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Detailed Solutions

ESE-2025 Mains Test Series

E & T Engineering Test No : 5

Section A : Material Science + Basic Electrical Engineering + Electronic Measurements and Instrumentation

Q.1 (a) Solution:

Given, Crystal structure of strontium : FCC

$$\text{Atomic radius, } r = 0.215 \text{ nm}$$

$$\text{Atomic weight, } M = 87.62 \text{ g/mol}$$

For FCC, the number of atoms per unit cell is $n = 4$.

The relation between atomic radius r and lattice paramter a for FCC crystal structure is $a = 2r\sqrt{2}$.

$$\text{Avogadro's number, } N_A = 6.022 \times 10^{23} \frac{\text{atoms}}{\text{mol}}$$

$$\text{The density, } \rho = \frac{nM}{V_c N_A}$$

where, volume of unit cell, $V_c = a^3$

We have, $a = 2 \times 0.215 \times \sqrt{2} \text{ nm} = 0.608 \text{ nm}$

$\therefore$ $V_c = a^3 = (0.608 \text{ nm})^3 = 0.225 \text{ nm}^3$

(or) $V_c = 0.225 \text{ nm}^3 \times \left(\frac{1 \text{ cm}}{10^7 \text{ nm}}\right)^3$

$\therefore \quad V_c = 2.25 \times 10^{-22}\ \text{cm}^3$

The density, $\rho = \dfrac{nM}{V_c N_A}$

$= \dfrac{4\times 87.62}{2.25\times 10^{-22}\times 6.022\times 10^{23}}$

$\therefore \quad \rho = 2.587\ \text{g/cm}^3$

Q.1 (b) Solution:

Definition of Commutation:

In DC machines, commutation refers to the process of current reversal in an armature coil as it passes through the Magnetic Neutral Axis (MNA). Since the emf induced in the armature conductors is alternating in nature, but the external circuit requires direct current, commutation ensures the conversion of this alternating nature to a unidirectional output.

During commutation, the coil gets short-circuited by the brush across two adjacent commutator segments. The current in this coil must reverse completely and smoothly during this brief short-circuit period. If reversal is not complete or is delayed, it causes sparking at the brushes, damaging both brushes and commutator.

Need for Improving Commutation:

Improper commutation results in:

- Sparking and overheating at the commutator
- Damage to commutator segments and brushes
- Electromagnetic interference (EMI)
- Reduced efficiency and life of the machine

Hence, to ensure sparkless (ideal) commutation, certain design and operational improvements are implemented.

Methods to Improve Commutation:

There are three main methods employed in DC machines to improve commutation:

1. **Resistance Commutation:** This method involves the use of carbon brushes instead of metallic brushes.

   Carbon brushes offer higher contact resistance at the interface between the brush and the commutator segments.

   The increased resistance reduces the circulating current in the short-circuited coil and slows down the rate of current reversal.

This helps the current to reverse gradually and completely within the commutation period, thereby reducing or eliminating sparking.

Advantages:

- Simple to implement
- Enhances brush and commutator life

2. **Voltage Commutation:** In this method, a commutating emf is induced in the coil undergoing commutation. This emf is opposite to the reactance voltage caused by the self-inductance of the coil and helps in forcing the current to reverse quickly during commutation.

   Ways to generate commutating emf:

   (a) **Brush Shift:** Due to armature reaction, the Magnetic Neutral Axis (MNA) shifts.

   By shifting the brush along the direction of rotation (generator) or against it (in motor), we align the brush with the new MNA.

   This allows a small emf to assist the current reversal in the coil.

   However, brush shift is not reliable under varying loads.

   (b) **Commutating Poles (Interpoles):** Small auxiliary poles placed between the main poles and connected in series with the armature winding.

   They generate an emf proportional to the armature current and exactly opposite to the reactance voltage in the coil undergoing commutation.

   This ensures sparkless commutation even under varying load conditions.

3. **Compensating Windings:** These windings are embedded in the pole faces of the main poles and connected in series with the armature.

   Their purpose is to neutralize the cross-magnetizing effect of armature reaction under the pole face.

   This helps to maintain uniform flux density and keeps the MNA fixed even under load.

   As a result, commutation remains stable and sparkless, particularly in machines with large armature current or fluctuating loads.

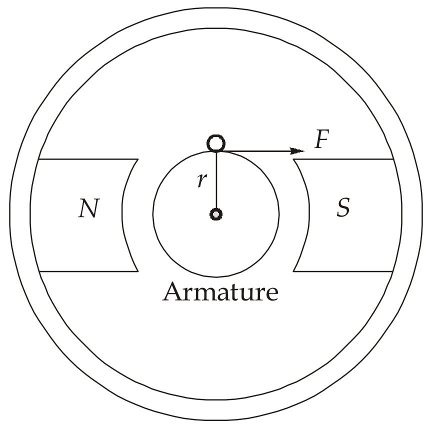
Conclusion:

Commutation is a crucial process in DC machines that ensures the delivery of direct current despite the alternating nature of induced emf in armature conductors. Sparkless commutation not only improves performance but also enhances the reliability and lifespan of the machine.

Methods like resistance commutation, voltage commutation and compensating windings are essential design features to achieve this objective effectively.

Q.1 (c) Solution:

In a DC motor, a circumferential force (F) at a distance r which is the radius of the armature is acted on each conductor, tending to rotate the armature. The sum of the torques due to all the armature conductors is known as armature torque (T_a).



Assume,

P = Number of Poles

r = Radius of The Armature

l = Effective Length of Each Conductor

Z = Total Number of Armature Conductors

A = Number of Parallel Paths

i = Current in Each Conductor

B = magnetic Flux Density

ϕ = Flux Per Pole

Force on each conductor, $F = Bil$

Torque developed by each conductor = $F \times r$

Thus, Total armature torque,

$$T_a = Z \times (F \times r) = ZBilr$$

We have, current in each conductor, $i = \frac{I_a}{A}$, where A is the number of parallel paths.

$$\text{Magnetic flux density, } B = \frac{\phi}{a}$$

Where a is the cross-sectional area of flux-path at radius $r = \frac{2\pi rl}{P}$

Therefore,
$$T_a = Z\times\frac{\phi}{a}\times\frac{I_a}{A}\times l\times r = Z\times\frac{\phi}{2\pi rl/P}\times\frac{I_a}{A}\times l\times r$$
$$T_a = \frac{PZ}{2\pi A}\phi I_a$$

For a given DC motor, $\frac{PZ}{2\pi A}$ is constant. Hence,
$$T_a \propto \phi I_a$$

i.e. the torque developed by a d.c. motor is directly proportional to the flux per pole and armature current.

Q.1 (d) Solution:

Given, Resistance, $R = (1000 \pm 10\%)\ \Omega$

Current $I = 10$ mA

Accuracy in current $= \pm2\%$ of Full scale reading $= \pm2\%$ of (20 mA)

$= \pm0.4$ mA

$\therefore$ $I = 10 \text{ mA} \pm 0.4 \text{ mA} = (10 \pm 4\%)$ mA

Power dissipated in the resistor,
$$P = I^2R \quad \text{...(i)}$$
$$P = (10 \times 10^{-3})^2 \times (1000) = 0.1 \text{ W}$$

Taking 'log' on both sides in equation (i),
$$\log P = \log (I^2R)$$
$$\log P = 2\log I + \log R$$

By differentiating both sides,
$$\frac{1}{P}\partial P = 2\frac{\partial I}{I}+\frac{\partial R}{R}$$
$$\frac{\partial P}{P} = 2 \times 4\% + 10\%$$
$$\frac{\partial P}{P} = 8\% + 10\%$$
$$\frac{\partial P}{P} = 18\%$$
$$P = 0.1 \text{ W} \pm 18\%$$

Thus, the accuracy in the measurement of power dissipated in the resistor is 18%.

Q.1 (e) Solution:

For a thermometer, using the Newton's Law of cooling,

$$\theta(t) = \theta_f(1-e^{-t/\tau})+\theta_i e^{-t/\tau}$$

where,

$$\theta_f = \text{final temperature (in °C)}$$

$$\theta(t) = \text{temperature at time } t \text{ (in °C)}$$

$$\theta_i = \text{initial temperature (in °C)}$$

Given,

$$\theta(30) = 97°\text{C}$$

$$\theta_f = 100°\text{C}$$

$$\theta_i = 30°\text{C}$$

$$t = 30 \text{ sec}$$

$$\therefore \quad 97 = 100(1 - e^{-30/\tau}) + 30e^{-30/\tau}$$

$$= 100 - 100e^{-30/\tau} + 30e^{-30/\tau}$$

$$97 = 100 - 70e^{-30/\tau}$$

$$\therefore \quad 70e^{-30/\tau} = 3$$

$$e^{-30/\tau} = \frac{3}{70} = 0.043$$

By taking 'ln' on both sides,

$$\frac{-30}{\tau} = \ln(0.043) = -3.15$$

$$\therefore \quad \tau = \frac{30}{3.15} = 9.52 \text{ sec}$$

$$\text{For } \theta(t) = 98°\text{C},$$

$$98 = 100(1 - e^{-t/9.52}) + 30e^{-t/9.52}$$

$$98 = 100 - 100e^{-t/9.52} + 30e^{-t/9.52}$$

$$98 = 100 - 70e^{-t/9.52}$$

$$70e^{-t/9.52} = 2$$

$$e^{-t/9.52} = \frac{2}{70} = 0.0285$$

By taking 'ln' on both sides,

$$\frac{-t}{9.52} = -3.55$$

$$\therefore \quad t = 33.8 \text{ sec}$$

Q.2 (a) Solution:

(i)

$$\text{Area of plates, } A = 5 \times 5 \times 10^{-6} = 25 \times 10^{-6} \text{ m}^2$$

$$\text{Pressure, } P = \frac{\text{Force}}{\text{Area}} = \frac{5}{(25 * 10^{-6})} = 0.2 \text{ MN/m}^2$$

$$\text{Voltage sensitivity, } g = \frac{d}{\epsilon_0 \epsilon_r} = \frac{150 \times 10^{-12}}{12.5 \times 10^{-9}} = 12 \times 10^{-3} \text{ Vm/N},$$

$$\text{Voltage generated, } E_0 = gtP = 12 \times 10^{-3} \times 1.25 \times 10^{-3} \times 0.2 \times 10^6 = 3 \text{ V}$$

$$\text{Strain, } \epsilon = \Delta t = \frac{\text{stress}}{\text{young's modulus}} = \frac{0.2 \times 10^6}{12 \times 10^6} = 0.0167$$

$$\text{Charge, } Q = dF = 150 \times 10^{-12} \times 5C = 750 \text{ pC}$$

$$\text{Capacitance, } C_p = \frac{Q}{E_0} = \frac{750 \times 10^{-12}}{3} \text{F} = 250 \text{ pF}$$

(ii) The deflecting torque in a moving iron instrument is given by

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

The rate of change of inductance with deflection is,

$$\frac{dL}{d\theta} = \frac{d}{d\theta}(12 + 6\theta - \theta^2)$$

$$= 6 - 2\theta \ \mu\text{H/radians} = (6 - 2\theta) \times 10^{-6} \text{ H/radians}$$

When the pointer is at steady-state, the deflecting torque is equal to the controlling torque. Thus,

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} = K\theta$$

where θ is the deflection of the pointer and K is the spring constant. We get,

$$\theta = \frac{1}{2} \frac{I^2}{K} \frac{dL}{d\theta}$$

$$\therefore \quad \theta = \frac{1}{2} \times \frac{(8)^2}{12 \times 10^{-6}} \times [6 - 2\theta] \times 10^{-6}$$

$$0.375\,\theta = 6 - 2\theta$$

$$\therefore \quad \theta = 2.526 \text{ radians} = 144.74°$$

(iii) Inductance with air gap length 1 m, $L = 2$ mH

Air gap length when coil is displayed by 0.02 mm = 1 – 0.02 = 0.98 mm

Since inductance is inversely proportional to the length of air gap, assume ΔL is the increase in inductance. Thus,

$$L + \Delta L = 2 \times \frac{1}{0.98} = 2.04 \text{ mH}$$

Thus, the value of inductance becomes 2.04 mH when a displacement of 0.02 mm is applied towards the core.

In general, assume $\pm x$ is the displacement and change in inductance as $\pm\Delta L$. Thus, we have

$$L \pm \Delta L = L \times \frac{l}{l \pm x} = L\left[1 \pm \frac{x}{l}\right]^{-1} \approx L(1 \pm x)$$

assuming $x << l$. Thus, we get

$$\Delta L \approx \pm Lx$$

Thus, the change in inductance is linearly proportional to the displacement.

Q.2 (b) Solution:

We have,

$$\text{Number of poles, } P = 4$$

$$\text{Number of conductors, } Z = 1200$$

$$\text{Rotational speed, } N = 500 \text{ r.p.m.}$$

$$\text{Diameter of the pole shoe, } D = 0.35 \text{ m}$$

$$\frac{\text{Pole arc}}{\text{Pole pitch}} = 0.7$$

$$\text{Length of the shoe, } l = 0.2 \text{ m}$$

$$\text{Flux density, } B = 0.75 \text{ T}$$

Now, we know that

$$\text{Generated EMF, } E_a = \frac{NP\phi}{60} \times \frac{Z}{A} \quad \text{...(i)}$$

where Flux per pole, $\phi = B.a$ ...(ii)

As, Pole arc = 0.7 × Pole pitch

$$\text{Pole arc} = 0.7 \times \frac{\pi D}{P}$$

$$\text{Pole arc} = 0.7 \times \frac{\pi \times 0.35}{4}$$

$$\text{Pole arc} = 0.19 \text{ m}$$

Area under pole,
$$a = \text{Pole arc} \times \text{length } (l)$$

$$a = 0.19 \times 0.2$$

$$a = 0.038 \text{ m}^2$$

Now from equation (ii), we get

$$\phi = 0.75 \times 0.038$$

$$\phi = 0.029 \text{ Wb} \qquad \text{...(iii)}$$

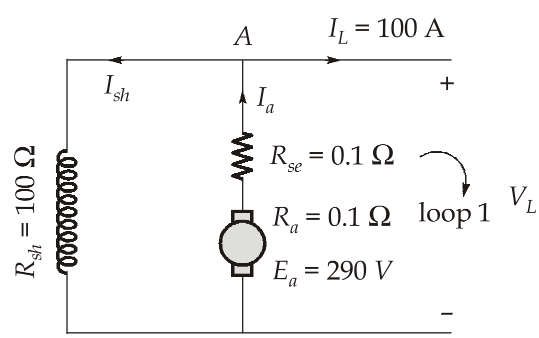
(i) If Armature Winding is Lap Connected, *A* = *P*

Using equation (i) and (iii), we get

$$E_a = \frac{500 \times 0.029 \times 1200}{60} \times \frac{A}{A}$$

$$E_a = 290 \text{ Volt}$$

The equivalent circuit of a cumulatively compounded dc generator with a long-shunt connection is shown below:



Given, $I_L = 100\text{A}$

On applying KCL at node *A*, we get

$$I_a = I_{sh} + I_L$$

$$I_a = \frac{V_L}{100} + 100$$

$$I_a = 0.01\, V_L + 100 \qquad \text{...(iv)}$$

On applying KVL in loop 1, we get

$$-290 + 2 \text{ (brush drop)} + I_a [0.1 + 0.1] + V_L = 0$$

$$V_L = 288 - I_a [0.2]$$

Using equation (iv), we get

$$V_L = 288 - [0.01\ V_L + 100]\ 0.2$$

$$V_L = 288 - 0.002\ V_L - 20$$

$$V_L = 268 - 0.002\ V_L$$

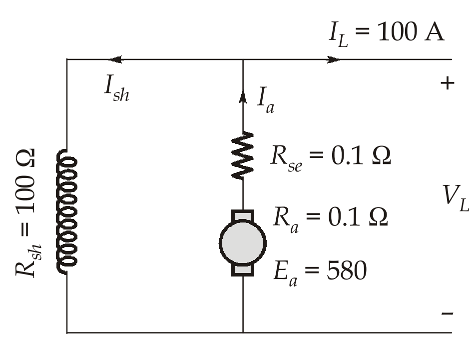
$$V_L = \frac{268}{1.002} = 267.465 \text{ volt}$$

(ii) If Armature Winding is Wave Connected, $A = 2$

Using equation (i) and (iii), we get

$$E_a = \frac{500 \times 0.029 \times 1200}{60} \times \frac{4}{2} = 580 \text{ Volt}$$

The equivalent circuit of a cumulatively compounded dc generator with a long-shunt connection is shown below:



On applying KCL at node A, we get

$$I_a = I_{sh} + I_L$$

$$I_a = \frac{V_L}{100} + 100$$

$$I_a = 0.01\ V_L + 100 \quad \text{...(v)}$$

On applying KVL in loop (i), we get

$$-580 + 2 \text{ (brush drop)} + I_a [0.1 + 0.1] + V_L = 0$$

$$V_L = 578 - I_a [0.2]$$

$$V_L = 578 - [0.01\ V_L + 100]0.2$$

$$V_L = 578 - 0.002\ V_L - 20$$

$$V_L + 0.002\ V_L = 558$$

$$V_L = 556.886 \text{ Volt}$$

Q.2 (c) Solution:

Given, $V_L = 100$ V, $I_L = 9$ A and $\cos\phi = 0.1$

$$\text{Load power} = V_L I_L \cos\phi = 100 \times 9 \times 0.1 = 90 \text{ W}$$

$$\text{Load power factor, } \cos\phi = 0.1$$

$$\therefore \quad \phi = 84.26°$$

Thus, $\sin\phi = 0.995$

and $\tan\phi = 9.95$

Resistance of pressure coil circuit,

$$R_p = 3000\ \Omega$$

Reactance of pressure coil circuit,

$$X_p = 2\pi f L_p = 2\pi \times 50 \times 30 \times 10^{-3} = 9.42\ \Omega$$

If β is the angle by which the current in pressure coil lags behind the voltage, we have

$$\tan\beta = \frac{9.42}{3000} = 0.00314$$

(i) When the pressure coil is connected on the load side, the wattmeter measures power loss in pressure coil circuit in addition to load power,

$$\text{True power} = VI\cos\phi = 100 \times 9 \times 0.1 = 90 \text{ W}$$

Considering only effect of inductance:

$$\text{Reading of wattmeter} = \text{true power } (1 + \tan\phi \tan\beta)$$

$$= 90(1 + 9.95 \times 0.00314) = 92.81 \text{ W}$$

Power loss in pressure coil circuit,

$$= \frac{V^2}{R_P} = \frac{(100)^2}{3000} = 3.33 \text{ W}$$

∴ Reading of wattmeter considering the power loss in pressure coil circuit

$$= 92.81 + 3.33 = 96.14 \text{ W}$$

$$\% \text{ error} = \frac{96.14 - 90}{90} \times 100 = 6.82\%$$

(ii) When the pressure coil is on the source side, the wattmeter measures the power in the load plus the power loss in the current coil. In fact, the current coil acts as a load.

$$\therefore \quad \text{Total power} = \text{power consumed in load} + I^2 R_c$$

$$= 90 + (9)^2 \times 0.1 = 98.1 \text{ W}$$

$$\text{Impedance of load} = \frac{V_L}{I_L} = \frac{100}{9} = 11.1\ \Omega$$

$$\text{Resistance of load} = Z\cos\phi = 11.1 \times 0.1 = 1.11\ \Omega$$

$$\text{Reactance of load} = Z\sin\phi = 11.1 \times 0.995 = 11.05\ \Omega$$

Resistance of load plus resistance of current coil

$$= 1.11 + 0.1 = 1.21\ \Omega$$

Reactance of load plus reactance of current coil,

$$= 11.05\ \Omega$$

Impedance of load including impedance of current coil,

$$= \sqrt{(1.21)^2 + (11.05)^2} = 11.1\ \Omega$$

Power factor of load including impedance of current coil,

$$\cos\phi' = \frac{1.21}{11.1} = 0.109$$

$$\therefore \quad \phi' = 83.74° \text{ and } \tan\phi' = 9.12$$

$$\therefore \quad \text{Reading of wattmeter} = 98.1(1 + 9.12 \times 0.00314) = 100.9\ \text{W}$$

$$\text{Percentage error} = \frac{100.9 - 90}{90} \times 100 = 12.1\%$$

It is clear from above results that in low power factor circuits, we should not use a connection wherein the current coil is on the load side as it results in greater errors.

Q.3 (a) Solution:

(i) 1. Moment of Inertia of cantilever,

$$I = \frac{1}{12}bt^3 = \frac{1}{12} \times (0.02) \times (0.004)^3$$

$$= 0.107 \times 10^{-9}\ \text{m}$$

$$\text{Deflection, } x = \frac{Fl^3}{3EI} = \frac{25 \times (0.25)^3}{3 \times 200 \times 10^9 \times 0.107 \times 10^{-9}} = 6.08\ \text{mm}$$

2. Deflection per unit force,

$$\frac{x}{F} = \frac{6.08}{25} = 0.2432\ \text{mm/N}$$

Overall sensitivity of measurement system:

$$= (0.2432\ \text{mm/N}) \times (0.5\ \text{V/mm}) = 0.1216\ \text{V/N}$$

For a voltmeter,

$$1 \text{ scale division} = \left(\frac{10}{1000}\right) = 0.1 \text{ V}$$

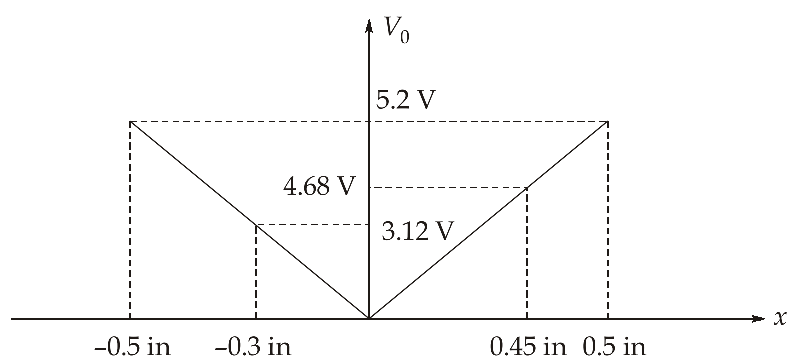
Since two-tenths of a scale division can be read with certainty, hence

$$\text{Resolution of voltmeter} = \left(\frac{2}{10}\right) \times 0.1 = 0.02 \text{ V}$$

$$\text{Minimum force that can be measured} = \frac{0.02}{0.1216} = 0.1645 \text{ N}$$

$$\text{Maximum force that can be measured} = \frac{10}{0.1216} = 82.2 \text{ N}$$

In LVDT, a zero-differential output voltage is produced when the core is in the center, or null position, where the induced voltages in the two secondary coils are equal. The induced voltages in the secondary coils become unequal as the core moves away from the null position, and the differential output voltage increases proportionately. The output voltage varies with the core movement as shown below:



(i) **1.** 0.5 in. core displacement produces 5.2 V, therefore a 0.45 in. core movement produces (0.45 × 5.2)/0.5 = 4.68 V

Similarly *a* –0.30 in. core movement produces

$$(0.30 \times 5.2)/(0.5) = 3.12 \text{ V}$$

2. A –0.25 in. core movement from the centre produces

$$(0.25 \times 5.2)/(0.5) = 2.6 \text{ V}$$

Q.3 (b) Solution:

(i) Applications of Synchronous Motors:

Synchronous motors are rarely used below 40 kW output in the medium speed range because of their much higher initial cost in comparison to that of induction motors. In addition they need a dc excitation source, and the starting and control devices which are usually more expensive—especially where automatic operation is required. However, there is a kW output and speed range where the disadvantage of higher initial cost vanishes, even to the point of putting the synchronous motors to the advantage. Where low speeds and high kW outputs are involved, the induction motors are no longer cheaper because they need large amounts of iron in order to restrict the air gap flux density to 0.7 T. In the synchronous machines, on the other hand, a value twice this figure is permissible because of the separate excitation.

The various classes of service for which synchronous motors are employed may be classified as (i) power factor correction (ii) voltage regulation and (iii) constant speed, constant-load drives.

1. They are used in power houses and substations in parallel to the bus-bars to **improve the power factor**. For this purpose, they are run **without mechanical load** on them and overexcited.
2. In factories having a large number of induction motors, or other power apparatus operating at lagging power factor, they are employed to improve the power factor.
3. Such motors are also used to regulate the voltage at the end of transmission lines.
4. Because of the higher efficiency possible with synchronous motors, they can be employed advantageously for the loads where constant speed is required. Typical applications of high-speed synchronous motors **(above 500 rpm)** are drives such as **fans, blowers, dc generators, line shafts, centrifugal pumps** and **compressors, reciprocating pumps** and **compressors, constant speed frequency changers, rubber** and **paper mills** etc.

The fields of applications of low speed synchronous motors **(below 500 rpm)** are drives such as reciprocating compressors when started unloaded, dc generators, centrifugal and screw type pumps, vacuum pumps, electroplating generators, line shafts, rubber and band mills, ball and tube mills, chippers, metal rolling mills etc. Flywheel is used for pulsating loads.

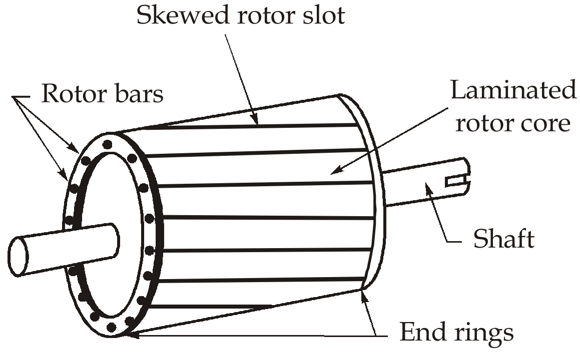
Synchronous motors can be run at ultra-low speeds by using high power electronic converters which generate very low frequencies. Synchronous motors in very large sizes (as high as 10 MW size) operating at ultra low speeds are employed to drive crushers, rotary kilns and variable speed ball mills.

Basis of Difference	Synchronous motor	Induction Motor
Type of Excitation	A synchronous motor is a doubly excited machine because both its rotor and stator are excited.	An induction motor is a singly excited machine.
Supply System	Its armature winding is energized from an AC source and field winding from a DC source.	Its stator winding is energized from an AC source.
Speed	It always runs at synchronous speed. The speed is independent of load.	If the load is increased, the speed of the induction motor decreases. It is always less than the synchronous speed.
Starting	It is not self starting. It has to be run-up to synchronous speed by any means before it can be synchronized to AC supply.	Induction motor has self starting torque.
Operation	A synchronous motor can be operated with lagging and leading power by changing its excitation.	An induction motor operates only at a lagging power factor. At high loads, the power factor becomes very poor.
Usage	It can be used for power factor correction in addition to supplying torque to drive mechanical loads.	An induction motor is used for driving mechanical loads only.
Efficiency	It is more efficient than an induction motor of the same output and voltage rating.	Its efficiency is lesser than that of the synchronous motor of the same output and the voltage rating.
Cost	A synchronous motor is costlier than an induction motor of the same output and voltage rating.	An induction motor is cheaper than the synchronous motor of the same output and voltage rating.

(ii) The rotors employed in 3-phase induction motors, according to the type of windings used, are of two types, viz; **squirrel cage rotor** and the **wound rotor** or **slip ring rotor**.

1. **Squirrel Cage Rotor:** Almost 90 per cent of induction motors are provided with squirrel cage rotor because of its very simple, robust and almost indestructible construction.

   In cage construction, **copper**, **brass**, or **aluminium** bars are placed, as the rotor conductors, approximately parallel to the shaft (one bar in each slot) and close to the rotor surface. The conductors are not insulated from the core, since the rotor currents naturally follow the path of least resistance, i.e., the rotor conductors. At both ends of the rotor, the rotor conductors are all short circuited by the continuous end rings of similar material to that of the rotor conductors. The rotor conductors and their end rings form a complete closed circuit in itself, resembling a squirrel cage, thus explaining the name. The rotor has a smaller number of slots than the stator and must be a non-integral multiple of stator slots so as to prevent Magnetic Locking of rotor and Stator teeth at the starting (Cogging). The slots on the rotor are always not parallel to the motor shaft but are usually skewed in order to obtain a uniform torque, avoid the magnetic locking of the stator and rotor and reduce the magnetic humming noise while running. In motors with ratings up to 100 kW, the squirrel cage structure is formed by aluminium cast (under pressure) into the slots of the rotor. In large motors, the rotor bars instead of being cast, are wedged into the rotor slots and are then welded securely to the end rings. The slots on rotor are either of semi-closed type or of totally closed type, because there is little difficulty in inserting the rotor bars in such slots. The advantage of semi-closed and totally closed slots is that the effective cross-sectional area of the air gap is increased, therefore, magnetizing current is reduced.



The squirrel cage rotor windings are perfectly symmetrical and have the advantage of being adaptable to **any number of pole pairs**. The distribution of current due to electromagnetic induction in the rotor bars varies from bar to bar sinusoidally and depends upon the position and time, assuming sinusoidal distribution of radial flux density in space and also the applied voltage to be varying sinusoidally with time.

Since the rotor winding is permanently short circuited in cage construction, there is no possibility of adding any external resistance in the rotor circuit.

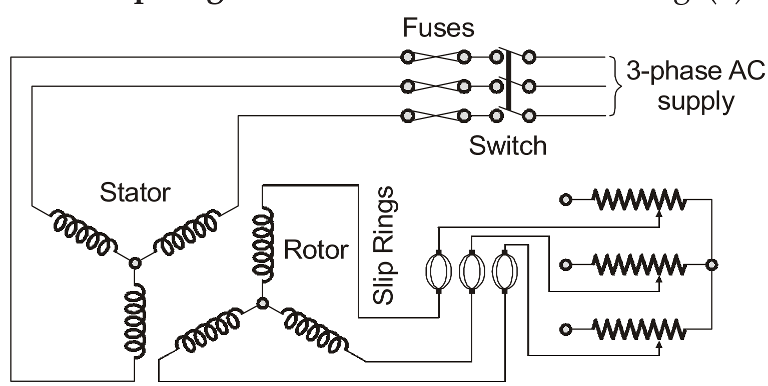
2. **Wound Rotor or Slip Ring Rotor:** As the name implies, such a rotor is wound with an insulated winding similar to that of the stator except that the number of slots is smaller and fewer turns per phase of a heavier conductor are used. **Bar**, **strap**, or **wire** is used for rotor windings, the last being used where many turns are desired. A large number of rotor turns increases the secondary voltage and reduces the current that flows through the slip rings. The secondary voltage

determines the insulation that must be provided; furthermore, the voltage and current influence the value of the resistance to be employed across the slip rings. The motor operation is not influenced by the number of rotor turns, but the ratio of transformation is determined by consideration of secondary current, danger of high secondary emf at starting, and distance to secondary resistors. The standstill open-circuit slip-ring voltage is usually **100 to 400 V** for small machines using hand operated gear and maximum up to 1 kV for large machines.

The rotor is wound for the same number of poles as that of the stator. **The rotor winding is always 3-phase winding even when the stator is wound for two phases. The rotor winding may be star or delta connected but star-connection is usually preferred.**

The three finish terminals are connected together to form star point and the three start terminals are connected to three phosphor-bronze (or brass) slip rings mounted on but insulated from the rotor shaft. The brushes, which carry the current from and to the rotor windings are held in box type holders mounted on insulated steel rods, securely bolted to the end shield. Each brush is fed forward by a lever held in tension by an adjustable spring. These brushes are further externally connected to a 3-phase star-connected rheostat for the purpose of starting and speed control.

At the time of starting, the entire resistance is included in the rotor circuit and this resistance is gradually cut out as the rotor picks up the speed. For the normal running condition, the entire external resistance is cut out and the rotor windings are short circuited automatically through the slip rings by means of a metal collar which is pushed along the shaft and connects all the rings together. The rotor is skewed in this case also. Since the connection of the wound secondary to the external terminals is made through slip rings and brushes, wound secondary motors often are called slip-ring induction motors. A sectional diagram of a **slip-ring induction motor** is shown in Fig. (b).



(b) Slip-Ring Induction Motor with Starting Rheostat

Squirrel cage Induction motor Advantages	Slip ring Induction motor Advantages
(i) Rugged in construction (ii) No slip rings, brush gears, etc. (iii) Minimum maintenance (iv) Trouble-free performance (v) Cheaper (vi) Comparatively higher efficiency (vii) Possible to obtain medium starting torque by using double cage rotor or deep bar rotor (viii) Relatively better cooling conditions (ix) Comparatively better pull out torque and overload capacity.	(i) Much higher starting torque (by inserting resistance in rotor circuit) (ii) Comparatively lesser starting current (2 to 3 times the full load current) (iii) Capable of starting with load demanding high starting torque (iv) speed control (by varying resistance in the rotor circuit) (v) Can be started directly on lines, (resistance in the rotor circuit acts like a starter and reduces the starting current)
Disadvantages	**Disadvantages**
(i) Low starting torque (ii) Higher starting current (5 to 6 times the full load current) (iii) No speed control (iv) Needs a starter (v) Cannot be used for loads demanding high starting torque	(i) Higher cost (ii) Comparatively lower efficiency (iii) Higher degree of maintenance (iv) Extra losses in external resistance, specially when operated at reduced speed (v) Extra slip ring, brush gears, etc.

Q.3 (c) Solution:

(i) The magnetic field produced by the solenoid,

$$H = \frac{NI}{l} = \frac{1000 \times 2.5}{0.25} = 10000 \text{ A/m}$$

The increase in magnetic induction when placed in oxygen

$$= 1.04 \times 10^{-8} \text{ Wb/m}^2 = \mu_0 M$$

This increase is due to magnetization (M),

$$M = \frac{1.04 \times 10^{-8}}{4\pi \times 10^{-7}} = 8.276 \times 10^{-3} \text{ A/m}$$

Magnetic susceptibility, $\chi_m = \frac{M}{H} = \frac{8.276 \times 10^{-3}}{10^4} = 8.276 \times 10^{-7}$

(ii) A nanoparticle of Si can be made by laser evaporation of a Si substrate in the region of a helium gas pulse. The beam of neutral clusters is photolyzed by a UV laser producing ionized clusters whose mass to charge ratio is then measured in a mass spectrometer. The most striking property of nanoparticles made of semiconducting elements is the pronounced changes in their optical properties compared to those of bulk material. There is a significant shift in the optical absorption spectra toward the blue (shorter wavelength) as the particle size is reduced.

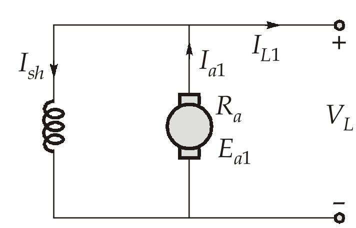
In a bulk semiconductor a bound electron-hole pair, called an exciton, can be produced by a photon having an energy greater than that of the bandgap of material. The photon excites an electron from the filled band to the unfilled band above. Because of the Coulomb attraction between the positive hole and the negative electron, a bound pair, called an exciton is formed that can move through the lattice. The separation between the hole and the electron is many lattice parameters.

The existence of the exciton has a strong influence on the electronic properties of the semiconductor and its optical absorption. The exciton can be modeled as a hydrogen-like atom and has energy levels with relative spacings analogous to the energy levels of the hydrogen atom but lower actual energies. Light induced transitions between these hydrogen like energy levels produce a series of optical absorptions.

When the nanoparticle becomes smaller than or comparable to the radius of the orbit of the electron-hole pair, there are two situations, called weak-confinement and the strong-confinement regimes. In the weak regime, the particle radius is larger than the radius of the electron-hole pair, but the range of motion of the exciton is limited, which causes the blue shift of the absorption spectrum. When the radius of the particle is smaller than the orbit radius of the electron-hole pair, the motion of the electron and the hole become independent and the exciton does not exist. The hole and electron have their own set of energy levels. Here there is also a blue shift, and the emergence of a new set of absorption lines.

Q.4 (a) Solution:

As generator,



Load current, $I_{L1} = \dfrac{110\times1000}{220} = 500\ A$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{55} = 4\text{ A}$$

Armature current, $I_{a1} = I_{L1} + I_{sh}$

$= 500 + 4 = 504$ A

Generated voltage, $E_1 = V + I_{a1}R_a$ + brush contact drop

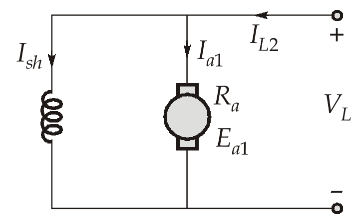
$= 220 + (504 \times 0.025) + 2 = 234.6$ V

As Motor

When the belt breaks and the terminals of the generator remain connected across the brushes, the machine continues to run as a motor. The direction of the armature current is reversed compared to the direction of current in the armature while running as generator.

However, the direction of rotation remains the same.

The line input current to the motor.



$$I_{L2} = \frac{11\times1000}{220} = 50 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{55} = 4 \text{ A}$$

Armature current, $I_{a2} = I_{L2} - I_{sh} = 50 - 4 = 46 \text{ A}$

Back e.m.f of the motor,

$$E_2 = V - I_{a2} R_a\text{-brush drop}$$
$$= 220 - (46 \times 0.025) - 2$$
$$= 216.85 \text{ V}$$

We know that, $E = kN\phi \Rightarrow N = \dfrac{E}{K\phi}$

Speed of the motor,

$$N_2 = \frac{E_2\phi_1}{E_1\phi_2}\times N_1$$

For the same machine, $\phi_2 = \phi_1$

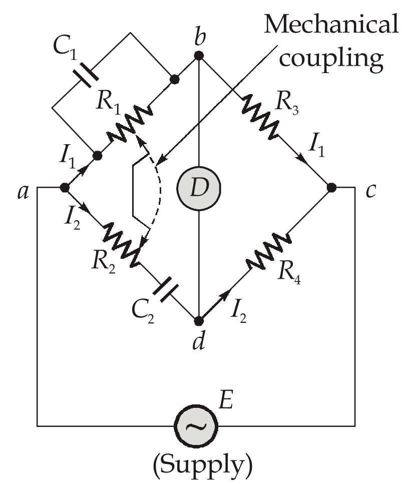
$$N_2 = \frac{E_2}{E_1}\times N_1$$

$$N_2 = \frac{216.85}{234.6}\times 400$$

$$N_2 = 369.7 \text{ rpm}$$

Q.4 (b) Solution:

The circuit diagram of Wein's bridge is shown below,



At balance condition,

$$Z_{ab} \cdot Z_{cd} = Z_{ad} \cdot Z_{bc}$$

$$\left(R_1 \,||\, \frac{1}{j\omega C_1}\right) R_4 = \left(R_2 + \frac{1}{j\omega C_2}\right) R_3$$

$$\Rightarrow \qquad \frac{R_1 R_4}{1 + j\omega C_1 R_1} = R_3\left[R_2 + \frac{1}{j\omega C_2}\right]$$

$$\Rightarrow \qquad R_1 R_4 = R_3\left[R_2 + \frac{C_1}{C_2}R_1\right] + jR_3\left[\omega C_1 R_1 R_2 - \frac{1}{\omega C_2}\right]$$

Equating real part, we get

$$R_1 R_4 = R_2 R_3 + \frac{C_1}{C_2} R_1 R_3$$

$$\therefore \qquad \frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2}$$

Equating imaginary part, we get

$$\omega C_1 R_1 R_2 - \frac{1}{\omega C_2} = 0$$

$$\therefore \qquad \omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \text{ rad/sec}$$

Thus, $$f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \text{ Hz}$$

Generally, in most of the Wein's bridges,

$$R_1 = R_2 = R$$

and $$C_1 = C_2 = C$$

then, equation becomes,

$$R_4 = 2R_3$$

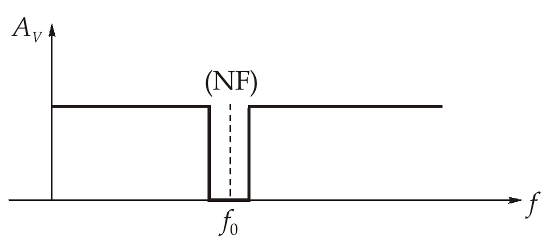
and, $$\omega = \frac{1}{RC} \text{ and } f = \frac{1}{2\pi RC}$$

The bridge components are adjusted to achieve the balance condition. When the bridge is in balance condition, the frequency of the input AC source can be calculated using the above formula.

Wein's Bridge Applications:

- It may be employed in a "Harmonic distortion analyzer" where it is used as "Notch filter".
- It also finds applications in Audio and High frequency oscillators as the frequency determining device (100 Hz-100 KHz).
- The bridge may be used in "Frequency-determining device" balanced by a single control and this control may be calibrated directly in terms of frequency.
- It may also be used for the measurement of "Capacitance".

Wein's Bridge as "Notch Filter"



- "Wein's Bridge" can reject only one particular frequency "f_0" signal for which it is tuned, while it passes all other frequencies. Hence, Wein's Bridge could be used as a "Notch Filter".

Q.4 (c) Solution:

In dielectric materials, all electrons are bound: the only motion possible in the presence of an electric field is a minute displacement of positive and negative charges in opposite directions. The displacement is usually small compared to atomic dimensions. A dielectric in which this charge displacement takes place is said to be polarized, and its molecules are said to possess induced dipole moment. These dipoles produce their own field, which adds to that of external fields. In addition to displacing the positive and negative charges, an applied electric field can also polarize a dielectric by orienting molecules that possess a permanent dipole moment.

The four basic *polarization mechanism:*

(a) Electronic or induced polarization: It occurs due to the displacement of electron cloud relative to atomic nuclei.

(b) Ionic Polarization: It occurs due to the displacement of cations (positive ions) and anions (negative ions) in opposite directions.

(c) Orientational polarization: It occurs to orientation of molecular dipoles in the direction of applied field which would otherwise be randomly distributed due to thermal randomization.

(d) Interfacial or space charge polarization: It occurs due to accumulation of charges at an interface between two materials.

For the given material,

Polarization is given by, $\vec{p} = \epsilon_0 (\epsilon_r - 1)\vec{E}$

Given, $\epsilon_r = 2.4$

and $\vec{E} = 10^4 \vec{a}_x$ V/m

Thus,
$$\vec{p} = 8.85 \times 10^{-12} \times (2.4 - 1)10^4 \vec{a}_x$$
$$= 8.85 \times 10^{-12} \times 1.4 \times 10^4$$
$$= 12.39 \times 10^{-8} \hat{a}_x \text{C/m}^2$$

Now for individual dipole moment,
$$\vec{P} = N\vec{p}$$
$$12.39 \times 10^{-8} \vec{a}_x = 3.2 \times 10^{19} \vec{p}$$

Dipole moment,
$$\vec{p} = \frac{12.39 \times 10^{-8} \vec{a}_x}{3.2 \times 10^{19}} = 3.872 \times 10^{-27} \vec{a}_x \text{C-m.}$$

Section B : Material Science + Basic Electrical Engineering + Electronic Measurements and Instrumentation

Q.5 (a) Solution:

(i) Given,

diameter of specimen, $d = 7.0 \times 10^{-3}$ m

length of specimen, $L = 57 \times 10^{-3}$ m

Current, $I = 0.25$ A

Voltage, $V = 24$ V

Distance between the probes,, l = 45 mm

Electrical conductivity of the specimen,

$$\sigma = \frac{Il}{VA} \qquad \left[\because \text{Current density, } J = \frac{I}{A} = \sigma E = \sigma\frac{V}{l}\right]$$

where, area, $A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2$

$$= \pi\left(\frac{7.0\times10^{-3}\text{m}}{2}\right)^2$$

$\therefore$ $A = 38.48 \times 10^{-6}\ \text{m}^2$

$$\therefore \quad \sigma = \frac{(0.25)(45\times10^{-3})}{(24)(38.48\times10^{-6})}$$

$$\sigma = 12.18\ (\Omega\text{m})^{-1}$$

(ii) Resistance of the entire specimen,

$$R = \frac{V'}{I}$$

where V' : Voltage across the entire length. Here, V = 24 V is measured for a length of 45 mm.

Since material is homogeneous,

$$\frac{V'}{L} = \frac{V}{l}$$

$$V' = \frac{VL}{l} = \frac{24\times57\times10^{-3}}{45\times10^{-3}}$$

$$V' = \frac{1368}{45}\text{V}$$

$\therefore$ $V' = 30.4$ V

$$\therefore \quad R = \frac{V'}{I} = \frac{30.4}{0.25} = 121.6\ \Omega$$

Q.5 (b) Solution:

Given, Voltage, $V = 0.6 + 0.4 \sin \omega t - 0.1 \sin 2\omega t$

At $\omega = 10^6$ rad/sec,

$$V = 0.6 + 0.4 \sin 10^6 t - 0.1 \sin 2 \times 10^6 t \text{ Volts}$$

Electrostatic instrument measures RMS value.

We have,

$$i_c = C\frac{dV}{dt}$$

$$= 10^{-6}\left[\frac{d}{dt}(0.6+0.4\sin\omega t-0.1\sin 2\omega t)\right]$$

$$= 10^{-6}\frac{d}{dt}\left[0.6+0.4\sin 10^6 t-0.1\sin 2\times 10^6 t\right]$$

$$i_c = 10^{-6}[0.4 \times 10^6 \cos 10^6 t - 0.1 \times 2 \times 10^6 \cos 2 \times 10^6 t]$$

$$i_c = 0.4 \cos 10^6 t - 0.2 \cos 2 \times 10^6 t$$

∴ RMS value read by Electrostatic ammeter is,

$$i_{c(\text{rms})} = \sqrt{\left(\frac{0.4}{\sqrt{2}}\right)^2+\left(\frac{0.2}{\sqrt{2}}\right)^2} = \sqrt{\frac{0.4^2}{2}+\frac{0.2^2}{2}}$$

∴ $$i_{c(\text{rms})} = 0.316 \text{ A}$$

The moving coil voltmeter reads average value i.e. the D.C. value,

$$V_{mc} = 0.6 \text{ V}$$

The resistor's current

$$I_R = \frac{V}{R} = \frac{0.6+0.4\sin 10^6 t-0.1\times\sin 2\times 10^6 t}{1000}$$

$$= 0.6 \times 10^{-3} + 0.4 \times 10^{-3} \sin 10^6 t - 0.1 \times 10^{-3} \sin 2 \times 10^6 t$$

Since electrostatic ammeter reads RMS current,

$$I_{R(\text{rms})} = \sqrt{(0.6\times 10^{-3})^2+\left(\frac{0.4\times 10^{-3}}{\sqrt{2}}\right)^2+\left(\frac{0.1\times 10^{-3}}{\sqrt{2}}\right)^2}$$

$$= 10^{-3}\sqrt{(0.6)^2+\left(\frac{0.4}{\sqrt{2}}\right)^2+\left(\frac{0.1}{\sqrt{2}}\right)^2}$$

∴ $$I_{R(\text{rms})} = \sqrt{0.36+0.08+0.005} = 0.667 \text{ mA}$$

The moving Iron instrument reads RMS value

∴ $$V_{\text{rms}} = \sqrt{(0.6)^2+\left(\frac{0.4}{\sqrt{2}}\right)^2+\left(\frac{0.1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{0.36+0.08+0.005}$$

∴ $$V_{\text{rms}} = 0.667 \text{ V}$$

Q.5 (c) Solution:

(i) The resistance at normal position (slider is at center) = $\frac{10000}{2} = 5000\ \Omega$

Resistance of potentiometer per unit length = $\frac{10000}{50} = 200\ \Omega/\text{mm}$

1. Change of resistance from normal position

$$= 5000 - 3850 = 1150\ \Omega$$

Therefore, Displacement of slider from its normal position

$$= \frac{1150}{200} = 5.75 \text{ mm}$$

2. For a resistance of 7560 Ω,

$$\text{Displacement} = \frac{7560 - 5000}{200} = 12.8 \text{ mm}$$

$$\text{Resolution} = \frac{\text{Minimum resistance that can be measured}}{200 \text{ mV/mm}}$$

$$= \frac{10}{200} = 0.05 \text{ mm}$$

(ii) For plane *A*,

The fractional intercept along *x*, *y* and *z* crystallographic direction are, $\left(\frac{1}{2}, 1, -1\right)$

The rationalized receiprocals of the fractional intercept are,

$$(hkl) = (2\ 1\ {-1})$$

(or) $$(hkl) = (2\,1\,\bar{1})$$

$\therefore$ The Miller indices for plane *A* is $(2\,1\,\bar{1})$

For plane '*B*':

The fractional intercept along *x*, *y* and *z* crystallographic direction are

$$\left(\infty, \frac{1}{2}, -1\right)$$

The rationalized reciprocals of the fractional intercept are,

$$(hkl) = \left(\infty\ \frac{1}{2} - 1\right) = \left(\frac{1}{\infty}\ \ 2\ \ -1\right)$$

$$= (0\ \ 2\ \ {-1})$$

(or) Miller indices $(hkl) = (0\ \ 2\ \ \bar{1})$

Q.5 (d) Solution:

Most alternators have the rotating field and the stationary armature. The rotating-field type alternator has several advantages over the rotating-armature type alternator:

1. A stationary armature is more easily insulated for the high voltage for which the alternator is designed. This generated voltage may be as high as 33 kV.
2. The armature windings can be braced better mechanically against high electromagnetic forces due to large short-circuit currents when the armature windings are in the stator.
3. The armature windings, being stationary, are not subjected to vibration and centrifugal forces.
4. The output current can be taken directly from fixed terminals on the stationary armature without using slip rings, brushes, etc.
5. The rotating field is supplied with direct current. Usually the field voltage is between 100 to 500 volts. Only two slip rings are required to provide direct current for the rotating field, while at least three slip rings would be required for a rotating armature. The insulation of the two relatively low voltage slip rings from the shaft can be provided easily.
6. The bulk and weight of the armature windings are substantially greater than the windings of the rotating field poles. Therefore, the rotating field type alternator has a smaller size than a rotating armature type alternator.
7. Rotating field is comparatively light and can be constructed for high speed rotation.
8. The stationary armature may be cooled more easily because the armature can be made large to provide a number of cooling ducts.

Q.5 (e) Solution:

Machine A

- Error specification is 0.2% of reading + 10 counts
- Error on reading 100 V = $100 \times \frac{0.2}{100} = 0.2$ V
- Error due to counts: A 4 1/2 digit DVM counts from 00000 to 19999. Thus,

$$\frac{\text{Volt}}{\text{Count}} = \frac{200}{20000} = 0.01$$

So, error due to 10 counts = 0.01 × 10 = 0.1 V

$$\text{Total error} = 0.2\% \text{ of } 100\text{ V} + 0.1$$
$$= 0.2 + 0.1 = 0.3\text{ V}$$

$$\% \text{ Error} = \frac{0.3}{100} \times 100 = 0.3\%$$

Machine B

- Error specification is 0.2% of reading +2 digit

$$\text{Sensitivity on 200 V range} = \frac{FSR}{10^N} = \frac{200}{10^4} = 0.02 \text{ volt/digit}$$

$$\text{Error due to digits} = 0.02 \times 2 = 0.04 \text{ V}$$

So, $$\text{Total error} = 0.2\% \text{ of reading} + 2 \text{ digit}$$

$$= \frac{0.2}{100} \times 100 + 0.04$$

$$\text{Total error} = 0.24$$

$$\% \text{ error} = \frac{0.24}{100} \times 100 = 0.24\%$$

As machine B has less % error as compared to machine A. Therefore, machine B is more suitable for the customer.

Q.6 (a) Solution:

(i) Given, $S = 100 \text{ kVA} = 100 \times 10^3 \text{ VA}$

We know that,

$$\text{Efficiency, } \eta = \frac{xS\cos\phi}{xS\cos\phi + P_i + x^2 P_{cfl}}$$

Given at full load ($x = 1$) and power factor $\cos\phi = 0.83$, efficiency is 97.23%. Assume P_i as the core loss and P_{cfl} as the full load copper loss. Thus,

$$\eta = \frac{(1 \times 100 \times 10^3 \times 0.83)}{(100 \times 10^3 \times 0.83) + P_i + P_{cfl}} = \frac{97.23}{100}$$

$$83 \times 10^5 = 97.23\,(83 \times 10^3 + P_i + P_{cfl})$$

$$P_i + P_{cfl} = 2364.6 \text{ Watts} \qquad \text{...(i)}$$

At half full load $\left(x = \frac{1}{2}\right) \cos\phi = 1, \eta = 98.72\%$

$$\eta_{hl} = \frac{\left(\frac{1}{2} \times 100 \times 10^3 \times 1\right)}{\left(\frac{1}{2} \times 100 \times 10^3 \times 1\right) + P_i + \left(\frac{1}{2}\right)^2 P_{fl}} = \frac{98.72}{100}$$

$$\frac{98.72}{100} = \frac{50\times10^3}{50\times10^3 + P_i + \frac{P_{cfl}}{4}}$$

$$\frac{98.72}{100} = \frac{50\times10^3\times4}{200\times10^3 + 4P_i + P_{cfl}}$$

$$(98.72)(200 \times 10^3 + 4P_i + P_{cfl}) = 100 \times 50 \times 4 \times 10^3$$

$$4P_i + P_{clf} = 2593.19 \text{ Watts} \quad ...(ii)$$

On solving equation (i) and (ii), we get

$$P_i = 76.2 \text{ Watt}$$

$$P_{cfl} = 2288.403 \text{ Watt}$$

At maximum efficiency, the iron loss is equal to copper loss. Assume maximum efficiency occurs at fraction 'x' of the full load. We have,

$$P_i = x^2 P_{cfl} \Rightarrow x = \sqrt{\frac{P_i}{P_{cfl}}}$$

Full load current on the secondary side = $\frac{100\times1000}{11000} = 9.09$ A

At maximum efficiency, $I_{2m} = I_{2fl}\sqrt{\frac{P_i}{P_{cfl}}} = 9.09\sqrt{\frac{76.2}{2288.403}} = 1.66$ A

(ii) 1. The synchronous speed of the motor is,

$$N_s = \frac{120f}{P} = \frac{120\times50}{2} = 3000 \text{ rpm}$$

We know that, slip, $s = \frac{N_s - N_m}{N_s}\times100\%$

$$= \frac{3000-2950}{3000}\times100\%$$

$$s = 1.67\%$$

2. As frictional and windage losses are zero, it implies,

$$P_m = P_{ind} = P_{\text{shaft}}$$

$$T_{ind} = \frac{P_m}{\omega_m} = \frac{15\times10^3}{\left(\frac{2\pi N_m}{60}\right)}$$

$$T_{ind} = \frac{15000\times60}{2\pi\times2950} = 48.56 \text{ N-m}$$

3. Considering linear torque-speed characteristics in low slip region, torque is directly proportional to slip.

Thus, if the torque is doubled, then the new slip will be

$$s' = 2 \times s = 2 \times 1.67 = 3.34\%$$

Therefore,

$$\text{Motor speed, } N'_m = (1 - s')N_s$$

$$N'_m = \left(\frac{1-3.34}{100}\right) \times 3000 = 2899.8 \text{ rpm} = 2900 \text{ rpm}$$

4. The power supplied by the motor,

$$P'_m = T'_{ind} \cdot \omega_m$$

For $T'_{ind} = 2T_{ind}$,

$$P'_m = (2 \times 48.56) \cdot \frac{2\pi N'_m}{60} = \frac{2 \times 48.56 \times 2\pi \times 2900}{60}$$

$$P'_m = 29.5 \text{ kW}$$

Q.6 (b) Solution:

(i) The voltage across the resistance R_b without either meter connected, is calculated using the voltage division rule.

Therefore, $$V_{Rb} = \frac{30 \times 5 \text{ k}\Omega}{30 \text{ k}\Omega} = 5 \text{ V}$$

(ii) Starting with meter 1, having sensitivity $S = 1$ kΩ/V,

Resistance of meter 1,

$$R_{m1} = \text{Sensitivity} \times \text{Full-scale voltage range}$$

$$= \frac{1000\ \Omega}{V} \times 10 \text{ V}$$

$$R_{m1} = 10 \text{ k}\Omega$$

The meter resistance R_{m1} appears in parallel with R_b. Thus, the equivalent resistance,

$$R_{eq} = \frac{R_b \times R_{m1}}{R_b + R_{m1}} = \frac{5k \times 10k}{5k + 10k} = 3.33 \text{ k}\Omega$$

Therefore, the voltage reading obtained with meter 1 using the voltage division equation is,

$$V_{Rb} = \frac{R_{eq}}{R_{eq} + R_a} \cdot V = \frac{3.33k}{3.33k + 25k} \times 30 = 3.53 \text{ V}$$

(iii) The total resistance that meter 2 presents to the circuit is

$$R_{m2} = S \times \text{Range} = 20\ \text{k}\Omega/\text{V} \times 10\ \text{V}$$
$$= 200\ \text{k}\Omega$$

The parallel combination of R_b and meter 2 gives,

$$R_{eq} = \frac{R_b \times R_{m2}}{R_b + R_{m2}} = \frac{5k \times 200k}{5k + 200k} = 4.88\ \text{k}\Omega$$

Therefore the voltage reading obtained with meter-2, using the voltage division equation is

$$V_{Rb} = \frac{4.88k}{25k + 4.88k} \times 30 = 4.9\ \text{V}$$

(iv) The error in the reading of the voltmeter is given as:

$$\%\ \text{error} = \frac{\text{Measured value} - \text{True value}}{\text{True value}} \times 100$$

$$\%\ \text{error for meter 1} = \frac{3.33\,\text{V} - 5\,\text{V}}{5\,\text{V}} \times 100 = -33.4\%$$

Similarly,

$$\%\ \text{error for meter 2} = \frac{4.9\,\text{V} - 5\,\text{V}}{5\,\text{V}} \times 100 = -2\%$$

Q.6 (c) Solution:

(i) Relationship between polarizability and permittivity:

In a dielectric, the displacement flux density

$$D = \epsilon_0 E + P \quad \text{...(i)}$$

where, E is the electric field strength and P is the polarization.

Also, we know that $D = \epsilon_0 \epsilon_r E$ ...(ii)

From equation (i) and (ii),

$$\epsilon_0 \epsilon_r E = \epsilon_0 E + P$$

$$\Rightarrow \quad P = \epsilon_0(\epsilon_r - 1)E$$

But polarization, $P = N\alpha E$

where, N is the number of dipoles per unit volume and α is called polarizability

So, $\epsilon_0(\epsilon_r - 1)E = N\alpha E_i$ ...(iii)

where E_i is the internal electric field in the dielectric.

This is the desired relationship between the polarizability and permittivity.

In a cubic crystal, the internal field seen by an atom is

$$E_i = E + \frac{P}{3\epsilon_0}$$

or $$E_i = E + \frac{\epsilon_0(\epsilon_r - 1)E}{3\epsilon_0}$$

$\Rightarrow$ $$E_i = \left(\frac{\epsilon_r + 2}{3}\right)E$$

Using equation (iii),

$$\epsilon_0(\epsilon_r - 1)E = N\alpha\left(\frac{\epsilon_r + 2}{3}\right)E$$

$\Rightarrow$ $$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0} \qquad \text{...(iv)}$$

This equation is the Clausius Mossotti equation.

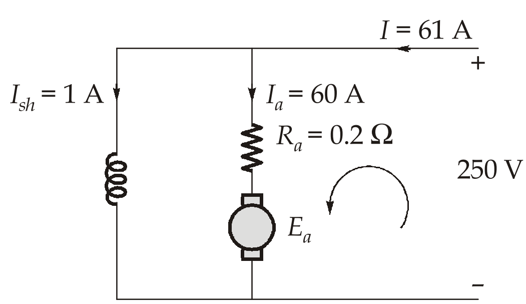
(ii) Polarization, $$P = \varepsilon_0(\varepsilon_r - 1)E$$

$$\varepsilon_r = 1 + \frac{P}{\varepsilon_0 E} = 1 + \frac{4.3\times10^{-8}\,\text{C/m}^2}{(8.854\times10^{-12}\,\text{F/m})(1000\,\text{V/m})}$$

$$\varepsilon_r \approx 5.86$$

Q.7 (a) Solution:

We can draw electrical equivalent of motor as,



On applying KVL, we get

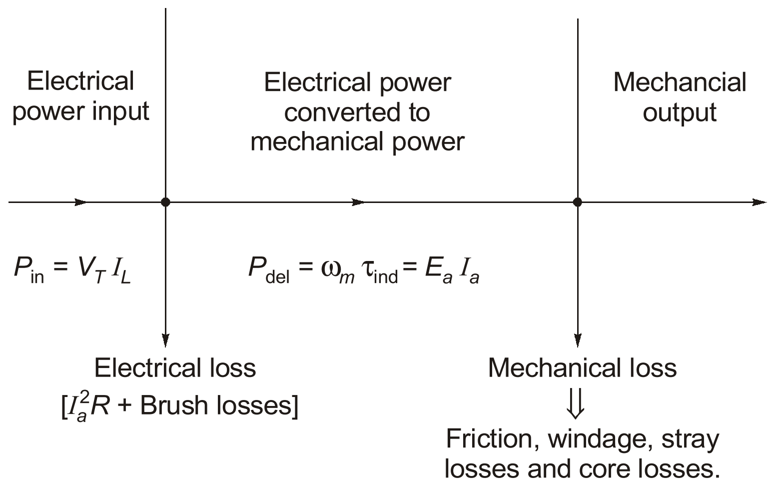
$$-250 + I_aR_a + 2[\text{brush drop}] + E_a = 0$$

$$E_a = 250 - 2 - I_aR_a$$

$$E_a = 248 - (60 \times 0.2)$$

$$E_a = 236\text{ V}$$

The power flow diagram of DC motor is given as below,



$$P_{in} = V_T I_L = 250 \times 61 = 15.25 \text{ kW}$$

$$\text{Electrical loss} = \left[I_a^2 R_a + \text{Brush losses}\right]$$

$$= (60)^2 \times 0.2 + (2 \times 60)$$

$$= 840 \text{ W} = 0.840 \text{ kW}$$

(i) Total torque:

$$\tau_{ind} = \frac{E_a I_a}{\omega_m}$$

$$\tau_{ind} = \frac{236 \times 60}{\frac{2\pi N}{60}} = \frac{236 \times 60 \times 60}{2\pi \times 1000} = 135.22 \text{ Nm}$$

(ii) We have,

$$P_{out} = 10 \text{ kW}$$

$$\Rightarrow \quad \tau_{useful} \cdot \omega = 10 \text{ kW}$$

$$\tau_{useful} = \frac{10 \times 10^3 \times 60}{2\pi N}$$

$$\tau_{useful} = \frac{10 \times 10^3 \times 60}{2\pi \times 1000}$$

$$\text{Useful Torque, } \tau_{useful} = 95.5 \text{ Nm}$$

(iii) As,

$$E_a = \frac{NP\phi}{60} \times \frac{Z}{A}$$

For wave connected winding, $A = 2$. Thus,

$$236 = \frac{4 \times 1000 \times \phi}{60} \times \frac{560}{2}$$

$$\phi = \frac{236 \times 60 \times 2}{560 \times 4000}$$

$$\phi = 0.0126 \text{ Wb}$$

(iv) Power developed $= E_a I_a = 236 \times 60 = 14160$ W

It is given that the motor gives the 10 kW output.

∴ Total power output + Rotational losses = Power developed

Rotational losses $= 14160 - 10000 = 4160$ W

(v) Total input to motor $= VI$

$$= V(I_a + I_{sh})$$

$$= 250(60 + 1)$$

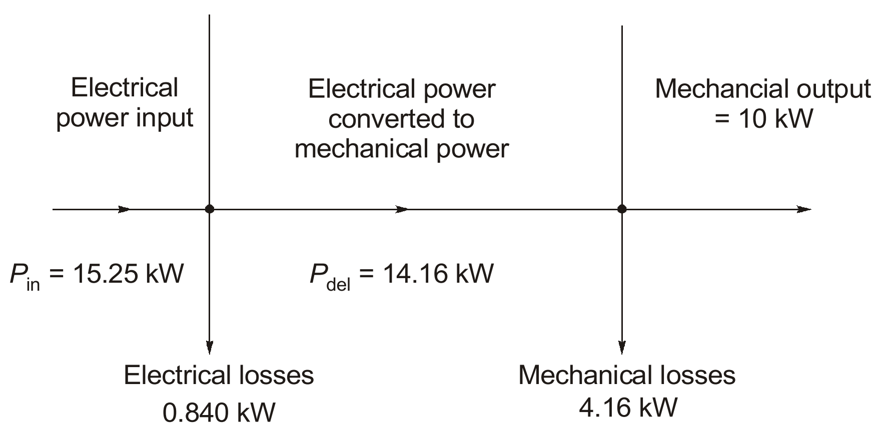
$$= 15250 \text{ W}$$

$$\eta\% = \frac{\text{motor output}}{\text{motor input}} \times 100\%$$

$$= \frac{10000}{15250} \times 100 = 65.57\%$$

Now,

Complete power-flow diagram of the motor is drawn as below:



Q.7 (b) Solution:

(i) Let, E_a = Voltage of pre-accelerating anode

m = Mass of electron (in kg)

E_d = Potential difference between deflecting plates (Volt)

d = Distance between deflecting plates (in m)

l_d = Length of deflecting plates (in m)

L = Distance between screen and the centre of the deflecting plates (in m)

D = Deflection of electron beam on the screen in y-direction

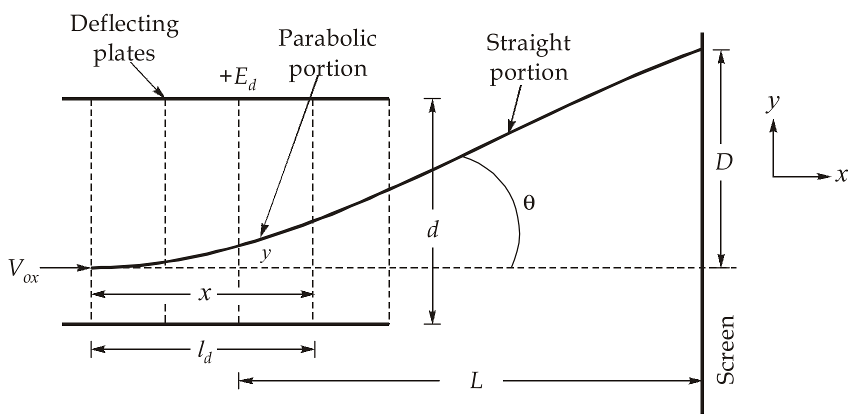
a_y = Acceleration of electron in y-direction
q = e = Charge on electron
V = Velocity of electron

The electron beam is accelerated before entering the deflecting plates to achieve the velocity V by accelerating voltage E_a.

The loss in potential energy of $e^-(P.E.) = q \times E_a$

Also, gain in kinetic energy, $(K.E.) = \frac{1}{2}mV^2$ where V is the velocity of electron when entering field of deflecting plates.

We have, Loss in $P.E.$ = Gain in $K.E.$



$$\therefore \qquad qE_a = \frac{1}{2}mV^2$$

$$\Rightarrow \qquad V = \sqrt{\frac{2qE_a}{m}}$$

Deflection of electron beam in CRT is given by,

$$D = \frac{Ll_dE_d}{2dE_a}$$

For a deflection of 3 cm, voltage applied to deflecting plates

$$E_d = \frac{2dE_aD}{Ll_d} = \frac{2\times5\times10^{-3}\times2000\times3\times10^{-2}}{0.3\times2\times10^{-2}} = 100 \text{ V}$$

$\therefore$ Input voltage required for a deflection of 3 cm,

$$V_i = \frac{E_d}{\text{Amplifier Gain}} = \frac{100}{100} = 1 \text{ V}$$

(ii) Moment of inertia of beam,

$$I = \frac{1}{12}bt^3 = \frac{1}{12}\times 0.02\times(0.003)^3$$
$$= 45 \times 10^{-12} \text{ m}^4$$

$$\text{Deflection, } x = \frac{Fl^3}{3EI}$$

$$\therefore \quad \text{Force, } F = \frac{3EIx}{l^3}$$
$$= \frac{3\times 200\times 10^9\times 45\times 10^{-12}\times 12.7\times 10^{-3}}{(0.25)^3} = 22 \text{ N}$$

Bending moment at 0.15 m from free end,

$$M = Fx = 22 \times 0.15 = 3.3 \text{ Nm}$$

Stress at 0.15 m from free end,

$$S = \frac{M}{I}\cdot\frac{t}{2} = \frac{3.3}{45\times 10^{-12}}\times\frac{0.003}{2} = 110 \text{ MN/m}^2$$

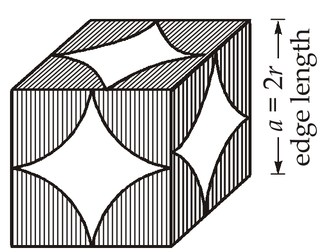
$$\text{Strain, } \varepsilon = \frac{\Delta L}{L} = \frac{S}{E} = \frac{110\times 10^6}{200\times 10^9} = 0.55 \times 10^{-3}$$

$$\therefore \quad \text{Gauge factor} = \frac{\Delta R / R}{\Delta L / L} = \frac{0.152/120}{0.55\times 10^{-3}} = 2.3$$

Q.7 (c) Solution:

There are three types of cubic crystal structures:

- **Simple Cubic Crystal structure (SCC):**
  In this structure, there is one Lattice point at each of the eight corners of the unit cell. It has a coordination number of six.



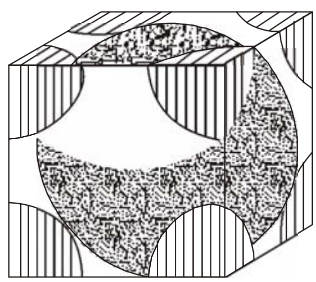
Atomic Packing Factor (APF) of the crystal structure is defined as the ratio of total volume of the atoms per unit cell to the volume of the unit cell. It is also known as packing efficiency (η)

Number of atoms per unit cell = $\frac{1}{8}\times 8 = 1$

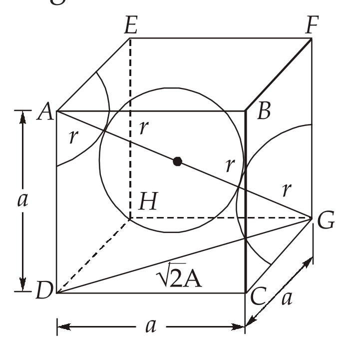
Here, $\quad \text{APF} = \dfrac{1\times\frac{4}{3}\pi r^3}{a^3} = \dfrac{1\times(\pi/6)a^3}{a^3} = \dfrac{\pi}{6} = 0.524$

$\therefore \quad$ % APF = 52.4%

- **Body Centered Cubic structure (BCC):**
  In this structure, there are eight atoms at corners and one atom at the center of the unit cell. It has a coordination number of eight.



(a)



(b)

Body centered cubic structure

From the figure,

$$\left(\frac{r}{2}+2r+\frac{r}{2}\right)^2 = (\sqrt{2}a)^2 + a^2 \quad \Rightarrow \quad r = \frac{a\sqrt{3}}{4}$$

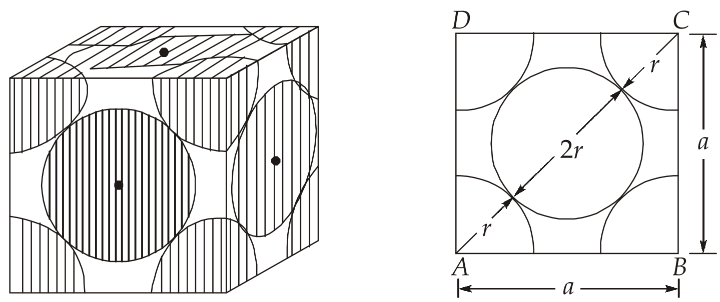
and for BCC, $\quad N = \frac{1}{8}\times 8 + 1 = 2$

Now, $\quad (\text{APF})_{BCC} = \dfrac{N\times \text{Volume of atom}}{\text{Volume of unit cell}}$

$$= \frac{2\times\frac{4}{3}\pi\left(\frac{a\sqrt{3}}{4}\right)^3}{a^3} = \frac{\frac{2\times\pi\sqrt{3}}{16}a^3}{a^3} = \frac{\pi\sqrt{3}}{8} = 0.68$$

$\therefore \quad$ % APF = 68%

- **Face Centered Cubic structure (FCC):**
  In this structure, one atom lies at each corner of the cube and one atom at the center of each face. The coordination number of FCC structure is (4 + 4 + 4 = 12)



(a) (b)

Face centered cubic structure

From the figure,

$$16r^2 = a^2 + a^2$$

$\Rightarrow$ Atomic radius, $r = \dfrac{a\sqrt{2}}{4}$

and for FCC, $N = \dfrac{1}{8}\times 8+\dfrac{1}{2}\times 6 = 4$

Now, $(\text{APF})_{FCC} = \dfrac{N \times \text{Volume of atom}}{\text{Volume of unit cell}}$

$$= \frac{4\times\frac{4}{3}\pi\times\left(\frac{a\sqrt{2}}{4}\right)^3}{a^3} = \frac{4\times\frac{\sqrt{2}\pi a^3}{24}}{a^3} = \frac{\sqrt{2}\pi}{6} = \frac{\pi}{3\sqrt{2}} = 0.74$$

$\therefore$ % APF = 74 %

Q.8 (a) Solution:

Principle of operation: The signal waveform is converted to a train of pulses and is applied continuously to an AND gate, as shown in figure below. A pulse of 1s is applied to the other terminal and the number of pulses counted during this period indicates the frequency.

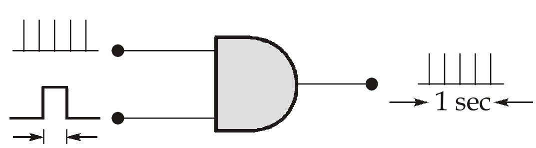
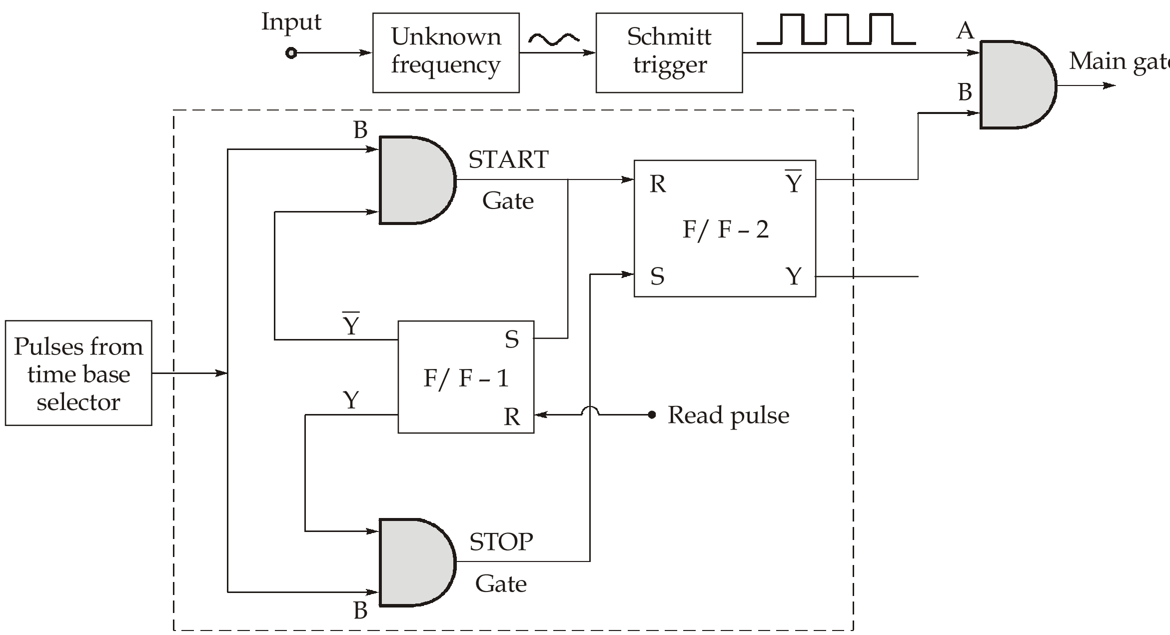


Fig: Principle of digital frequency measurement

The signal whose frequency is to be measured is converted into a train of pulses, one pulse for each cycle of the signal. The number of pulses occurring in adequate interval of time is then counted by an electronic counter. Since each pulse represents the cycle of the unknown signal, the number of counter is a direct indication of the frequency of the signal.

Basic circuit for frequency measurement: The basic circuit or frequency measurement is as shown in figure below. The input unknown frequency is applied to a Schmitt trigger, producing positive pulses at the output. These pulses are called the counter signals and are present at point A of the main gate. Positive pulses from the time base selector are present at input of the start gate and the stop gate.



Gate control F/F

Initially, the flip-flop (FF – 1) is at its logic 1 state. The resulting voltage from output Y is applied to point A of the STOP gate and enables this gate. The logic 0 stage at the output $\overline{Y}$ of the F/F – 1 is applied to the input A of the START gate and disables the gate.

As the STOP gate is enabled, the positive pulses from the time base pass through the STOP gate to the set (S) input of the F/F – 2 thereby setting F/F – 2 to the 1 state and keeping it there.

The resulting 0 output level from $\overline{Y}$ of F/F – 2 is applied to terminal B of the main gate. Hence, no pulses from the unknown frequency source can pass through the main gate.

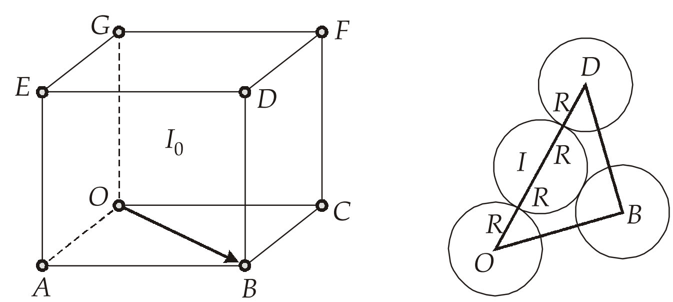
In order to start the operation, a positive pulse is applied to (read input) reset input of F/F – 1, thereby causing its state to change. Hence $\overline{Y}$ = 1, Y = 0 and as a result the STOP gate is disabled and the START gate is enabled.

The pulse from the unknown frequency source pass through the main gate to the counter and the counter starts counting. This same pulse from the START gate is applied to the set input of F/F – 1, changing its state from 0 to 1. This disables the START gate and enables the stop gate. However, till the main gate is enabled, pulses from the unknown frequency continue to pass through the main gate to the counter.

The counter counts the number of pulses occurring between two successive pulses from the time base selector. If the time interval between this two successive pulses from the time base selector is one second, then the number of pulses counted within this interval is the frequency of the unknown frequency source, in hertz.

Q.8 (b) Solution:

(i) BCC unit cell: In a BCC unit cell, atoms are present at each of the eight corners of the cube and one atom at the center of the cube.



Vector OB is along [1 1 0] direction.

$$\therefore \quad OD = (R + 2R + R) = 4R$$

$$BD = a$$

For BCC,

$$a = \frac{4R}{\sqrt{3}}$$

$$OB = \sqrt{OD^2 - BD^2} = \sqrt{16R^2 - \frac{16R^2}{3}} = 4R\sqrt{\frac{2}{3}}$$

Linear atomic density, $LD_{[1\,1\,0]} = \dfrac{\text{Number of atoms centred along on [1 1 0] direction}}{\text{Length of [1 1 0] direction vector}}$

$$= \frac{\left(\frac{1}{2}+\frac{1}{2}\right)\text{atom}}{OB} = \frac{1 \text{ atom}}{4R\times\sqrt{\frac{2}{3}}}$$

We have,

$$R = \frac{\sqrt{3}a}{4}$$

$$LD_{[1\,1\,0]} = \frac{1\text{ atom}}{4\times\frac{\sqrt{3}a}{4}\times\sqrt{\frac{2}{3}}} = \frac{1\text{ atom}}{\sqrt{2}a}$$

$$LD_{[1\,1\,0]} = \frac{1\text{ atom}}{\sqrt{2}\times 2.89\times 10^{-7}\text{ mm}}$$

$$\therefore \quad LD_{[1\,1\,0]} = 2.446 \times 10^{6}\text{ atoms/mm}$$

(ii) Given, critical magnetic field at $T = 8$ K,

$$H_C(T) = 10^5\text{ A/m}$$

Critical magnetic field at $T = 0$ K, $H_0 = 2 \times 10^5$ A/m

The critical magnetic field varies with temperature as,

$$H_C(T) = H_0\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$\therefore \quad 1-\left(\frac{T}{T_C}\right)^2 = \frac{H_C(T)}{H_0}$$

$$1-\frac{H_C(T)}{H_0} = \left(\frac{T}{T_C}\right)^2$$

$$\therefore \quad \left(\frac{T}{T_C}\right)^2 = 1-\frac{10^5}{2\times 10^5} = 1-\frac{1}{2}$$

$$\left(\frac{T}{T_C}\right)^2 = \frac{1}{2}$$

$$T^2 = \frac{T_C^2}{2}$$

$$\therefore \quad T_C^2 = 2 \times T^2$$

$$\therefore \quad T_C = \sqrt{2}\times T$$

$$= \sqrt{2}\times 8$$

$$T_C = 11.3\text{ K}$$

(iii) 1. Mean of the voltage:

$$\bar{X} = 41+\frac{0.7+1+0.8+1+1.1+0.9+1.5+1+0.9+0.8}{10}$$

$$= 41+\frac{9.7}{10} = 41.97$$

2. Standard deviation:

X	$\bar{X}$	$d = (X - \bar{X})$	d^2
41.7	41.97	–0.27	0.0729
42	41.97	+0.03	0.0009
41.8	41.97	–0.17	0.0289
42	41.97	+0.03	0.0009
42.1	41.97	+0.13	0.0169
41.9	41.97	–0.07	0.0049
42.5	41.97	+0.53	0.2809
42	41.97	+0.03	0.0009
41.9	41.97	–0.07	0.0049
41.8	41.97	–0.17	0.0289
			0.4410

$$\sigma = \sqrt{\frac{\sum_{i=1}^{10} d_i^2}{n-1}} = \sqrt{\frac{0.4410}{9}} = 0.2214 \text{ V}$$

3. Probable error:

$$\text{Probable error} = 0.6745\,\sigma = 0.2214 \times 0.6745$$
$$= 0.1493$$

Q.8 (c) Solution:

(i) We have,

Alternator with,

Number of poles, $P = 12$

The alternator runs at synchronous speed, thus

$$N_s = 500 \text{ rpm} = \frac{120 f}{P}$$

The frequency of alternator,

$$f = \frac{PN_s}{120} = \frac{12 \times 500}{120} = 50 \text{ Hz}$$

Given, the speed of the induction motor is 1440 rpm. Under normal operating conditions, an induction motor operates at slightly less than its synchronous speed.

The supply frequency for induction motor is 50 Hz. The possible synchronous speed for 50 Hz are $\frac{120f}{P_m} = \frac{6000}{P_m}$ i.e. 3000 rpm, 1500 rpm, 1000 rpm etc. So, the closest synchronous speed corresponding to the actual speed of 1440 rpm is 1500 rpm i.e., N_{sm} = 1500 rpm

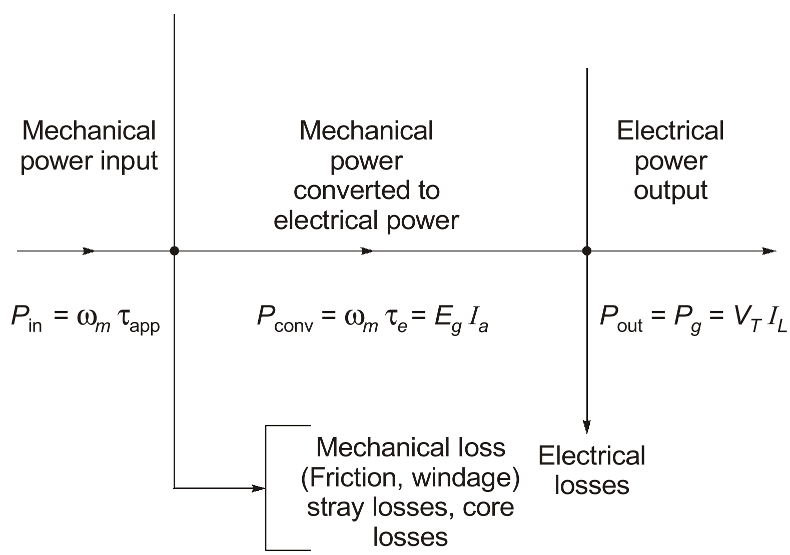
$$\text{Slip of the motor, } s = \frac{N_{sm} - N_r}{N_{sm}} = \frac{1500-1440}{1500} = 0.04 = 4\%$$

Number of poles of the motor,

$$P_m = \frac{120f}{N_{sm}} = \frac{120 \times 50}{1500}$$

$$P_m = 4$$

(ii) The Power Flow Diagram is used to determine the efficiency of a generator or motor. The power flow diagram of DC Generator is shown below. In DC generator, the mechanical power is given as an input which is converted into electrical power, and the output is obtained in the form of electrical power. There are various losses such as friction, windage, stray losses and core losses.



Power Flow Diagram of a DC Generator

The mechanical power input to the DC generator is given by the below equation,

$$P_{in} = \omega_m \tau_{app} \quad \text{...(i)}$$

where,

ω_m is the angular speed of the armature in radian per second.

τ_{app} is the applied torque in Newton-meter.

The sum of stray losses, mechanical losses and core losses are subtracted from the input power, i.e. P_{in} to get the net mechanical power converted to electrical power by Electro-Mechanical conversion.

$$P_{conv} = P_{in} - \text{Stray loss} - \text{Mechanical Loss} - \text{Core Losses}$$

$$P_{conv} = \omega_m \tau_e \quad \text{...(ii)}$$

Where τ_e is the electromagnetic torque. The resulting electric power produced is given by the equation:

$$P_{conv} = E_g I_a \quad \text{...(iii)}$$

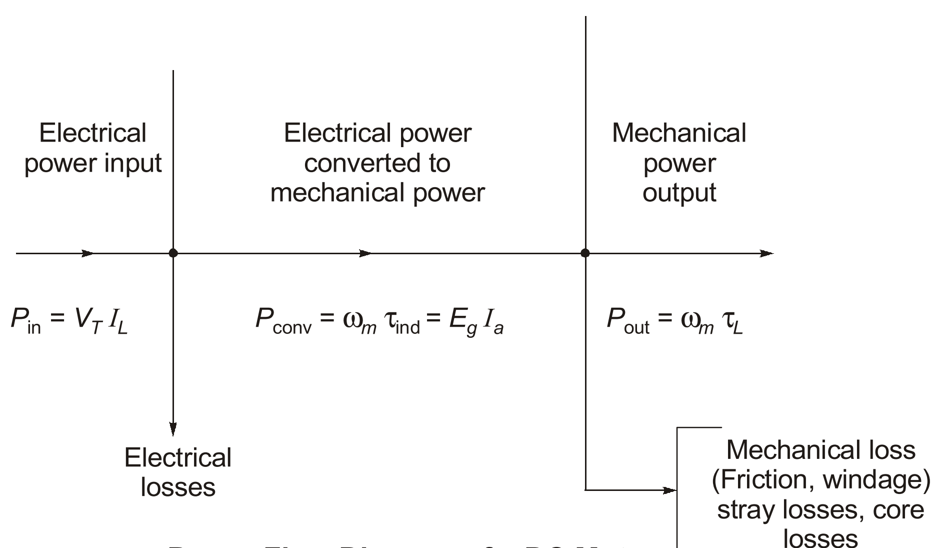
The net electrical power output is obtained by subtracting electrical power I^2R losses and brush losses from P_{conv}.

$$P_{out} = P_{conv} - \text{Electrical } I^2R \text{ loss} - \text{Brush losses} \quad \text{...(iv)}$$

$$P_{out} = V_T I_L \quad \text{...(v)}$$

where, V_T is the terminal voltage, and I_L is the current delivered to the load.

The power flow diagram of DC Motor is shown below:



Power Flow Diagram of a DC Motor

From the power flow diagram of DC Motor, it is clear that the input which is given to the motor is in the electrical form which is converted into mechanical power in the second stage. The output is in the form of mechanical power.

In a DC motor, the input electrical power P_{in} is given by the equation shown below:

$$P_{in} = V_T I_L \quad \text{...(vi)}$$

$$P_{conv} = P_i - \text{Copper Losses} \quad \text{...(vii)}$$

Power output is given by the equation shown below:

$$P_{\text{out}} = \omega_m \tau_L \quad \text{...(viii)}$$

Also, $P_{\text{out}} = P_{\text{conv}}$ – Core losses – Mechanical losses – Stray losses

and τ_L is the load torque in Newton-meter

Thus, the power flow diagram gives an overview, that how one form of energy is converted into another form.

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