Try to avoid calculation mistake

Revise concepts

MADE EASY

Leading Institute for ESE, GATE & PSUs

ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-2 : Digital Electronics + Microprocessors + Systems and Signal Processing

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
Pune ☐	Kolkata ☐	Hyderabad ☐	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	26
Q.2	45
Q.3	27
Q.4	
Section-B	
Q.5	53
Q.6	32
Q.7	
Q.8	
Total Marks Obtained	183

Signature of Evaluator: Sourabh kumar

Cross Checked by: ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Digital Electronics + Microprocessors

Q.1 (a) Perform the BCD operation on the following number system:

(i) (010001101001.0110) + (100000010111.1000)

(ii) (011001010100.0101) – (100101111001.0110)

(iii) (011001100110.0110) – (010010011001.1001)

[4 + 4 + 4 marks]

(i)

0100 0110 1001 . 0110
1000 0001 0111 . 1000

1100 0111 0000 1110
+0110 (1) 0110 + 0110

1 0010 0111 (1) 0110 (1) 0100

So, sum value is

⇒ 0010 0111 0110 0100

(ii)

0110 0101 0100 . 0101
– 1001 0111 1001 . 0110

Go through the made easy solution

~~A+B~~ A + (–B) ⇒ A + 2's complement of B

1001 0111 1001 . 0110

$\xrightarrow{1's}$ 0110 1000 0110 . 1001
+ 1

0110 1000 0110 . 1010

0110 0101 0100 . 0101
+0110 1000 0110 . 1010

1100 1101 0110 . 1111
0110 +0110   0110

(1) 0010 (1) 0011 0110 0101 (1)

⇒ 0010 0011 0110 . 0101

Ans

(iii) 0 1 1 0 0 1 1 0 0 1 1 0 . 0 1 1 0
− 0 1 0 0 1 0 0 1 1 0 0 1 . 1 0 0 1

⇒ ~~1's comp~~

0 1 0 0 1 0 0 1 1 0 0 1 . 1 0 0 1 $\xrightarrow{1's}$

1 0 1 1 0 1 1 0 0 1 1 0 . 0 1 1 0
+ 1

1 0 1 1 0 1 1 0 0 1 1 0 . 0 1 1 1

0 1 1 0 0 1 1 0 0 1 1 0 . 0 1 1 0
+ 1 0 1 1 0 1 1 0 0 1 1 0 . 0 1 1 1

0 0 0 1 1 1 0 0 1 1 0 0 1 1 0 1

(1) 0 1 1 0 + 0 1 1 0 + 0 1 1 0 + 0 1 1 0
0 1 1 1 0 0 1 0 0 0 1 0 0 0 1 1
(1) (1)

Sum value → 0 1 1 1 0 0 1 0 0 0 1 0 . 0 0 1 1

Ans

4

Do not write in this margin

Q.1 (b) The given below decimal numbers are stored in 6 digit register in sign-magnitude form. Convert them to signed 10's complement form and perform the following operations. Also convert the result into equivalent hexadecimal number.

(i) (+9286) + (+801)   (ii) (+9286) + (-801)

(iii) (-9286) + (-801)   (iv) (-9286) + (-801)

[12 marks]

(b) (i)

$$\begin{array}{r} +9286 \\ +\ 801 \\ \hline 010087 \end{array}$$

~~010087 →~~

~~0000 0001 0000 0000 1000 0111~~

~~0 1 0 0 8 7~~

~~In Hexa → $(010087)_{16}$~~

(ii) $-801 \rightarrow$ 10's complement of 801

$\rightarrow$ 9's complement of $(801) + 1$

$$\begin{array}{r} 999 \\ 801 \\ \hline 198 \end{array} \xrightarrow{+1} 199$$

$$\begin{array}{r} +9286 \\ +\ 199 \\ \hline 9485 \end{array}$$

~~$\Rightarrow +9485$~~

$= (009485)_{16}$

(iii). $(-9286) \Rightarrow 10$'s complement of 9286

$$\begin{array}{r} 9999 \\ 9286 \\ \hline 0713 \\ +\ 1 \\ \hline 0714 \end{array} \Rightarrow \begin{array}{r} 0714 \\ 0801 \\ \hline (001515) \end{array}$$ Ans

(iv) $(-9286) + (-801)$

$$\begin{array}{r} 9999 \\ 9286 \\ \hline 0713 \\ +\ 1 \\ \hline 0714 \end{array} \quad \begin{array}{r} 999 \\ 801 \\ \hline 198 \\ +\ 1 \\ \hline 199 \end{array} \quad \begin{array}{r} 0714 \\ 199 \\ \hline 10913 \end{array}$$ Ans

5

Q.1 (c) Determine the minimized expression for the given below Boolean function in SOP form using K-map:

$F(A, B, C, D) = \Pi(0, 1, 4, 5, 6, 7, 9, 13)$.

Implement the minimized function using NAND gate only.

[12 marks]

Soln.-

Given $F(A,B,C,D) = \Pi(0,1,4,5,6,7,9,13)$

$= \Sigma(2,3,8,10,11,12,14,15)$

K-Map

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3 (b) 1	2 1
$\bar{A}B$	4	5	7	6
AB	(c) 12 1	13	(a) 15 1	14 1
$A\bar{B}$	8 1	9	11 1	10 1

Pair (a)

$= ABCD + ABC\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D}$

$= ABC + A\bar{B}C$

$= AC$

Pair (b)

$= \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D}$

$= \bar{A}\bar{B}C + A\bar{B}C$

$= \bar{B}C$

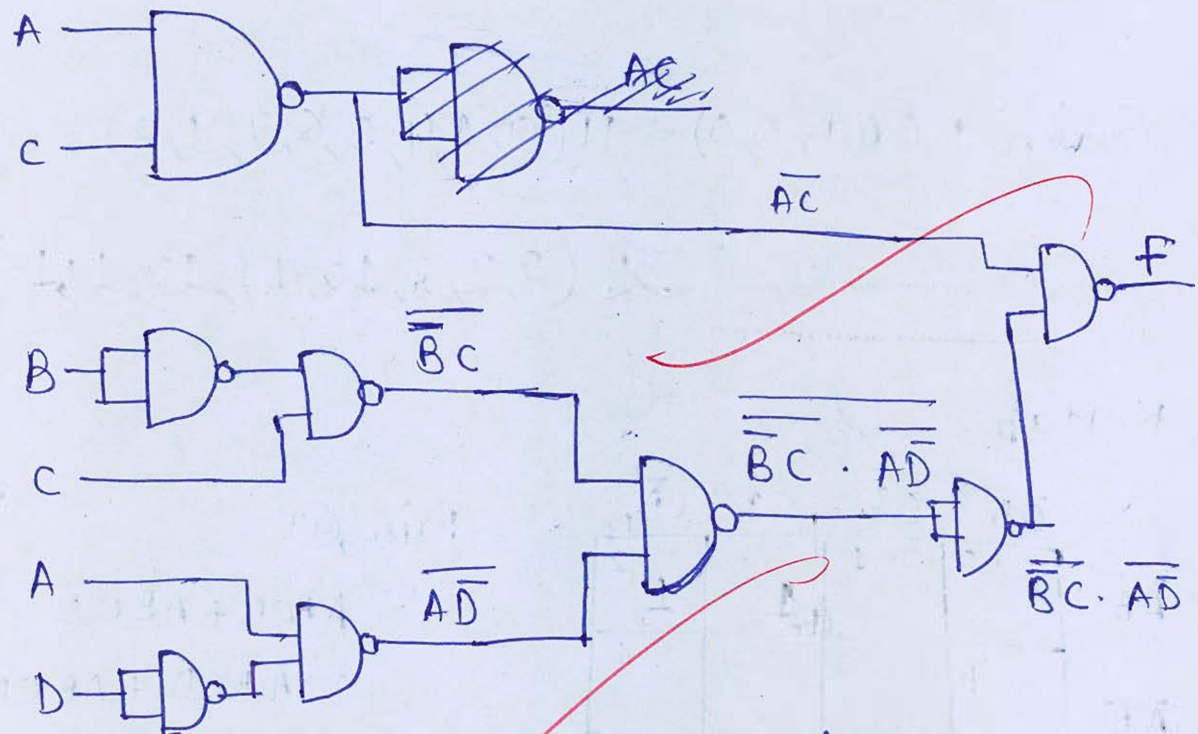
Pair (c)

$= AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D}$

$= A\bar{C}\bar{D} + AC\bar{D} = A\bar{D}$

So; $\boxed{F(A,B,C,D) = AC + \bar{B}C + A\bar{D}}$

$$F = \overline{AC + \bar{B}C + A\bar{D}} = \overline{\overline{AC} \cdot \overline{\bar{B}C} \cdot \overline{A\bar{D}}}$$

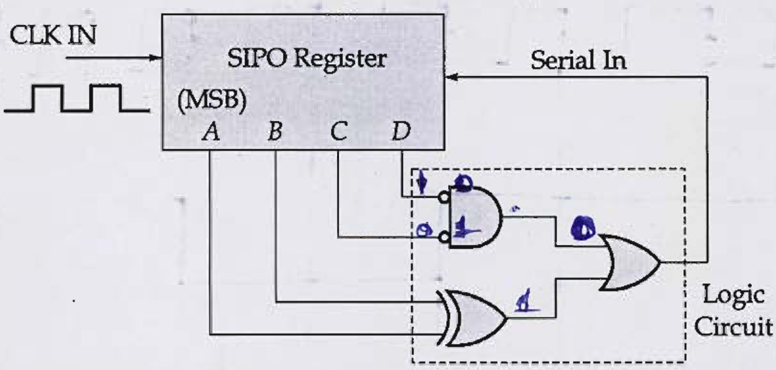


Ans

11

Good Approach

Q.1 (d) A shift register with associated combinational logic circuit is shown in figure below:



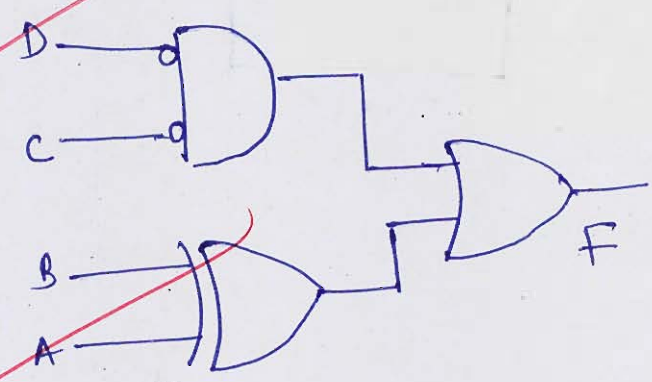
Explain its operation by drawing waveforms for the output of shift register (*ABCD*). Assume that initially shift register is in state *ABCD* = 1111.

[12 marks]

Ans. -

A B C D	Clk
1 1 1 1	0
1 1 1 0	1
1 1 0 0	2
1 0 0 1	3
0 0 1 1	4
0 1 1 0	5
1 1 0 1	6
1 0 1 0	7
0 1 0 1	8
1 0 1 1	9
0 1 1 1	10
1 1 1 1	11

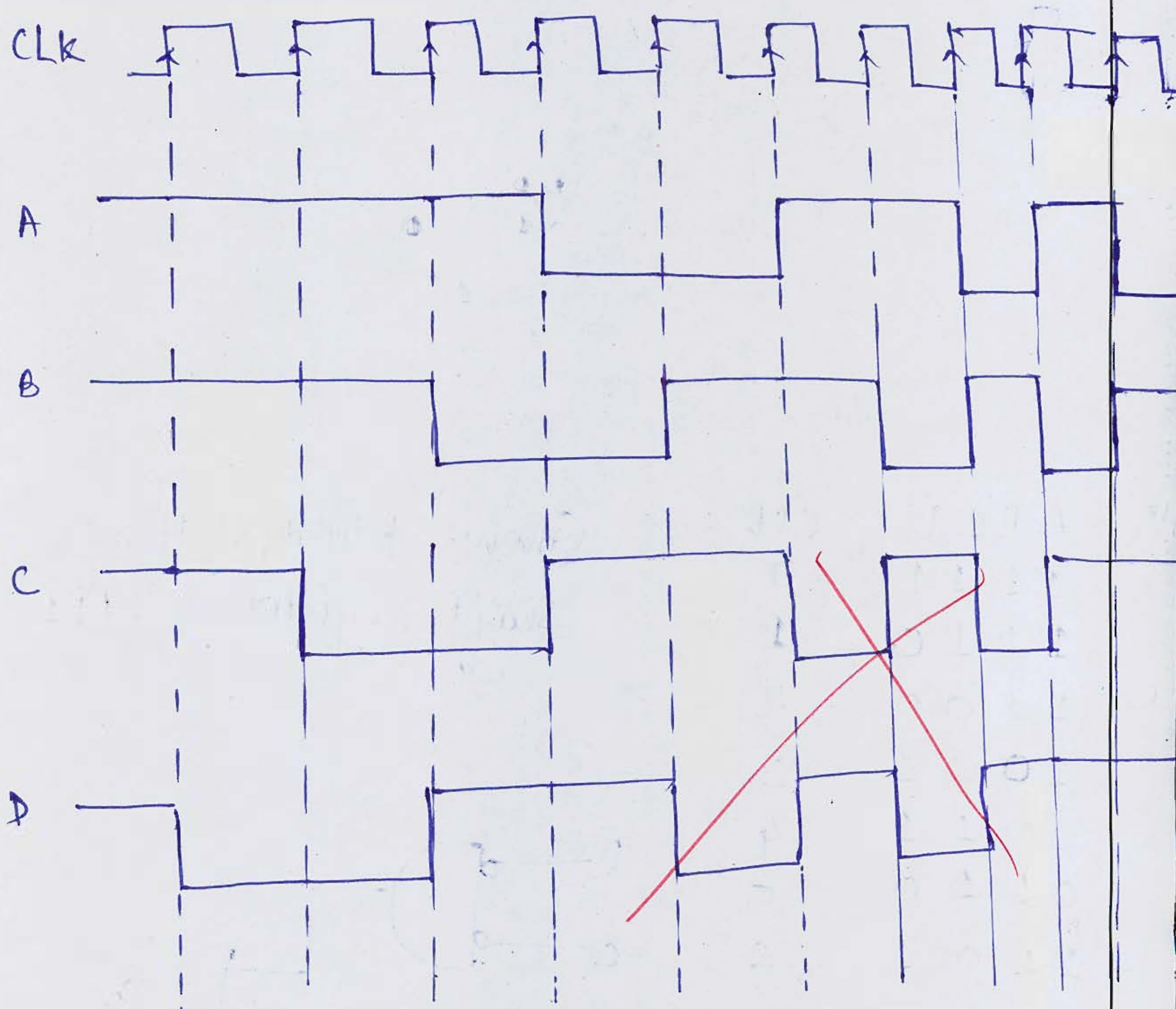
Given:- Initial state of shift register = 1111



$F = \bar{D} \cdot \bar{C} + A \oplus B$

This o/p shall be fed to the serial i/p LSB bit i.e D.

Waveforms of the o/p obtained in each clock pulse is as follows →



Ans

6

Do not write in this margin

Q.1 (e) For an 8085 microprocessor write an assembly language program to move a block of 10 bytes of data from one section of memory to another section of memory. Write comments for selected instructions.

[12 marks]

~~MOV~~

LDAX 0024H

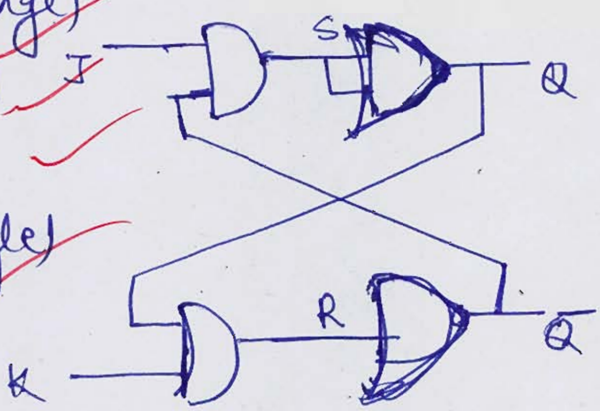
In complete solution

Q.2 (a) (i) Write truth table and logic symbol of J-K flip-flop and also draw J-K flip-flop using NAND and NOR latch.

[10 marks]

Soln:- Truth table of J-K flip flop -

J	K	Q^+
0	0	Q (No change)
0	1	0 (Reset)
1	0	1 (Set)
1	1	$\bar{Q}$ (Toggle)



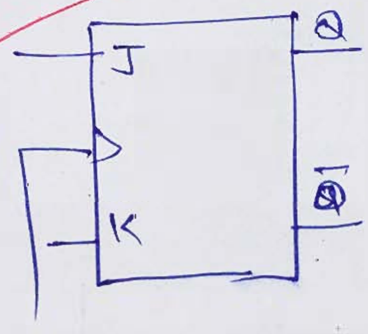
$S = J\bar{Q}$ $R = KQ$

Try to avoid over writing

6

J	K	Q	Q^+
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Logic symbol-



Q.2 (a) (ii) Convert SR flip-flop into JK flip-flop and explain race around condition in J-K flip-flop.

[10 marks]

Sol^n:-

J	K	Q	Q^+	S	R
0	0	0	0	0	X
0	0	1	1	X	0
0	1	0	0	0	X
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	1	X	0
1	1	0	1	1	0
1	1	1	0	0	1

K-Map

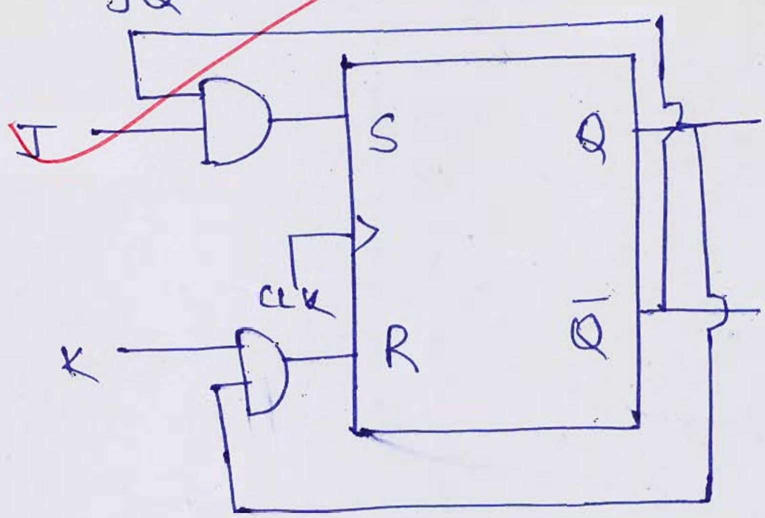
S	$\bar{K}\bar{Q}$	$\bar{K}Q$	KQ	$K\bar{Q}$
$\bar{J}$	0 (0)	X (1)	0 (3)	0 (2)
J	1 (4)	X (5)	0 (7)	1 (6)

R	$\bar{K}\bar{Q}$	$\bar{K}Q$	KQ	$K\bar{Q}$
$\bar{J}$	X	0	1	X
J	0	0	1	0

$S = J\bar{K}\bar{Q} + JK\bar{Q}$

$= J\bar{Q}$

$R = KQ$.



Race- around condition→

Race-around condition is usually observed in J-K Flip flop when length of the clock pulse is greater than the propagation delay of the gates. Hence, a different O/P is usually obtained before the next clock pulse arrives. To prevent race-around condition –

1.) Edge triggering is preferred,
2.) Master -slave Flip flop is used.
3.) length of the clock pulse should be less than the propagation delay.

Ans

9

Good Approach

Q.2 (b) Design a 3 × 8 decoder with active low enable pin and also draw its logic circuit.

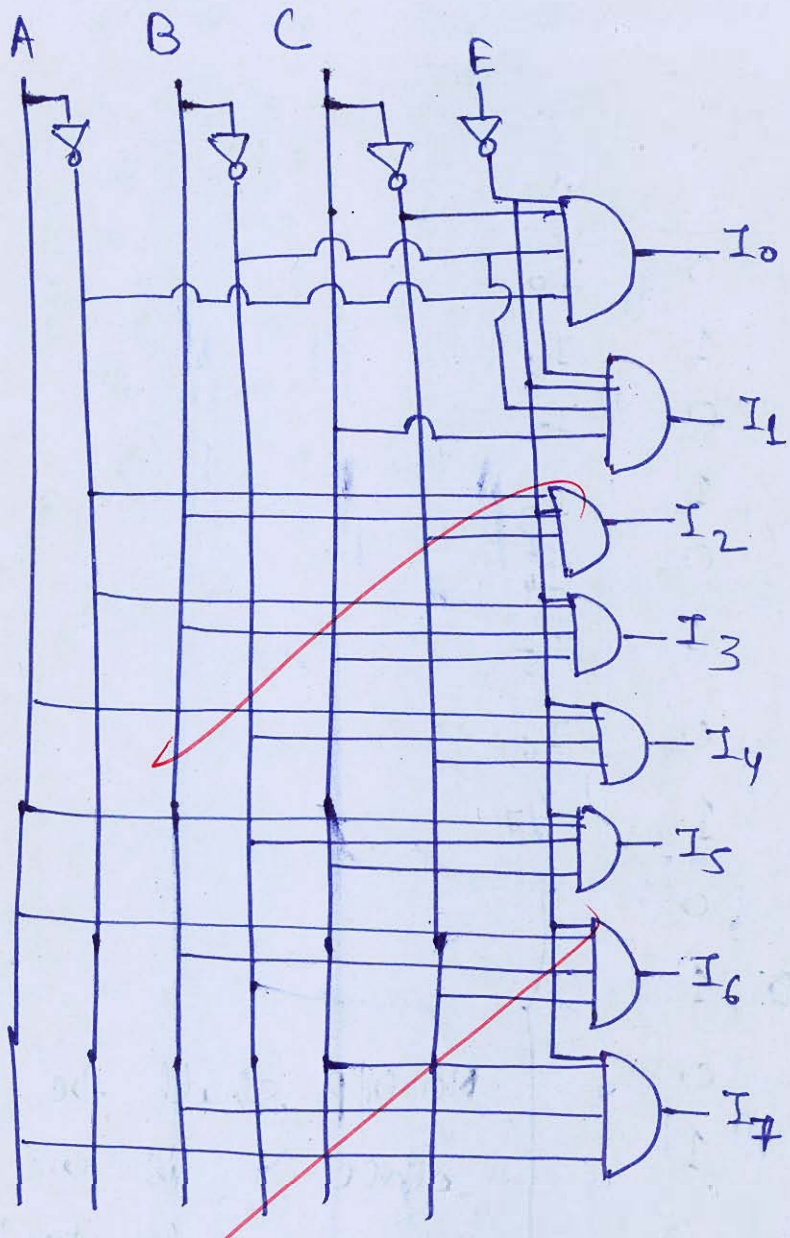
[20 marks]

Sol^n.

E	A	B	C	Y
0	0	0	0	I_0
0	0	0	1	I_1
0	0	1	0	I_2
0	0	1	1	I_3
0	1	0	0	I_4
0	1	0	1	I_5
0	1	1	0	I_6
0	1	1	1	I_7
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

No O/P shall be obtained since it is an active low enable device.

Do not write in this margin



Logic ckt of a 3x8 decoder

15

Q.2 (c) Explain the following instructions of 8085 microprocessor:

(i) RLC (ii) RAR (iii) CALL 16-bit address

(iv) RET (v) CMP

[20 marks]

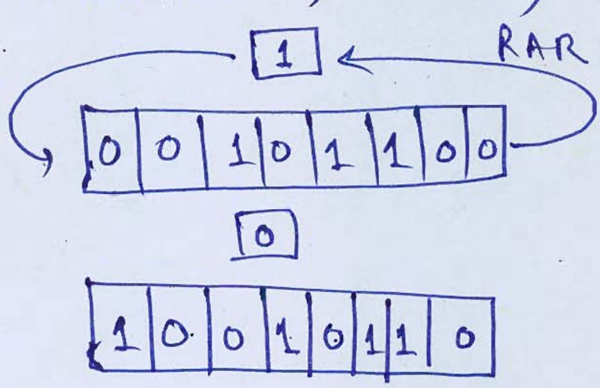
Solⁿ:

(i) RLC → The instruction RLC rotates the accumulator content left without carry. It is a 1 bit instruction which acts on the content of the accumulator. It is used to check the sign bit of the contents of the accumulator and also used for multiplication operation.

1 bit instruction, 1 M/C, 4 T-states.

(ii) RAR → It is a 1 bit instruction which acts on the con~~tents~~ of the accumulator, rotating it rightwards with carry. It is used to check LSB bit i.e whether a number is even or odd.

1 bit instruction, 1M/C, 4T-states



(iii) CALL 16 bit address →

CALL is a conditional / unconditional instruction used to call a sub-routine. A sub-routine is smaller programmes written so as to be accessed again & again using CALL instruction. When a CALL instruction is executed, the 16 bit address in pasted on the program counter and the corresponding subroutine is run.

3 bit instruction, 5 M/C, 18 T-states.

(iv) RET → A RET i.e return instruction is always used in 8085 programming at the end of sub-routine. When a return command is executed, the address of the main program which was stored in stack is accessed back and the execution of the main program starts from the point where it was initially stopped (when CALL was executed).

At the end of RET instruction SP → SP-2.

15

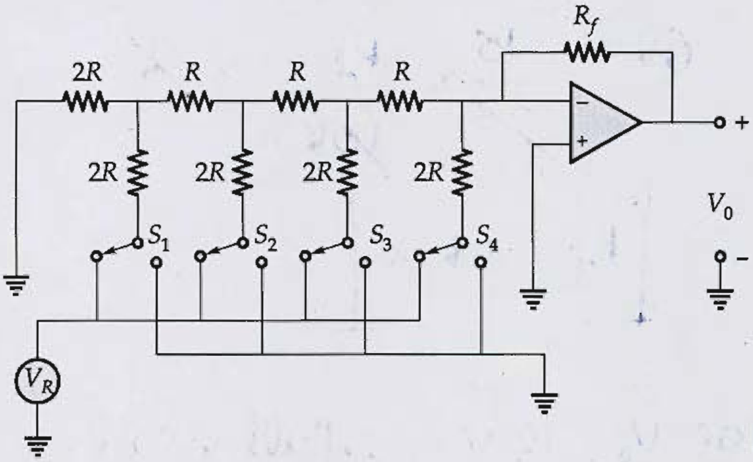
(v) CMP → CMP is a logical instruction which compares the data of a register/ Memory location with the contents of the accumulator. Only Z, & CY flag gets affected in CMP instruction.

Content of data = Content of Acc ⇒ CY = 0, Z = 1

\> ⇒ CY = 0, Z = 0

< ⇒ CY = 1, Z = 0

Q.3 (a) (i) Consider a 4-bit R-2R DAC shown in figure. Assume the feedback resistance R_f of the op-amp is variable, the resistance $R = 10\ k\Omega$ and $V_R = 10$ V.



Determine the values of R_f that should be connected to achieve the following output conditions:

1. The value of 1 LSB at the output is 0.5 V.
2. An analog output of 6 V for a binary input of 1000.
3. The full scale output voltage of 12 V.
4. The actual maximum output voltage of 10 V.

(ii) Explain the working of flash type *ADC* with the help of truth table.

[10 + 10 marks]

Ans- (i)

For a R-2R ladder type DAC.

$$V_o = \left|\frac{V_R}{2^{n-1}}\right| \left(\frac{R_f}{R}\right) \sum_{i=0}^{n} 2^i b_i$$

i) for 1 LSB i/p $\qquad V_R = 10V, \quad R = 10k\Omega$

Given $V_o = 0.5V$

$$0.5 = \frac{10}{2^3} \times \frac{R_f}{10k\Omega} \times 1$$

$$\boxed{\cancel{R_f = 4k\Omega}}$$

(ii) For binary i/p of 1000

$$V_0 = 6V = \frac{\cancel{10}}{\cancel{2^3}} \times \frac{R_f}{\cancel{10}k\Omega} \times \cancel{8}$$

$$\boxed{R_f = 6k\Omega}$$

(iii) For $V_0 = 12V$, full scale.

1111
$2^3\, 2^2\, 2^1\, 2^0$

8
+4
+2
+1

$$12 = \frac{10}{2^3} \times \frac{R_f}{10k\Omega} \times (15)$$

$$\boxed{R_f = 6.4\,k\Omega}$$

4.) for $V_0 = 10V$

$$\cancel{10} = \frac{\cancel{10}}{2^3} \times \frac{R_f}{10k\Omega} \times 15$$

$$\boxed{R_f = 5.33\,k\Omega}$$

3

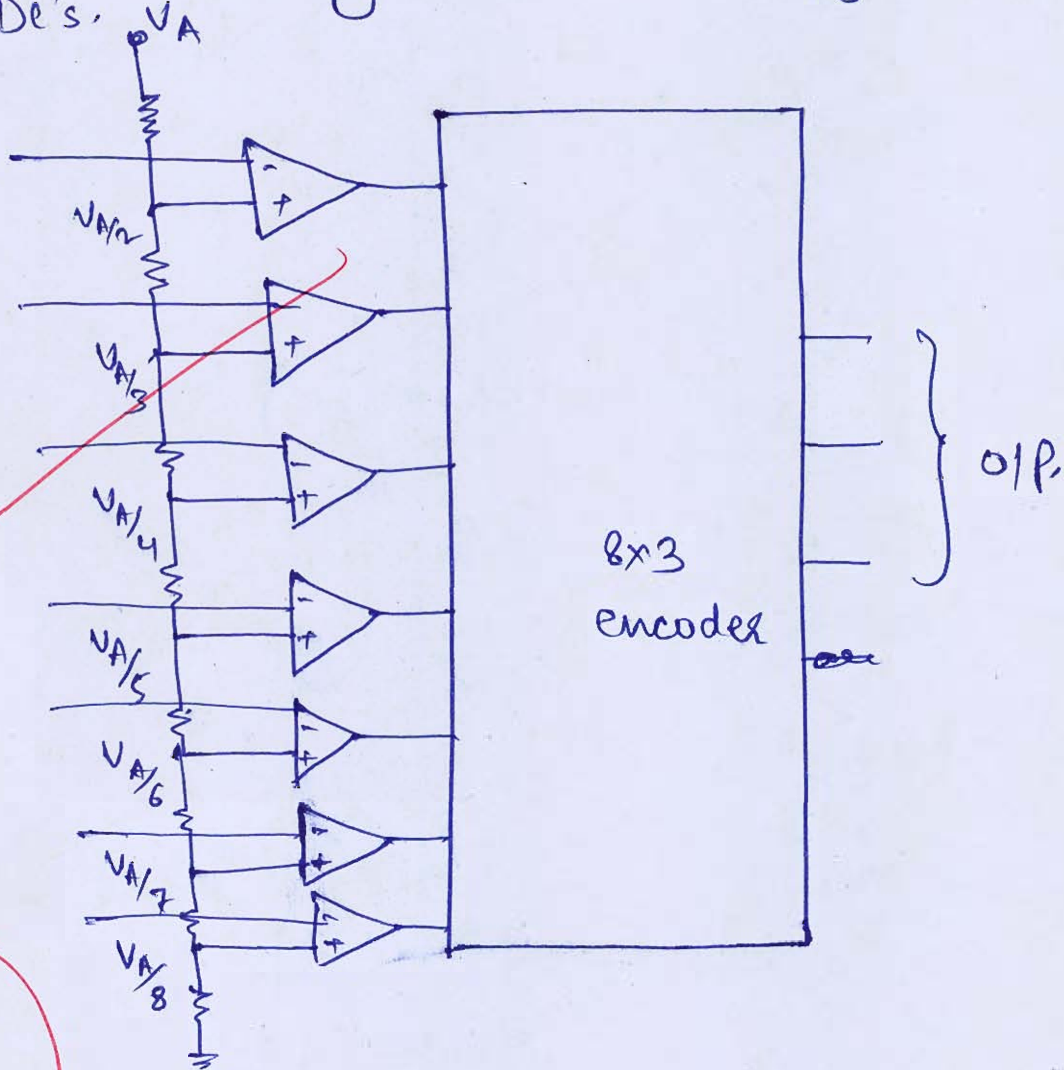
(ii) Flash type ADC

→ Fastest ADC

→ Uses comparator and resistors for comparison.

→ Uses PIPO register.

→ It's accuracy is worst among all the ADC's.

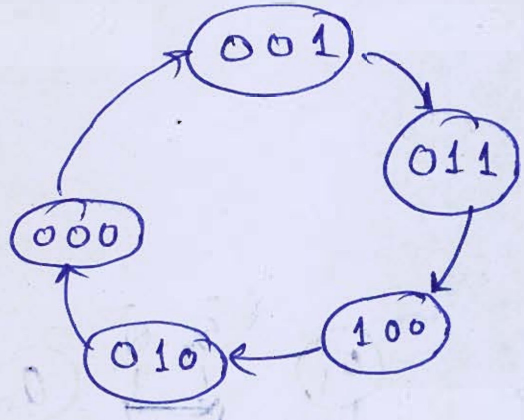


3

Q.3 (b) Design a sequential circuit using J-K flip flop followed by T-FF followed by S-R flip flop to count a sequence $1 \to 3 \to 4 \to 2 \to 0 \to 1$ (Use J-K FF for MSB).

[20 marks]

4(b)



	Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	J_2	K_2	T_1	S_0	R_0
1	0	0	1	0	1	1	0	X	1	X	0
3	0	1	1	1	0	0	1	X	0	0	1
4	1	0	0	0	1	0	X	1	1	0	X
2	0	1	0	0	0	0	0	X	0	0	X
0	0	0	0	0	0	1	0	X	0	1	0

J_2	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	0 (0)	0 (1)	1 (3)	0 (2)
Q_2	X (4)	X (5)	X (7)	X (6)

$J_2 = Q_1 Q_0$

K_2	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	0 (0)	X (1)	X (3)	0 (2)
Q_2	1 (4)	X (5)	X (7)	X (6)

$K_2 = Q_2$

T	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	0 (0)	1 (1)	0 (3)	0 (2)
Q_2	1 (4)	X (5)	X (7)	X (6)

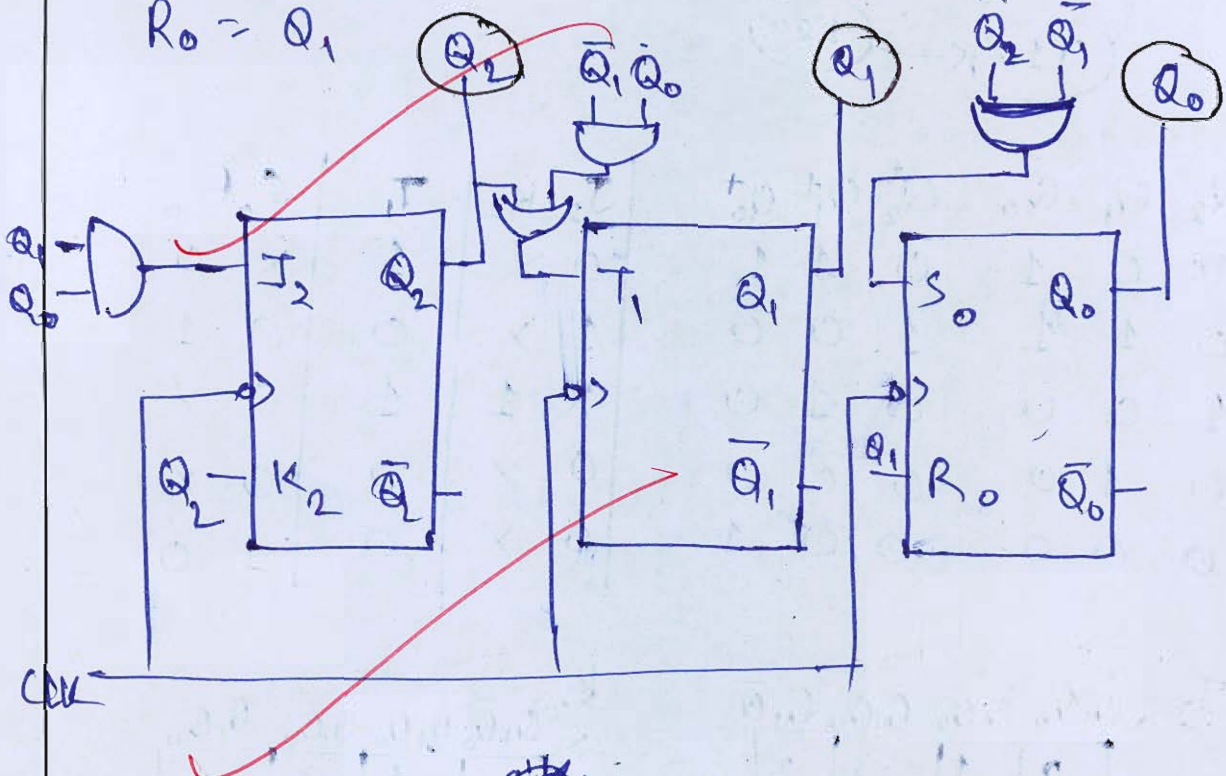
$T = Q_2 + \bar{Q}_1 Q_0$

S_0	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	1 (0)	X (1)	0 (3)	0 (2)
Q_2	0 (4)	X (5)	X (7)	X (6)

$S_0 = \bar{Q}_2 \bar{Q}_1$

R_0	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	0 (0)	0 (1)	1 (3)	X (2)
Q_2	X (4)	X (5)	X (7)	X (6)

$R_0 = Q_1$



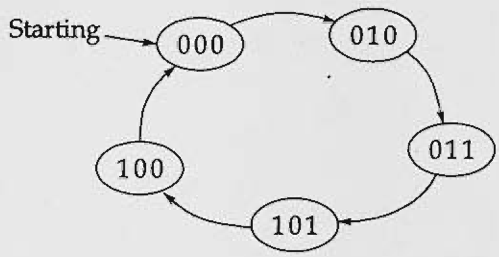
circuit diagram

16

Do not write in this margin

Do not write in this margin

Q.3 (c) (i) The state transition diagram of a synchronous counter is given in figure. Design the counter circuit using S-R flip flops.



(ii) Draw EX-OR gate using transmission gates.

[14 + 6 marks]

	Present State Q_2	Q_1	Q_0	Next State Q_2^+	Q_1^+	Q_0^+	S_2	R_2	S_1	R_1	S_0	R_0
0	0	0	0	0	1	0	0	X	1	0	0	X
2	0	1	0	0	1	1	0	X	X	0	1	0
3	0	1	1	1	0	1	1	0	0	1	X	0
5	1	0	1	1	0	0	X	0	0	X	0	1
4	1	0	0	0	0	0	0	1	0	X	0	X

K-Map

S_2

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	0	X	1	0
Q_2	0	X	X	X

$S_2 = Q_0$

R_2

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	X	X	0	X
Q_2	1	0	X	X

$R_2 = \bar{Q}_0$

S_1

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	1	X	0	X
Q_2	0	0	X	X

$S_1 = \bar{Q}_2\bar{Q}_0$

R_1

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	0	X	1	0
Q_2	X	X	X	X

$R_1 = Q_0$

S_0

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	0 (0)	X (1)	X (3)	1 (2)
Q_2	0 (4)	0 (5)	X (7)	X (6)

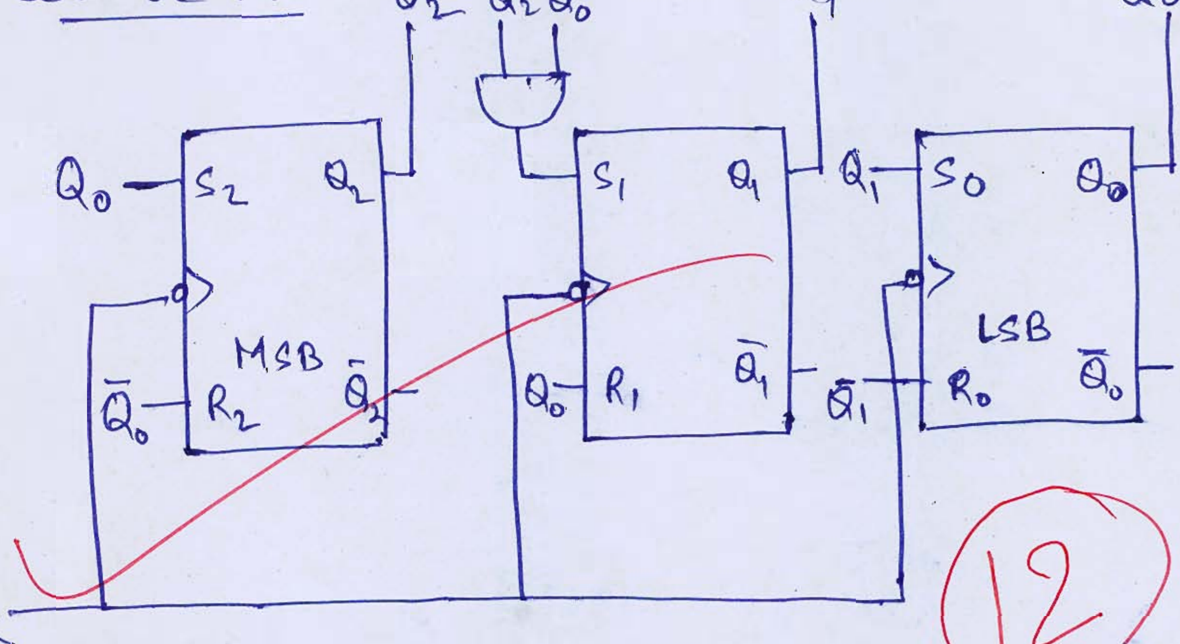
$S_0 = Q_1$

R_0

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	X (0)	X (1)	0 (3)	0 (2)
Q_2	X (4)	1 (5)	X (7)	X (6)

$R_0 = \bar{Q}_1$

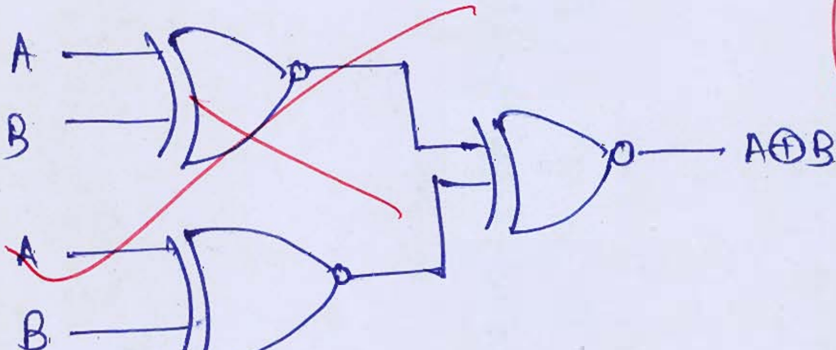
<u>Counter ckt</u> -



12

(ii) $A \oplus B = A\bar{B} + \bar{A}B$

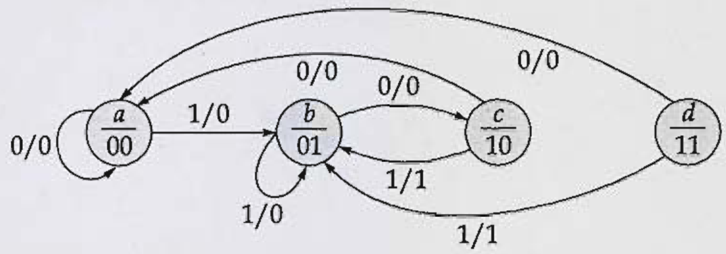
$A \odot B = \bar{A}\bar{B} + AB$



Elaborate it more

Do not write in this margin

Q.4 (a) Consider the state diagram shown below:

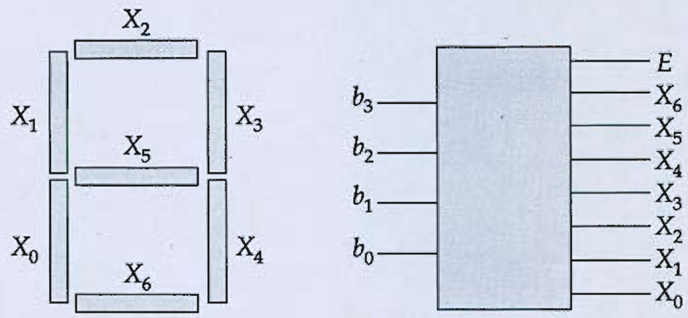


(i) Obtain the state reduction diagram.

(ii) Design a clocked sequential machine using JK FFs for the given sequential circuit.

[8 + 12 marks]

Q.4 (b) A seven segment display consists of seven light-emitting diodes (LEDs) as shown below:



7-Segment display

where, $E = 0$ for valid digit with the display showing the corresponding digit.

$E = 1$ for invalid digit with the display showing the symbol 'E'.

Implement the above given seven segment display using a ROM.

[20 marks]

Q.4 (c) For an 8085 microprocessor, assume a sensor connected at port 30H. Read 10 bytes of data from port and if the value is +ve then store the corresponding bytes from memory location 4000H. If the data is -ve then switch 'ON' LED by sending FFH to port 50H. Write an Assembly Language Program for above condition.

[20 marks]

Section B : Systems and Signal Processing

Q.5 (a) A stable system with zero initial conditions is described by the difference equation $y(n) = x(n) - 2x(n-1) + x(n+1)$.

(i) Find the impulse response $h(n)$ of the system.

(ii) Plot the output $y[n]$ for an input, $x[n] = u[n-2]$

(iii) Find the value of $\sum_{n=-\infty}^{\infty} y[n]$ for input $x[n] = u[n-2]$.

[4 + 4 + 4 marks]

Solⁿ: ① $y(n) = x(n) - 2x(n-1) + x(n+1)$

Taking z Trf -

$$Y(z) = X(z) - 2z^{-1}X(z) + Z\{X(z)$$

$$\frac{Y(z)}{X(z)} = 1 - 2z^{-1} + z$$

$$H(z) = 1 - 2z^{-1} + z$$

Taking inverse z-trf

$$\boxed{h(n) = \delta(n) - 2\delta(n-1) + \delta(n+1)}$$

② $y(n) = x(n) - 2x(n-1) + x(n+1)$

for $x[n] = u[n-2]$

$$y(n) = u(n-2) - 2u(n-3) + u(n-1)$$

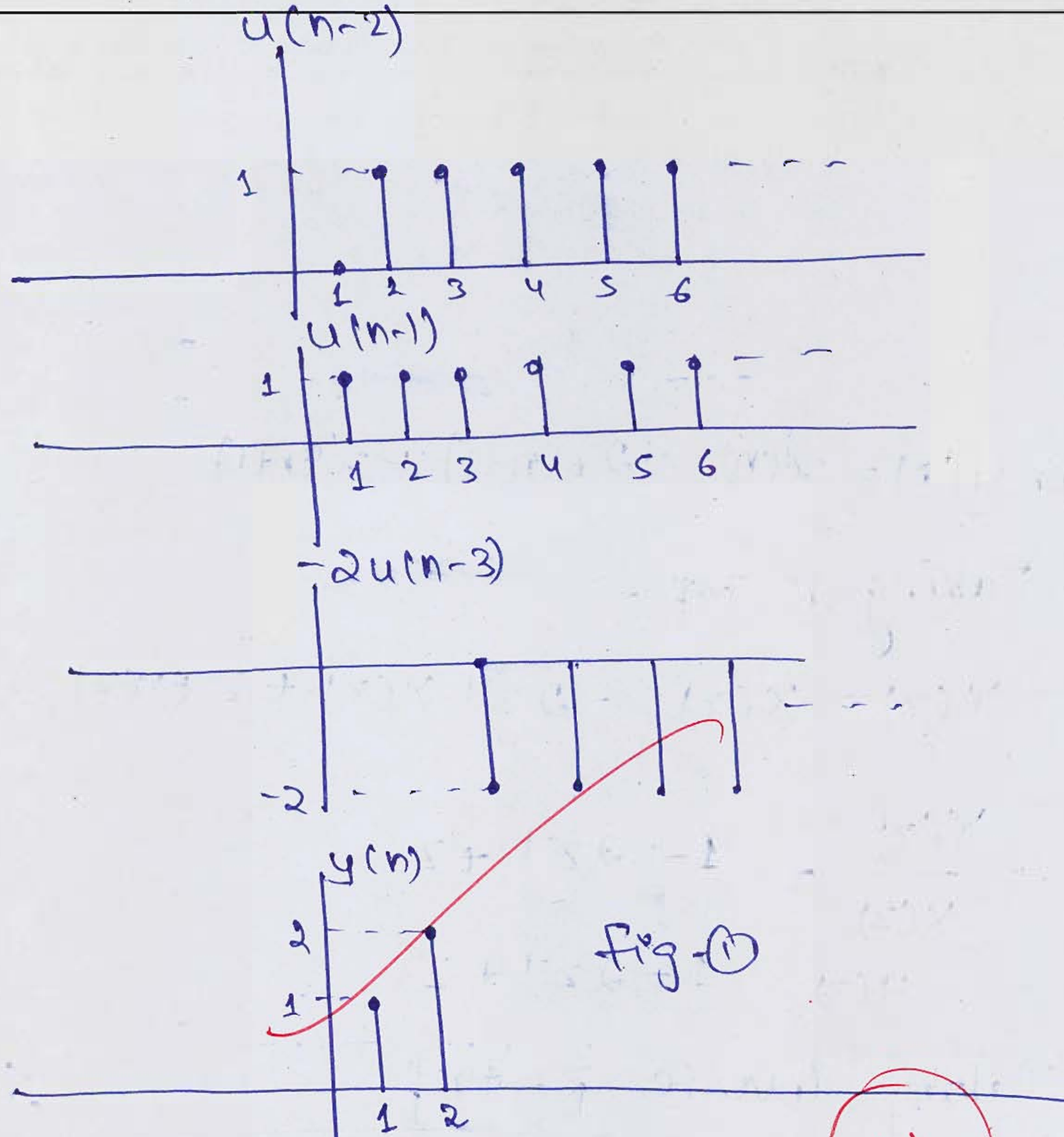


fig-①

11

Good Approach

(iii) For $x[n] = u[n-2]$

$$y[n] = u(n-2) + u(n-1) - 2u(n-3)$$

Plot as in fig-①

$$\sum_{n=-\infty}^{\infty} y[n] = \sum \delta(n-1) + 2\,\delta(n-2)$$

$$= \underline{\underline{3}}$$

$$\boxed{\sum_{n=-\infty}^{\infty} y(n) = 3}$$ Ans

Q.5 (b) Determine the inverse z-transform of the following $X(z)$ by the partial fraction expansion method,

$$X(z) = \frac{z+2}{2z^2-7z+3}$$

if the ROCs are (a) $|z| > 3$, (b) $|z| < 1/2$ and (c) $1/2 < |z| < 3$.

[12 marks]

Solⁿ:

$$X(z) = \frac{z+2}{2z^2-7z+3} = \frac{z^{\cancel{2}}\left[1+2z^{-1}\right]}{z^2\left[2-7z^{-1}+3z^{-2}\right]}$$

$$= \frac{z^{-1}\left[1+2z^{-1}\right]}{\left[2-7z^{-1}+3z^{-2}\right]}$$

$$\frac{z+2}{2z^2-7z+3} = \frac{A}{(z-3)} + \frac{B}{z-\frac{1}{2}}$$

$$z+2 = Az - \frac{A}{2} + Bz - 3B$$

$$\left.\begin{aligned} A+B &= 1 \\ -\frac{A}{2} - 3B &= 2 \end{aligned}\right] \rightarrow A = 2, \; B = -1$$

$$\text{So } X(z) = \frac{2}{(z-3)} + \frac{(-1)}{z-\frac{1}{2}}$$

(a) $|z| > 3 \longrightarrow$ Right sided.

$$X(z) = \frac{2z^{-1}}{(1-3z^{-1})} - \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})}$$

$x(n) \rightarrow$

$$\boxed{2(3)^{n-1}u(n-1) - \left(\frac{1}{2}\right)^{n-1}u(n-1)}$$

Ans.

(b) $|z| < \frac{1}{2}$ $\rightarrow$ Left sided.

$$X(z) = \frac{2z^{-1}}{(1-3z^{-1})} - \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})}$$

Taking IZT $\rightarrow$

$$\boxed{x(n) = 2(-(3)^{n-1}u(-n-1)) + (\tfrac{1}{2})^{n-1}u(-n-1)}$$

(c) $\frac{1}{2} < |z| < 3$

$$X(z) = \frac{2z^{-1}}{(1-3z^{-1})} - \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})}$$

Taking IZT $\rightarrow$

$$\boxed{x(n) = -2(3)^{n-1}u(-n-1) - (\tfrac{1}{2})^{n-1}u(n-1)}$$

10

Do not write in this margin

Q.5 (c) Determine the z-transform and the ROC of the signal, $x(n) = a^n u(n) - a^n u(n-1)$.

[12 marks]

Sol^n:

Given $x(n) = a^n u(n) - a^n u(n-1)$

$$a^n u(n) \overset{Z.T}{\rightleftharpoons} \frac{1}{(1-az^{-1})} \; ; \; |z| > |a|$$

By property, we know that

$$x(n-n_0) \rightleftharpoons z^{-n_0} X(z)$$

$$a^n u(n) \overset{Z.T}{\rightleftharpoons} \frac{1}{1-az^{-1}}$$

$$a^{n-1} u(n-1) \overset{Z.T}{\rightleftharpoons} \frac{z^{-1}}{1-az^{-1}}$$

$$a^n u(n-1) \overset{Z.T}{\rightleftharpoons} \frac{az^{-1}}{1-az^{-1}}$$

So $X(z) = \frac{1}{1-az^{-1}} - \frac{az^{-1}}{1-az^{-1}}$

$$= \frac{1-az^{-1}}{1-az^{-1}} = 1$$

So, $\boxed{X(z) = 1}$

ROC $\rightarrow$ entire z plane.

11

Q.5 (d) An LTI system has a unit step response given by $s(t) = (1 - e^{-t} - te^{-t})\,u(t)$. For a certain input $x(t)$, the output is observed to be equal to $y(t) = (2 - 3e^{-t} + e^{-3t})\,u(t)$. What is $x(t)$?

[12 marks]

Solⁿ:-

$$S(t) = 1 - e^{-t} - te^{-t}$$

$$\frac{Y(s)}{X(s)} = \text{T/Function} \Rightarrow \frac{S(s)}{U(s)} = \frac{1 - \frac{1}{s+1} - \frac{1}{(s+1)^2}}{\frac{1}{s}}$$

$$= \frac{\frac{(s+1)^2 - (s+1) - 1}{(s+1)^2}}{\frac{1}{s}} = \frac{s\left[\begin{array}{l} s^2+2s+\cancel{1} \\ -s-\cancel{1}-1 \end{array}\right]}{(s+1)^2}$$

Given $y(t) = \left(2 - 3e^{-t} + e^{-3t}\right)$

$$= \frac{s\left[s^2+s-1\right]}{(s+1)^2}$$

$$Y(s) = \frac{2}{s} - \frac{3}{(s+1)} + \frac{1}{s+3}$$

$$H(s) = \frac{1}{(s+1)^2}$$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{1}{(s+1)^2}$$

$$\frac{2(s+1)(s+3) - 3s(s+3) + s(s+1)}{s\,\cancel{(s+1)}\,(s+3)\cdot X(s)} = \frac{1}{(s+1)^{\cancel{2}}}$$

$$\frac{2\left[s^2+4s+3\right] - 3s^2 - 9s + s^2 + s}{s\,(s+3)\;X(s)} = \frac{1}{(s+1)}$$

Also →

Step response $= 1 - e^{-t} - te^{-t}$

Impulse response $= \frac{d}{dt}(\text{step response})$

$= 0 + e^{-t} - \left[-te^{-t} + e^{-t}\right]$

$= \cancel{e^{-t}} + te^{-t} - \cancel{e^{-t}}$

$= te^{-t}$

$$H(s) = \frac{1}{(s+1)^2}$$

$$\frac{\cancel{2s^2}+\cancel{8s}+6-\cancel{3s^2}-\cancel{9s}+\cancel{s^2}+\cancel{s}}{(s)\ (s+3)\quad X(s)} = \frac{1}{(s+1)}$$

$$\frac{6(s+1)}{s(s+3)} = X(s) = \frac{A}{s} + \frac{B}{s+3}$$

$$6s+6 = As+3A+Bs$$

$$A+B=6$$

$$\cancel{3}A=\cancel{6}\ 2 \Rightarrow A=2,\ B=4$$

$$X(s) = \frac{2}{s} + \frac{4}{s+3}$$

Taking inverse laplace txf.

$$\boxed{x(t) = (2+4e^{-3t})u(t)}$$

11

Good Approach

Q.5 (e) Consider the signal $y(t) = e^{-2t}u(t)$ is the output of a causal all-pass system for which the system function is

$$H(s) = \frac{s-1}{s+1}$$

(i) Find and sketch at least two possible inputs $x(t)$ that could produce $y(t)$.

(ii) From the solutions obtained in part (i), what is the input $x(t)$ if it known that a stable system exists that will have $x(t)$ as an output and $y(t)$ as the input? Find the impulse response $h(t)$ for this system.

[12 marks]

Sol^n:

$$y(t) = e^{-2t}u(t) \quad ; \quad H(s) = \frac{s-1}{s+1}$$

$$\Rightarrow Y(s) = \frac{1}{s+2}$$

$$Y(s) = X(s) \cdot H(s) \Rightarrow X(s) = \frac{Y(s)}{H(s)} = \left[\frac{\frac{1}{s+2}}{\frac{(s-1)}{(s+1)}}\right] = \frac{s+1}{(s+2)(s-1)}$$

$$X(s) = \frac{s+1}{(s+2)(s-1)} = \frac{A}{(s+2)} + \frac{B}{(s-1)}$$

$$s+1 = As - A + Bs + 2B$$

$$\left.\begin{aligned} A+B &= 1 \\ -A+2B &= 1 \end{aligned}\right\} A = \frac{1}{3}, \quad B = \frac{2}{3}$$

So; $X(s) = \frac{1}{3(s+2)} + \frac{2}{3(s-1)}$

for $x(t)$ right sided.

$$x(t) = \left(\frac{1}{3}e^{-2t} + \frac{2}{3}e^{t}\right)u(t)$$

Do not write in this margin

For $x(t)$ left sided.

$$x(t) = -\frac{1}{3}e^{-2t}u(-t) - \frac{2}{3}e^{t}u(-t)$$

Causal Signal

(ii) For stable system, $x(t)$ should be right sided and the right most pole should be on left side of $j\omega$ axis.

$$x(t) = \frac{1}{3}e^{-2t} - \frac{2}{3}e^{t}u(-t)$$

$$\boxed{-2 < \sigma < 1}$$

10

Given $H(s) = \frac{s-1}{s+1} = \frac{s-1+2-2}{s+1}$

$$= \frac{s+1-2}{s+1}$$

$$= 1 - \frac{2}{s+1}$$

Taking ILT $\rightarrow$ $\boxed{h(t) = \delta(t) - 2e^{-t}u(t)}$

Do not write in this margin

Q.6 (a) (i) Compute: $y(t) = x(t) * h(t)$ where, $x(t) = \text{sinc}\,\alpha t$, $h(t) = \text{sinc}\,\beta t$; $\alpha > \beta$

(ii) Compute : $y(t) = x(t) * h(t)$

$$x(t) = \text{rect}\left(\frac{t-6}{2}\right), h(t) = \text{rect}\left(\frac{t}{2}-3\right)$$

[10 + 10 marks]

(a)

$x(t) = \text{sinc}\,\alpha t = Sa(\alpha\pi t)$

$h(t) = \text{sinc}\,\beta t = Sa(\pi\beta t)$

we know that →

$$A_0\, Sa(Kt) \xrightarrow{F.T} \frac{A_0\pi}{K} \text{rect}(\omega/2K)$$

$$\underset{K}{Sa(\alpha\pi t)} \xrightarrow{F.T} \frac{1\times\pi}{\alpha\pi} \text{rect}\left(\frac{\omega}{2\pi\alpha}\right)$$

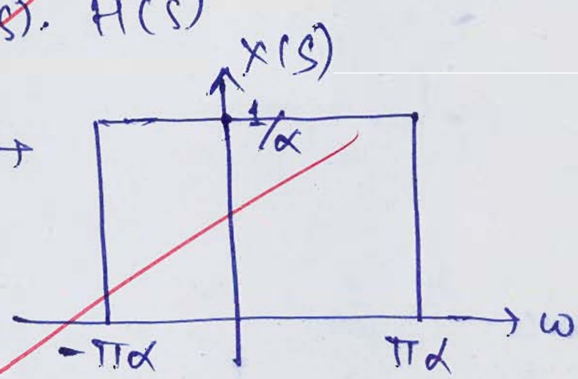
$$= \frac{1}{\alpha} \text{rect}\left(\frac{\omega}{2\pi\alpha}\right)$$

$$\text{IIIly } Sa(\beta\pi t) \rightarrow \frac{1}{\beta} \text{rect}\left(\frac{\omega}{2\pi\beta}\right)$$

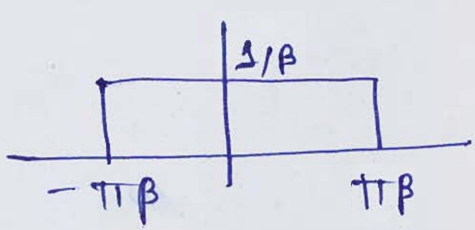
$y(t) = x(t) * h(t)$

$Y(S) = X(S) \cdot H(S)$

$X(S) \rightarrow$

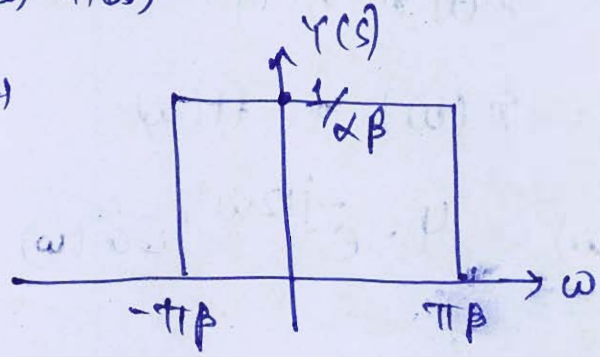


$H(S) \rightarrow$



So; $Y(\omega) = X(\omega) \cdot H(\omega)$

$\Rightarrow$



So; $Y(\omega) = \frac{1}{\alpha\beta} \, rect\left(\frac{\omega}{2\pi\beta}\right)$

Taking inverse fourier transform

$Y(\omega) = \frac{1}{\alpha\beta} \, rect\left(\frac{\omega}{2\pi\beta}\right) \longrightarrow y(t) = \frac{1}{\alpha} Sa(\pi\beta t)$

or $\boxed{y(t) = \frac{1}{\alpha} \, Sinc\, \beta t}$

Ans.

(ii) $x(t) = rect\left(\frac{t-6}{2}\right)$

We know that

$A \, rect(t/\tau) \xrightarrow{F.T} A\tau \, Sa\left(\frac{\omega\tau}{2}\right)$

& $x(t-t_0) \xrightarrow{F.T} e^{-j\omega t_0} X(\omega)$

$rect(t/2) \longrightarrow 2 \, Sa(\omega)$

$rect\left(\frac{t-6}{2}\right) \longrightarrow 2 e^{-j6\omega} \, Sa(\omega)$

Illy $h(t) = rect\left(\frac{t-6}{2}\right) \xrightarrow{F.T} 2e^{-j6\omega} Sa(\omega)$

Do not write in this margin

$$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

$$Y(\omega) = 4 \cdot e^{-j12\omega} Sa^2(\omega)$$

we know that

$$A\,tri\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau\, Sa^2\left(\frac{\omega\tau}{2}\right)$$

$$4\, Sa^2(\omega) \longrightarrow 2\,tri\left(\frac{t}{2}\right)$$

$$4\, e^{-j12\omega} Sa^2(\omega) \longrightarrow 2\,tri\left(\frac{t-12}{2}\right)$$

$$\boxed{y(t) = 2\,tri\left(\frac{t-12}{2}\right)}$$

$\tau/2 = 1$

$\tau = 2$

$A\tau = 4$

$A = 2$

18

Go through the made easy solution

Q.6 (b) Given below are the impulse response of FIR filter. Identify the type of filter implemented using this impulse response.

(i) $h[n] = 5\delta[n] - 7\delta[n-1] + 7\delta[n-3] - 5\delta[n-4]$

(ii) $h[n] = 5[\delta[n] - \delta[n-2]]$

[10 + 10 marks]

(i) $h[n] = 5\delta[n] - 7\delta[n-1] + 7\delta[n-3] - 5\delta[n-4]$

Taking z-txf.

$$H(z) = 5 - 7z^{-1} + 7z^{-3} - 5z^{-4}$$

$$= 5 - \frac{7}{z} + \frac{7}{z^3} - \frac{5}{z^4}$$

$$= \frac{5z^4 - 7z^3 + 7z - 5}{z^4}$$

at $z = 1$

$$H(z) = \frac{5 - 7 + 7 - 5}{1} = 0$$

at $z = -1$

$$H(z) = \frac{5 + 7 - 7 - 5}{1} = 0$$

Since, it is zero at both low & high frequency, , it could be ~~either~~ band pass ~~or band reject / notch filter.~~ filter.

(ii) $h(n) = 5[\delta[n] - \delta[n-2]]$

Taking Z-txf

$H(z) = 5[1 - z^{-2}]$

at $z=1$

$H(z) = 5[1-1] = 0$

at $z = -1$

$H(z) = 0$

10

Since, H(z) is zero at both low and high frequency, it will be passing only a band of frequency. So, the filter shall be band pass filter.

Q.6 (c) (i) Using impulse invariant method, convert the given analog filter transfer function $H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$ to digital transfer function with a sampling period 1 sec.

(ii) Find the Fourier transform of Gaussian modulated signal $x(t) = e^{-at^2} \cos \omega_c t$.

[10 + 10 marks]

(i) $H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$

$T = 1$ sec

In bilinear trf. $\Rightarrow$ $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

$= \frac{2}{1}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

$$H(s)\Big|_{s=2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \frac{1}{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 2\sqrt{2}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 1}$$

4

$$= \frac{(1+z^{-1})^2}{2(1-z^{-1})^2 + 2\sqrt{2}(1-z^{-1})(1+z^{-1}) + (1+z^{-1})^2}$$

$$= \frac{1 + z^{-2} + 2z^{-1}}{2(1 + z^{-2} - 2z^{-1}) + 2\sqrt{2}(1 - z^{-2}) + (1 + z^{-2} + 2z^{-1})}$$

$$= \frac{1 + z^{-2} + 2z^{-1}}{(2 + 2\sqrt{2} + 1) + z^{-2}[2 - 2\sqrt{2} + 1] + z^{-1}[-4 + 2]}$$

$$\boxed{H(z) = \frac{1 + z^{-2} + 2z^{-1}}{5.828 + 0.1715 z^{-2} - 2z^{-1}}}$$

Ans

Q.7 (a) (i) Find the magnitude and phase response for the system characterized by the difference equation

$$y(n)=\frac{1}{6}x(n)+\frac{1}{3}x(n-1)+\frac{1}{6}x(n-2)$$

[8 marks]

Q.7 (a) **(ii)** A filter is to be designed with the following desired frequency response using rectangular window:

$$H_d(e^{j\omega}) = \begin{cases} 0, & -\pi/4 \le \omega \le \pi/4 \\ e^{-j2\omega}, & \pi/4 \le |\omega| \le \pi \end{cases}$$

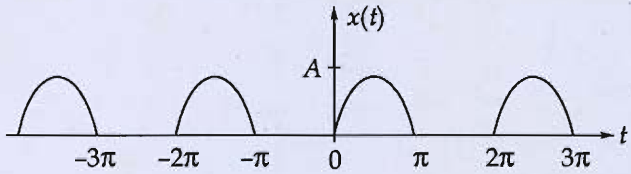
Determine the filter coefficients $h_d(n)$ if the window function is defined as

$$w(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Also, determine the frequency response $H(e^{j\omega})$ of the designed filter.

[12 marks]

Q.7 (b) Find the exponential Fourier series for the half wave rectified sine wave shown in figure.



[20 marks]

Q.7 (c) (i) The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation

$$\frac{dy(t)}{dt}+10y(t)=\int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau - x(t)$$

where, $z(t) = e^{-t}u(t) + \delta(t)$.

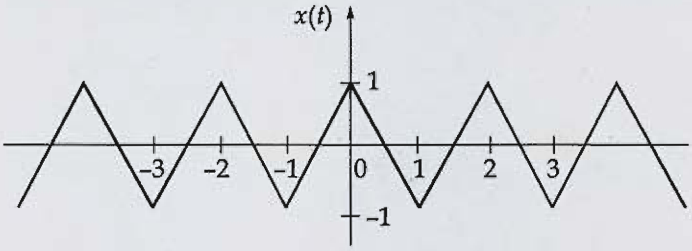
Determine the impulse response of the system.

[10 marks]

Q.7 (c) **(ii)** Convert the analog filter with system transfer function $H(s) = \dfrac{s+0.1}{(s+0.1)^2+9}$ into a digital IIR filter using bilinear transformation. The digital filter should have a resonant frequency of $\dfrac{\pi}{4}$.

[10 marks]

Q.8 (a) (i) Write down the exponential Fourier series representation of the signal $x(t)$.



(ii) Find the amount of power (in mW) contained in the 7[th] harmonic of the Fourier series representation of $x(t)$.

[10 + 10 marks]

Do not write in this margin

.8 (b) Let $g(t) = x(t)\cos^2 t * \dfrac{\sin t}{\pi t}$, where $*$ is convolution. Assuming that $x(t)$ is real and $X(\omega) = 0$ for $|\omega| \geq 1$, show that there exists an LTI system 'S' such that

$$x(t) \longrightarrow \boxed{S} \longrightarrow g(t)$$

[20 marks]

.8 (c) Determine the values of P_x and E_x for each of the following signals:

(i) $x_1(t) = e^{j(2t + \pi/4)}$

(ii) $x_2(n) = \left(\frac{1}{2}\right)^n u(n)$

(iii) $x_3(n) = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{8}\right)}$

(iv) $x_4(n) = \cos\left(\frac{n\pi}{4}\right)$

(Where P_x = Power and E_x = Energy)

[20 marks]

OOOO

Space for Rough Work

Space for Rough Work

Space for Rough Work