Detailed Solutions

ESE-2025 Mains Test Series

Civil Engineering Test No : 6

Section A : Flow of Fluids, Hydraulic Machines and Hydro Power

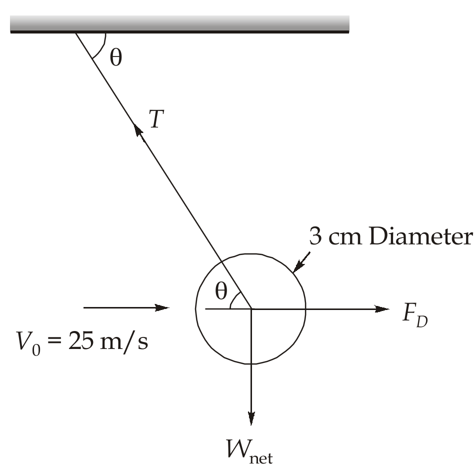
Q.1 (a) Solution:

Given:

Diameter of sphere, d = 3 cm = 0.03 m

Relative density of sphere, G = 2.5

Velocity of air, V_o = 25 m/sec.



For equilibrium of sphere.

$$T \cos \theta = F_D = C_D A \frac{\rho V_0^2}{2}$$

$$T \sin \theta = W_{net}$$

∴ $$\tan\theta = \frac{W_{net}}{C_D A \rho V_0^2 / 2}$$

Also, $$R_e = \frac{V_0 D}{\nu} = \frac{25 \times 0.03}{1.4 \times 10^{-5}} = 53571.43 \simeq 53571$$

As $10^4 < Re < 3 \times 10^5$; $C_D = 0.5$

$$W_{net} = W = \gamma_{sphere}(\text{Volume}) = 2.5 \times 10^3 \times 9.81 \times \frac{4}{3}\pi\left(\frac{0.03}{2}\right)^2$$

$$= 0.347 \text{ N}$$

Now, drag force on sphere

$$F_D = \frac{1}{2} C_D \rho_{air} A V^2$$

⇒ $$F_D = \frac{1}{2} \times 0.5 \times 1.25 \times \frac{\pi}{4}(0.03)^2 (25)^2$$

⇒ $$F_D = 0.138 \text{ N}$$

$$\tan\theta = \frac{W_{net}}{F_D} = \frac{0.347}{0.138} = 2.514$$

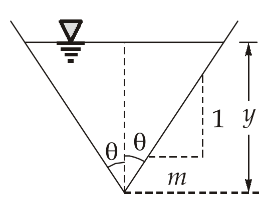
$$\theta = \tan^{-1}(2.514) = 68.31°$$

∴ Tension in the string $$= T = \frac{W_{net}}{\sin\theta} = \frac{0.347}{\sin(68.31)°} = 0.3734 \text{ N}$$

Q.1 (b) Solution:

For a triangular channel section, if θ is the angle of inclination of each of the sloping sides with the vertical and y is the depth of flow, then the wetted area 'A' and wetted perimeter 'P' can be respectively given by

$$A = my^2 \qquad [\because \quad m = \tan\theta]$$



⇒ $$A = y^2 \tan\theta$$

⇒ $$y = \sqrt{A / \tan\theta} \qquad \text{...(i)}$$

and $$P = 2y \sec\theta \qquad \text{...(ii)}$$

Substituting value of y from (i) in (ii), we get

$$P = 2\sec\theta\sqrt{\frac{A}{\tan\theta}} \quad ...(iii)$$

Assuming wetted area, A to be constant, equation (iii) can be differentiated with respect to θ and equated to zero for obtaining the condition for minimum wetted perimeter.

$$\therefore \quad \frac{dP}{d\theta} = 2\sqrt{A}\left[\frac{\sec\theta\tan\theta}{\sqrt{\tan\theta}} - \frac{\sec^3\theta}{2(\tan\theta)^{3/2}}\right] = 0$$

$$\Rightarrow \quad \sec\theta\,(2\tan^2\theta - \sec^2\theta) = 0 \quad [\because \sec\theta \neq 0]$$

$$\therefore \quad 2\tan^2\theta - \sec^2\theta = 0$$

$$\Rightarrow \quad \sqrt{2}\tan\theta = \sec\theta$$

$$\Rightarrow \quad \theta = 45°$$

$$\therefore \quad m = \tan\theta = 1$$

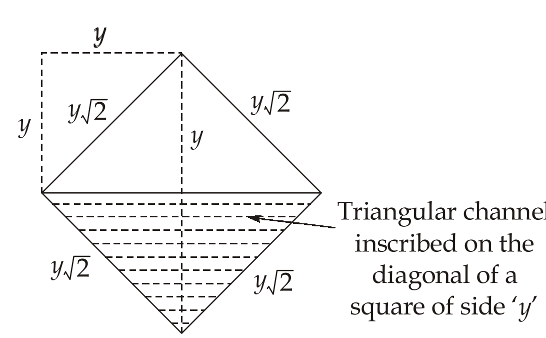
Hence a triangular channel will be most efficient when each of its sloping sides makes an angle of 45° with the vertical.

Hydraulic radius, $$R = \frac{A}{P} = \frac{y^2\tan\theta}{2y\sec\theta} \quad (\theta = 45°)$$

$$\Rightarrow \quad R = \frac{y^2\tan 45°}{2y\sec\theta}$$

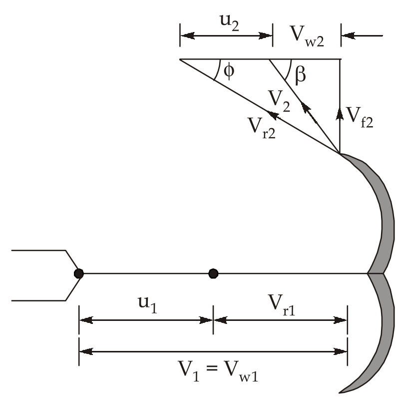
$$\Rightarrow \quad R = \frac{y}{2\sqrt{2}}$$

Thus it can be seen that the most efficient triangular channel section will be half the square inscribed on a diagonal and having equal sloping sides.



Q.1 (c) Solution:

The figure below shows a section through a bucket which is being acted on by a jet. The plane of the section is parallel to the axis of the wheel and contains the axis of the jet.



For Pelton turbine,

$$u_1 = u_2 = u = \frac{\pi DN}{60}$$

Force exerted by the jet of water on the bucket in the direction of motion of blade is,

$$F_x = \rho A V_1 \left(V_{w_1} \pm V_{w_2}\right)$$

Cross-section of jet, $A = \frac{\pi}{4} d^2$; d = Diameter of jet

Work done, $W.D. = F_x u = \rho A V_1 \left(V_{w_1} \pm V_{w_2}\right) u$

Kinetic energy of jet per second $= \frac{1}{2} m V_1^2 = \frac{1}{2} \times (\rho A V_1) V_1^2$

$\Rightarrow$ $$K.E. = \frac{1}{2} \rho A V_1^3$$

$$\text{Hydraulic efficiency, } \eta_h = \frac{\rho A V_1 \left(V_{w_1} \pm V_{w_2}\right) u}{\frac{1}{2} \rho A V_1^3}$$

But for Pelton wheel, $V_{w_1} = V_1$, $V_{r_1} = V_1 - u = V_{r_2}$

Also from outlet velocity triangle,

$$V_{w_2} = V_{r_2} \cos\phi - u$$

$$= V_{r_1} \cos\phi - u$$

$$\therefore \qquad \eta_h = 2\frac{[V_1 + (V_1 - u)\cos\phi - u]u}{V_1^2}$$

$$\Rightarrow \qquad \eta_h = 2\frac{[(V_1 - u)(1 + \cos\phi)]u}{V_1^2}$$

For maximum efficiency,

$$\frac{d}{du}\left[\frac{2(V_1 - u)(1+\cos\phi)}{V_1^2} \times u\right] = 0$$

$$\Rightarrow \qquad \frac{2(1+\cos\phi)}{V_1^2}\frac{d}{du}(V_1 u - u^2) = 0$$

$$\therefore \qquad u = \frac{V_1}{2}$$

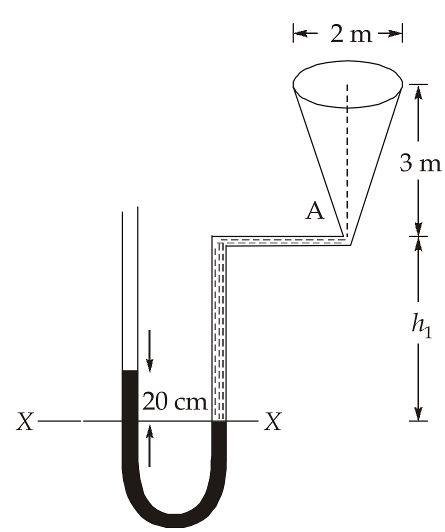
$\therefore$ Maximum hydraulic efficiency,

$$(\eta_h)_{max} = \left[\frac{2(V_1 - u)(1+\cos\phi)\times u}{V_1^2}\right]_{u=\frac{V_1}{2}}$$

$$= \left[\frac{2\times\frac{V_1}{2}(1+\cos\phi)\frac{V_1}{2}}{V_1^2}\right] = \frac{1+\cos\phi}{2}$$

Q.1 (d) Solution:

Vessel is empty:



Given:

Let h_1 = Height of water above X-X

Difference of mercury level h_2 = 20 cm

Specific gravity of mercury, S_2 = 13.6

Specific gravity of water, S_1 = 1.0

Density of mercury, $\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$

Density of water, $\rho_1 = 1000 \text{ kg/m}^3$

Equating the pressure above datum line X-X, we have,

$$\rho_2 g h_2 = \rho_1 g h_1$$

$$\Rightarrow \qquad 13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$$

$$\Rightarrow \qquad h_1 = 2.72 \text{ m of water.}$$

Vessel is full of water:

When vessel is full of water, the pressure in the right limb will increase and mercury level in the right limb will go down. Let the distance through which mercury level goes down in the right limb be, y cm as shown in figure. The mercury will rise in the left by a distance of y cm. Now the datum line is Z-Z. Equating the pressure above the datum line Z-Z,

Pressure in left limb = Pressure in right limb

$$\Rightarrow \quad 13.6 \times 1000 \times 9.81 \times \left(0.2 + \frac{2y}{100}\right) = 1000 \times 9.81 \times \left(3 + h_1 + \frac{y}{100}\right)$$

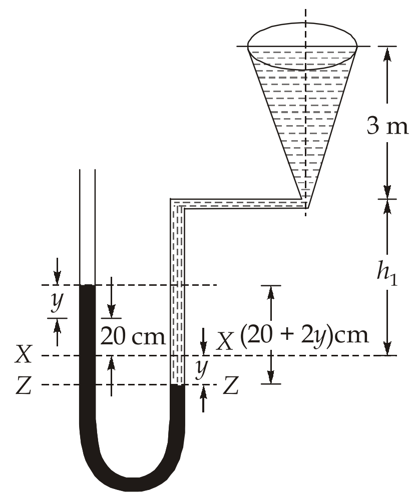
$$\Rightarrow \quad 13.6 \times \left(0.2 + \frac{2y}{100}\right) = \left(3 + 2.72 + \frac{y}{100}\right) \qquad (\because \; h_1 = 2.72 \text{ m})$$

$$\Rightarrow \quad 2.72 + \frac{27.2y}{100} = 3 + 2.72 + \frac{y}{100}$$

$$\Rightarrow \quad \frac{(27.2y - y)}{100} = 3.0$$

$$\Rightarrow \quad 26.2y = 3 \times 100 = 300$$

$$\Rightarrow \quad y = \frac{300}{26.2} = 11.45 \text{ cm}$$



The difference of mercury level in two limbs

$$= (20 + 2y) \text{ cm of mercury}$$
$$= 20 + 2 \times 11.45 = 20 + 22.90$$
$$= 42.90 \text{ cm of mercury}$$

∴ Reading of manometer = 42.90 cm.

Q.1 (e) Solution:

Given:

Diameter at inlet, $d_1 = 30$ cm

∴ Area of cross-section of pipe, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Diameter at throat, $d_2 = 15$ cm

∴ Area of cross-section of throat, $a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Let section (1) represents inlet and section (2) represents throat. Then $z_2 - z_1 = 30$ cm

Specific gravity of oil, $S_o = 0.9$

Specific gravity of mercury, $S_g = 13.6$

Reading of differential U-tube manometer, x = 25cm

Differential head, h is given by

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$$

$$= x\left(\frac{S_g}{S_o} - 1\right) = 25\left(\frac{136}{0.9} - 1\right) = 352.78 \text{ cm of oil.}$$

(i) Discharge of oil,

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

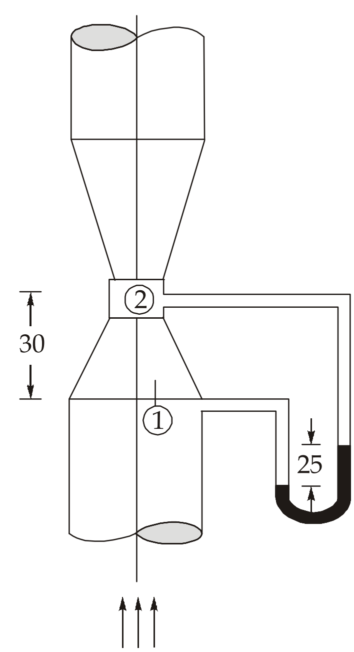
$$= \frac{0.98 \times 706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 352.78}$$

$$= \frac{101833663.3}{684.4} = 148792.61 \text{ cm}^3/\text{s}$$

$$= 148.79 \text{ lt/s}$$

(ii) Pressure difference between entrance and throat section,

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 352.78 \text{ cm of oil}$$



$$\Rightarrow \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2 = 352.78$$

But $z_2 - z_1 = 30$ cm

$$\therefore \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) - 30 = 352.78$$

$$\therefore \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 352.78 + 30$$

$$= 382.78 \text{ cm of oil}$$

$= 3.8278$ m of oil **Ans.**

$$\therefore (p_1 - p_2) = 3.8277 \times \rho g$$

But density of oil = Sp. gr. of oil × 1000 kg/m^3

= 0.9 × 1000 = 900 kg/cm^3

$$\therefore (p_1 - p_2) = 3.8278 \times 900 \times 9.81 \frac{N}{m^2}$$

$$= \frac{33795.6}{10^4} \text{N/cm}^2 = 3.3796 \text{ N/cm}^2$$

Q.2 (a) Solution:

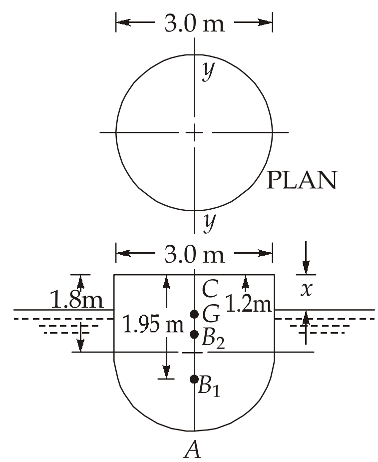
Given:

Diameter of body = 3.0 m

Depth of body = 1.8 m

Volume displaced by curved portion = 0.6 m^3 of water

Let B_1 is the centre of buoyancy of the curved surface and G is the centre of gravity of, the whole body.



Then CB_1 = 1.95 m

CG = 1.2 m

Total weight of water displaced by body = 3.9 tonnes

= 3.9 × 1000 = 3900 kgf

= 3900 × 9.81N = 38259 N

Let the height of the body above the water surface be x m. Total weight of water displaced by body

= Weight density of water × [Volume of water displaced]

= 1000 × 9.81 × [Volume of the body in water]

= 9810 [Volume of cylindrical part in water + Volume of curved portion]

= 9810 [$\frac{\pi}{4} \times D^2$ × Depth of cylindrical part in water + Volume displaced by curved portion]

$$\therefore \quad 38259 = 9810\left[\frac{\pi}{4}\times(3)^2\times(1.8-x)+0.6\right]$$

$$\Rightarrow \quad \frac{\pi}{4}(3)^2 \times (1.8 - x) + 0.6 = \frac{38259}{9810} = 3.9$$

$$\Rightarrow \quad \frac{\pi}{4} \times 3^2 \times (1.8 - x) = 3.9 - 0.6 = 3.3$$

$$\Rightarrow \quad 1.8 - x = \frac{3.3\times4}{\pi\times3\times3} = 0.46685$$

$$\Rightarrow \quad x = 1.8 - 0.46685 = 1.33 \text{ m}$$

Let B_2 be the centre of buoyancy of cylindrical part and B is the centre of buoyancy of the whole body.

Then depth of cylindrical part in water = 1.8 – x = 1.8 – 1.33 = 0.47 m

$$\therefore \qquad CB_2 = x + \frac{0.47}{2} = 1.33 + 0.235 = 1.565 \text{ m}$$

The distance of the centre of buoyancy of the whole body from the top of the cylindrical part is given as

$$CB = \frac{(\text{Volume of curved portion} \times \text{CB}_1 + \text{Volume of cylindrical part in water} \times \text{CB}_2)}{\text{Total volume of water displaced}}$$

$$= \frac{0.6 \times 1.95 + 3.3 \times 1.565}{(0.6 + 3.3)} = \frac{1.17 + 5.1645}{3.9} = 1.624 \text{ m}$$

Then $\qquad BG = CB - CG = 1.624 - 1.20 = 0.424$ m.

Meta-centric height, GM, is given by

$$GM = \frac{I}{\forall} - BG$$

where $\qquad I$ = M.O.I. of the plan of the body at water surface about y-y

$$= \frac{\pi}{64} \times D^4 = \frac{\pi}{64} \times 3^4 \text{ m}^4$$

$\forall$ = Volume of the body in water = 3.9 m^3

$$\therefore \qquad GM = \frac{\pi}{64} \times \frac{3^4}{3.9} - 0.424 = 1.0195 - 0.424 = 0.5955 \text{ m}$$

Q.2 (b) Solution:

(i) Given velocity profile, $\dfrac{U}{U_\infty} = \sin\left(\dfrac{\pi}{2} \cdot \dfrac{y}{\delta}\right)$

$$\Rightarrow \qquad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x} \int_0^\delta \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right) dy$$

$$\Rightarrow \qquad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x}\left\{\int_0^\delta \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\left[1 - \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right] dy\right\}$$

$$\Rightarrow \qquad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x}\left[\frac{-\cos\frac{\pi y}{2\delta}}{\frac{\pi}{2\delta}} - \left(\frac{1}{2}\frac{2\delta}{\pi} - \frac{\sin 2\left(\frac{\pi y}{2\delta}\right)}{4 \times \frac{\pi}{2\delta}}\right)\right]_0^\delta$$

$$\Rightarrow \quad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x}\left[\frac{-\cos\frac{\pi}{2}}{\frac{\pi}{2\delta}} + \frac{\sin\pi}{\frac{4\pi}{2\delta}} - \left(\frac{-1}{\frac{\pi}{2\delta}}\right) - \frac{\delta}{2}\right]$$

$$\Rightarrow \quad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x}\left[\frac{-\delta}{2} + \frac{2\delta}{\pi}\right]$$

$$\Rightarrow \quad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x}\left[\frac{2\delta}{\pi} - \frac{\delta}{2}\right]$$

$$\therefore \quad \frac{\tau}{\rho U_\infty^2} = \frac{4-\pi}{2\pi}\frac{\partial\delta}{\partial x} \quad \text{...(i)}$$

As $$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$$U = U_\infty \sin\left(\frac{\pi y}{2\delta}\right)$$

$$\therefore \quad \frac{du}{dy} = U_\infty \cos\left(\frac{\pi y}{2\delta}\right) \times \frac{\pi}{2\delta} \quad \text{...(ii)}$$

$$\therefore \quad \tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0} = \frac{mU_o\pi}{2\delta}$$

Equating τ_0 in (i),

$$\rho U_\infty^2 \left(\frac{4-\pi}{2\pi}\right)\frac{\partial\delta}{\partial x} = \frac{\mu U_\infty \pi}{2\delta}$$

$$\Rightarrow \quad (\delta)d\delta = \frac{\mu U_\infty \pi}{2\rho U_\infty^2}\left(\frac{2\pi}{4-\pi}\right)dx$$

By integrating,

$$\frac{\delta^2}{2} = \frac{\mu}{2\rho U_\infty}\left(\frac{2\pi^2}{4-\pi}\right)x + C$$

$$\Rightarrow \quad \frac{\delta^2}{2} = \frac{\mu}{\rho U_\infty}\left(\frac{\pi^2}{4-\pi}\right)x + C$$

At $x = 0$, $\delta = 0$ and so $C = 0$

$$\therefore \qquad \delta = \sqrt{\frac{2\mu}{\rho U_\infty}\left(\frac{\pi^2}{4-\pi}\right)x}$$

$$\Rightarrow \qquad \delta = \sqrt{\frac{2v}{U_\infty}\left(\frac{\pi^2}{4-x}\right)x} = \sqrt{\frac{2\times 0.1\times 10^{-4}}{5}\times\left(\frac{\pi^2}{4-\pi}\right)\times 0.5}$$

$$= 4.795 \times 10^{-3} \text{ m} = 4.795 \text{ mm}$$

(ii) Shear stress at $x = 500$ mm

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$$U = U_\infty \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

$$\therefore \qquad \left(\frac{du}{dy}\right) = U_\infty \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)\cdot\frac{\pi}{2\delta}$$

$$\therefore \qquad \left(\frac{du}{dy}\right)_{y=0} = \frac{U_\infty \pi}{2\delta}$$

$$\therefore \qquad \tau_0 = \frac{\mu U_\infty \pi}{2\delta} = \frac{(\rho v)U_\infty \pi}{2\delta}$$

$$= \frac{1.2\times 0.1\times 10^{-4}\times 5\times\pi}{2\times 4.795\times 10^{-3}} = 0.019655 \text{ N/m}^2$$

Q.2 (c) Solution:

(i)

$$\phi = (x^2 - y^2) + 3xy \qquad \text{...(1)}$$

$$u = \frac{-\partial\phi}{\partial x} = -(2x+3y)$$

But
$$u = \frac{-\partial\psi}{\partial y}$$

$$\Rightarrow \qquad \frac{-\partial\psi}{\partial y} = -(2x+3y)$$

$$\Rightarrow \qquad \frac{\partial\psi}{\partial y} = 2x + 3y$$

$$\psi = 2xy + \frac{3y^2}{2} + f(x) \quad ...(2)$$

$$v = \frac{\partial \psi}{\partial x} = 2y + f'(x) \quad ...(3)$$

But $v = \frac{-\partial \phi}{\partial y}$

$\Rightarrow$ $v = -[-2y + 3x]$

$\Rightarrow$ $v = 2y - 3x \quad ...(4)$

From (3) and (4)

$$2y + f'(x) = 2y - 3x$$

$\Rightarrow$ $f'(x) = -3x$

$\Rightarrow$ $f(x) = \frac{-3}{2}x^2$

Using (2)

$$\psi = 2xy + \frac{3}{2}y^2 + f(x)$$

Since $f(x) = \frac{-3}{2}x^2$

$\therefore$ $\psi = 2xy + \frac{3}{2}y^2 - \frac{3}{2}x^2$

$\Rightarrow$ $\psi = 2xy + \frac{3}{2}\left[y^2 - x^2\right]$

(ii) At point (1, 1) $\psi_A = \left(2\times1\times1 + \frac{3}{2}(1^2 - 1^2)\right)$

$\Rightarrow$ $\psi_A = 2$ units

At point (1, 2) $\psi_B = \left(2\times(1)\times(2) + \frac{3}{2}(2^2 - 1^2)\right)$

$\Rightarrow$ $\psi_B = \left(4 + \frac{9}{2}\right) = \frac{17}{2}$ units

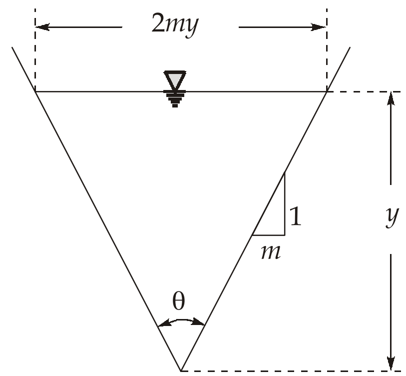
Flow rate between the stream lines passing through (1, 1) and (1, 2)

$$\Delta\psi = |\psi_A - \psi_B|$$

$$= \left|2 - \frac{17}{2}\right| = \frac{13}{2} = 6.5 \text{ units}$$

(ii)

Triangular channel section is shown in figure below:



Area of cross-section, $A = \frac{1}{2} \times 2my \times y = my^2$

Top width of the section, $T = 2my$

Given y_1 and y_2 are the alternate depths in the channel and let be the discharge through the channel be Q.

$\therefore$ Specific energy, $E_1 = y_1 + \frac{V_1^2}{2g}$

$$E_2 = y_2 + \frac{V_2^2}{2g}$$

$\therefore$ $E_1 = y_1 + \frac{Q^2}{2gA_1^2}$ and $E_2 = y_2 + \frac{Q^2}{2gA_2^2}$

$\Rightarrow$ $E_1 = y_1 + \frac{Q^2}{2g\left(m^2 y_1^4\right)}$ and $E_2 = y_2 + \frac{Q^2}{2g\left(m^2 y_2^4\right)}$

We know that, $E_1 = E_2$

$$\Rightarrow \quad y_1 + \frac{Q^2}{2g\left(m^2 y_1^4\right)} = y_2 + \frac{Q^2}{2g\left(m^2 y_2^4\right)} \quad \text{...(1)}$$

Dividing above equation with y_1,

$$1 + \frac{Q^2}{2g\left(m^2 y_1^5\right)} = \frac{y_2}{y_1} + \frac{Q^2}{2g\left(m^2 y_2^4\right) y_1}$$

$$\Rightarrow \quad 1+\frac{Q^2}{2g\left(m^2y_1^5\right)} = \frac{y_2}{y_1}+\frac{Q^2}{2g\left(m^2y_2^5\right)}\left(\frac{y_2}{y_1}\right)$$

$$\Rightarrow \quad 1+\frac{Q^2}{2g\left(m^2y_1^5\right)} = \frac{y_2}{y_1}\left[1+\frac{Q^2}{2g\left(m^2y_2^5\right)}\right] \quad \text{...(2)}$$

Now

Froude number is given by $F = \frac{V}{\sqrt{gD}}$

where V = Velocity of flow

g = Acceleration due to gravity

D = Hydraulic depth

$$\therefore \quad F_1{}^2 = \frac{V_1^2}{gD_1} = \frac{V_1^2}{g\,A_1/T_1}$$

$$\Rightarrow \quad F_1{}^2 = \frac{\left(\frac{Q^2}{m^2y_1^4}\right)}{g\times\frac{my_1^2}{2my_1}} = \frac{2Q^2}{gm^2y_1^5} \quad \text{...(3)}$$

Similarly

$$F_2{}^2 = \frac{\left(\frac{Q^2}{m^2y_2^4}\right)}{g\times\frac{my_2^2}{2my_2}} = \frac{2Q^2}{gm^2y_2^5} \quad \text{...(4)}$$

From (3) and (4)

$$\frac{F_1^2}{F_2^2} = \frac{y_2^5}{y_1^5}$$

$$\Rightarrow \quad \frac{y_2}{y_1} = \left(\frac{F_1}{F_2}\right)^{2/5} \quad \text{...(5)}$$

Substituting (3) and (4) in (2)

$$1+\frac{F_1^2}{4} = \frac{y_2}{y_1}\left[1+\frac{F_2^2}{4}\right]$$

$$\Rightarrow \qquad \frac{y_2}{y_1} = \frac{1+\frac{F_1^2}{4}}{1+\frac{F_2^2}{4}} \qquad \text{...(6)}$$

From eq. (5) and (6)

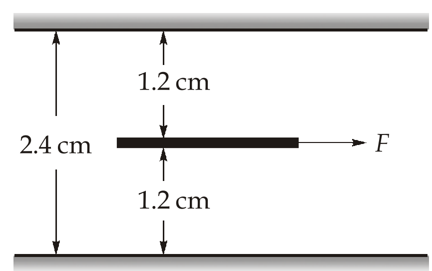
$$\left(\frac{F_1}{F_2}\right)^{2/5} = \frac{1+\frac{F_1^2}{4}}{1+\frac{F_2^2}{4}}$$

$$\Rightarrow \qquad \left(\frac{F_1}{F_2}\right)^{2/5} = \frac{4+F_1^2}{4+F_2^2}$$

$$\Rightarrow \qquad \left(\frac{F_1}{F_2}\right)^{2} = \left(\frac{4+F_1^2}{4+F_2^2}\right)^5$$

(Hence proved)

Q.3 (a) Solution:



Distance between two large plane surfaces = 2.4 cm

Area of thin plate, $A = 0.5 \text{ m}^2$

Velocity of thin plate, $u = 0.6 \text{ m/s}$

Viscosity of glycerine, $\mu = 8.10 \times 10^{-1} \text{ N s/m}^2$

Case I: When the thin plate is in the middle of the two plane surfaces [Refer to Figure]

Let, F_1 = Shear force on the upper side of the thin plate

F_2 = Shear force on the lower side of the thin plate

F = Total force required to drag the plate

Then $F = F_1 + F_2$

The shear stress (τ_2) on the upper side of the thin plate is given by equation,

$$\tau_1 = \mu\left(\frac{du}{dy}\right)_1$$

Where du = Relative velocity between thin plate and upper large plane surface

= 0.6 m/sec

dy = Distance between thin plate and upper large plane surface

= 1.2 cm = 0.012 m (plate is thin and hence thickness of plate can be neglected)

$\therefore$ $$\tau_1 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

Now shear force, F_1 = Shear stress × Area

$= \tau_1 \times A = 40.5 \times 0.5 = 20.25$ N

Similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\tau_2 = \mu\left(\frac{du}{dy}\right)_2 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

$\therefore$ Shear force, $F_2 = \tau_2 \times A = 40.5 \times 0.5 = 20.25$ N

$\therefore$ Total force, $F = F_1 + F_2 = 20.25 + 20.25 = 40.5$ N. **Ans.**

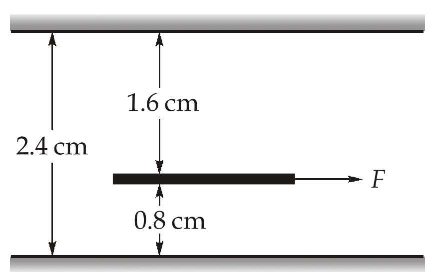
Case II: When the thin plate is at a distance of 0.8 cm from one of the plane surfaces.

Let the thin plate is at a distance of 0.8 cm from the lower plane surface.

Then distance of the plate from the upper plane surface

$= 2.4 - 0.8 = 1.6$ cm $= 0.016$ m

(Neglecting thickness of the plate)



$\therefore$ Shear force on the upper side of the thin plate,

$$F_1 = \text{Shear stress} \times \text{Area} = \tau_1 \times A$$

$$= \mu\left(\frac{du}{dy}\right)_1 \times A = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.016}\right) \times 0.5 = 15.1875 \text{ N}$$

Shear force on the lower side of the thin plate,

$$F_2 = \tau_2 \times A = \mu\left(\frac{du}{dy}\right)_2 \times A$$

$$= 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.8/100}\right) \times 0.5 = 30.375 \text{ N}$$

$\therefore$ Total force required $= F_1 + F_2 = 15.1875 + 30.375 = 45.5625$ N.

Q.3 (b) Solution:

Given:

Total length of pipe, $L = 40$ m

Length of 1st pipe, $L_1 = 25$ m

Diameter of 1st pipe, $d_1 = 150$ mm = 0.15 m

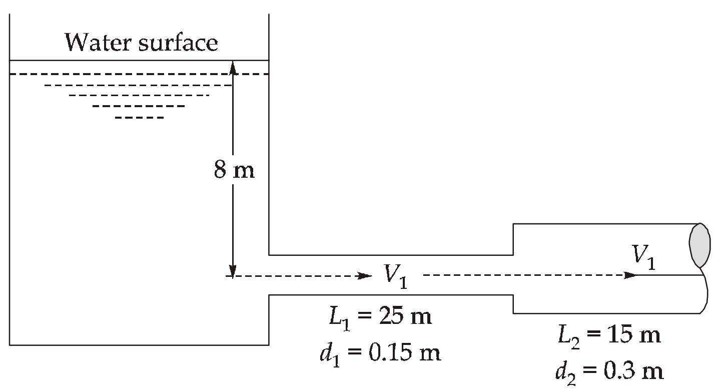
Length of 2nd pipe, $L_2 = 40 - 25 = 15$ m

Diameter of 2nd pipe, $d_2 = 300$ mm = 0.3 m

Height of water, $H = 8$ m

Coefficient of friction, $f = 0.01$

Applying Bernoulli's theorem to the free surface of water in the tank and outlet of pipe as shown in figure and taking reference line passing through the centre line of pipe,



$$0 + 0 + 8 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + \text{All losses}$$

$$\Rightarrow \quad 8.0 = 0 + \frac{V_2^2}{2g} + h_i + h_{f_1} + h_e + h_{f_2} \quad \text{...(i)}$$

where h_i = loss of head at entrance = $0.5\dfrac{V_1^2}{2g}$

h_{f_1} = head lost due to friction in pipe 1 = $\dfrac{4fL_1V_1^2}{2d_1g}$

h_e = head lost due to sudden enlargement = $\dfrac{(V_1 - V_2)^2}{2g}$

h_{f_2} = head lost due to friction in pipe 2 = $\dfrac{4fL_2V_2^2}{2d_2g}$

But from continuity equation, we have

$$A_1V_1 = A_2V_2$$

$$\Rightarrow \quad V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}d_2^2 \times V_2}{\frac{\pi}{4}d_1^2} = \left(\frac{d_2}{d_1}\right)^2 V_2 = \left(\frac{0.3}{0.15}\right)^2 V_2 = 4V_2$$

Substituting the value of V_1 in various head losses, we have

$$h_1 = \frac{0.5V_1^2}{2g} = \frac{0.5\times(4V_2)^2}{2g} = \frac{8V_2^2}{2g}$$

$$h_{f_1} = \frac{4\times 0.01\times 25\times(4V_2)^2}{0.15\times 2\times g} = \frac{4\times 0.01\times 25\times 16}{0.15}\times\frac{V_2^2}{2g} = 106.67\frac{V_2^2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

$$h_{f_2} = \frac{4\times 0.01\times 15\times V_2^2}{0.3\times 2g} = \frac{4\times 0.01\times 15}{0.3}\times\frac{V_2^2}{2g} = 2\left(\frac{V_2^2}{2g}\right)$$

Substituting the values of these losses in equation (i), we get

$$8.0 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67\frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2\frac{V_2^2}{2g}$$

$$\Rightarrow \quad 8.0 = \frac{V_2^2}{2g}[1 + 8 + 106.67 + 9 + 2] = 126.67\frac{V_2^2}{2g}$$

$$\Rightarrow \quad V_2 = \sqrt{\frac{8.0\times 2\times g}{126.67}} = \sqrt{\frac{8.0\times 2\times 9.81}{126.67}} = \sqrt{1.2391} = 1.113 \text{ m/s}$$

$$\therefore \quad \text{Rate of flow, } Q = A_2V_2 = \frac{\pi}{4}(0.3)^2 \times 1.113 = 0.07867 \text{ m}^3\text{/s} = 78.67 \text{ litres/s}$$

Q.3 (c) Solution:

Given parameters:

1. Velocity, V
2. Length of the body, l
3. Dynamic viscosity of the fluid, μ
4. Density of the fluid, ρ

5. Gravitational acceleration, g
6. Resistance, R

There are 6 physical quantities with 3 fundamental units.

Hence, number of p-terms = 6 – 3 = 3

Choosing length l, velocity V and density ρ as the 3 repeating variables and establishing the π-terms.

(i) $$\pi_1 = l^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot R$$

$$\Rightarrow \quad [M^\circ][L^\circ][T^\circ] = [L]^{a_1}\left[\frac{L}{T}\right]^{b_1}\left[\frac{M}{L^3}\right]^{C_1}\left[MLT^{-2}\right]^1$$

Equating exponents of M, L and T respectively,

Mass: M $\quad c_1 + 1 = 0$

$\Rightarrow \quad c_1 = -1 \quad$ ...(1)

Length: L $\quad a_1 + b_1 - 3c_1 + 1 = 0 \quad$ ...(2)

Time: T $\quad -b_1 - 2 = 0$

$\Rightarrow \quad b_1 = -2 \quad$ ...(3)

Substituting $\quad c_1 = -1$ and $b_1 = -2$ in equation (2)

$$a_1 - 2 - 3(-1) + 1 = 0$$

$\Rightarrow \quad a_1 = -2$

$$\therefore \quad \pi_1 = l^{-2} \cdot V^{-2} \cdot \rho^{-1} \cdot R = \frac{R}{l^2V^2\rho}$$

(ii) $$\pi_2 = l^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot \mu$$

$$\Rightarrow \quad [M^\circ][L^\circ][T^\circ] = [L]^{a_2}\left[\frac{L}{T}\right]^{b_2}\left[\frac{M}{L^3}\right]^{C_2}\left[\frac{M}{LT}\right]^1$$

Equating exponents of M, L and T respectively,

Mass: M $\quad c_2 + 1 = 0$

$\Rightarrow \quad c_2 = -1 \quad$ ...(4)

Length: L $\quad a_2 + b_2 - 3c_2 - 1 = 0 \quad$ ...(5)

Time T $\quad -b_2 - 1 = 0$

$\Rightarrow \quad b_2 = -1 \quad$ ...(6)

Putting the values of b_2 and c_2 in equation (5), we get

$$a_2 + (-1) - 3(-1) - 1 = 0$$

$$\therefore \quad a_2 = -1, c_2 = -1, b_2 = -1$$

$$\therefore \quad \pi_2 = \frac{\mu}{lV\rho}$$

(iii) $$\pi_3 = l^{a_3} \cdot V^{b_3} \cdot \rho^{c_3} \cdot g$$

$$\Rightarrow \quad [M^\circ][L^\circ][T^\circ] = [L]^{a_3}\left[\frac{L}{T}\right]^{b_3}\left[\frac{M}{L^3}\right]^{C_3}\left[\frac{L}{T^2}\right]^{1}$$

Equating exponents of M, L and T respectively,

Mass: M $\quad c_3 = 0$ ...(7)

Length: L $\quad a_3 + b_3 - 3c_3 + 1 = 0$ ...(8)

Time: T $\quad b_3 = -2$ ...(9)

On putting the value of c_3 and b_3 in equation (8), we get

$$a_3 = 1, b_3 = -2, c_3 = 0$$

$$\therefore \quad \pi_3 = l^1 \cdot V^{-2} \cdot \rho^0 \cdot g$$

$$\therefore \quad \pi_3 = \frac{lg}{V^2}$$

Thus the functional relationship becomes as following

$$\phi\left(\frac{R}{l^2V^2\rho}, \frac{\mu}{\rho V l}, \frac{lg}{V^2}\right) = 0$$

$$\Rightarrow \quad \frac{R}{l^2V^2\rho} = \phi\left(\frac{\mu}{\rho V l}, \frac{lg}{V^2}\right)$$

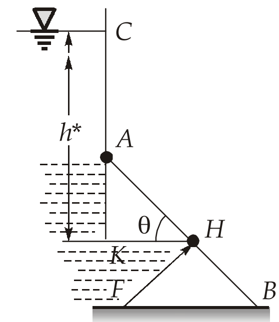
$$\Rightarrow \quad \frac{R}{l^2V^2\rho} = \phi\left(\frac{\rho V l}{\mu}, \frac{V^2}{lg}\right)$$

The above step has been made on the postulate that reciprocal of π-term and its square root is non-dimensional.

$$\therefore \quad R = l^2V^2\rho\phi\left(\frac{\rho V l}{\mu}, \frac{V}{\sqrt{lg}}\right)$$

The resistance R is a function of reynolds number $\left(\frac{\rho V l}{\mu}\right)$ and Froude's number $\left(\frac{V}{\sqrt{lg}}\right)$

Q.4 (a) Solution:



Given:

Width of gate, $B = 2$ m; Length of gate $L = 3$ m

$\therefore$ Area, $A = 2 \times 3 = 6\ \text{m}^2$

Weight of gate and fixed weight $W = 343350$ N

Angle of inclination, $\theta = 45°$

Let h is the required height of water.

Depth of C.G. of the gate and weight = $\bar{h}$

From figure $\bar{h} = h - ED = h - (AD - AE)$

$$= h - (AB \sin\theta - EG \tan\theta) \qquad \left\{\because \tan\theta = \frac{AE}{EG} \therefore AE = EG\tan\theta\right\}$$

$$= h - (3 \sin 45° - 0.6 \tan 45°)$$

$$= h - (2.121 - 0.6) = (h - 1.521) \text{ m}$$

The total pressure force, F is given by

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 6 \times (h - 1.521)$$

$$= 58860\ (h - 1.521) \text{ N.}$$

The total force F is acting at the centre of pressure as shown in figure at H. The depth of H from free surface is given by h^* which is equal to

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}, \text{ where } I_G = \frac{bd^3}{12} = \frac{2\times 3^3}{12} = \frac{54}{12} = 4.5 \text{ m}^4$$

$$\therefore \qquad h^* = \frac{4.5\times\sin^2 45}{6\times(h-1.521)} + (h-1.521) = \frac{0.375}{(h-1.521)} + (h-1.521)\text{m}$$

Now taking moments about hinge A, we get

$$343350 \times EG = F \times AH$$

$$\Rightarrow \qquad 343350 \times 0.6 = F\times\frac{AK}{\sin 45°}$$

$$\left[\text{From } \Delta AKH,\ AK = AH \sin\theta = AH \sin 45° \ \therefore\ AH = \frac{AK}{\sin 45°}\right]$$

$$= \frac{58860(h-1.521)\times AK}{\sin 45°}$$

$\therefore \quad AK = \dfrac{343350 \times 0.6 \times \sin 45°}{58860(h-1.521)} = \dfrac{0.3535 \times 7}{(h-1.521)}$ ...(i)

But $\quad AK = h^* - AC = \dfrac{0.375}{(h-1.521)} + (h-1.521) - AC$ ...(ii)

But $\quad AC = CD - AD = h - AB \sin 45° = h - 3 \times \sin 45° = h - 2.121$

$\therefore$ Substituting this value of AC in (ii), we get

$$AK = \frac{0.375}{h-1.521} + (h-1.521) - (h-2.121)$$

$$= \frac{0.375}{h-1.521} + 2.121 - 1.521 = \frac{0.375}{h-1.521} + 0.6 \quad \text{...(iii)}$$

Equating the two values of AK from (i) and (iii)

$$\frac{0.3535 \times 7}{h-1.521} = \frac{0.375}{h-1.521} + 0.6$$

$\Rightarrow \quad 0.3535 \times 7 = 0.375 + 0.6\,(h - 1.521) = 0.375 + 0.6\,h - 0.6 \times 1.521$

$\Rightarrow \quad 0.6h = 2.4745 - 0.375 + 0.6 \times 1.521 = 2.0995 + 0.9126 = 3.0121$

$\Rightarrow \quad h = \dfrac{3.0121}{0.6} = 5.02 \text{ m}$ **Ans.**

Q.4 (b) Solution:

(i)

Radial flow reaction turbine: Radial flow turbines are those turbines in which the water flows in the radial direction. The water may flow radially from outwards to inwards (i.e., towards the axis of rotation) or from inwards to outwards (i.e. away from axis of rotation). If the water flows from outwards to inwards through the runner, the turbine is known as inward radial flow turbine. And if the water flows from inwards to outwards, the turbine is known as outward radial flow turbine.

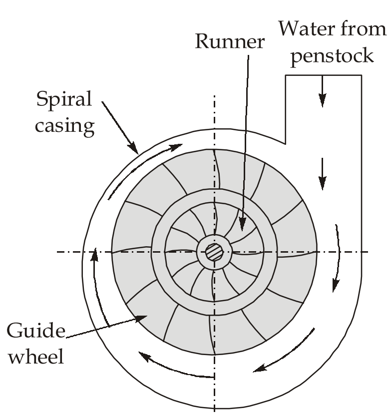
Reaction turbine means that the water at the inlet of the turbine possesses kinetic energy as well as pressure energy. As the water flows through the runner, a part of pressure energy goes on changing into kinetic energy. Thus the water through the runner is under pressure. The runner is completely enclosed in an air-tight casing and casing and the runner is always full of water.

Main parts of a radial flow reaction turbine: The main parts of a radial flow reaction turbine are:

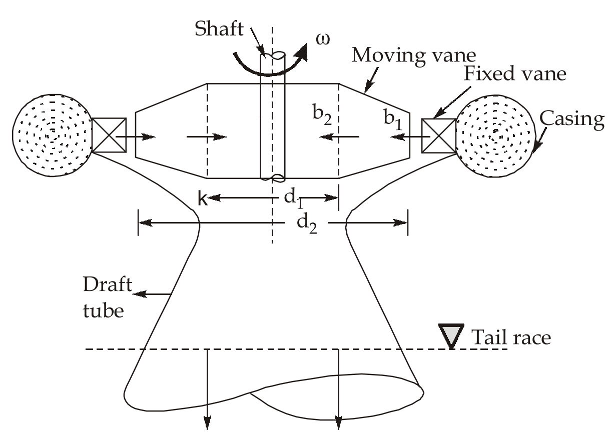
1. **Casing:** In case of reaction turbine, casing and runner are always full of water. The water from the penstock enters the casing which is of spiral shape in which area of

cross-section of the casing goes on decreasing gradually. The casing completely surrounds the runner of the turbine. The casing as shown in figure is made of spiral shape, so that the water may enter the runner at constant velocity throughout the circumference of the runner. The casing is made of concrete, cast steel or plate steel.

2. **Guide Mechanism:** It consists of a stationary circular wheel all round the runner of the turbine. The stationary guide vanes are fixed on the guide mechanism. The guide vanes allow the water to strike the vanes fixed on the runner without shock at inlet. Also by a suitable arrangement, the width between two adjacent vanes of guide mechanism can be altered so that the amount of water striking the runner can be varied.
3. **Runner:** It is a circular wheel on which a series of radial curved vanes are fixed. The surface of the vanes are made very smooth. The radial curved vanes are so shaped that the water enters and leaves the runner without shock. The runners are made of cast steel, cast iron or stainless steel. They are keyed to the shaft.
4. **Draft-tube:** The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure. The water at exit cannot be directly discharged to the tail race. A tube or pipe of gradually increasing area is used for discharging water from the exit of the turbine to the tail race. This tube of gradually increasing area is called as draft tube.



Spiral Casing



Cross-sectional view of reaction turbine

(ii)

Given:

$\eta_0 = 0.75$, S.P. = 150 kW, $H = 7.5$ m, $N = 160$ rpm, hydraulic lossers = 20% of available energy

$$\text{Peripheral velocity, } u_1 = 0.25\sqrt{2 \times 9.81 \times 7.5} = 3.033 \text{ m/s}$$

Velocity of flow at inlet, $V_{f_1} = 0.95\sqrt{2\times 9.81\times 7.5} = 11.524 \text{ m/s}$

Since the discharge at the outlet is radial,

$$\therefore \quad V_{w_2} = 0 \text{ and } V_{f_2} = V_2$$

Hydraulic efficiency is given as

$$\eta_h = \frac{\text{Total head at inlet – Hydraulic loss}}{\text{Head at inlet}} = \frac{H - 0.2H}{H}$$

$$\Rightarrow \quad \eta_h = 0.8$$

But

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

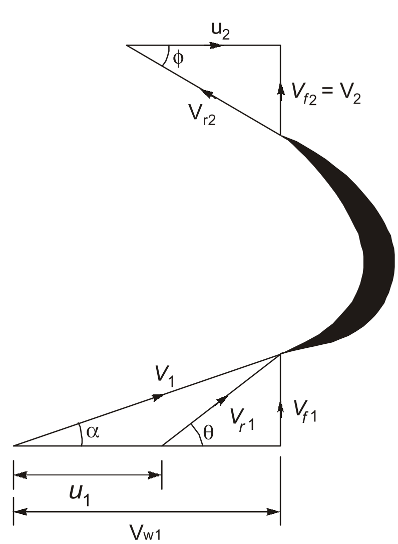
$$\Rightarrow \quad 0.8 = \frac{V_{w_1}(3.033)}{9.81\times 7.5}$$

$$\Rightarrow \quad V_{w_1} = 19.4 \text{ m/s}$$

1. The guide blade angle i.e., α

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{11.524}{19.4}$$

$$\therefore \quad \alpha = 30.711°$$



2. The wheel vane angle at inlet *i.e.* θ

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{11.524}{19.4 - 3.033}$$

$$\therefore \quad \theta = 35.15°$$

3. Diameter of wheel at inlet (D_1)

We know
$$u_1 = \frac{\pi D_1 N}{60}$$

$$\Rightarrow \quad 3.033 = \frac{\pi \times D_1 \times (160)}{60}$$

$$\Rightarrow \quad D_1 = 0.362 \text{ m} = 36.2 \text{ cm}$$

4. Width of the wheel at inlet (B_1)

$$\eta_0 = \frac{S.P.}{W.P.} = \frac{150 \times 1000}{\rho g Q H}$$

$$\Rightarrow \quad 0.75 = \frac{150 \times 1000}{1000 \times 9.81 \times Q \times 7.5}$$

$$\Rightarrow \quad Q = 2.718 \text{ m}^3/\text{s}$$

Also,
$$Q = \pi D_1 B_1 V_{f_1}$$

$$\Rightarrow \quad 2.718 = \pi \times 0.362 \times B_1 \times 11.524$$

$$\Rightarrow \quad B_1 = 0.207 \text{ m} = 20.7 \text{ cm}$$

Q.4 (c) Solution:

Given:

$$D_1 = 0.4 \text{ m}; D_2 = 0.2 \text{ m}, L = 2 \text{ m}, Q = 20 \text{ lit/s} = 0.02 \text{m}^3/\text{s}$$

(i) **Convective acceleration at middle when Q = 20 lit/s.**

In this case, the rate of flow is constant and equal to $0.02\text{m}^3/\text{s}$. The velocity of flow is in x-direction only. Hence this is one-dimensional flow and velocity components in y and z directions are zero i.e., $v = 0$ $w = 0$.

$$\therefore \quad \text{Convective acceleration} = u\frac{\partial u}{\partial x} \quad \text{...(i)}$$

Let us find the value of u and $\frac{\partial u}{\partial x}$ at a distance x from inlet.

The diameter (D_x) at a distance x from inlet or at section X-X is given by,

$$D_x = 0.4 - \frac{0.4 - 0.2}{2} \times x = (0.4 - 0.1x)\text{m}$$

The area of cross-section (A_x) at section X-X is given by,

$$A_x = \frac{\pi}{4}D_x^2 = \frac{\pi}{4}(0.4 - 0.1x)^2$$

Velocity (u) at the section X-X in terms of Q (i.e., in terms of rate of flow) is,

$$u = \frac{Q}{\text{Area}} = \frac{Q}{A_x} = \frac{Q}{\frac{\pi}{4}D_x^2} = \frac{4Q}{\pi(0.4-0.1x)^2}$$

$$= \frac{1.273Q}{(0.4-0.1x)^2} = 1.273Q(0.4-0.1x)^{-2} \text{ m/s} \quad ...(ii)$$

To find $\frac{\partial u}{\partial x}$, we must differentiate equation (ii) with respect to x.

$$\therefore \quad \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left[1.273Q(0.4-0.1x)^{-2}\right]$$

$$= 1.273 \times (-2)Q[0.4 - 0.1x]^{-3} \times (-0.1)$$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = 0.2546Q[0.4 - 0.1x]^{-3} \quad ...(iii)$$

So, convective acceleration at $x = 1$ m is,

$$\frac{u\partial u}{\partial x} = 1.273Q\,[0.4 - 0.1x]^{-2} \times 0.2546Q[0.4 - 0.1x]^{-3}$$

$$= 1.273\,(0.02)\,(0.4 - 0.1)^{-2} \times 0.2546\,(0.02)\,(0.4 - 0.1)^{-3}$$

$$= 0.05335 \text{ m/sec}^2$$

Case II : Total acceleration at middle of pipe at 15th second.

Here Q changes from 0.02 m^3/s to 0.04 m^3/s in 30 seconds and we need to find the total acceleration at x = 1 m and t = 15 seconds.

Total acceleration = Convective acceleration + Local acceleration (at t = 15 seconds)

Rate of flow at t = 15 seconds is given by

$$Q = Q_1 + \frac{Q_2 - Q_1}{30} \times 15 \text{ where } Q_2 = 0.04 \text{ m}^3\text{/s and } Q_1 = 0.02 \text{ m}^3\text{/s}$$

$$= 0.02 + \frac{(0.04-0.02)}{30} \times 15 = 0.03 \text{ m}^3\text{/s}$$

The velocity (u) and velocity gradient $\left(\frac{\partial u}{\partial x}\right)$ in terms of Q are given by equations (ii) and (iii) respectively.

$$\therefore \quad \text{Convective acceleration} = u \cdot \frac{\partial u}{\partial x}$$

$$= [1.273Q\,(0.4 - 0.1x)^{-2}] \times [0.2546Q\,(0.4 - 0.1x)^{-3}]$$

$$= 1.273 \times 0.2546Q^2 \times (0.4 - 0.1 \times 1)^{-5}$$

∴ Convection acceleration (when Q = 0.03 m^3/s and x = 1 m)

$$= 0.12 \text{m/s}^2 \quad \text{...(iv)}$$

$$\text{Local acceleration} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}[1.273Q\,(0.4 - 0.1x)^{-2}]$$

[$\because$ u from equation (ii) is $u = 1.273Q\,(0.4 - 0.1x)^{-2}$]

$$= 1.273 \times (0.4 - 0.1x)^{-2}\frac{\partial Q}{\partial t}$$

$$= 1.273 \times (0.3)^{-2} \times \frac{0.02}{30} = 0.00943 \text{m/s}^2 \quad \text{...(v)}$$

$$\left[\because \frac{\partial Q}{\partial t} = \frac{Q_2 - Q_1}{t} = \frac{0.04 - 0.02}{30} = \frac{0.02}{30}\text{m}^3/\text{s}^2\right]$$

Hence adding equations (iv) and (v), we get total acceleration.

$\therefore$ Total acceleration = Convective acceleration + Local acceleration

$$= 0.12 + 0.00943 = 0.1294 \text{ m/s}^2$$

Section B : Water Resource Engineering and Hydrology

Q.5 (a) Solution:

Total precipitation = 20 + 40 + 0 + 30 + 50 + 40 + 5 = 185 mm

Total discharge observed = 0 + 8 + 19 + 34 + 68 + 58 + 48 + 40 + 25 + 19 + 15 + 11 + 6 + 3 + 0 = 354 m^3/s

$\therefore$ Volume of rainfall = 3600 × 354 m^3

ϕ-index:

Assuming the value of ϕ-index to be more than 5 mm/h

$$\therefore \left[\frac{(20-\phi)+(40-\phi)+(30-\phi)+(50-\phi)+(40-\phi)}{1000}\right]\times 12\times 10^6 = 354 \times 3600$$

$$\Rightarrow \phi = 14.76 \text{ mm/hr} > 5 \text{ mm/hr}$$

$\therefore$ The assumption is correct.

W-index:

Evaporation loss + Seepage loss,

$$= 1.5 \times 3 = 4.5 \text{ mm}$$

$$\therefore \frac{180 - 4.5 - 5W}{1000}\times 12\times 10^6 = 354 \times 3600$$

$$\Rightarrow \text{W-index} = 13.86 \text{ mm/hr}$$

Q.5 (b) Solution:

Given:

$$R = 300 \text{ m}$$
$$r = 20 \text{ cm} = 0.20 \text{ m}$$
$$S = 4 \text{ m}$$
$$b = 25 \text{ m}$$
$$T = 125 \times 10^{-4} \text{ m}^2/\text{sec}$$

We know,

Coefficient of transmissibility, $T = K\,b$

$$\Rightarrow \quad K = \frac{T}{b} = \frac{125\times10^{-4}}{25} = 5 \times 10^{-4} \text{ m/sec}$$

So, coefficient of permeability is 5×10^{-4} m/sec

Now,

$$Q = \frac{2\pi Kb(H-h)}{\log_e\left(\frac{R}{r}\right)} = \frac{2\pi Kb(H-h)}{2.303\log_{10}\left(\frac{R}{r}\right)} = \frac{2.72\,Kb(H-h)}{\log_{10}\left(\frac{R}{r}\right)}$$

But $\quad S = (H-h)$

$$\therefore \quad Q = \frac{2.72\,KbS}{\log_{10}\left(\frac{R}{r}\right)}$$

$$\Rightarrow \quad Q = \frac{2.72\times5\times10^{-4}\times25\times4}{\log_{10}\left(\frac{300}{0.2}\right)}$$

$$\Rightarrow \quad Q = \frac{1360\times10^{-4}}{3.1761} = 428.198 \times 10^{-4} \text{ m}^3/\text{sec}$$

$$\Rightarrow \quad Q = 0.0428 \text{ m}^3/\text{sec} = 42.8 \text{ lt/s}$$

Q.5 (c) Solution:

Advantages:

1. It reduces the loss of water due to seepage and hence the duty is enhanced.
2. It controls the water logging and hence the ill effects of water-logging are eliminated.
3. It provides smooth surface and hence the velocity of flow can be increased.

4. Due to the increased velocity the discharge capacity of canal also gets increased.
5. Due to the increased velocity, the evaporation loss also gets reduced.
6. It eliminates the effect of scouring in the canal bed.
7. The increased velocity eliminates the possibility of silting in the canal bed.
8. It controls the growth of weeds along the canal sides and bed.
9. It provides the stable section of the canal.
10. It reduces the requirement of large lands for the canal, because smaller section of the canal can produce greater discharge.
11. It prevents the sub-soil salt to come in contact with the canal water.
12. It reduces the maintenance cost of the canals.

Disadvantages

1. The initial cost of the canal lining is very high. So, it makes the project very expensive with respect to the output.
2. It involves much difficulties for repairing the demaged section of lining.
3. It takes too much time to complete the project work.
4. It becomes difficult, if the outlets are required to be shifted or new outlets are required to be provided, because the dismantling of the lined section is difficult.

Q.5 (d) Solution:

Available moisture = Field capacity – Permanent wilting point = 35 – 18 = 17%.

Readily available moisture is 75% of the available moisture,

$$\text{Readily available moisture} = 17 \times 0.75 = 12.75\%$$

$$\text{Optimum moisture content} = 35 - 12.75 = 22.25\%$$

Now, by applying irrigation water, the moisture content is to be raised from 22.25% to 35%.

From $$d_w = \frac{\gamma_S \times d}{\gamma_W} \times [F_c - M_o]$$

Here, $$\gamma_S = 1.5 \text{ g/cm}^3, \gamma_w = 1 \text{ g/cm}^3$$

$$d = 70 \text{ cm} = 0.70 \text{ m}$$

$$F_c = 35\% = 0.35, M_o = 22.25\% = 0.2225$$

Depth of water, $$d_w = \frac{1.5 \times 0.70}{1} \times [0.35 - 0.225]$$

$$\Rightarrow \quad d_w = 1.05 \times 0.1275$$

$$\Rightarrow \quad d_w = 0.133875 \text{ m} = 13.39 \text{ cm}$$

Daily consumptive use of water (C_u) = 17 mm = 1.7 cm

From, $$f_w = \frac{d_w}{C_u}$$

Frequency of irrigation, $$f_w = \frac{13.39}{1.7} = 7.87 \simeq 7 \text{ days (say)}$$

Irrigation water will be required before 7.87 days i.e. 7 days (say)
Hence, water should be applied in the field at an interval of 7 days.

Q.5 (e) Solution:

Comparison between Kennedy's theory and Lacey's yheory:

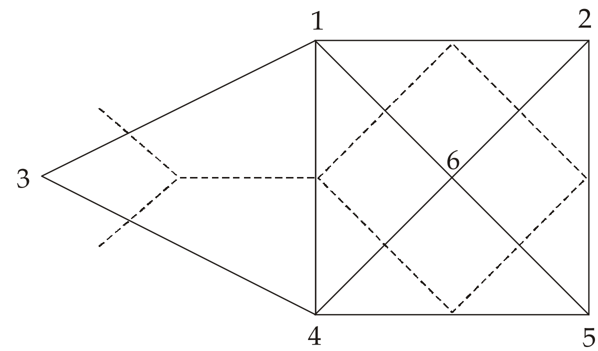
Kennedy's theory	Lacey's theory
1. It states that the silt carried by the flowing water is kept in suspension by the vertical component of eddies which are generated from the bed of the channel.	1. It states that the silt carried by the flowing water is kept in suspension by the vertical component of eddies which are generated from the entire wetted perimeter of the channel.
2. It gives relation between 'V' and 'D'.	2. It gives relation between 'V' and 'R'.
3. In this theory, a factor known as critical velocity ratio 'm' is introduced to make the equation applicable to different channels with different silt grades.	3. In this theory, a factor known as silt factor 'f' is introduced to make the equation applicable to different channels with different silt grades.
4. In this theory, Kutter's equation is used for finding the mean velocity of flow.	4. This theory gives an equation for finding the mean velocity of flow.
5. This theory gives no equation for bed slope.	5. This theory gives an equation for bed slope.
6. In this theory, the design is based on trial-and-error method.	6. This theory does not involve trial and error method.

Drawbacks in Lacey's Theory: The followings are the drawbacks of Lacey's theory:

1. The concept of true regime is theoretical and cannot be achieved practically.
2. The various equations are derived by considering the silt factor ⇒ which is not at all constant.
3. The concentration of silt is not taken into account.
4. Silt grade and silt charge are not clearly defined.
5. The equations are empirical and based on the available data from a particular type of channel. So, it may not be true for a different type of channel.
6. The characteristics of regime channel may not be same for all cases.

Q.6 (a) Solution:

(i)



Station-1:

$$\text{Precipitation, } P_1 = 10 \text{ cm}$$

$$\text{Thiessen's polygon area, } A_1 = \frac{1}{2}\times\frac{b}{2}\times\frac{b}{2}+\frac{1}{3}\left[\frac{1}{2}\times b\times b\sin 60°\right]$$

$$\Rightarrow \quad A_1 = \frac{1}{2}\times 4\times 4+\frac{1}{3}\left[\frac{1}{2}\times 8\times\frac{8\sqrt{3}}{2}\right] = 17.238 \text{ km}^2$$

Section-2:

$$\text{Precipitation, } P_2 = 6 \text{ cm}$$

$$A_2 = \frac{1}{2}\times 4\times 4 = 8\,\text{km}^2$$

Section-3:

$$\text{Precipitation, } P_3 = 3 \text{ cm}$$

$$A_3 = \frac{1}{3}\times\left(\frac{1}{2}\times 8\times\frac{8\sqrt{3}}{2}\right) = 9.238\,\text{km}^2$$

Section-4:

$$\text{Precipitation, } P_4 = 12 \text{ cm}$$

$$A_4 = A_1 = 17.238 \text{ km}^2$$

Section-5:

$$\text{Precipitation, } P_5 = 3.6 \text{ cm}$$

$$A_5 = A_2 = 8 \text{ km}^2$$

Section-6:

$$\text{Precipitation, } P_6 = 8.4 \text{ cm}$$

$$A_6 = (8\times 8) - (4\times 8) = 32 \text{ km}^2$$

$$\text{Mean precipitation} = \frac{\sum_{i=t}^{6} P_i A_i}{\sum_{i=t}^{6} A_i}$$

$$= \frac{(10\times17.238)+(6\times8)+(3\times9.238)+(12\times17.238)+(3.6\times8)+(8.4\times32)}{17.238+8+9.238+17.238+8+32}$$

$$= 8.205 \text{ cm}$$

(ii) **Rain:** It is the principal form of precipitation in India. The term rainfall is used to describe precipitation in the form of water drops of size larger than 0.5 mm. The maximum size of a raindrop is about 6 mm. Any drop larger in size than this tends to break up into drops of smaller sizes during its fall from the clouds. On the basis of its intencity, rainfall is classified as:

Type	Intersity
Light rain	Trace to 2.5 mm/h
Moderate rain	2.5 mm/h to 7.5 mm/h
Heavy rain	> 7.5 mm/h

- **Snow:** Snow is another important form of precipitation. Snow consists of ice crystals which usually combine to form flakes. When fresh, snow has an initial density varying from 0.06 to 0.15 g/cm^3 and it is usual to assume an average density of 0.1 g/cm^3. In India, snow occurs only in this Himalayan regions.
- **Drizzle:** A fine sprinkle ot numerous water droplets of size lees than 0.5 mm and intensity less than 1 mm/h is known as drizzle. In this, the drops are so small that they appear to float in the air.
- **Glaze:** When rain or drizzle comes in contact with cold ground at around 0°C, the water drops freeze to form an ice coating called glaze or freezing rain.
- **Sleet:** It is frozen raindrops of transparent grains which form when rain falls through air at subfreezing temperature. In Britain, sleet denotes precipitation of snow and rain simultaneously.
- **Hail:** It is a showery precipitation the form of irregular pellets or lumps of ice of size more man 8 mm. Hails occur in violent thunderstorms in which vertical currents are very strong.

Measurement of precipitation:

Precipitation is expressed in terms of the depth to which rainfall water would stand on an area if all the rain were collected on it. Thus 1 cm of rainfall over a catchment area of 1 km^2 represents a volume of water equal to 10^4 m^3. In the case of snowfall, an equivalent depth of water is used as the depth of precipitation. The precipitation is collected and measured in a raingauge. Terms such as pluviometer, ombrometer and hyetometer are also sometimes used to designate a raingauge.

A raingauge essentially consists of a cylindrical-vessel assembly kept in the open to collect rain. The rainfall catch of the raingauge is affected by its exposure conditions. To enable the catch of raingauge to accurately represent the rainfall in the area surrounding the raingauge, standard settings are adopted. For placing a raingauge the following considerations are important:

- The ground must be level and in the open and the instrument must present a horizontal catch surface.
- The rain gauge must be set as near the ground as possible to reduce wind effects but it must be sufficiently high to prevent splashing, flooding, etc.
- The instrument must be surrounded by an open fenced area of at least 5.5 m × 5.5 m. No object should be nearer to the instrument than 30 m or twice the height of the obstruction.

Raingauge can be broadly classified into two categories as (i) non-recording raingauges and (ii) recording gauges.

Q.6 (b) Solution:

(i)

$$C_a = 1200 \text{ ppm at 0.6 m above bed}$$

$$V_s = 4 \text{ cm/s} = 0.04 \text{ m/s}$$

$$a = 0.6 \text{ m}, D = 6 \text{ m}$$

$$y = 6 - 2 = 4 \text{ m above the bottom}$$

$$R = D = 6 \text{ m (wide stream)}$$

Shear friction velocity,

$$V_* = \sqrt{gRS} = \sqrt{9.81 \times 6 \times \frac{1}{5000}} = 0.1085 \text{ m/s}$$

$$\frac{V_s}{V_* K} = \frac{0.04}{0.1085 \times 0.4} = 0.922$$

$$\frac{C}{C_a} = \left[\frac{a}{y} \times \frac{D-y}{D-a}\right]^{\frac{V_s}{V_* K}}$$

$$\Rightarrow \quad C = 1200\left[\frac{0.6}{4} \times \frac{6-4}{6-0.6}\right]^{0.922}$$

$$\Rightarrow \quad C = 83.526 \text{ mg}/l$$

(ii)

The consumptive use is computed from the Blaney-Criddle equation is :

$$C_u = K\Sigma f$$

Where, $$f = \frac{p}{40}(1.8t+32)$$

Month	$t°(C)$	$t°(F) = 1.8t + 32$	$p(\%)$	C_u
August	22	71.6	7.20	10.31
September	19	66.2	7.18	9.506
October	18.5	65.3	7.50	9.795
November	16	60.8	7.30	8.876

$\Sigma C_u = 38.487$ cm

$\therefore$ $C_u = 38.487$ cm

$R_e = 1.5 + 0.6 = 2.1$ cm

$CIR = C_u - R_e = 38.487 - 2.1 = 36.387$ cm

$NIR = CIR$ (since no water is used for deep percolation)

$$\therefore \quad FIR = \frac{NIR}{\mu_a} = \frac{36.387}{0.75} = 48.516 \text{ cm}$$

(iii) A good irrigation water is the one which performs the intended functions without any side effects which retard the plant growth. Irrigation water may be said to be unsatisfactory for its intended use if it contains: (1) chemicals toxic to plants or the persons using plant as food (2) chemicals which reacts with the soil to produce unsatisfactory moisture characteristics and (3) bacteria injurious to persons or animals eating plants irrigated with the water.

A good irrigation water must serve the following purposes:

- It acts as a solvent for the nutrients. Water forms the solution of the nutrients, and this solution is absorbed by the roots. Thus, water acts as the nutrient carrier.
- The irrigation water supplies moisture which is essential for the life of bacteria beneficial to the plant growth.
- Irrigation water supplies moisture which is essential for the chemical action within the plant leading to its growth.
- Some salts present in soil react to produce nourishing food products only in the presence of water.
- Water cools the soil and the atmosphere, and thus makes more favourable environment for healthy plant growth.
- Irrigation water, with controlled supplies, washes out or dilutes salts in the soil.

- It reduces the hazard of soil piping.
- It softens the tillage pans.

Q.6 (c) Solution:

Total rainfall in 12 hours = 2 + 2.5 + 7.6 + 3.8 + 10.6 + 5 + 7 + 10 + 6.4 + 3.8 + 1.4 + 1.4 = 61.5 cm

Total run-off in 12 hours = 27.5 cm

∴ Total infiltration in 12 hours,

= 61.5 – 27.5 = 34 cm

∴ Average infiltration rate = $\frac{34}{12} = 2.833$ cm/hr

It will be observed from the data that rainfall intensity is less than infiltration rate in 1st, 2nd, 11th and 12th hour

Hence during these hours, rate of infiltration will be equal to rainfall. In rest of period, the rainfall is more than infiltration.

Hence if 'f' is average rate of infiltration during remaining 8 hrs then

$$8f + 2 + 2.5 + 1.4 + 1.4 = 34$$

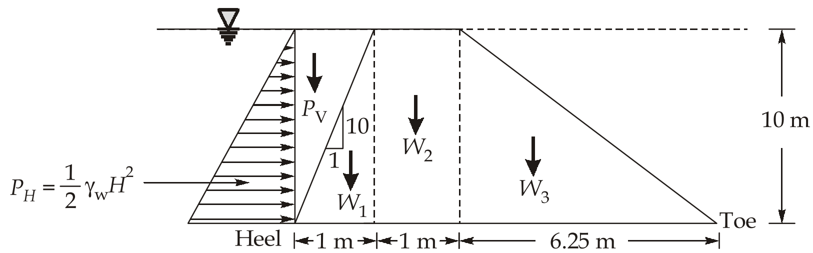
$$\Rightarrow \quad f = 3.3375 \text{ cm/hour} \simeq 3.34 \text{ cm/hour}$$

Average depth of hourly rainfall excess is computed as below.

Time (hr)	Rainfall (cm)	Rainfall excess from			Avg. rainfall excess			Total hourly rainfall
		A_1 $\phi = 7.5$ cm/h	A_2 $\phi = 4$ cm/h	A_3 $\phi = 0.8$ cm/h	Col. (3)×4/15	Col. (4)×6/15	Col. (5)×5/15	Excess in cm/hr over entire basin
1	2.0	–	–	1.2	–	–	0.4	0.4
2	2.5	–	–	1.7	–	–	0.567	0.567
3	7.6	0.1	3.6	6.8	0.0267	1.44	2.267	3.73
4	3.8	–	–	3	–	–	1	1
5	10.6	3.1	6.6	9.8	0.8267	2.64	3.267	6.73
6	5	–	1	4.2	–	0.4	1.4	1.8
7	7	–	3	6.2	–	1.2	2.067	3.267
8	10	2.5	6	9.2	0.667	2.4	3.067	6.134
9	6.4	–	2.4	5.6	–	0.96	1.867	2.827
10	3.8	–	–	3	–	–	1	1
11	1.4	–	–	0.6	–	–	0.2	0.2
12	1.4	–	–	0.6	–	–	0.2	0.2

Q.7 (a) Solution:

The cross section of the masonry dam is shown in the figure below



All the forces will be calculated by considering unit length of masonry dam. Let W be the self weight of the masonry dam, P_H be the horizontal force due to water and P_V be the vertical force due to triangular block of water. Neglecting uplift forces, we calculate the respective parameters as tabulated below:

Force	Description	F_V (tonnes)	F_H (tonnes)	LA from toe(m)	M_R (t.m)	M_0(t.m)
W_1	$(+)\frac{1}{2}\times 1\times 10\times 1\times 2240$	11.2 t	-	7.583	84.93	-
W_2	$(+)\,1\times 10\times 1\times 2240$	22.4 t	-	6.75	151.20	-
W_3	$(+)\frac{1}{2}\times 6.25\times 10\times 2240$	70.0 t	-	4.17	291.90	-
P_V	$(+)\frac{1}{2}\times 1\times 10\times 1000$	5.0 t	-	7.92	39.6	-
P_H	$(-)\frac{1}{2}\times 1000\times 10^2\times 1$	-	(–) 50 t	3.33	-	(–) 166.5
		$\Sigma F_V = 108.6$ t	$\Sigma F_H = -50$ t		$\Sigma M_R = 567.63$ t-m	$\Sigma M_0 = -166.5$ t-m

1. Factor of safety against overturning about toe is given by

$$F_0 = \frac{\Sigma M_R}{\Sigma M_0} = \frac{567.63}{166.5} = 3.409 > 1.5 \quad \text{(Hence OK)}$$

Thus, the masonry dam is safe against overturning.

2. Factor of safety against sliding is given by

$$F_S = \frac{\mu \Sigma F_V}{\Sigma F_H} = \frac{0.75\times 108.6}{50} = 1.63 > 1 \quad \text{(Hence OK)}$$

Thus, the masonry dam is safe against sliding too.

3. Shear friction factor = $\frac{\mu \Sigma F_V + bq}{\Sigma F_H}$

where b = width of dam at the joint = 8.25 m

q = permissible shear stress at the joint = 14 kg/cm^2 = 14 × 10^{-3} × 10^4 t/m^2

$$\therefore \quad \text{Shear friction factor} = \frac{(0.75 \times 108.6) + (8.25 \times 14 \times 10^{-3} \times 10^4)}{50} = 24.73 > 3$$

Thus, the dam is safe in shear friction factor criterion too.

Q.7 (b) Solution:

(i)

Following method of surface irrigation are generally used:

Surface irrigation: In all the surface methods of irrigation, water is either ponded on the soil or allowed to flow continuously over the soil surface for the duration of irrigation.

(a) Uncontrolled (or wild or free) flooding method: When water is applied to the cropland without any preparation of land and without any levees to guide or restrict the flow of water on the field, the method is called ‘uncontrolled’, ‘wild’ or 'free' flooding.

(b) Border strip method: Border strip irrigation (or simply 'border irrigation') is a controlled surface flooding method of applying irrigation water. In this method, the farm is divided into a number of strips. These strips are separated by low levees (or borders).

(c) Check method: The check method of irrigation is based on rapid application of irrigation water to a level or nearly level area completely enclosed by dikes. In this method, the entire field is divided into a number of almost levelled plots surrounded by levees.

(d) Basin method : In this method, a field is levelled in all directions and is encompassed by a dyke to prevent runoff. This provides undirected flow of water onto the field.

(e) Furrow method: As an alternative to flooding. The entire land surface is divided in small channels along the primary direction of the movement of water and letting the water flow through these channels which are termed 'furrows', 'creases' or 'corrugation'. Furrows necessitate the wetting of only about half to one-fifth of the field surface.

Advantages and disadvantages of various methods are as following:

Method	Advantages	Disadvantages
Surface irrigation (Flooding)	(1) Usable on shallow soils. (2) Usable if expense of leveling is high (3) Low cost (4) Resistant to livestock	(1) Runoff and deep percolation are high (2) Soil erosion is high on step farms (3) Fertilizer is eroded from soil.
Surface irrigation (Border)	(1) Usable for growing crops	(1) Large amount of water is needed (2) Land leveling is required (3) Usable for soils with low disperse (4) Damage is necessary
Surface irrigation (Basin)	(1) Varying amount of water (2) No runoff (3) Usable for rapid irrigation (4) No loss in fertilizers conditions	(1) Costs may be high if no land leveling. (2) Not usable for soils that disperse easily from a crust (except rice).
Surface irrigation (Furrow)	(1) High irrigation efficiency (2) Easy installation (3) Easy to maintain (4) Usable for most soils.	(1) Skilled labor is required (2) It is not suitable for operation of machinery (3) Drainage is necessary

(ii)

Using Gumbel's equation, we have

$$X_{(T)} = \overline{X} + K\sigma$$

where K is given by general equation

$$K = \frac{y_T - \overline{y}_n}{s_n}$$

where $\overline{y}_n$ and s_n remain the same for one analysis, because n is fixed in one analysis.

$$\therefore \quad X_{100} = \overline{X} + \left[\frac{y_{100}}{s_n} - \frac{\overline{y}_n}{s_n}\right]\sigma = 485 \text{ m}^3/\text{s} \text{ (given)} \quad ...(i)$$

and

$$X_{50} = \overline{X} + \left[\frac{y_{50}}{s_n} - \frac{\overline{y}_n}{s_n}\right]\sigma = 445 \text{ m}^3/\text{s} \text{ (given)} \quad ...(ii)$$

Subtracting (ii) from (i), we get

$$\left[\frac{y_{100}}{s_n} - \frac{y_{50}}{s_n}\right]\sigma = 40 \text{ m}^3/\text{s} \quad ...(iii)$$

y_T is given by equation as

$$y_T = -\ln\ln\left(\frac{T}{T-1}\right)$$

$$\therefore \quad y_{100} = -\left[\ln\ln\cdot\frac{100}{99}\right] = 4.60015$$

$$\text{and} \quad y_{50} = -\left[\ln\ln\cdot\frac{50}{49}\right] = 3.90194$$

$\therefore$ Substituting y_{100} and y_{50} in (iii), we get

$$(4.60015 - 3.90194)\cdot\frac{\sigma}{s_n} = 40$$

$$\Rightarrow \quad \frac{\sigma}{s_n} = 57.2894$$

Also, for given 1000 years return period, we have

$$y_{1000} = -\left[\ln\ln\frac{1000}{999}\right] = 6.90726$$

$$\therefore \quad [y_{1000} - y_{100}]\frac{\sigma}{s_n} = X_{1000} - X_{100}$$

Substituting the values, we get

$$[6.90726 - 4.60015]\, 57.2894 = X_{1000} - 485$$

$$\Rightarrow \quad 132.17 = X_{1000} - 485$$

$$\Rightarrow \quad X_{1000} = 617.17 \text{ m}^3/\text{s}$$

Q.7 (c) Solution:

(i)

Stage is defined as the height of the water surface above a datum plane, whereas gauge height can be defined as the water surface elevation to some predetermined gauge datum.

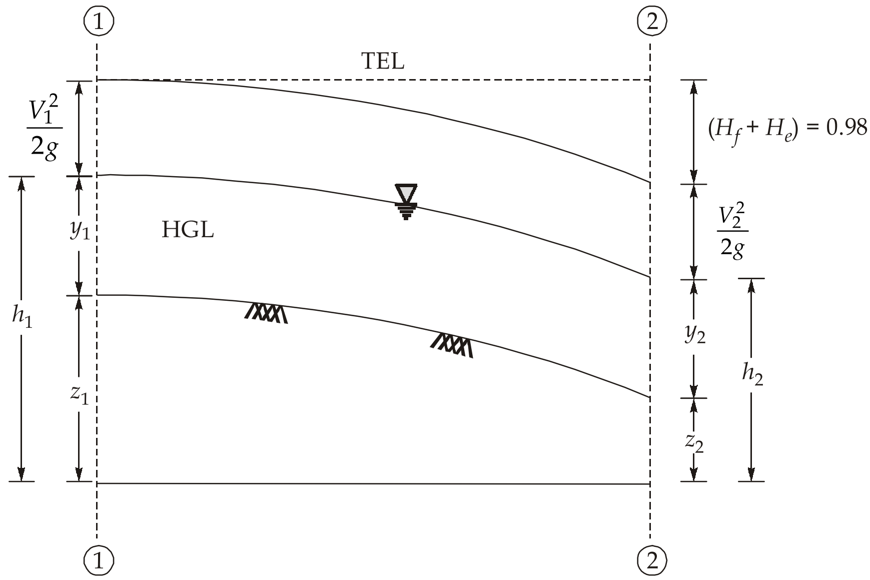
Methods of measurement of stage of a channel:

- Actual wading through the stream with a staff gauge.
- Fixing gauges along the cross-section.
- By suspending the gauge from a structure.
- Recording type of gauge.
- Automatic water stage recorder.

(ii)

Discharge is given by Manning's formula as,

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$



This formula is further modified to take average conveyance $\left(K = \frac{Q}{\sqrt{S}}\right)$ into account as follows:

$$Q = \sqrt{K_1 K_2 S}$$

where, $K = \frac{1}{n} A R^{2/3}$ for each section

Conveyance for section 1-1,

$$K_1 = \frac{1}{0.045}\left(\frac{A}{P}\right)^{2/3} \times A$$

$\Rightarrow$

$$K_1 = \frac{1}{0.045}\left(\frac{206}{65}\right)^{2/3} \times 206 = 9876.97 \simeq 9877 \text{ m}^3/\text{s}$$

Conveyance for section 2-2

$$K_2 = \frac{1}{0.045}\left(\frac{200}{53.8}\right)^{2/3} \times 200 = 10665.53 \simeq 10666 \text{ m}^3/\text{s}$$

As a first approximation,

$$S = \text{Slope of energy line}$$

$$S = \frac{\text{Fall in reach}}{\text{Length of fall}} = \frac{h_f}{L} = \frac{0.98}{125} = 7.84 \times 10^{-3}$$

$$\text{Peak discharge, } Q = \sqrt{9877 \times 10666 \times 7.84 \times 10^{-3}} = 908.806 \text{ cumecs}$$

$$\simeq 908.81 \text{ cumecs}$$

With this discharge, average velocity of flow at both the sections can be calculated. Energy loss due to contraction/expansion is computed as :

$$V_1 = \frac{908.81}{206} = 4.41 \text{ m/s}$$

$$V_2 = \frac{908.81}{200} = 4.54 \text{ m/s}$$

$$\therefore \quad \Delta h_v = \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) = \frac{4.41^2}{2 \times 9.81} - \frac{4.54^2}{2 \times 9.81} = -0.059 \text{ m}$$

Since Δh_v is negative,

$$S = \frac{\Delta h + \Delta h_v}{L} = \left(\frac{0.98 + 0.059}{125}\right) = 0.008312$$

$$\therefore \quad Q = \sqrt{9877 \times 10666 \times 0.008312} = 935.8 \text{ cumecs}$$

2nd Iteration

$$V_1 = \frac{935.8}{206} = 4.54 \text{ m/s}$$

$$V_2 = \frac{935.8}{200} = 4.68 \text{ m/s}$$

$$\Delta h_v = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{4.54^2}{2 \times 9.81} - \frac{4.68^2}{2 \times 9.81} = -0.0657$$

Since Δh_v is negative,

$$\therefore \quad S = \frac{\Delta h + \Delta h_v}{L} = \frac{0.98 + 0.0657}{125} = 8.365 \times 10^{-3}$$

$$\therefore \quad Q = 938.74 \text{ cumecs}$$

3rd iteration

$$V_1 = \frac{938.74}{206} = 4.56 \text{ m/s}$$

$$V_2 = \frac{938.74}{200} = 4.69 \text{ m/s}$$

$$\Delta h_v = -0.0613$$

$$\therefore \quad S = 8.3304 \times 10^{-3}$$

$$Q = 936.8 \text{ cumecs}$$

Q.8 (a) Solution:

(i) **Wheat :**

$$\text{Discharge required} = \frac{4800}{1600} = 3 \text{ cumec}$$

$$\text{Volume of water required} = 3 \times 110 = 330 \text{ cumec-day}$$

Sugarcane :

$$\text{Discharge required} = \frac{5800}{720} = 8.056 \text{ cumec}$$

$$\text{Volume of water required} = 8.056 \times 360$$
$$= 2900 \text{ cumec-day}$$

Cotton :

$$\text{Discharge required} = \frac{2500}{1800} = 1.389 \text{ cumec}$$

$$\text{Volume of water required} = 1.389 \times 200$$
$$= 277.8 \text{ cumec-day}$$

Rice :

$$\text{Discharge required} = \frac{3600}{1000} = 3.6 \text{ cumec}$$

$$\text{Volume of water required} = 3.6 \times 140 = 504 \text{ cumec-day}$$

Vegetable :

$$\text{Discharge required} = \frac{1500}{800} = 1.875 \text{ cumec}$$

$$\text{Volume of water required} = 1.875 \times 180 = 337.5 \text{ cumec-day}$$

Hence, total volume of water required on the field for all crops

$$= 330 + 2900 + 277.8 + 504 + 337.5$$
$$= 4349.3 \text{ cumec-day}$$

$\therefore$ Total volume of water required on the field in one day

$$= 4349.3 \times 8.64 \times 10^4 \text{ m}^3$$
$$= 37577.952 \text{ ha.m}$$

Since, losses in the canal system are 25%. The volume of water required at the head of the canal

$$= 37577.952 \times \frac{100}{75} = 50103.936 \text{ ha.m}$$

Allowing for 10% reservoir losses, the storage capacity of the reservoir

$$= 50103.936 \times \frac{100}{90} = 55671.04 \text{ ha.m}$$

(ii) 1. Effective rainfall: Effective rainfall is that part of the precipitation falling during the growing period of a crop that is available to meet the evapo-transpiration needs of the crop.

2. Consumptive irrigation requirement: Consumptive irrigation requirement is defined as the amount of irrigation water that is required to met the evapo-transpiration needs of the crop during its full growth. Therefore,

$$\text{CIR} = C_u - R_e$$

where, C_u is the consumptive use of water.

3. Net irrigation requirement: Net irrigation requirement is defined as the amount of irrigation water required at the field to meet the evapo-transpiration needs of crop as well as other needs such as leaching etc. Thus

NIR = $C_u - R_e$ + Water lost in deep percolation for the purpose of leaching etc.

4. Field irrigation requirement: Field irrigation requirement is the amount of water required to meet 'net irrigation requirements' plus the water lost in percolation in the field water courses, field channels and in field applications of water. If η_a is water application efficiency, we have

$$\text{FIR} = \frac{NIR}{\eta_a}$$

5. Gross irrigation requirement: Gross irrigation requirement is the sum of water required to satisfy the field irrigation requirement and the water lost as conveyance losses in distributaries upto the field. If η_c is the water conveyance efficiency, we have

$$\text{GIR} = \frac{FIR}{\eta_c}$$

Q.8 (b) Solution:

(i)

The seepage water exerts a force at each point in the direction of flow and tangential to the stream lines. This force has a maximum disturbing tendency at the exit because at this point, its direction is vertically upwards. This gradient of pressure of water at the end is called "Exit Gradient".

Exit gradient, $$G_E = \frac{H}{d} \times \frac{1}{\pi\sqrt{\lambda}}$$

where, $$\lambda = \frac{1+\sqrt{1+\alpha^2}}{2}$$

and $$\alpha = \frac{b}{d}$$

Given, b = 10 m, d = 1.5 m, H = 4 m

$$\therefore \quad \alpha = \frac{10}{1.5} = 6.67$$

$$\lambda = \frac{1+\sqrt{1+6.67^2}}{2} = 3.87$$

$$\therefore \quad G_E = \frac{4}{1.5\pi}\frac{1}{\sqrt{3.87}} = 0.43$$

(ii)

River training works covers all those engineering works which are constructed on a river so as to guide and confine the flow of rivers and to requlate the river bed configuration.

Objectives of river training are :

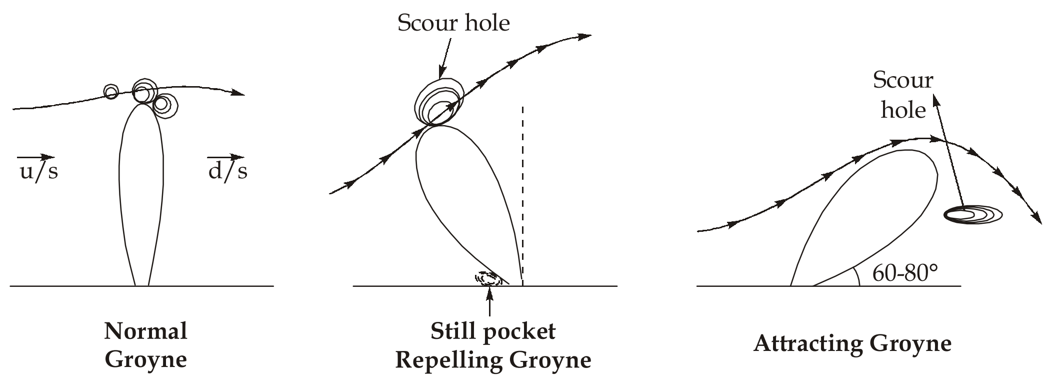
1. To prevent the river from changing its course.
2. To prevent flooding of the surrounding area.
3. To protect the river banks.
4. To ensure effective disposal of sediment load.

Groynes: These are embankment type structures constructed transverse to the river flow, extending from the bank into the river. These are constructed to protect the bank from which they are extended, by deflecting the river current away from the bank.

Groynes are of three types:

1. Normal groyne
2. Repelling groyne

3. Attracting groyne



(iii)

Given $Q = 50\ m^3/s, f = 1.1$

$$\therefore \quad V = \left(\frac{Qf^2}{140}\right)^{1/6} = \left(\frac{50\times 1.1^2}{140}\right)^{\frac{1}{6}} = 0.8695\ m/s$$

$$\text{Flow area, } A = \frac{Q}{V} = \frac{50}{0.8695} = 57.50\ m^2$$

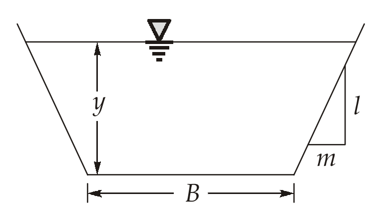
$$\text{Hydraulic radius, } R = \frac{5}{2}\frac{V^2}{f} = \frac{5}{2}\cdot\frac{(0.8695)^2}{1.1} = 1.718\ m$$

$$\text{Wetted perimeter, } P = 4.75\sqrt{Q} = 4.75\sqrt{50} = 33.59\,m$$

Consider a trapezoidal channel with 1 H : 2 V side slopes. (Hence, m = 1/2)

$$\therefore \quad P = B + 2y\sqrt{1+m^2} = B + \sqrt{5}y$$

$$\Rightarrow \quad 33.59 = B + \sqrt{5}y \quad ...(i)$$



$$A = (B+my)y = \left(B + \frac{y}{2}\right)y$$

$$\Rightarrow \quad 57.5 = By + \frac{y^2}{2} \quad ...(ii)$$

From (i) and (ii),

$$y = 1.898\ m \simeq 1.9\ m$$

$$\therefore \quad B = 33.59 - \sqrt{5}y = 29.345\ m \simeq 29.35\ m\ (say)$$

$$\text{Bed slope, } S = \frac{f^{5/3}}{3340Q^{1/6}} = \frac{(1.1)^{5/3}}{3340\times(50)^{1/6}} \simeq \frac{1}{5469}$$

Q.8 (c) Solution:

Given:

Storage equation is as under:

$$S = K[xI + (1 - x)O] \quad ...(i)$$

Also, the basic routing equation is

$$\left(\frac{I_1 + I_2}{2}\right)t - \left(\frac{O_1 + O_2}{2}\right)t = (S_2 - S_1) \quad ...(ii)$$

Substituting the values of S_1 and S_2 from (i) into (ii), we have

$$S_1 = K\left[xI_1 + (1-x)O_1\right]$$

$$S_2 = K\left[xI_2 + (1-x)O_2\right]$$

$$\therefore \quad \left(\frac{I_1 + I_2}{2}\right)t - \left(\frac{O_1 + O_2}{2}\right)t = K\left[xI_2 + (1-x)O_2\right] - K\left[xI_1 + (1-x)O_1\right]$$

$$\Rightarrow \left(\frac{I_1 + I_2}{2}\right)t + K\left[xI_1 + (1-x)O_1\right] = \left(\frac{O_1 + O_2}{2}\right)t + K\left[xI_2 + (1-x)O_2\right]$$

$$\Rightarrow (I_1 + I_2) + \frac{2K}{t}\left[xI_1 + (1-x)O_1\right] = (O_1 + O_2) + \frac{2K}{t}\left[xI_2 + (1-x)O_2\right]$$

$$\Rightarrow \left[I_1 + \frac{KxI_1}{0.5t}\right] + \left[I_2 - \frac{KxI_2}{0.5t}\right] + \left[\frac{K(1-x)O_1}{0.5t} - O_1\right] = \left[O_2 + \frac{K(1-x)}{0.5t}O_2\right]$$

$$\Rightarrow O_2\left[\frac{0.5t + K(1-x)}{0.5t}\right] = I_1\left[\frac{0.5t + Kx}{0.5t}\right] + I_2\left[\frac{0.5t - Kx}{0.5t}\right] + O_1\left[\frac{K(1-x) - 0.5t}{0.5t}\right]$$

$$\Rightarrow \quad O_2(K - Kx + 0.5t) = I_1(Kx + 0.5t) + I_2(0.5t - Kx) + O_1(K - Kx - 0.5t)$$

$$\Rightarrow \quad O_2 = I_1\left[\frac{Kx + 0.5t}{K - Kx + 0.5t}\right] + I_2\left[\frac{-Kx + 0.5t}{K - Kx + 0.5t}\right] + O_1\left[\frac{K - Kx - 0.5t}{K - Kx + 0.5t}\right]$$

$$\Rightarrow \quad O_2 = C_0I_2 + C_1I_1 + C_2O_1$$

which is the required Muskingum equation, where

$$C_0 = \frac{-Kx + 0.5t}{K - Kx + 0.5t}, \; C_1 = \frac{Kx + 0.5t}{K - Kx + 0.5t} \text{ and } C_2 = \frac{K - Kx - 0.5t}{K - Kx + 0.5t}$$

Sum of coefficients,

$$C_0 + C_1 + C_2 = \frac{1}{K - Kx + 0.5t}[-Kx + 0.5t + Kx + 0.5t + K - Kx - 0.5t] = 1$$

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