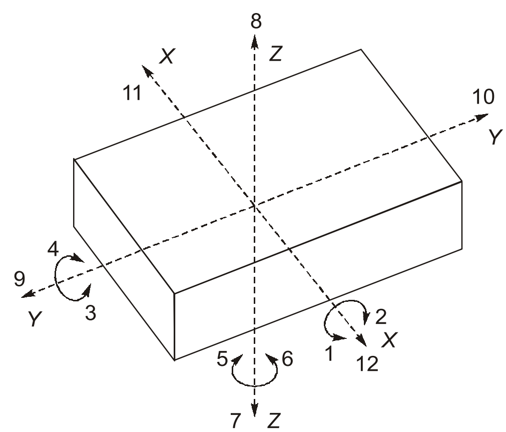
6. (c) (ii)

Jig and Fixture: A device which holds and positions a work, locates or guides the cutting tool relative to work is called jig. It is usually not fixed with machine table and is lighter in construction.

Generally jigs are used on drilling, reaming, tapping and counter-boring.

A fixture is a work holding device, which holds and positions the work but does not itself guide the cutting tool. The setting of tool is done separately. It is usually bolted or clamped with table and is heavier in construction. Generally fixtures are used in turning, milling, grinding, shaping, planning, etc.

Various degrees of freedom for a body in space: Consider a body with true plane surfaces in space as shown.



This body can move or rotate in any direction as shown. The six direction of movements along X-Y & Z direction are called movements of translation as

X-translation = 11, 12

Y-translation = 9, 10

Z-translation = 7, 8

The six direction of rotation about X, Y and Z-direction are called relational movements as

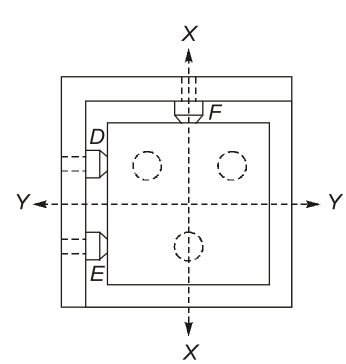
about X-axis, 1, 2

about Y-axis, 4, 3

about Z-axis, 5, 6

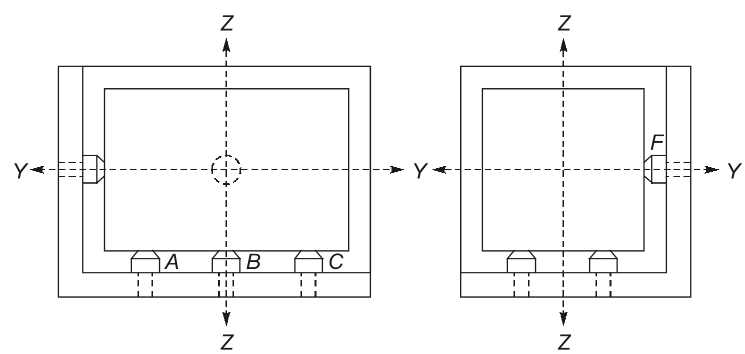
These 12 movements (translational and rotational) are called 12 degree of freedom in space.

To confine the body/workpiece, accurately, in another body (jig or fixture) the movements of these twelve degree of freedom are restricted successively as shown in figure below.



(i) The work piece is placed on three pins *A*, *B* and *C* on the base. These three pins restrict the rotation about *X*-*X* and *Y*-*Y* direction and one movement (down ward) along *Z*-axis i.e. total 5 out of 12.

(ii) The two more pins *D* & *E* in a vertical plane. These two pins restricts rotation about *Z*-*Z*-axis and one movement about *Y*-axis i.e. total three out 7 left above.

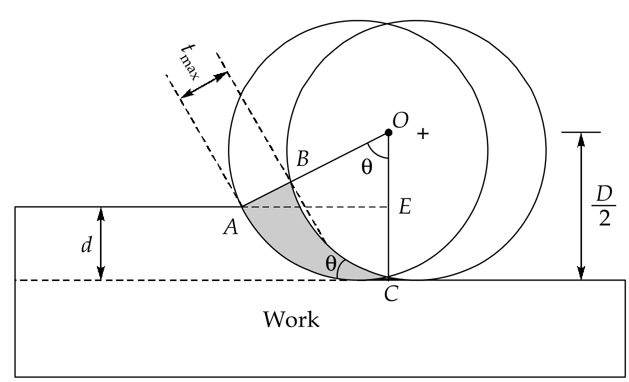


(iii) One more pin in another vertical plane which restrict one rotation along *X* axis out of 4 left above.

These 6 pins restricts nine degrees of freedom (dof) of total twelve dof. These 3-dof left above are restricted by bolting or clamping in order to loading and unloading the workpiece without destruction easily.

3-2-1 principle or six point location principle: The locating of work piece on six pins in order to restrict extra movement of jig or fixture is called 3-2-1 or six point location principle.

Q.7 (a)(i)



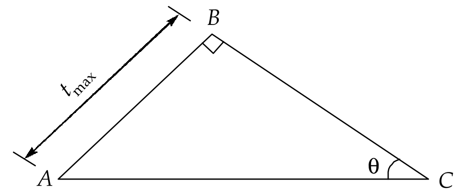
Let f = Table feed (in mm/min)

N = Cutter rpm

Z = Number of teeth

∴ Feed per tooth, $f_t = \dfrac{f}{NZ}$

From the Geometry :



$$AC = \frac{\text{Feed}}{\text{Tooth}} = f_t = \frac{f}{NZ}$$

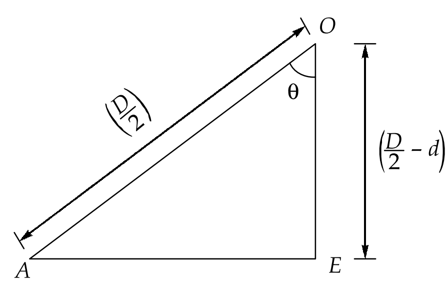
Maximum uncut thickness, t_{max} = AB

Now, in right angle triangle ABC, right angle at B.

$$AB = AC \sin\theta \qquad [\angle ACB = \theta]$$

and in ΔAED, $\cos\theta = \dfrac{OE}{OA} = \dfrac{\frac{D}{2} - d}{\frac{D}{2}} = 1 - \dfrac{2d}{D}$

∴ $\sin\theta = \sqrt{1 - \left(1 - \dfrac{2d}{D}\right)^2} = \sqrt{1 - 1 - \left(\dfrac{2d}{D}\right)^2 + \dfrac{2\times 2d}{D}}$



$$\sin\theta = \sqrt{\frac{4d}{D} - \frac{4d^2}{D^2}} = 2\sqrt{\frac{d}{D}\left(1 - \frac{d}{D}\right)}$$

∴ $t_{max} = AB = AC\sin\theta = f_t \times 2\sqrt{\dfrac{d}{D}\left(1 - \dfrac{d}{D}\right)}$

Hence, $t_{max} = \dfrac{2f}{NZ}\sqrt{\dfrac{d}{D}\left(1 - \dfrac{d}{D}\right)}$

(ii)

Given : Z = 15 teeth, Rake angle, $\alpha = 10°$, N = 200 rpm

Cutter diameter, D = 80 mm, Table feed, f = 75 mm/min

Depth of cut, d = 5 mm, Ultimate shear strength, τ_{us} = 420 N/mm^2

Width, b = 50 mm, Coefficient of friction, μ = 0.7

Now, Friction angle, $\beta = \tan^{-1}(\mu) = \tan^{-1}(0.7) = 34.992°$

According to Lee and Shaffer,

$$\phi = 45° + \alpha - \beta$$

$$= 45° + 10° - 34.992° = 20.008°$$

Maximum uncut thickness,

$$t_{max} = \frac{2f}{NZ}\sqrt{\frac{d}{D}\left(1-\frac{d}{D}\right)} = \frac{2\times 75}{200\times 15}\sqrt{\frac{5}{80}\left(1-\frac{5}{80}\right)}$$

$$= 0.0121 \text{ mm}$$

Maximum shear force, $(F_s)_{max} = \tau_s \times \frac{bt}{\sin\phi} = 420\times\frac{50\times 0.0121}{\sin(20.008)°} = 742.654$ N

Maximum cutting force, $(F_c)_{max} = R\cos(\beta - \alpha) = \frac{(F_s)_{max}\cos(\beta-\alpha)}{\cos(\phi+\beta-\alpha)}$

$$(F_c)_{max} = \frac{742.654\times\cos(34.992-10)°}{\cos(20.008+34.992-10)°} = 951.93 \text{ N}$$

Now, angle between two consecutive teeth,

$$\frac{360°}{Z} = \frac{360°}{15} = 24°$$

$$\cos\theta = 1-\frac{2d}{D}$$

$$\theta = \cos^{-1}\left(1-\frac{2\times 5}{80}\right)$$

$$\theta = 28.955°$$

Since, $\theta > \frac{360°}{z}$

Maximum torque due to one teeth,

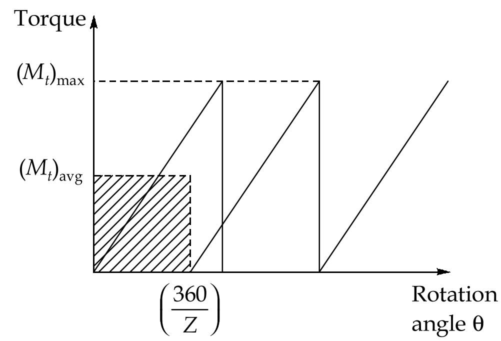
$$(M_t)_{max} = F_C \times \frac{D}{2}$$

$$= \frac{951.93\times 80}{2\times 1000} = 38.08 \text{ N-m}$$

Average Torque : $(M_t)_{avg} = \dfrac{\frac{1}{2}(M_t)_{max} \times \theta}{\left(\frac{360}{Z}\right)}$

$= \dfrac{1}{2} \times \dfrac{38.08 \times 28.955}{24}$

$(M_t)_{avg} = 22.97$ N-m



Power consumption, $P = (M_t)_{avg} \times \left(\dfrac{2\pi N}{60}\right)$

$= 22.97 \times \dfrac{2\pi \times 200}{60}$

$P = 481.08$ W

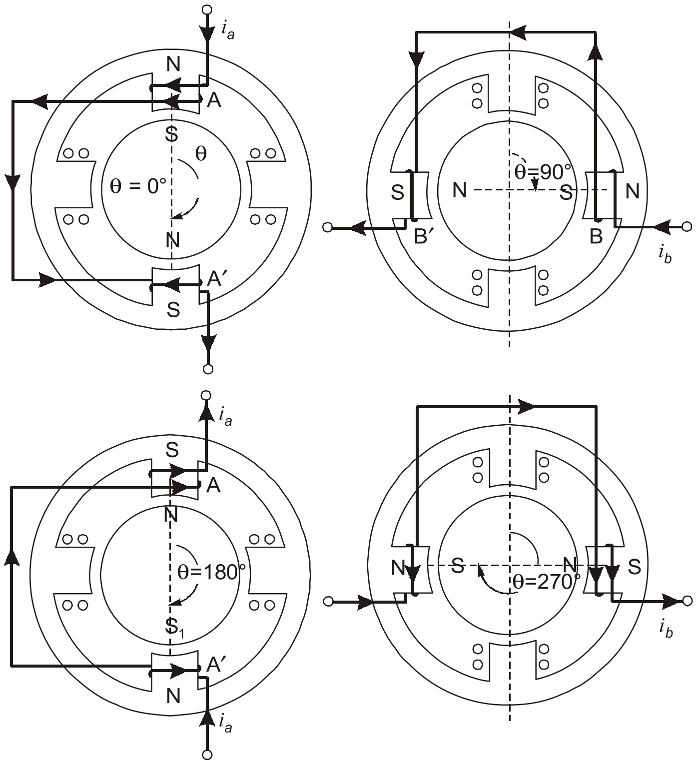
Q.7 (b)

A stepper motor is a pulse-driven motor that changes the angular position of the rotor in steps. Due to this nature of a stepper motor it is widely used in low cost, open loop position control systems. A stepper motor is basically a brushless DC- motor whose rotor rotates in discrete angular movements when its winding is energized in a programmed manner. In stepper motor a full rotation is divided into a number of equal steps.

Permanent magnet (PM) stepper motor: The stator of permanent magnet stepper motor consists of salient poles with concentrated windings. The rotor as the name of this motor suggest, consists of permanent magnet poles. For the illustration of the working principle of PMSM an elementary form of 2-phase 4/2 pole stepper motor is considered here. The concentrated winding on diametrically opposite poles are connected in series so as to result in 2-phase winding on the stator. The rotor is magnetized to give two permanent magnets.

Working: Two coils *AA*' connected in series constitute phase *A* winding. When this winding is excited with current i_a, the stator produced poles attract the rotor permanent magnet poles so that their magnetic axis coincides. Let this exciting of phase A winding be denoted by +*A*.

Now the current in phase *A* winding is reduced to zero while phase *B* winding is excited with current i_b. Stator produced poles now attract the rotor poles, causing a CW step rotation through θ = 90°. Let the exciting of phase *B* winding be denoted by +*B*.



Internal structure of Permanent Magnet (PM) Stepper Motor

Now the phase winding *A* is again excited but with current opposite to i_a that is $-i_a$ this time. Now rotor poles further move through a step of 90° CW so that θ =180°. This step of exciting phase winding be denoted by $-A$.

Now the phase winding *B* is made to carry exciting current opposite to that of i_b that is $-i_b$ this time. The rotor again executes further step of 90° CW so that θ = 270°. This method of exciting phase *B* winding be designated as $-B$.

For further 90° CW step phase winding *B* is de-energized and phase winding *A* is energized. This shows that four steps complete one revolution of the rotor movement.

So here by the application of each current pulse to the stator winding in proper sequence, the rotor can be made to execute discrete angular steps of 90°. Sequence of exciting the stator phase winding is $+A$, $+B$, $-A$, $-B$, $+A$ for CW rotor movement. For CCW rotor rotation, sequence of exciting stator phase winding is $+A$, $-B$, $-A$, $+B$, $-B$.

If both the stator windings are excited in the sequence $+A$ together with $+B$, then the resultant stator field is along the interpolar axis, the rotor therefore moves a step of 45° CW. This shows for obtaining the angular step of 45° CW the switching sequence should be as $+A$, $(+A +B)$, $+B$, $(+B -A)$, $-A$, $(-A - B)$, $-B$, $(-B +A)$, $+A$.

This method of reducing step angle to half the normal step is called half step mode of excitation.

Advantages of stepper motors:

- Low cost
- Ruggedness
- Simplicity of construction
- Low maintenance
- Less likely to stall or slip
- Will work in any environment
- Excellent start stop and reversing responses

Disadvantages of stepper motor:

- Low torque capacity compared to DC motors.
- Limited speed.
- During overloading, the synchronization will be broken. Vibration and noise occur when running at high speed.

7. (c)

For complete substitutional solubility the following criteria must be met :

1. The difference in atomic radii between Cu and the other element ($\Delta R\%$) must be less than ±15%.
2. The crystal structures must be the same.
3. The electronegativities must be almost same.
4. The valency should be the same or nearly the same.

Element	Atomic Radius (R-nm)	$\Delta R\% = \left\lvert \frac{R - R_{Cu}}{R_{Cu}} \right\rvert \times 100$	Crystal Structure	Electronegativity (E)	ΔE (Electronegativity difference) $\lvert E - E_{Cu} \rvert$	Valency
Cu	0.1278	0	FCC	1.9	0	+2
C	0.071	44.44				
H	0.046	64.00				
O	0.060	53.05				
Ag	0.1445	13.067	FCC	1.9	0	+1
Al	0.1431	11.971	FCC	1.5	0.4	+3
Co	0.1253	1.956	HCP	1.8	0.1	+2
Cr	0.1249	2.269	BCC	1.6	0.3	+3
Fe	0.1241	2.895	BCC	1.8	0.1	+2
Ni	0.1246	2.5039	FCC	1.8	0.1	+2
Pd	0.1376	7.668	FCC	2.2	0.3	+2
Pt	0.1387	8.5289	FCC	2.2	0.3	+2
Zn	0.1332	4.225	HCP	1.6	0.3	+2

1. For a substitutional solid solution having complete solubility must satisfied following criteria as discussed earlier :

   - First criteria qualified by following elements [$\Delta R\% \leq \pm 15\%$]

     Ag, Al, Co, Cr, Fe, Ni, Pd, Pt, Zn

   - Second criteria qualified by following elements (The crystal structure must be same).

     Ag, Al, Ni, Pd and Pt

   - Third criteria qualified by following elements among the abvoe elements (Electronegativity must be similar or nearly similar).

     Ag and Ni

   - Fourth criteria qualified by following elements among the abvoe elements (valency should be same).

     Ni

   Therefore element Ni meet all of the criteria and thus form substitutional solid solution having complete solubility.

2. For a substitutional solid solution of incomplete solubility at least any one of the above criteria should not be satisfied:

   Elements Co, Cr, Fe and Zn are having either BCC or HCP crystal structure and Pd, Pt, Al having much electronegativity difference and Ag having valency different than +2. Therefore, elements Pd, Pt, Co, Cr, Fe, Zn, Al and Ag form substitutional solid solutions of incomplete solubility.

3. C, H and O form interstitial solid solutions. These elements have atomic radii that are significantly smaller than the atomic radius of Cu.

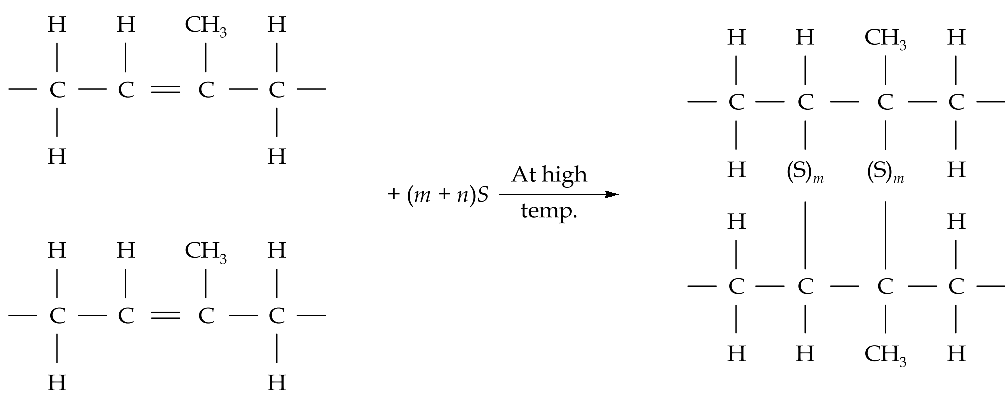
8. (a) (i)

The process of crosslinking in elastomers is known as vulcanization. It can be achieved by a non-reversible chemical reaction, generally it is carried out at an elevated temperature.

In most of the vulcanization process, we add sulfur compounds to the heated elastomer.

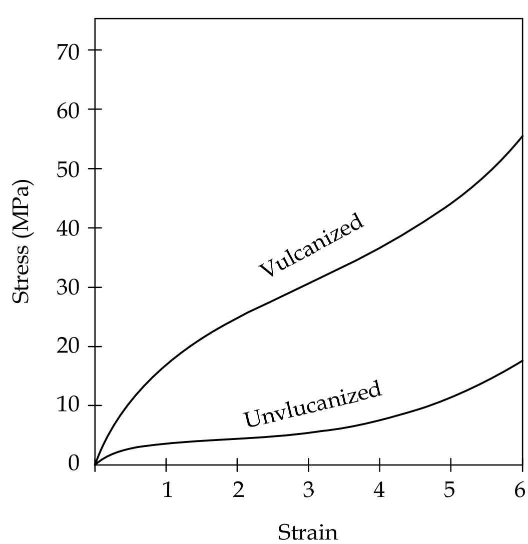
In this process chains of sulfur atoms bond with adjacent polymer backbone chains and it is used for crosslinking of them.

Example :



For crosslinking main sites are doubly bonded carbon atoms before vulcanization. But after vulcanization process these bond becomes singly bonded. With the increase in extent of vulcanization hardness, abrasion resistance improved. Modulus of elasticity, tensile strength and resistance to degradation will improve by vulcanization.

With the increase in sulfur content above a limit, hardness increases but limit of extensibility decreases. Because vulcanized materials are cross-linked, so these materials are thermosetting in nature.



(Fig.: Stress and strain curves for vulcanized and unvulcanized natural rubber)

8. (a) (ii)

Classification based on number of dimensions:

(i) Zero-dimensional (nanoparticle) [0 – D; d < 100 nm]

(ii) One-dimensional (e.g. Nano rods, nanowives and nanotubes)

$$[1\text{–}D;\ d < 100\text{ nm}]$$

(iii) Two-dimensional (e.g. nanofilms, nanosheets and nanocoatings)

$$[2\text{–}D;\ t < 100\text{ nm}]$$

(iv) Three-dimensional (e.g. nanosized crystals in a bulk polycrystalline material)

$$[3\text{–}D;\ L >> 100\text{ nm}]$$

Classification based on materials:

(i) Carbon based nanomaterials

(ii) Metal based nanomaterials

(iii) Dendrimers

(iv) Composites

There are many molecules that can be used to make nanodevices and nanostructures but the most promising and powerful are carbon nanotubes (CNTs) because of their unique properties. The CNT is a tubular form of carbon with a diameter that can be as small as 1 nm and length of few nanometers to micro range. There are several techniques for making carbon nanotubes which require expensive equipment and other related catalyst materials. Currently, several methods are in use for growing carbon nanotubes:

(i) Arc process

(ii) Laser ablation

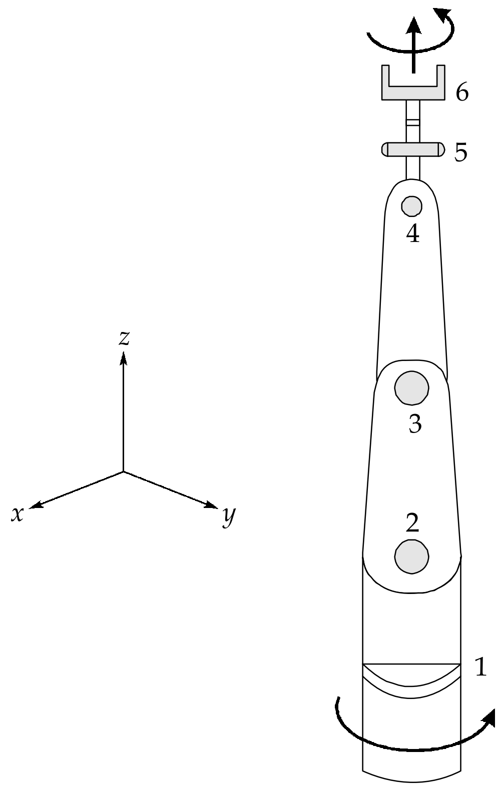
(iii) Chemical vapour deposition (CVD)

Among there, CVD is the most widely used method for the production of carbon nanotubes. It is used to grow nanotubes on patterned or unpatterned substrates. During CVD, a substrate is prepared with a layer of metal catalyst particles, most commonly cobalt, nickel, iron or a combination. The metal nanoparticles can also be produced by other ways, including reduction of oxides or oxides solid solutions. The diameters of the nanotubes that arc to by grown are related to the size of the metal particles. This can be controlled by patterned (or masked) deposition of the metal, annealing or by plasma etching of metal layer. The substrate is heated to approximately at 700°C. To initiate the growth of nanotubes, two gases are bled into the reactor a process gas (such as ammonia, nitrogen or hydrogen) and a carbon containing gas (such as C_2H_2, ethylene, ethanol or methane). Nanotubes grow at the sites of the metal catalyst, the carbon containing gas is broken apart at the surface of the catalyst particle and the carbon is transported to the edges of the particle, where it forms the nanotubes.

8. (b)

(i)

Degeneracy occurs when the robot loses a degree of freedom and therefore, cannot perform as desired. This occurs under two conditions : (1) when the robot's joints reach their physical limits and as a result, cannot move any further, (2) a robot may become degenerate in the middle of its workspace if the z-axes of two similar joints become collinear. This means that, at this instant, whichever joint moves, the same motion will result, and consequently, the controller does not know which joint to move. Since in either case the total number of degrees of freedom available is less than six, there is no solution for the robot. In the case of collinear joints, the determinant of the position matrixed is zero as well. Figure shows a simple robot in a vertical configuration, where joints 1 and 6 are collinear. As you can see, whether joint 1 or joint 6 rotate, the end effector will rotate the same amount. In practice, it is important to direct the controller to take an emergency action; otherwise the robot will stop. Please note that this condition occurs if the two joints are similar. Otherwise, if one joint is prismatic and one is revolute (as in joints 3 and 4 of the Stanford arm), although the *z*-axes are collinear, the robot will not be in degenerate condition. Paul has shown that if sin α_4, sin α_5 or sin α_6 are zero, the robot will be degenerate (this occurs if joints 4 and 5, or 5 and 6 are parallel, and therefore, result in similar motions). Obviously, α_4 and α_5 can be designed to prevent the degeneracy of the robot. However, anytime θ_5 approaches zero or 180°, the robot will become degenerate.



An example of a robot in a degenerate position

We should be able to position and orientate a 6-DOF robot at any desired location within its work envelope by specifying the position and the orientation of the hand. However, as the robot gets increasingly closer to the limits of its workspace, it will get a point where, although it is possible to locate it at a desired point, it will be impossible to orientate it at desired orientations. The volume of points where we can position the robot as desired but not orientate it is called nondexterous volume.

(ii)

We have,

$$T = \left[\begin{array}{ccc|c} 0.527 & -0.574 & 0.628 & 2 \\ 0.369 & 0.819 & 0.439 & 5 \\ -0.766 & 0 & 0.643 & 3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

We know,
$$T = \left[\begin{array}{c|c} R & P_{avg} \\ \hline 0 & 1 \end{array}\right]$$

So, $$T^{-1} = \left[\begin{array}{c|c} R^T & -R^T P_{avg} \\ \hline 0 & 1 \end{array}\right]$$

Now, $$-R^T P_{avg} = \begin{bmatrix} -0.527 & -0.369 & +0.766 \\ +0.574 & -0.819 & 0 \\ -0.628 & -0.439 & -0.643 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -0.601 \\ -2.947 \\ -5.38 \end{bmatrix}$$

and $$R^T = \begin{bmatrix} 0.527 & 0.369 & -0.766 \\ -0.574 & 0.819 & 0 \\ 0.628 & 0.439 & 0.643 \end{bmatrix}$$

Then, $$T^{-1} = \begin{bmatrix} 0.527 & 0.369 & -0.766 & -0.601 \\ -0.574 & 0.819 & 0 & -2.947 \\ 0.628 & 0.439 & 0.643 & -5.38 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

8. (c)

Given machining cost C_m = ₹ 300 per hour = ₹ 5 per min

Tool cost, C_t = ₹ 50

Tool change time, T_c = 1 min

Idle time (loading and unloading) for component change, t_0 = 2 min

Setup time, t_i = 2 hour

Number of parts, p = 1000

Taylor's tool life equation, $VT^{0.25} = 650$

$n = 0.25$, $C = 650$

Optimum tool life for minimum cost

$$T_0 = \left(T_c + \frac{C_t}{C_m}\right)\left(\frac{1-n}{n}\right) = \left(1 + \frac{50}{5}\right)\left(\frac{1-0.25}{0.25}\right)$$

$$T_0 = 33 \text{ min}$$ Answer (2)

Optimum cutting speed, $$V_0 = \frac{C}{T_0^n} = \frac{650}{(33)^{0.25}}$$

$$V_0 = 271.196 \text{ m/min}$$ Answer (1)

$$\text{Machining time, } T_m = \frac{\pi DL}{1000 fV_0} = \frac{\pi \times 100 \times 250}{1000 \times 0.2 \times 271.196}$$

$$T_m = 1.448 \text{ min}$$

Total time per piece

$$T_{\text{total}} = \text{Idle time } (t_0) + \frac{\text{Initial setup time for a batch } (t_i)}{\text{Number of parts produced per batch } P} +$$

$$\text{Machining time } (T_m) + \frac{\text{total change time } (T_c) \times \text{Machining time } (T_m)}{\text{Optimum tool life } (T_0)}$$

$$= 2 + \frac{2 \times 60}{1000} + 1.448 + 1 \times \frac{1.448}{33}$$

$$= 2 + 0.12 + 1.448 + 0.043878$$

$$T_{\text{total}} = 3.611878 \text{ min per piece}$$

Total production time for a batch (1000 component)

$$T_{\text{total}} = 3611.878 \text{ min}$$ Answer(4)

$$\text{Total cost per piece} = C_m \left(t_0 + \frac{t_i}{p} + t_m \right) + (C_t + T_c \times C_m) \times \frac{T_m}{T_0}$$

$$= 5 \left(2 + \frac{2 \times 60}{1000} + 1.448 \right) + (50 + 1 \times 5) \times \frac{1.448}{33}$$

$$= 17.84 + 2.413 = ₹\ 20.253333 \text{ per piece}$$

Total production cost for a batch (1000 component)

$$= ₹\ 20253.44$$ Answer(3)

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