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Detailed Solutions

ESE-2025 Mains Test Series

Electrical Engineering Test No : 5

Section A : Basic Electronics Engineering + Computer Fundamentals + Electromagnetic Field Theory

Q.1 (a) Solution:

$$I = \int \vec{J} \cdot d\vec{S},$$

Where, $$d\vec{S} = r^2 \sin\theta \, d\phi \, d\theta \, \hat{a}_r$$

(i) $$I = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \frac{1}{r^3} 2\cos\theta r^2 \sin\theta \, d\phi \, d\theta \Big|_{r=0.2}$$

$$= \frac{2}{r} 2\pi \int_{\theta=0}^{\pi/2} \sin\theta \, d(\sin\theta) \Big|_{r=0.2}$$

$$= \frac{4\pi}{0.2} \frac{\sin^2\theta}{2} \Big|_0^{\pi/2} = 10\pi = 31.4 \text{ A}$$

(ii) The only difference here is that we have $0 \le \theta \le \pi$ instead of $0 \le \theta \le \frac{\pi}{2}$ and $r = 0.1$ m.

Hence, $$I = \frac{4\pi}{0.1} \frac{\sin^2\theta}{2} \Big|_0^{\pi} = 0 \text{ A}$$

Alternatively, for this case

$$I = \oint \vec{J} \cdot d\vec{S} = \int \nabla \cdot \vec{J} \cdot dv = 0$$

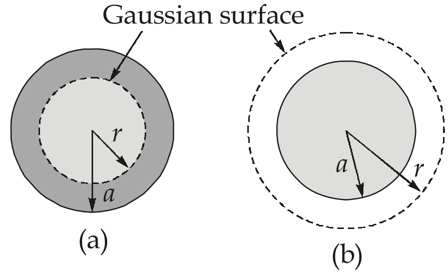
Since $\nabla \cdot \vec{J} = 0$, we can show this

$$\nabla \cdot \vec{J} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{2}{r}\cos\theta\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\frac{1}{r^3}\sin^2\theta\right]$$

$$= \frac{-2}{r^4}\cos\theta + \frac{2}{r^4}\cos\theta = 0$$

Q.1 (b) Solution:

The charge distribution is similar to that in figure,



Gaussian surface for a uniformly charged sphere when (a) $r \le a$ and (b) $r \ge a$.

Since symmetry exists, we can apply Gauss's law of find $\vec{E}$.

$$\varepsilon_0 \oint_s \vec{E} \cdot d\vec{S} = Q_{enc} = \int_v \rho_v dv$$

(i) For $r < R$,

$$\varepsilon_0 E_r 4\pi r^2 = Q_{enc} = \int_0^r \int_0^\pi \int_0^{2\pi} \rho_v r^2 \sin\theta d\phi d\theta dr$$

$$= \int_0^r 4\pi r^2 \frac{\rho_0 r}{R} dr = \frac{\rho_0 \pi r^4}{R}$$

or $$\vec{E} = \frac{\rho_0 r^2}{4\varepsilon_0 R}\hat{a}_r$$

(ii) For $r > R$,

$$\varepsilon_0 E_r 4\pi r^2 = Q_{enc} = \int_0^R \int_0^\pi \int_0^{2\pi} \rho_v r^2 \sin\theta d\phi d\theta dr$$

$$= \int_0^R \frac{\rho_0 r}{R} 4\pi r^2 dr + \int_R^r 0 \cdot 4\pi r^2 dr = \rho_0 \pi R^3$$

or $$\vec{E} = \frac{\rho_0 R^3}{4\varepsilon_0 r^2}\hat{a}_r$$

Q.1 (c) Solution:

For sliding window protocol with Go-Back-N

Data rate = 1.544 Mbps

Frame size = 64 bytes = 64 × 8 bits = 512 bits

Propagation speed = 0.16×10^6 km/s

Distance = 3000 km

Utilization = 100%

$$\Rightarrow \quad U = \frac{W \times T_t}{T_t + 2T_p}$$

$$T_p = \frac{3000\,\text{Km}}{0.16 \times 10^6\,\text{Km}}\text{sec} = 18.75\text{ msec}$$

$$T_t = \frac{512\text{ bits}}{1.544\text{ Mbps}} = 0.33\text{ msec}$$

$$\Rightarrow \quad 1 = \frac{W \times 0.33}{0.33 + 2 \times 18.75}$$

$$\Rightarrow \quad W = \frac{37.83}{0.33} = 114.64$$

Q.1 (d) Solution:

Computer architecture is a set of rules and method that describe the functionality organization and implementation of computer system. It is description of capabilities and programming model of a computer but not a particular implementation.

Reduced Instruction Set Computer architecture (RISC) has following properties:

1. RISC has simpler instruction, hence simpler instruction decoding is involved in process.
2. Instructions size is less than size of one word.
3. Instruction take single clock cycle to be executed.
4. RISC has more number of general purpose registers.
5. RISC has simple addressing models.
6. RISC has less data type involved.
7. It has hardwired unit of programming.
8. RISC processors are highly pipelined.
9. Execution time in RISC is very less.
10. RISC has no requirement of external memory for calculations.
11. RISC has applications in video processing telecommunication and image processing.

Q.1 (e) Solution:

(i) For circular cylindrical coordinate system,

$$\nabla\times\vec{A} = \frac{1}{\rho}\begin{vmatrix}\hat{\rho} & \rho\hat{\phi} & \hat{z}\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z}\\ A_\rho & \rho A_\phi & A_z\end{vmatrix} = \frac{1}{\rho}\begin{vmatrix}\hat{\rho} & \rho\hat{\phi} & \hat{z}\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z}\\ \rho z^2 & \rho^2\sin^2\phi & 2\rho z\sin^2\phi\end{vmatrix}$$

$$= \frac{1}{\rho}\left[(2\rho z\sin 2\phi)\hat{\rho} - \rho(2z\sin^2\phi - 2\rho z)\hat{\phi} + (2\rho\sin^2\phi)\hat{z}\right]$$

$$= 2z\sin 2\phi\hat{\rho} + 2z(\rho - \sin^2\phi)\hat{\phi} + 2\sin^2\phi\hat{z}$$

(ii) For spherical coordinate system,

$$\nabla\times\vec{B} = \frac{1}{r^2\sin\theta}\begin{vmatrix}\hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ A_r & rA_\theta & r\sin\theta A_\phi\end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\begin{vmatrix}\hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ r & 0 & r^2\sin\theta\cos^2\theta\end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta}(r^2\sin\theta\cos^2\theta)\hat{r} - 2r^2\sin\theta\cos^2\theta\hat{\theta}\right]$$

$$= \frac{\cos^3\theta - 2\cos\theta\sin^2\theta}{\sin\theta}\hat{r} - 2\cos^2\theta\hat{\theta}$$

Q.2 (a) Solution:

Given :

$$V_T = 0.8\text{ V}, k = \frac{\mu_n C_{ox} W}{2L} = 1.2\text{ mA/V}^2$$

$$\lambda = 0, I_{DQ} = 0.6\text{ mA}$$

$$I_{DQ} = k(V_{GSQ} - V_T)^2$$

$$0.6\times10^{-3} = 1.2\times10^{-3}(V_{GSQ} - 0.8)^2$$

$$V_{GSQ} = \sqrt{\frac{0.6\times10^{-3}}{1.2\times10^{-6}}} + 0.8$$

$$V_{GSQ} = 1.507\text{ V}$$

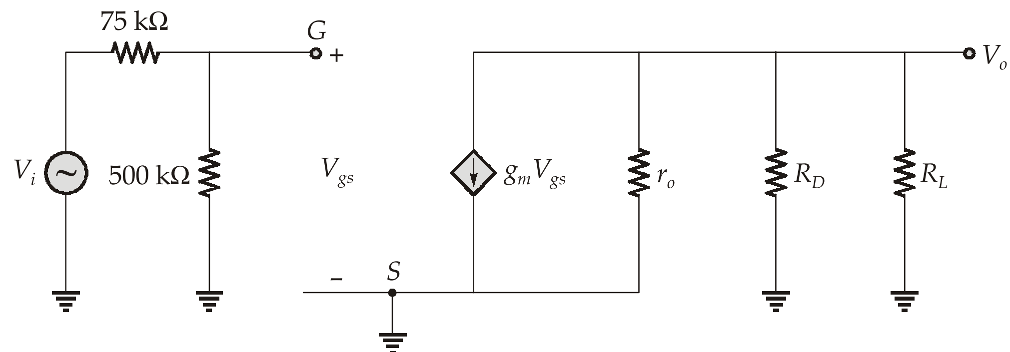
AC Analysis :

Trans-conductance, $g_m = 2k(V_{GSQ} - V_T)$

$$g_m = 2 \times 1.2 \times 10^{-3}(1.507 - 0.8) = 1.697\ \text{m℧}$$

Output resistance, $r_o = \infty$

Small signal equivalent circuit



Output voltage,

$$v_o = -g_m V_{gs}[r_o \parallel R_D \parallel R_L]$$

$$v_{gs} = \frac{500k}{(500+75)k} v_i$$

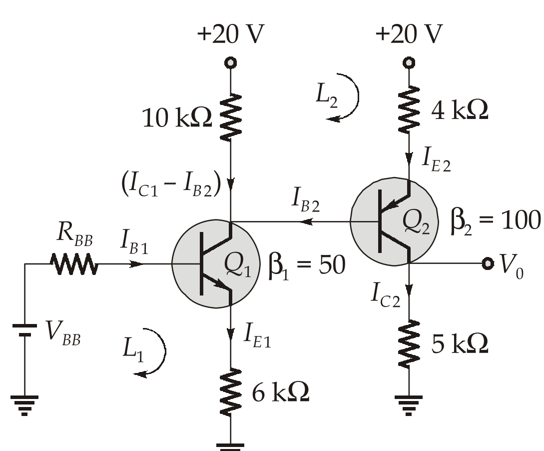
Voltage gain,

$$A_v = \frac{v_o}{v_i} = \frac{-g_m[r_o \parallel R_D \parallel R_L)500}{500+75}$$

$$A_v = \frac{-1.697 \times 10^{-3}[3.4 \times 10^3] \times 500}{575} = -5.017$$

Q.2 (b) Solution:

Given circuit can be redrawn,



$$V_{BB} = \frac{20\times100}{100+200} = 6.67 \text{ V or } \frac{20}{3} \text{ V}$$

$$R_{BB} = \frac{100\times200}{100+200} = 66.667 \text{ k}\Omega \text{ or } \frac{200}{3} \text{ k}\Omega$$

By applying KVL in loop-1,

$$6.67 - \frac{200}{3}I_{B1} - 0.7 - 6I_{E1} = 0$$

$$5.97 - \frac{200}{3}I_{B1} - 6(\beta_1 + 1)I_{B1} = 0$$

$$5.97 - \frac{200}{3}I_{B1} - 6(50 + 1)I_{B1} = 0$$

$$I_{B1} = \frac{5.97}{372666.67} = 16.02\ \mu\text{A}$$

$\therefore$
$$I_{C1} = \beta_1 I_{B1}$$
$$= 50 \times 16.02\ \mu\text{A} = 0.801 \text{ mA}$$

$$I_{E1} = I_{C1} + I_{B1}$$
$$= 0.801 \text{ mA} + 16.02\ \mu\text{A} = 0.817 \text{ mA}$$

Applying KVL in loop 2,

$$-20 + 4I_{E2} + 0.7 - (I_{C1} - I_{B2})\, 10I_{B2} + 20 = 0$$
$$4I_{E2} + 0.7 - 10I_{C1} + 10I_{B2} = 0$$
$$4(100 + 1)I_{B2} + 0.7 - 10I_{C1} + 10I_{B2} = 0$$
$$4(101)I_{B2} + 0.7 - 10(I_{C1}) + 10I_{B2} = 0$$
$$404\, I_{B2} + 0.7 - 10I_{C1} + 10I_{B2} = 0$$
$$414I_{B2} + 0.7 - 10[0.801] = 0$$
$$414\, I_{B2} = 7.31$$
$$I_{B2} = \frac{7.31}{414\times10^3} = 17.66\ \mu\text{A}$$
$$I_{C2} = 100 \times 17.66\ \mu\text{A}$$
$$= 1.766 \text{ mA}$$

Given,
$$R_L = 5 \text{ k}\Omega$$

$$\text{Output voltage} = I_{C2} \times 5 \times 10^3$$
$$= 1.766 \times 10^{-3} \times 5 \times 10^3$$
$$= 8.83 \text{ V}$$

Q.2 (c) (i) Solution:

By the principle of superposition, the potential at point P is given as

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r_1} - \frac{2Q}{r} + \frac{Q}{r_2}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\left[\frac{r}{r_1} + \frac{r}{r_2} - 2\right] \quad \text{...(i)}$$

Now, using trigonometric relation for the triangle with sides r, r_1 and l, we have

$$r_1^2 = r^2 + l^2 + 2rl\cos\theta$$

$$\therefore \quad \left(\frac{r_1}{r}\right)^2 = 1 + \frac{l^2}{r^2} + \frac{2l}{r}\cos\theta$$

$$\therefore \quad \left(\frac{r_1}{r}\right) = \left[1 + \frac{l^2}{r^2} + \frac{2l}{r}\cos\theta\right]^{1/2}$$

$$\therefore \quad \left(\frac{r}{r_1}\right) = \left[1 + \frac{l^2}{r^2} + \frac{2l}{r}\cos\theta\right]^{-1/2}$$

$$= 1 - \frac{1}{2}\left(\frac{l^2}{r^2} + \frac{2l}{r}\cos\theta\right) + \frac{-\frac{1}{2}\left(-\frac{1}{2} - 1\right)}{2}\left(\frac{l^2}{r^2} + \frac{2l}{r}\cos\theta\right)^2 + \ldots.$$

$$= 1 - \frac{l}{r}\cos\theta + \frac{l^2}{r^2}\left(\frac{3\cos^2\theta - 1}{2}\right)$$

Similarly, we have

$$\left(\frac{r}{r_2}\right) = 1 + \frac{l}{r}\cos\theta + \frac{l^2}{r^2}\left(\frac{3\cos^2\theta - 1}{2}\right)$$

Thus, from (i), we get

$$V = \frac{Q}{4\pi\varepsilon_0 r}\left[\frac{r}{r_1} + \frac{r}{r_2} - 2\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\left[1 - \frac{l}{r}\cos\theta + \frac{l^2}{r^2}\left(\frac{3\cos^2\theta - 1}{2}\right) + 1 + \frac{l}{r}\cos\theta + \frac{l^2}{r^2}\left(\frac{3\cos^2\theta - 1}{2}\right) - 1\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\left[\frac{l^2}{r^2}(3\cos^2\theta - 1)\right]$$

$$V = \frac{Ql^2}{4\pi\varepsilon_0 r^3}(3\cos^2\theta - 1)$$

Q.2 (c) (ii) Solution:

- Point $A(3, -2, 2)$ is in the conductor as

$$y = -2 < 0 \text{ at A}$$

$$\therefore \quad \vec{E} = \vec{D} = 0$$

- At point $B(-4, 1, 5)$,

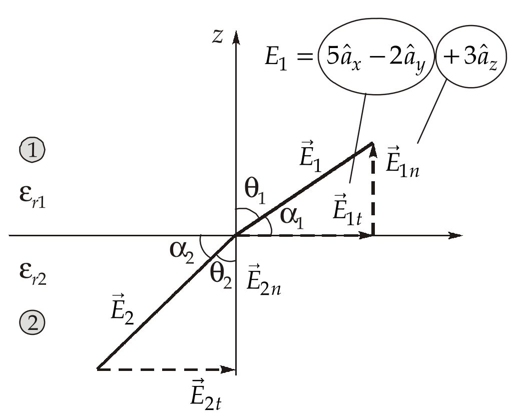
$$\rho_s = 2 \text{ nC/m}^2$$

$$\therefore \quad D_n = \rho_s = 2 \text{ nC/m}^2$$

$$\therefore \quad \vec{D} = 2\hat{a}_y \text{ nC/m}^2$$

$$\therefore \quad \vec{E} = \frac{\vec{D}}{\varepsilon_0\varepsilon_r} = \frac{2\times10^{-9}}{\frac{10^{-9}}{36\pi}\times 2} = 36\pi\,\hat{a}_y = 113.1\hat{a}_y \text{V/m}$$

Q.3 (a) Solution:



(i) Since $\hat{a}_z$ is normal to the boundary plane, we obtain the normal components as

$$E_{1n} = \vec{E}_1 \cdot \hat{a}_n = \vec{E}_1 \cdot \hat{a}_z = 3$$

$$\vec{E}_{1n} = 3\hat{a}_z$$

$$\vec{E}_{2n} = (\vec{E}_2 \cdot \hat{a}_z)\hat{a}_z$$

$$\vec{E} = \vec{E}_n + \vec{E}_t$$

Hence, $\vec{E}_{1t} = \vec{E}_1 - \vec{E}_{1n} = 5\hat{a}_x - 2\hat{a}_y$

Thus, $\vec{E}_{2t} = \vec{E}_{1t} = 5\hat{a}_x - 2\hat{a}_y$

Similarly, $\vec{D}_{2n} = \vec{D}_{1n} \rightarrow \varepsilon_{r2}\vec{E}_{2n} = \varepsilon_{r1}\vec{E}_{1n}$

or, $\vec{E}_{2n} = \dfrac{\varepsilon_{r1}}{\varepsilon_{r2}}\vec{E}_{1n} = \dfrac{4}{3}(3\hat{a}_z) = 4\hat{a}_z$

Thus, $\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n}$

$$= 5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z \text{ kV/m}$$

(ii) Let α_1 and α_2 be the angles $\vec{E}_1$ and $\vec{E}_2$ make with the interface while θ_1 and θ_2 are the angles they make with the normal to the interface as shown in figure,

$$\alpha_1 = 90 - \theta_1$$

$$\alpha_2 = 90 - \theta_2$$

Since, $E_{1n} = 3$

and $E_{1t} = \sqrt{25+4} = \sqrt{29}$

$$\tan\theta_1 = \frac{E_{1t}}{E_{1n}} = \frac{\sqrt{29}}{3} = 1.795 \rightarrow \theta_1 = 60.9°$$

Hence, $\alpha_1 = 29.1°$

Alternatively, $\vec{E}_1 \cdot \hat{a}_n = |E_1| \cdot 1 \cdot \cos\theta_1$

or $\cos\theta_1 = \dfrac{3}{\sqrt{38}} = 0.4867 \rightarrow \theta_1 = 60.9°$

Similarly, $E_{2n} = 4,$

$$E_{2t} = E_{1t} = \sqrt{29}$$

$$\tan\theta_2 = \frac{E_{2t}}{E_{2n}} = \frac{\sqrt{29}}{4} = 1.346 \rightarrow \theta_2 = 53.4°$$

Hence, $\alpha_2 = 36.6°$

Note that $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\varepsilon_{r1}}{\varepsilon_{r2}}$ is satisfied

(iii) The energy densities are given by

$$W_{E1} = \frac{1}{2}\varepsilon_1|\vec{E}_1|^2 = \frac{1}{2}\times 4\times\frac{10^{-9}}{36\pi}\times(25+4+9)\times 10^6$$

$$= 672\ \mu\text{J/m}^3$$

$$W_{E2} = \frac{1}{2}\varepsilon_2|\vec{E}_2|^2 = \frac{1}{2}\times 3\times\frac{10^{-9}}{36\pi}\times(25+4+9)\times 10^6$$

$$= 597\ \mu\text{J/m}^3$$

(iv) At the centre (3, 4 –5) of the cube of side 2 m, $z = -5 < 0$; that is, the cube is in region-2 with $2 \le x \le 4$, $3 \le y \le 5$, $-6 \le z \le -4$

Hence,
$$W_E = \int W_{E2}dv = \int_{x=2}^{4}\int_{y=3}^{5}\int_{z=-6}^{-4} W_{E2}\,dz\,dy\,dz = W_{E2}(2)(2)(2)$$

$$= 597 \times 8\ \mu\text{J} = 4.776\ \text{mJ}$$

Q.3 (b) Solution:

(i) Given :
$$H = 10^5\rho^2\hat{a}_\phi\ \text{A/m}$$

$$\rho = 5\ \text{mm},\ V = 0.1\ \text{V},\ L = 20\ \text{m}$$

From Maxwell's equation,

$$\vec{J} = \nabla\times\vec{H}$$

$$\vec{J} = \frac{1}{\rho}\begin{bmatrix}\hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z}\\ 0 & \rho 10^5\rho^2 & 0\end{bmatrix}$$

$$= \frac{1}{\rho}\left[0+0+a_z\left(\frac{\partial}{\partial\rho}(10^5\rho^3)\right)-0\right]$$

$$= \frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}(10^5\rho^3)\hat{a}_z$$

$$\vec{J} = \frac{3\times 10^5\rho^2}{\rho}\hat{a}_z = 3\times 10^5\rho\hat{a}_z$$

Electric field,

$$\vec{E} = \frac{V}{L} = \frac{0.1}{20} = 5\times 10^{-3}\ \text{V/m}$$

Also,
$$\vec{J} = \sigma\vec{E}$$
$$\sigma = \frac{\vec{J}}{\vec{E}} = \frac{3\times10^5\rho}{5\times10^{-3}}$$
$$\sigma = \frac{3\times10^5(5\times10^{-3})}{5\times10^{-3}} = 3\times10^5\ \text{S/m}$$

(ii) The current in the wire,
$$I = \int_s \vec{J}\cdot\vec{ds}$$
$$\vec{ds} = \rho d\rho d\phi \hat{a}_z$$
$$I = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{5\text{mm}} 3\times10^5\rho\cdot\rho d\rho d\phi$$
$$= 3\times10^5\left[\frac{\rho^3}{3}\right]_0^{5\text{mm}} \times[\phi]_0^{2\pi}$$
$$I = 1\times10^5\left[\rho^3\right]_0^{5\text{mm}} \times[\phi]_0^{2\pi}$$
$$= 1\times10^5[(5\times10^{-3})^3 - 0]\times 2\pi$$
$$I = 0.0785\ \text{A}$$

Resistance,
$$R = \frac{V}{I}$$
$$R = \frac{0.1}{0.0785} = 1.273\ \Omega$$
$$R = 1.273\ \Omega$$

Q.3 (c) Solution:

(i) The code is as follows:

```
# include <stdio.h>
int main ()
{
    char C;
    int lowercasevowel, uppercasevowel;
    printf ("Enter an alphabet");
```

```
    scanf("%c", &C);
    lowercasevowel = (C == 'a' || C == 'e' || C == 'i' || C = 'o' || C == 'u');
    uppercasevowel = (C == 'A' || C == 'E' || C == I || C == 'O' || C = = 'U');
    if(lowercasevowel || uppercasevowel)
    printf ("%c is a vowel", C);
    else
    printf("%c is a consonant", C);
    return 0;
}
```

Output : Enter an alphabet : G

G is a consonant

(ii) 1. While Loop:

(a) In while loop condition is tested first and statement are executed if condition is true.

(b) While loop is entry control loop.

(c) It does not run if given condition is false.

Do While Loop:

(a) In do while, the statements are executed for the first time and then the conditions are tested if condition turns out to be true the statements are again executed.

(b) do while is an exit control loop.

(c) It runs at least once even though the condition given is false.

2. Iteration:

(a) The iteration is when a loop repeatedly executes until the controlling condition becomes false.

(b) Its execution takes less time as it does not use the stack.

Recursion:

(a) Recursion is when a statement in a function calls itself repeatedly. Infinite recursion occurs if the recursion step does not reduce the problem in a manner that converges on some condition (base case) and infinite recursion can crash the system.

(b) It's execution takes more time due to the overhead of maintaining the stack.

The primary difference between recursion and iteration is that recursion is a process, always applied to a function and iteration is applied to the set of instructions which we want to get repeatedly executed.

Q.4 (a) Solution:

To obtain the value of k :

We know that,
$$k = \frac{I_{D(\text{on})}}{\left[V_{GS(\text{on})} - V_{GS(\text{Th})}\right]^2}$$

Substituting the values from figure, we get
$$k = \frac{3\times10^{-3}}{[10-5]^2} = 1.2 \times 10^{-4}\ \text{A/V}^2 \quad \text{...(i)}$$

To obtain the value of I_{DQ} :

We know that,
$$I_D = k\left[V_{GS} - V_{GS(\text{Th})}\right]^2$$
$$= 1.2 \times 10^{-4}\ [V_{GS} - 5]^2 \quad \text{...(ii)}$$

But we do not know the value of V_{GS} so let us obtain it. Consider input circuit of figure. Apply KVL to write,
$$V_G = V_{GS} + V_S = V_{GS} + I_D R_S$$
$$\therefore \quad V_{GS} = V_G - I_D R_S = \frac{R_2}{(R_1 + R_2)} \cdot V_{DD} - I_D R_S$$

Substituting the values we get,
$$V_{GS} = \frac{18}{(18+22)} \times 40 - (820 I_D) = 18 - 820 I_D \quad \text{...(iii)}$$

Substitute equation (iii) into (ii), we get
$$I_D = 1.2 \times 10^{-4}[18 - 820 I_D - 5]^2$$
$$\therefore \quad I_D = 1.2 \times 10^{-4}[13 - 820 I_D]^2$$
$$= 1.2 \times 10^{-4}\ [169 - 21.32 \times 10^3 I_D + 672.4 I_D{}^2]$$
$$\therefore \quad I_D = 0.02028 - 2.56 I_D + 80.68 I_D{}^2$$
$$\therefore \quad 80.68 I_D^2 - 3.56 I_D + 0.02028 = 0$$

This quadratic equation can be solved as,
$$I_D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values we get,
$$I_D = \frac{3.56 \pm \sqrt{(3.56)^2 - (4 \times 80.68 \times 0.02028)}}{2 \times 80.68}$$

$$\therefore \quad I_D = 6.72 \text{ mA}$$

or $$I_D = 37.4 \text{ mA}$$

Now substitute I_D = 37.4 mA in the following equation

$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$
$$= 40 - 37.4(3 + 0.820) = -102.88 \text{ volts}$$

A negative V_{DS} is practically not acceptable.

Hence, $$I_D \neq 37.4 \text{ mA}$$

$$\therefore \quad I_{DQ} = 6.72 \text{ mA}$$

To obtain the value of V_{GSQ} :

Substituting the value of I_{DQ} in the equation (iii), we get

$$V_{GSQ} = 18 - 820 \times 6.72 \text{ mA}$$
$$= 18 - 820 \times 6.72 \times 10^{-3}$$
$$= 18 - 5.51 = 12.49 \text{ Volts}$$

To obtain the value of V_{DSQ} :

$$V_{DSQ} = V_{DD} - I_{DQ}(R_D + R_S) \qquad \text{...(iv)}$$

Substituting the values we get,

$$V_{DSQ} = 40 - 6.72(3 + 0.820)$$
$$\therefore \quad = 14.32 \text{ Volts}$$

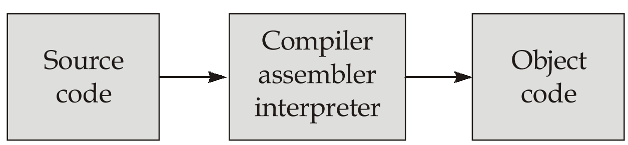
Q.4 (b) Solution:

(i)

```
# include<studio.h>
# include<conio.h>
int main()
{
    int count, temp, i, j, number[30];
    printf (No. of number of numerals to be sorted”);
    scanf(“%d” & count);
    printf(“enter%d numbers”, count)’
    for(i=0; i<count; i++)
    scanf(“%d”, number[i]);
    for(i=count–2; i>0, i++)
{
    for(j=0; j<=i; j++)
```

```
{
    if (number[j]>number[j+1])
{
    temp=number[j];
    number[j]=number[j+1],
    number[j+1]=temp
}}}
    printf("sorted elements")
    for(i=0; i<count; i++)
    printf("d", number[i]);
}
```

(ii) Translator software is used to convert a program written in high-level language and assembly language to a form that the computer can understand. Translator software converts a program written in assembly language, and high-level language to a machine-level language program. The translated program is called the object code.



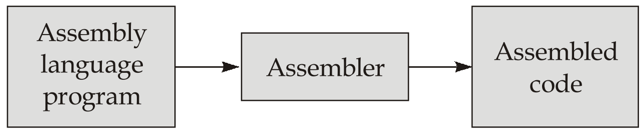
(Block diagram of translator software)

There are three different kinds of translator software:

- Assembler
- Compiler and
- Interpreter

Assembler converts a program written in assembly language to machine language. Compiler and interpreter convert a program written in high-level language to machine language.

Assembler: Assembly language is also referred to as a symbolic representation of the machine code. Assembler is a software that converts a program written in assembly language into machine code. There is usually a one-to-one correspondence between simple assembly statements and machine language instructions. The machine language is dependent on the processor architecture, though computers are generally able to carry out the same functionality in different wave. Thus the corresponding assembly language program also differ for different computer architectures.



Compiler: A program written in a high level language that the computer can understand, i.e., binary form. Compiler is the software that translates the program written in a high-level language to machine language. The program written in high-level language as referred to as the source code and compiled program is referred as the object code. The object code is the executable code, which can run as a stand-alone code. It does not require the compiler to be present during execution. Each programming language has its own compiler. Some languages that use a compiler are C++, COBOL, Pascal and FORTRAN.

The compilation process generally involves two parts: breaking down the source code into small pieces and creating an intermediate representation, and constructing the object code for the intermediate representation. The compiler also reports syntax errors, if any, in the source code.

Interpreter: The purpose of interpreter is similar to that of a compiler. The interpreter is used to convert the high-level language program into computer understandable form. However, the interpreter functions in a different way than a compiler. Interpreter performs line-by-line execution of the source code during program execution. Interpreter reads the source code line-by-line, converts it into machine understandable form, execute the line, and then proceeds to the next line. Some languages that use an interpreter are BASIC and Python.

Difference between a Compiler and an Interpreter: Compiler and interpreter are used to convert a program written in high-level language to machine language, however, they work differently. The key differences between a compiler and an interpreter are as follows:

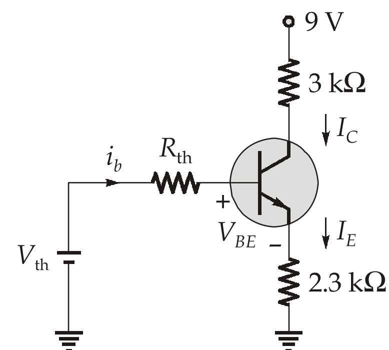
- Interpreter looks at a source code line-by-line. Compiler looks at the entire source code
- Interpreter converts a line into machine executable form, executes the line, and proceeds with the next line. Compiler converts the entire source code into object code and creates the object code. The object code is then executed by the user.
- For a given source code, once is compiled, the object code is created. This object code can be executed multiple number of times by the user. However, interpreter executes line-by-line, so executing the program using an interpreter

means that during each execution, the source code is first interpreted and then executed.

- During execution of an object code, the compiler is not required. However, for interpretation, both interpreter and the source code is required during execution (because source code is interpreted during execution).
- Since interpreter interprets line-by-line, the intepreted code runs slower than the compiled code.

Q.4 (c) (i) Solution:

The Thevenin equivalent circuit for DC model is



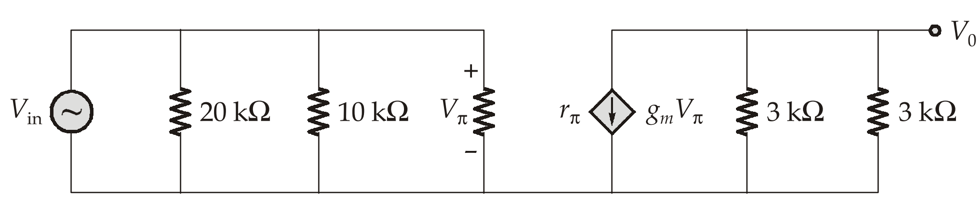
$$V_{th} = \frac{10}{10+20} \times 9 = 3 \text{ V}$$

$$R_{th} = \frac{10 \times 20}{10+20} = 6.67 \text{ k}\Omega$$

Assume β to be very high,

Therefore I_B is small an can be ignored

Thus, $$I_E = \frac{V_{th} - V_{BE}}{R_E} = \frac{3-0.7}{2.3\text{k}\Omega} = 1 \text{ mA}$$



The small signal model is

$$g_m = \frac{|I_C|}{V_T} = \frac{1 \text{ mA}}{25 \text{ mV}} = \frac{1}{25} \text{A/V}$$

$$V_0 = -g_m V_\pi \times (3\text{k} \,||\, 3\text{k})$$

$$= \frac{-1}{25} V_{in} (1.5\,k\Omega) = -60\ V_{in}$$

i.e. $$\frac{V_0}{V_{in}} = -60$$

Q.4 (c) (ii) Solution:

Direct Memory Access is a feature of computer systems that allows computer systems that allows certain hardware subsystems. e.g., I/O module to access main memory independently of the central processing unit. CPU is only involved at the beginning and end of the transfer and interrupted only after entire block has been transferred. DMA increases system concurrency by allowing the CPU to perform tasks while the DMA system transfers data via the system and memory buses.

Different types of DMA transfer modes are :

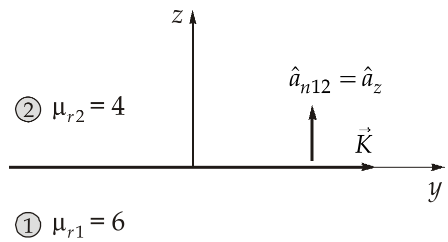
1. Burst or Block transfer DMA mode.
2. Cycle stealing transfer DMA mode.
3. Demand (continuous/complete data) transfer DMA mode.
4. Transparent DMA.

1. **Burst or Block Transfer DMA :** In burst mode, once the DMA controller takes the control of the system bus, it transfers all bytes of data block (without interruption) before releasing control of the system bus back to the processor. Burst mode is useful for loading programs or data files into memory because CPU needs data to carry on operation.
2. **Cycle Stealing Transfer DMA :** In cycle stealing mode, once the DMA controller takes the control of the system bus, it transfers only one byte of data (without interruption) before releasing control of the system bus back to the processor. DMA controller "steals" memory cycles from processor, though processor initiates most memory access.
3. **Demand Transfer DMA Mode :** In demand transfer mode, once the DMA controller takes the control of the system bus, it transfers entire data (without interruption) before releasing control of the system bus back to the processor. It can transfer all bytes of data transferred without interruption.
4. **Transparent (DMA) Mode :** In this mode, DMA controller monitors the CPU and system bus. If CPU is not using the system bus during any cycle then DMA controller uses the system bus. CPU may access the memory in the instruction fetch operand fetch and operand write cycles.

   In decode and execution cycles, the CPU can operate without accessing the memory in parallel with DMA controller.

Section B : Basic Electronics Engineering + Computer Fundamentals + Electromagnetic Field Theory

Q.5 (a) Solution:



$$\vec{B}_{1n} = \vec{B}_{2n} = 8\hat{a}_z \rightarrow B_z = 8 \quad \text{...(i)}$$

But
$$\vec{H}_2 = \frac{\vec{B}_2}{\mu_2} = \frac{1}{4\mu_0}(5\hat{a}_x + 8\hat{a}_z)\text{mA/m} \quad \text{...(ii)}$$

and
$$\vec{H}_1 = \frac{\vec{B}_1}{\mu_1} = \frac{1}{6\mu_0}(B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z)\text{mA/m} \quad \text{...(iii)}$$

Having found the normal components, we can find the tangential components by using

$$(\vec{H}_1 - \vec{H}_2)\times\hat{a}_{n12} = \vec{K}$$

or
$$\vec{H}_1 \times \hat{a}_{n12} = \vec{H}_2 \times \hat{a}_{n12} + K \quad \text{...(iv)}$$

Substituting equations (ii) and (iii) into equation (i) gives

$$\frac{1}{6\mu_0}(B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z)\times\hat{a}_z = \frac{1}{4\mu_0}(5\hat{a}_x + 8\hat{a}_z)\times\hat{a}_z + \frac{1}{\mu_0}\hat{a}_y$$

Equating components yields, $\vec{B}_y = 0,$

$$\frac{-B_x}{6} = \frac{-5}{4} + 1$$

or
$$\vec{B}_x = \frac{6}{4} = 1.5 \quad \text{...(v)}$$

From equation (i) and (v), we have

$$\vec{B}_1 = 1.5\hat{a}_x + 8\hat{a}_z \ \text{mWb/m}^2$$

$$\vec{H}_1 = \frac{\vec{B}_1}{\mu_1} = \frac{1}{\mu_0}(0.25\hat{a}_x + 1.33\hat{a}_z)\ \text{mA/m}$$

and
$$\vec{H}_2 = \frac{1}{\mu_0}(1.25\hat{a}_x + 2a_z)\text{mA/m}$$

Q.5 (b) Solution:

Given :
$$E = \frac{2r}{(r^2+a^2)^2}\hat{a}_r \text{ V/m}$$

We know the potential,

$$V(r) = -\int E \cdot dL$$

$$dL = dr\,\hat{a}_r$$

$$V(r) = -\int \frac{2r}{(r^2+a^2)^2}dr$$

$$V(r) = \frac{1}{(r^2+a^2)} + C$$

(i) $V = 0$ at infinity, i.e.,

$$V(\infty) = \frac{1}{\infty} + C = 0$$

$$C = 0$$

$$V(r) = \frac{1}{r^2+a^2}$$

(ii) $V = 0$ at $r = 0$

$$V(0) = \frac{1}{0+a^2} + C = 0$$

$$C = \frac{-1}{a^2}$$

$$V(r) = \frac{1}{r^2+a^2} + \left(\frac{-1}{a^2}\right)$$

$$V(r) = \frac{1}{r^2+a^2} - \frac{1}{a^2} = \frac{-r^2}{(r^2+a^2)a^2}$$

$$V(r) = \frac{-r^2}{a^2(r^2+a^2)}$$

(iii) $V = 100$ at $r = a$

$$V(a) = \frac{1}{a^2+a^2} + C = 100$$

$$C = 100 - \frac{1}{2a^2}$$

$$V(r) = \frac{1}{r^2 + a^2} + 100 - \frac{1}{2a^2}$$

$$V(r) = \frac{a^2 - r^2}{2a^2(r^2 + a^2)} + 100$$

Q.5 (c) Solution:

$$T_{(read)} = \text{Hit} \times \text{Read Time of } L_1 + (1 - \text{hit}) \times \text{Read Time of Memory}$$
$$= 0.8 \times 5 \text{ ns} + 0.2 \times 100 \text{ ns}$$
$$= 4 \text{ ns} + 20 \text{ ns} = 24 \text{ ns}$$
$$H_w = 1 \text{ So, memory organization is simultaneous.}$$
$$T_{(write)} = \text{Write time to main memory} = 200 \text{ ns}$$

[Since, in WRITE-THROUGH CPU writes in both cache and main memory. Hence writing time of main memory is considered.]

$$T_{avg} = F_{Read} \times T_{(read)} + F_{write} \times T_{(write)}$$
$$= 0.6 \times 24 \text{ ns} + 0.4 \times 200 \text{ ns}$$
$$= 14.4 \text{ ns} + 80 \text{ ns}$$
$$= 94.4 \text{ ns}$$

Q.5 (d) Solution:

From the given circuit,

$$V_S = 0; \quad V_D = V_G = 8 - I_D R_D \qquad \text{...(i)}$$

when $R_D = 3\text{ k}\Omega$ and $I_D = 2$ mA

$\therefore$
$$V_{GS} = V_G - V_S = V_G = 8 - I_D R_D$$
$$V_{GS} = 8 - 2 \times 3 = 2 \text{ V}$$

Now,
$$K = \frac{I_D}{(V_{GS} - V_T)^2} = \frac{2 \times 10^{-3}}{(2-1)^2} = 2 \text{ mA/V}^2$$

Now, R_D is reduced to 2 kΩ.

Let
$$R'_D = 2 \text{ k}\Omega$$
$$V_{GS} = 8 - 2I_D$$
$$I_D = K[V_{GS} - V_T]^2$$
$$= 2[8 - 2I_D - 1]^2$$

$$0.5I_D = (7 - 2I_D)^2$$

$$\Rightarrow \quad 4I_D^2 - 28I_D + 49 = 0.5I_D$$

$$\Rightarrow \quad 4I_D^2 - 28.5I_D + 49 = 0$$

On solving,

$$I_D = 4.22 \text{ mA}; \qquad I_D = 2.898 \text{ mA}$$

$$V_{GS} = 8 - 2 \times 4.226, \quad V_{GS} = 8 - 2 \times 2.898$$

$$= -0.452 \text{ V} \qquad = 2.204 \text{ V}$$

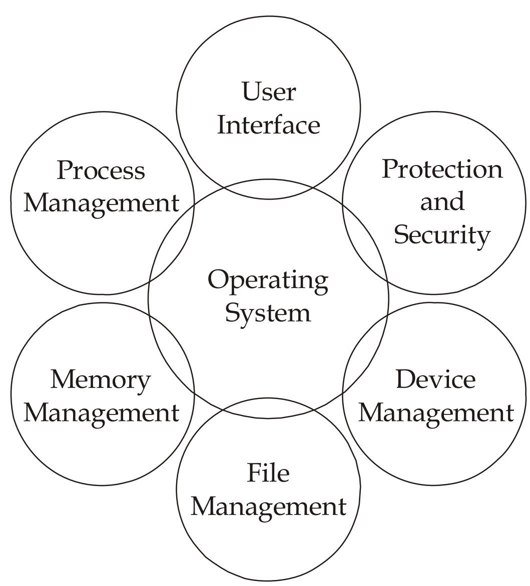
$$V_{GS} < V_T \qquad V_{GS} > V_T$$

Therefore, when R_D is decreased by 1 kΩ, the drain current is 2.898 mA.

Q.5 (e) Solution:

Operating system is a large and complex software consisting of several components. Each component of operating system has its own set of defined inputs and outputs. Different components of OS perform specific tasks to provide the overall functionality of operating system.

The main functions of OS are as follows :



Process Management : The process management activities handled by the OS are :

1. Control access to shared resources like file, memory, I/O and CPU.
2. Control execution of applications.
3. Create, execute and delete a process (system process of user process).
4. Cancel or resume process.

5. Schedule a process.
6. Synchronization.

Memory Management : The activities of memory management handled by OS are :

1. Allocate memory
2. Free memory
3. Re-allocate memory to a program when a used block is freed.
4. Keep track of memory usage.

File Management :

1. Create and delete both files and directories.
2. Provide access to files.
3. Allocate space for files.
4. Keep back-up of files.
5. Secure files

Device Management : The device management tasks handled by OS are :

1. Open, close and write device drivers.
2. Communicate, control and monitor the device driver.

Protection and Security : OS protects the resources of the system. User authentication, file attributes like read, write, encryption and back-up of data are used by OS to provide basic protection.

User Interface or Command Interpreter : Operating system provides an interface between the computer user and computer hardware. The user interface is a set of commands or a graphical interface via which the user interacts with the applications and the hardware.

Q.6 (a) Solution:

Looking at figure, we understand that,

$$R_S = 1\ \text{k}\Omega, \qquad h_{ie} = 1.8\ \text{k}\Omega,$$

$$R_L = 4\ \text{k}\Omega, \qquad h_{re} = 3 \times 10^{-4}$$

$$\frac{1}{h_{oe}} = 80\ \text{k}\Omega$$

(i) To obtain the current gain:

$$A_i = \frac{-h_{ie}}{1 + h_{oe}R_L} = \frac{-50}{1 + \frac{4}{80}} = -47.62$$

(ii) Input impedance (Z_i):

$$Z_i = h_{ie} + h_{re}A_iR_L = 1.8\text{k} + (3 \times 10^{-4} \times -47.62 \times 4 \times 10^3)$$

$$\therefore \quad Z_{in} = 1742.86\ \Omega$$

(iii) To calculate the voltage gain:

$$A_V = \frac{A_iR_L}{Z_{in}} = \frac{(-47.62)\times 4\text{k}}{1742.86\,\Omega} = -109.29$$

$$A_{VS} = \frac{A_VZ_{in}}{Z_{in}+R_S} = \frac{-109.29\times 1742.86}{1742.86+1000} = -69.45$$

(iv) To calculate the output resistance (R_0):

$$Y_0 = h_{oe} - \frac{h_{fe}h_{re}}{h_{ie}+R_S} = \frac{1}{80\text{k}} - \frac{50\times 3\times 10^{-4}}{2.8\text{k}}$$

$$= 0.0125 \times 10^{-3} - 53.57 \times 10^{-7} = 7.143 \times 10^{-6}$$

$$R_0 = \frac{1}{Y_0} = 0.14 \times 10^6\ \Omega = 0.14\ \text{M}\Omega$$

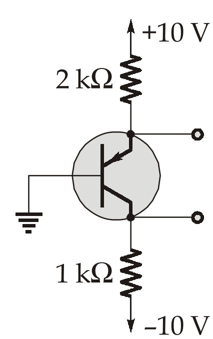
Q.6 (b) Solution:

(i) The base of this pnp transistor is grounded. While the emitter is connected to a positive supply (V^+ = +10 V) through R_E. It follows that the emitter-base junction will be forward biased with,

$$V_E = V_{EB} = 0.7\text{ V}$$

Thus the emitter current will be given by

$$I_E = \frac{V^+ - V_E}{R_E} = \frac{10-0.7}{2} = 4.65\text{ mA}$$



Since the collector is connected to a negative supply (more negative than the base voltage) through R_C, it is possible that this transistor is operating in the active mode. Assuming this to be the case, we obtain,

$$I_C = \alpha I_E$$

Given, β = 100, which results in α = 0.99,

$$V_C = V^- + I_CR_C$$

$$= -10 + 4.6 \times 1 = -5.4\text{ V}$$

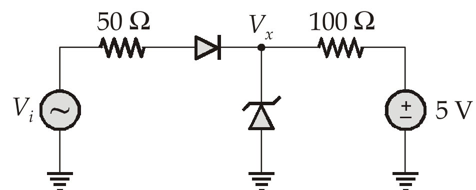
Thus the collector-base junction is reverse biased by 5.4 V, and the transistor is indeed in the active mode, which supports our original assumption.

It remains only to calculate the base current,

$$I_B = \frac{I_E}{\beta+1} = \frac{4.65}{101} = 0.05 \text{ mA}$$

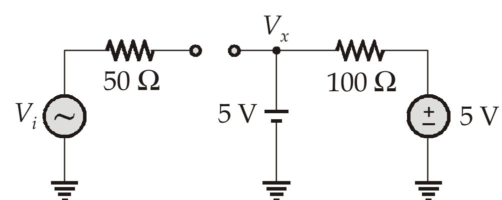
(ii) Let both diodes be OFF V_x = 5 V

If V_i < 5 V then both diodes are off



Therefore current through 100 Ω remains 0.

If V_i > 5 V then normal diode become on and zener diode undergoes breakdown.



Now also current through 100 Ω is zero.

Hence power dissipation in 100 Ω resistor is zero.

Q.6 (c) (i) Solution:

IEEE standard for floating-point arithmetic is a technical standard for floating-point computation.

The standard addressed many problems found in the diverse floating point implementations that made them difficult to use reliably and reduce their portability.

IEEE standard 754 floating point is the most common representation today for real numbers on computers, including Intel-based PCs, MACs and most common unix platforms.

There are several ways to represent floating point numbers IEEE 754 is the most efficient in the most cases. IEEE 754 has 3 basic components.

1. **The sign of Mantissa :** Here, 0 represents a positive number while 1 represents a negative number.
2. **The Biased Exponent :** The exponent field needs to represent both positive and negative exponents. A bias is added to the actual exponent in order to get the stored exponent.
3. **The Normalised Mantissa :** The Mantissa is a part of number in scientific notation or a floating point number, consisting of its significant digits. Here, we have only two digits, i.e., 0 and 1. So, a normalised Mantissa is one with only one to the left of the decimal.

IEEE 754 numbers are divided into two based on the three components. Single precision and double precision.

Single precision IEEE 754 floating-point standard :

32-bits

Sign	Exponent	Mantissa
1-bit	8-bits	23-bits

Double Precision : IEEE 754 floating-point integer

64-bits

Sign	Exponent	Mantissa
1-bit	11-bits	52-bits

Q.6 (c) (ii) Solution:

Capacity of main memory = 256 MB

Capacity of cache memory = 1 MB

Block size = 128 bytes

A set contain 8 blocks.

Since the address space of processor is 256 MB.

The processor will generate address of 28-bits to access a byte (word).

The number of blocks contained by main memory = $\frac{256 \text{ MB}}{128 \text{ B}} = 2^{21}$

Therefore, number of bits required to specify one block in main memory = 21.

Since the block size is 128 bytes.

The number of bits required access each word (byte) = 7.

For associative, the address format :

Tag	Word
21	7

The number of blocks contained by cache memory = $\frac{1 \text{ MB}}{128 \text{ B}} = 2^{13}$.

Therefore, the number of bits required to specify one block in cache memory = 13.

For direct cache, the address format is

Tag	Block	Word
8	13	7

(Block + Word: Index)

Q.7 (a) (i) Solution:

Instruction size = 16 bit

Type I instruction design:

16 bit

opcode	IR	IR	IR
4 bit	4 bit	4 bit	4 bit

Number of operations = $2^4 = 16$

$\downarrow$

Free opcodes = $16 - 4 = 12$

Type II instruction design:

16 bit

opcode	FR	FR
4 bit	6 bit	6 bit

Free opcodes after type 1 instruction

$= 12$

Free opcodes after type 2 instruction

$= 12 - 8 = 4$

Type III instruction design:

16 bit

opcode	IR	FR
6 bit	4 bit	4 bit

Expand opcode size = 6 bit

$\downarrow$

Number of opcodes = $4 \times 2^2 = 16$

$\therefore$ Number of free opcodes after type 3 instruction

$= 16 - 14 = 2$

Type IV instruction design:

16 bit

opcode	FR
10 bit	6 bit

Expand opcode size = 10 bit

$\downarrow$

Number of opcodes = 2×2^4

$= 2^5 = 32$

Q.7 (a) (ii) Solution:

$$\text{Average read time} = 0.9 \times 1 \text{ ns} + 0.1 \times 5 \text{ ns}$$
$$= 0.9 + 0.5 = 1.4 \text{ ns}$$

In the execution sequence number of read operations = 160

So total time required for read operation

$$= 160 \times 1.4 \text{ ns} = 224 \text{ ns}$$
$$\text{Average write time} = 0.9 \times 2 \text{ ns} + 0.1 \times 10 \text{ ns}$$
$$= 1.8 + 1 = 2.8 \text{ ns}$$

In the execution sequence number write operation = 40.

So, the total time required for write operation = 40 × 2.8 ns = 112 ns

Total time for instruction execution time for both read and write = 224 + 112 = 336 ns.

200 times, access takes 336 ns.

1 time, access takes

$$\text{Average memory access time} = \frac{336}{200} = 1.68 \text{ ns}$$

Q.7 (b) Solution:

The work done is given by

$$W = -Q\int_B^A \bar{E}\cdot\overline{dL}$$

Let us differential length $\overline{dL}$ in Cartesian co-ordinate system is,

$$\overline{dL} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

$$\bar{E}\cdot\overline{dL} = (-8xy\,\hat{a}_x - 4x^2\hat{a}_y + \hat{a}_z)\,(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$
$$= -8xy\,dx - 4x^2dy + dz$$

As, $\bar{a}_x\cdot\bar{a}_x = \bar{a}_y\cdot\bar{a}_y = \bar{a}_z\cdot\bar{a}_z = 1$ other dot products are zero.

$$\therefore \quad W = -Q\int_A^B -8xy\,dx - 4x^2dy + dz$$

$$= -Q\left[\int_B^A -8xy\,dx - \int_B^A 4x^2\,dy + \int_B^A dz\right]$$

Case-I: The path is $y = 3x^2 + z$, $z = x + 4$

$$\therefore \quad y = 3x^2 + x + 4$$

$$\frac{dy}{dx} = (6x + 1)$$

For, $\int_B^A -8xy\,dx \rightarrow$ The limits are $x = 1$ to $x = 2$

For, $\int_B^A -4x^2 y \rightarrow$ The limits are $y = 8$ to $y = 18$

For, $\int_B^A dz \rightarrow$ The limits are $z = 5$ to $z = 6$

$$\therefore \qquad W = -Q\left[\int_{x=1}^{2} -8xy\,dx - \int_{y=8}^{18} 4x^2\,dy + \int_{z=5}^{6} dz\right]$$

Using, $y = 3x^2 + x + 4$ and $dy = (6x + 1)\,dx$ and changing limits of y from 8 to 18 interms of x from 1 to 2 we get,

$$\therefore \qquad = -Q\left[\int_{x=1}^{2} -8x(3x^2 + x + 4)\,dx - \int_{x=1}^{2} 4x^2(6x+1)\,dx + \int_{z=5}^{6} dz\right]$$

$$= -Q\left[\left(-6x^4 - \frac{8}{3}x^3 - 16x^2 - 6x^4 - \frac{4}{3}x^3\right)_{x=1}^{2} + (z)_5^6\right]$$

$$= -Q[-256 + 1]$$

$$= -6 \times -255 = 1530\text{ J}$$

Case-2: Straight line path from B to A.

To obtain the equations of the straight line, any two of the following three equations of planes passing through the line are sufficient, $B(1, 8, 5)$ and $A(2, 18, 6)$.

Using the co-ordinates of A and B,

$$y - 8 = \frac{18-8}{2-1}(x-1)$$

$$\therefore \qquad y - 8 = 10(x - 1)$$

$$\therefore \qquad y = 10x - 2$$

$$\therefore \qquad dy = 10\,dx$$

and

$$z - 5 = \frac{6-5}{18-8}(y-8)$$

$\therefore$ $$10z = y + 42$$

Now,
$$W = -Q\left[\int_{x=1}^{2} -8xy\,dx - \int_{y=8}^{18} 4x^2 dy + \int_{z=5}^{6} dz\right]$$

$$= -Q\left[\int_{x=1}^{2} -8x(10x-2)\,dx - \int_{x=1}^{2} 4x^2(10\,dx) + \int_{z=5}^{6} dz\right]$$

$$= -Q\left\{\left[-\frac{80}{3}x^3 + \frac{16x^2}{2} - \frac{40x^3}{3}\right]_{x=1}^{2} + [z]_5^6\right\}$$

$$= -Q\{-213.33 + 32 - 106.667 + 26.667 - 8 + 13.33 + 1\}$$

$$= -Q[-255] = 1530 \text{ J}$$

This shows that irrespective of path selected, the work done in moving a charge from B to A remains same.

Q.7 (c) Solution:

(i) The linear velocity of inner conductor is,

$$\vec{v}_1 = \omega r_1 \hat{a}_\phi = 2\pi f r_1 \hat{a}_\phi$$

$$= 2\pi \times \frac{500}{60} \times 0.03\hat{a}_\phi = \frac{1}{2}\pi\hat{a}_\phi \text{ m/s}$$

The linear velocity of the outer conductor is,

$$\vec{v}_2 = \omega r_2 \hat{a}_\phi = 2\pi f r_2 \hat{a}_\phi = 2\pi \times \frac{500}{60} \times 0.05\hat{a}_\phi$$

$$= \frac{5}{6}\pi\hat{a}_\phi$$

Since the magnetic field is not time-varying, only motional emf will be induced.

The motional emf in the inner conductor is

$$V_1 = \oint (\vec{v}_1 \times \vec{B}_1)\cdot \vec{dl}_1$$

$$= \int_0^l \left(\frac{\pi}{2}\hat{a}_\phi \times \frac{1}{4}\hat{a}_r\right)\cdot(dz\hat{a}_z) = \frac{\pi}{8}\int_0^l (-\hat{a}_z)\cdot dz\hat{a}_z$$

$$= \frac{\pi}{8}\int_0^{0.5}(-dz) = -\frac{\pi}{16} \text{ Volts (in } z\text{-direction)}$$

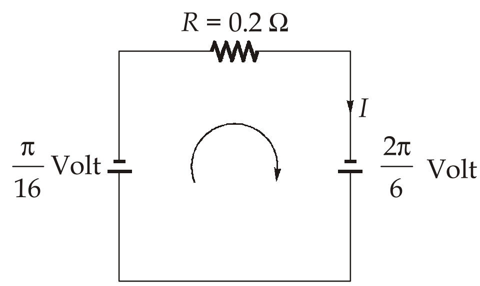
Similarly, the motional induced emf in the outer conductor is

$$V_2 = \oint_l (\vec{V}_2 \times \vec{B}_2)\cdot \vec{dl}_2$$

$$= \int_0^l \left(\frac{5\pi}{6}\hat{a}_\phi \times 0.8\hat{a}_r\right)\cdot(dz\hat{a}_z)$$

$$= \frac{4\pi}{6}\int_0^{0.5}(-dz) = -\frac{2\pi}{6} \text{ Volts (in } z\text{-direction)}$$

Therefore, the loop can redrawn as



$$\frac{\pi}{16} + 0.2i = \frac{2\pi}{6}$$

$$I = \frac{\frac{2\pi}{6} - \frac{\pi}{16}}{0.2} = 4.25 \text{ A}$$

(ii) Ampere's circuit law states that the line integral of $\vec{H}$ around a closed path is exactly the same as the current I_{enc} enclosed by the path.

In other words, the circulation of $\vec{H}$ equals I_{enc}; that is,

$$\oint \vec{H}\cdot d\vec{l} = I_{enc}$$

Using the Ampere's law, the magnetic field inside the conductor at arbitrary distance $r \le R$ can be obtained as

$$B \times 2\pi r = \mu I \times \frac{\pi r^2}{\pi R^2}$$

$$B = \frac{\mu I r}{2\pi R^2} \text{ T}$$

∴ The magnetic energy density is

$$W_m = \frac{1}{2}\cdot\frac{B^2}{\mu} = \frac{1}{2\mu}\left(\frac{\mu I r}{2\pi R^2}\right)^2 = \frac{\mu I^2 r^2}{8\pi^2 R^4}$$

Hence, total magnetic energy stored inside the wire is

$$E_m = \int W_m = \int_0^R \frac{\mu I^2 r^2}{8\pi^2 R^4}\times(2\pi r l)dr$$

$$= \frac{\mu I^2}{8\pi^2 R^4}\times 2\pi l \times \left.\frac{r^4}{4}\right|_0^R$$

$$E_m = \frac{\mu I^2 l}{16\pi}\text{ J}$$

Q.8 (a) Solution:

Step - 1:

Calculate I_B:

$$I_B = \frac{I_C}{\beta} = \frac{2\text{ mA}}{50} = 40\ \mu\text{A}$$

Step - 2:

Apply KVL to the collector circuit and calculate R_E .

$$V_{CC} = I_C R_C + V_{CE} + I_E R_E$$

$$= I_C R_C + V_{CE} + (I_C + I_B) R_E$$

$$\therefore \quad R_E = \frac{V_{CC} - V_{CE} - I_C R_C}{(I_C + I_B)}$$

$$= \frac{10 - 5 - (2\times 2)}{(2\text{ mA} + 40\mu\text{A})} = 490.19\ \Omega$$

Step - 3:

Calculate R_B :

The expression for stability factor S for the voltage divider bias circuit is given by

$$S = (1+\beta)\frac{1+\left(\frac{R_B}{R_E}\right)}{(1+\beta)+\left(\frac{R_B}{R_E}\right)}$$

$$\therefore \qquad 5 = (1+50)\times\frac{1+\left(\frac{R_B}{490.2}\right)}{51+\left(\frac{R_B}{490.2}\right)} = \frac{51\times(490.2+R_B)}{(51\times 490.2)+R_B}$$

$$= \frac{25000.2+51R_B}{25000.2+R_B}$$

$$\therefore \qquad R_B = 2.173 \text{ k}\Omega$$

But $$R_B = R_1 \parallel R_2 = \frac{R_1R_2}{(R_1+R_2)} = 2.173 \text{ k}\Omega$$

$$\therefore \qquad \frac{R_2}{R_1+R_2} = \frac{2.173K}{R_1} \qquad \text{...(i)}$$

Step - 4:

Calculate R_1 and R_2 :

Apply KVL to the input loop to get,

$$V_{R2} = V_{BE} + I_ER_E$$

Assume $V_{BE} = 0.7$ V

$$\therefore \qquad V_{R2} = 0.7 + (I_C + I_B)R_E = 0.7 + (2.04 \times 10^{-3} \times 490.19)$$

$$= 1.69 \text{ V} \qquad \text{...(ii)}$$

But $$V_{R2} = \frac{R_2}{R_1+R_2}V_{CC}$$

$$\therefore \qquad \frac{R_2}{R_1+R_2} = \frac{V_{R2}}{V_{CC}} = \frac{1.69}{10} = 0.169 \qquad \text{...(iii)}$$

Comparing equation (i) and (iii), we get

$$\frac{2.173\times 10^3}{R_1} = 0.169$$

$$\therefore \qquad R_1 = 12.78 \text{ k}\Omega$$

and $$\frac{R_2}{12.78+R_2} = 0.169$$

$$\therefore \qquad R_2 - 0.169R_2 = 12.78 \times 0.169$$

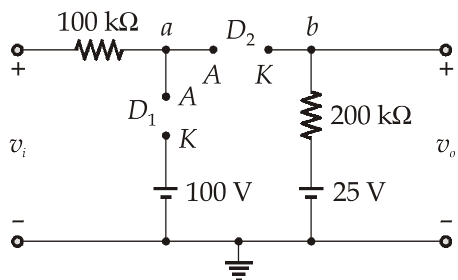
$$\therefore \qquad R_2 = 2.59 \text{ k}\Omega$$

Q.8 (b) Solution:

Case-I, when both the diodes are in OFF state:

In this case both the diodes act as open circuits and the equivalent circuit can be given by,

$$v_o = v_b = 25 \text{ V}$$
$$v_a = v_i$$



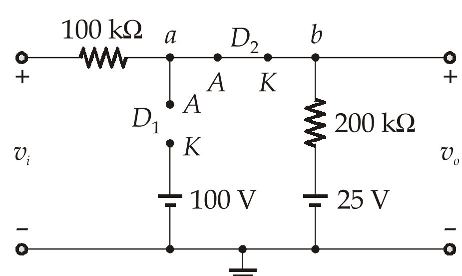
From the above diagram, it is clear that the diode D_2 conducts for $v_i > 25$ V and the diode D_1 conducts for v_i much greater than 25 V. So, both the diodes are in OFF state for $v_i < 25$ V.

Hence, $$v_o = 25 \text{ V} \;;\; \text{for } v_i \leq 25 \text{ V} \quad ...(i)$$

Case-II, when D_2 is in ON state and D_1 is in OFF state:

From case-I, it is clear that D_2 will be in ON state for $v_i > 25$ V and the equivalent circuit for this case can be given by,

$$v_o = v_a = v_b$$
$$= \frac{v_i - 25}{100 + 200} \times 200 + 25 \text{ V}$$
$$= \frac{2}{3}v_i + \frac{25}{3} \text{ V}$$



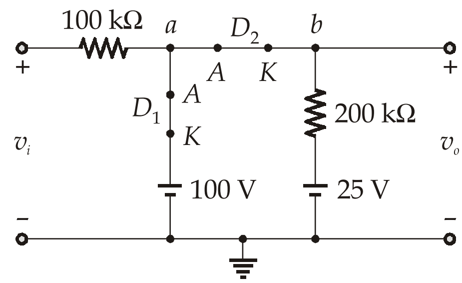
This case will sustain until D_1 starts conducting.

D_1 conducts for, $$v_a > 100 \text{ V}$$
$$\frac{2}{3}v_i + \frac{25}{3} \text{ V} > 100 \text{ V}$$
$$v_i > 137.5 \text{ V}$$

So, $$v_o = \frac{2}{3}v_i + \frac{25}{3} \text{ V} \;;\; \text{for } 25 \text{ V} < v_i \leq 137.5 \text{ V} \quad ...(ii)$$

Case-III, when both the diodes are in ON state:

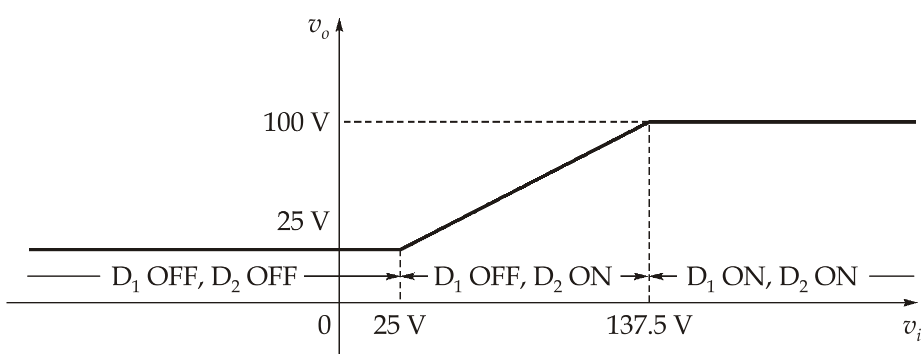
From case-II, it is clear that for $v_i > 137.5$ V both the diodes will be in ON state and the equivalent circuit for this case can be given by,



$$v_o = 100 \text{ V} \;;\; \text{for } v_i > 137.5 \text{ V} \qquad \text{...(iii)}$$

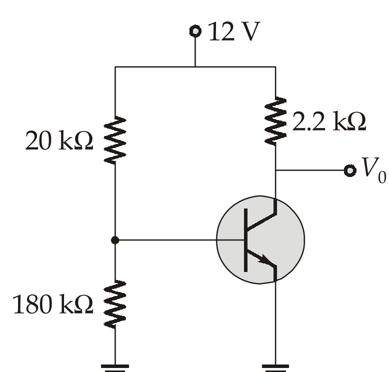
By summarizing all the three cases, the voltage transfer characteristic of the circuit can be plotted as shown below.

$$v_o = \begin{cases} 25 \text{ V} & ; \quad v_i \le 25 \text{ V} \\ \dfrac{2}{3}v_i + \dfrac{25}{3} \text{ V} & ; \quad 25 \text{ V} < v_i \le 137.5 \text{ V} \\ 100 \text{ V} & ; \quad v_i > 137.5 \text{ V} \end{cases}$$

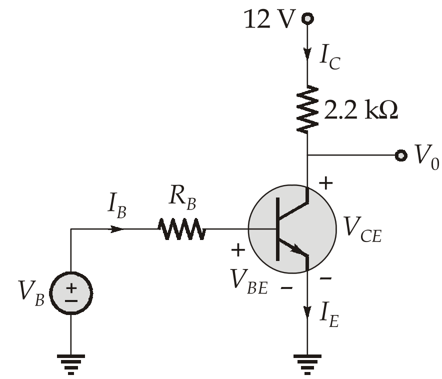


Q.8 (c) Solution:

(i) When V_i = 12 V, R = 20 kΩ



On taking the thevenin equivalent across the base terminal,



Where, $$V_B = 12 \times \frac{180}{180+20} = 10.8 \text{ V}$$

and $$R_B = (20 \,||\, 180)\text{k}\Omega = 18 \text{ k}\Omega$$

Apply KVL in base-emitter loop

$$-V_B + I_B R_B + V_{BE} = 0$$

$$I_B = \frac{10.8 - 0.7}{18} = 0.5611 \text{ mA}$$

$$I_C = \beta I_B + (1+\beta) I_{CBO}$$

$$= 30 \times 0.5611 + 31 \times 100 \times 10^{-6}$$

$$= 16.8361 \text{ mA}$$

Now, $$V_0 = 12 - I_C \times 2.2$$

$$= 12 - 16.83 \times 2.2 = -25.03 \text{ Volt} < V_{CE\,sat}$$

Therefore transistor is operating in saturation region and $V_0 = V_{CE\,sat} = 0.2$ V.

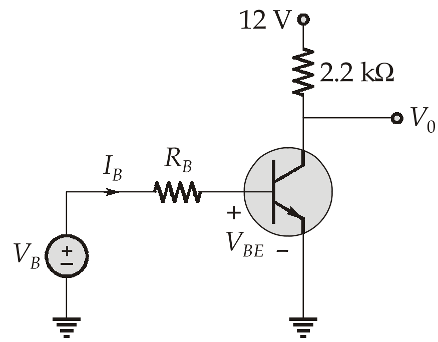
(ii) The minimum value of R for which the transistor to remain in active region, when $V_i = 12$ V.

We get, $$I_{C\,sat} = \frac{12 - 0.2}{2.2} = 5.36 \text{ mA} \qquad [\because V_{CE(sat)} = 0.2 \text{ V}]$$

$$I_{B\,(max)} = \frac{I_{C(sat)}}{\beta_{min}} = \frac{5.36}{30} = 0.1786 \text{ mA}$$

$$R_B = \frac{R \times 180}{180 + R}$$

and $$V_B = \frac{12 \times 180}{180 + R}$$



Apply KVL in base-emitter loop,

$$V_B = V_{BE} + I_B R_B$$

$$\frac{12 \times 180}{R + 180} = 0.7 + 0.1786 \times \frac{180R}{180 + R}$$

$$2160 = 126 + 0.7R + 32.148\ R$$

$$R = \frac{2034}{32.848} = 61.92\ \text{k}\Omega$$

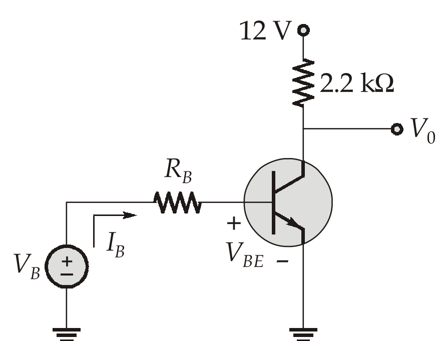
This $R = 61.92$ kΩ is corresponding to R_{min}

Therefore, $R_{\text{min}} = 61.92$ kΩ

(iii) When $V_i = 1$ V and $R = 15$ kΩ

$$R_B = (15 \,||\, 180)\text{k}\Omega = 13.846\ \text{k}\Omega$$

$$V_B = 1 \times \frac{180}{195} = 0.9230\ \text{V}$$



Apply KVL in base-emmiter loop,

$$I_B = \frac{V_B - V_{BE}}{R_B} = \frac{0.923 - 0.7}{13.846} = 0.0161\ \text{mA}$$

$$I_C = \beta I_B + (1 + \beta) I_{CBO}$$

$$= 30 \times 0.0161 + 31 \times 100 \times 10^{-6}$$

$$= 0.4862 \text{ mA}$$

$$V_0 = 12 - I_C \times 2.2$$

$$= 12 - 0.4862 \times 2.2 = 10.93 \text{ V}$$

$$\because \quad V_{CE} = 10.93 > V_{\text{CE sat}}$$

Therefore transistor operating in active region.

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