Try to avoid calculation mistake

Revise concepts

use standard notations

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ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-2 : Digital Electronics + Microprocessors + Systems and Signal Processing

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	32
Q.2	
Q.3	36
Q.4	
Section-B	
Q.5	53
Q.6	38
Q.7	
Q.8	31
Total Marks Obtained	190

Signature of Evaluator: Sourabh kumar

Cross Checked by: ....

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IMPORTANT INSTRUCTIONS

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1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Do not write in this margin

Section A : Digital Electronics + Microprocessors

Q.1 (a) Perform the BCD operation on the following number system:

(i) (010001101001.0110) + (100000010111.1000)

(ii) (011001010100.0101) – (100101111001.0110)

(iii) (011001100110.0110) – (010010011001.1001)

[4 + 4 + 4 marks]

(i)

$$\begin{array}{r} 0100\ 0110\ 1001 \cdot 0110 \\ +\ 1000\ 0001\ 0111 \cdot 1000 \\ \hline \end{array} \quad \begin{array}{l} = (469\cdot375)_{10} \\ = (817\cdot500)_{10} \\ \hline (1286\cdot875)_{10} \end{array}$$

Hence answer = (0001 0010 1000 0110 . 1110)

(ii)

$$\begin{array}{r} 0110\ 0101\ 0100 \cdot 0101 \\ -\ 1001\ 0111\ 1001 \cdot 0110 \end{array} \quad \begin{array}{l} = 654\cdot3125 \\ = -979\cdot375 \\ \hline -325\cdot0625 \end{array}$$

= (0011 0010 0101 . 1010)

=

(iii)

$$\begin{array}{r} 0110\ 0110\ 0110 \cdot 0110 \\ -\ 0100\ 1001\ 1001 \cdot 1001 \end{array} \quad \begin{array}{l} = (666\cdot375)_{10} \\ = -(499\cdot5625)_{10} \end{array}$$

$= (166 \cdot 8125)_{10}$

$= (0001\ 0110\ 0110 \cdot 1101)_2$

2

4

1 (b) The given below decimal numbers are stored in 6 digit register in sign-magnitude form. Convert them to signed 10's complement form and perform the following operations. Also convert the result into equivalent hexadecimal number.

(i) (+9286) + (+801) (ii) (+9286) + (-801)

(iii) (-9286) + (+801) (iv) (-9286) + (-801)

[12 marks]

(i) ~~9286~~ $\xrightarrow{10's}$ ~~01124~~

~~801~~ $\xrightarrow{10's}$ ~~00209~~

~~1333~~ $\xrightarrow{10's}$ ~~77~~

(ii) ~~9286~~ $\xrightarrow{}$ ~~0713~~ $\rightarrow$

9286 $\xrightarrow{10's}$ 0714

801 $\xrightarrow{10's}$ 0199

0913 $\xrightarrow{10's}$ (10087)

(iii) 9286 $\xrightarrow{10's}$ 0714

− 801 $\xrightarrow{10's}$ 0199

(0515) $\xrightarrow{10's}$ ~~485~~

3

1 (c) Determine the minimized expression for the given below Boolean function in SOP form using K-map:

$F(A, B, C, D) = \Pi(0, 1, 4, 5, 6, 7, 9, 13)$.

Implement the minimized function using NAND gate only.

[12 marks]

Given, $F(A,B,C,D) = \pi(0,1,4,5,6,7,9,13)$

$= \Sigma m(2,3,8,10,11,12,14,15)$

4-variable K-map

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	1	0	1	1
10	1	0	1	1

→ $\bar{B}C$

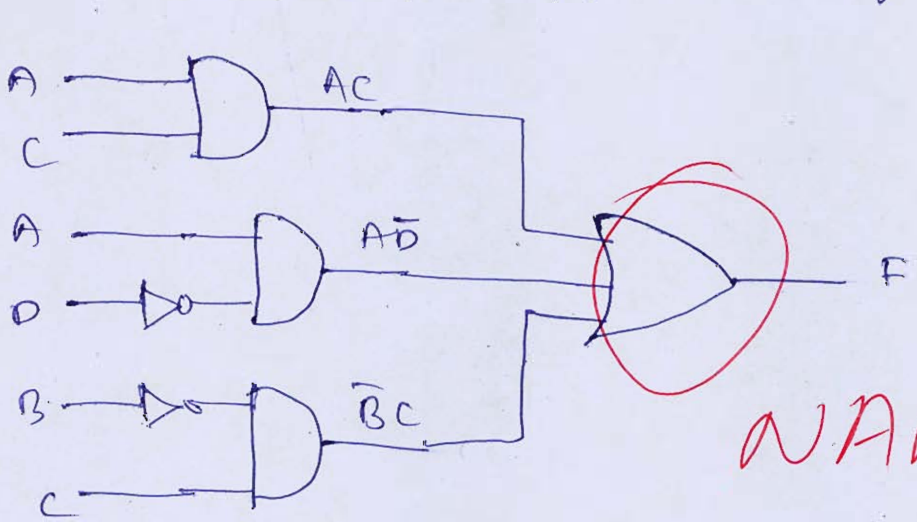
→ AC

$A\bar{D}$

3- sub group ~~are~~ with 4 minterms are formed

$\therefore$ $\boxed{F(A,B,C,D) = AC + A\bar{D} + \bar{B}C}$

The above expression can be formed ~~by~~ from the basic AND, NOT, and OR gates ~~as~~ -

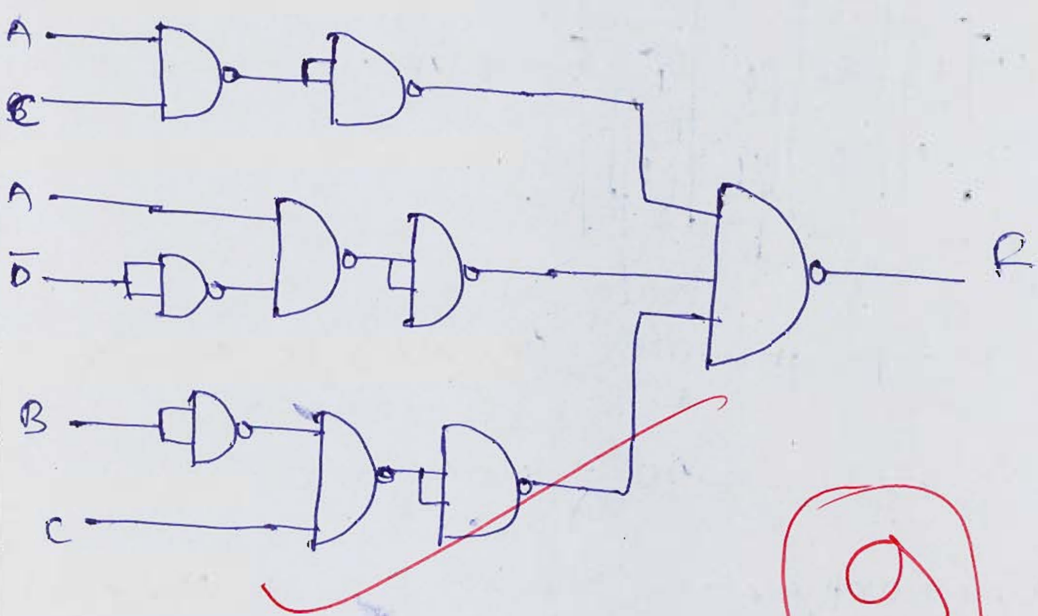


NAND gate

∴ AND gate is replace with 2 NAND,
NOT gate is replace with 1 NAND

∴ OR gate is replace using AND-OR equivalence - invert the i/p, OR → AND and invert the o/p.

∴ Therefore, the expression can be represented using NAND gates as given below —



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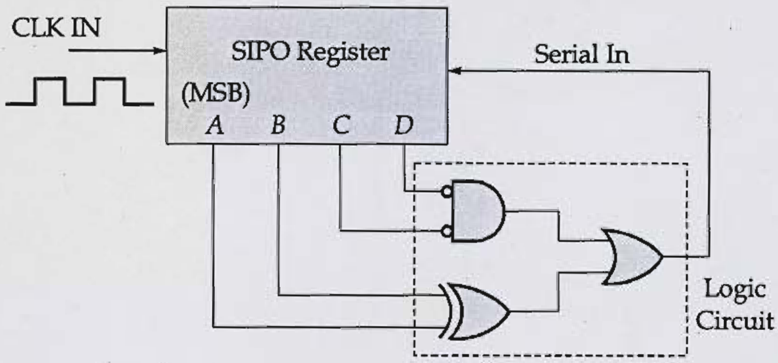
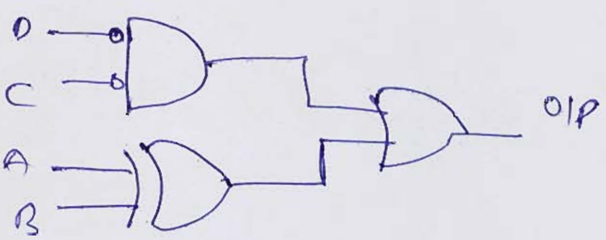
1 (d) A shift register with associated combinational logic circuit is shown in figure below:



Explain its operation by drawing waveforms for the output of shift register ($ABCD$). Assume that initially shift register is in state $ABCD = 1111$.

[12 marks]



$Y = O/p = A \oplus B + \bar{C}\bar{D}$

A	B	C	D	O/p = Y
1	1	1	1	0
0	1	1	1	1
1	0	1	1	0
0	0	0	1	0
0	0	0	0	1
1	0	0	0	1
1	1	0	0	1
1	1	1	0	0
1	1	1	1	1

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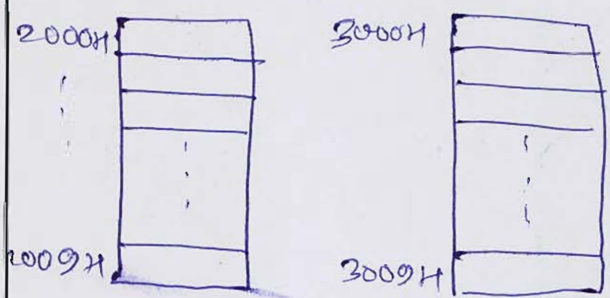
A	B	C	D	Y
1	1	1	1	0
0	1	1	1	1
1	0	1	1	1
1	1	0	1	0
0	1	1	0	1
1	0	1	1	1
1	1	0	1	0
0	1	1	0	1
1	0	1	1	1
1	1	0	1	0

Now it is repeating

1011 → 1101 — 0110

Q.1 (e) For an 8085 microprocessor write an assembly language program to move a block of 10 bytes of data from one section of memory to another section of memory. Write comments for selected instructions.

[12 marks]



Let a data is ~~from~~ move from 2000H to 2009H (10 byte block) data to 3000H to 3009H

```
       MVI  L, 09H    ; count the block of byte
       LXI  B, 2000H  ; starting address of 1st block
       LXI  D, 3000H  ; starting address of 2nd block
BACK:  LDAX B         ; move data from 2000H to Accumulator
       STAX D         ; move data from Accumulator to 3000H
       INX  B         ; increment 2000H to 2001H
       INX  D         ; increment 3000H to 3001H
       DCR  L         ; decrement the count
       JNZ  BACK
       HLT            ; end of program.
```

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.2 (a) (i) Write truth table and logic symbol of J-K flip-flop and also draw J-K flip-flop using NAND and NOR latch.

[10 marks]

.2 (a) **(ii)** Convert SR flip-flop into JK flip-flop and explain race around condition in J-K flip-flop.

[10 marks]

Do not write in this margin

.2 (b) Design a 3 × 8 decoder with active low enable pin and also draw its logic circuit.

[20 marks]

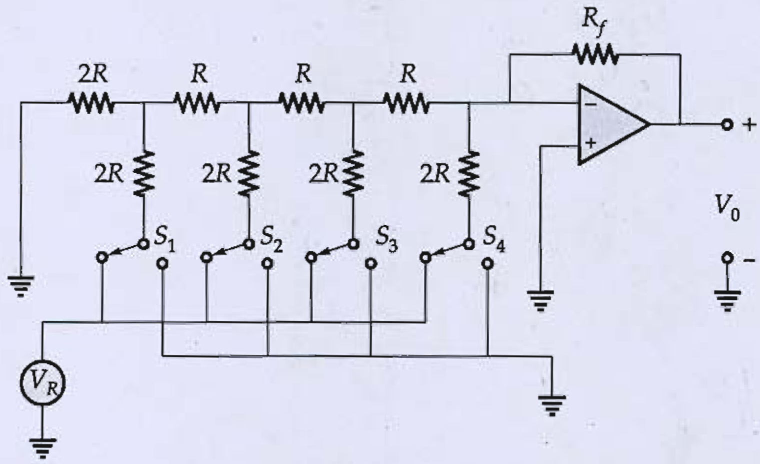
Q.2 (c) Explain the following instructions of 8085 microprocessor:

(i) RLC (ii) RAR (iii) CALL 16-bit address

(iv) RET (v) CMP

[20 marks]

.3 (a) (i) Consider a 4-bit R-2R DAC shown in figure. Assume the feedback resistance R_f of the op-amp is variable, the resistance $R = 10\ \text{k}\Omega$ and $V_R = 10$ V.



Determine the values of R_f that should be connected to achieve the following output conditions:

1. The value of 1 LSB at the output is 0.5 V.
2. An analog output of 6 V for a binary input of 1000.
3. The full scale output voltage of 12 V.
4. The actual maximum output voltage of 10 V.

(ii) Explain the working of flash type *ADC* with the help of truth table.

[10 + 10 marks]

(i) Given $R = 10\,\text{k}\Omega$ $V_R = 10\,\text{V}$

~~Case of~~ for 4-bit R-2R ladder output voltage

(1) $$V_0 = \frac{V_R}{2^4} \cdot \frac{R_f}{R}\left[2^3 b_3 + 2^2 b_2 + 2^1 b_1 + 2^0 b_0\right]$$

for a digital input = $b_3 b_2 b_1 b_0$

∴ for 1 LSB = 0001

Output voltage

$$V_0 = \frac{V_R \cdot R_f}{2^4 \cdot R}\left[2^3 \times 0 + 2^2 \times 0 + 2 \times 0 + 2^0 \times 1\right]$$

$$\Rightarrow 0.5 = \frac{10 \times R_f}{16 \times 10} \times 1$$

$$\Rightarrow \boxed{R_f = 5\,\text{k}\Omega}$$

(2) For 1000

$$V_0 = \frac{V_R \times R_f}{2^4 \times R} \left[2^3 \times 1 + 2^2 \times 0 + 2^1 \times 0 + 2^0 \times 1 \right]$$

$$\Rightarrow 6 = \frac{10 \times R_f}{16 \times 10} \times 8$$

$$\Rightarrow \boxed{R_f = 12\ k\Omega}$$

(3) Full scale o/p voltage = 1111

$$V_0 = \frac{V_R \times R_f}{2^4 \times R} \{ 2^3 \times 1 + 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 1 \} \rightarrow (1)$$

$$\Rightarrow 12 = \frac{10 \times R_f}{16 \times 10} \times 15$$

$$\boxed{R_f = 12.8\ k\Omega}$$

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(4) $(V_0)_{max} = 10\ V$

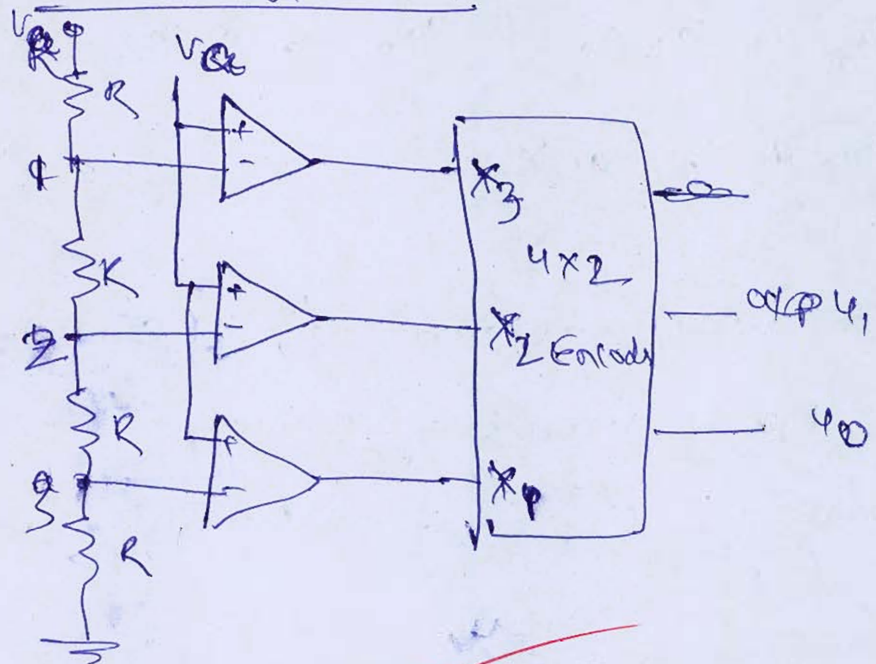
Good Approach

$$\Rightarrow 10 = \frac{10 \times R_f}{16 \times 10} \times 15 \quad [\text{from eq} - (1)]$$

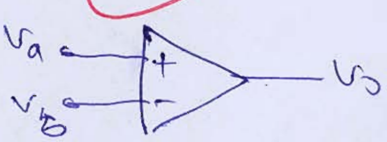
$$\Rightarrow \boxed{R_f = 10.67\ k\Omega}$$

Do not write in this margin

(ii) Flash type ADC



$V_1 = \frac{V}{4}$ $V_2 = \frac{V}{2}$ $V_3 < \frac{3V}{4}$



$V_a > V_b \Rightarrow V_0 = +ve$

$V_a < V_b \Rightarrow V_0 = +ve$

V_a	x_3	x_2	x_1	x_0	y_1	y_0
$0 < V_a < \frac{V_R}{4}$	0	0	0	0	0	0
$\frac{V_R}{4} < V_a < \frac{V_R}{2}$	0	0	0	1	0	1
$\frac{V_R}{2} < V_a < \frac{3V_R}{4}$	0	0	1	1	1	0
$\frac{3V_R}{4} < V_a < V_R$	0	1	1	1	1	1

$y_0 = \bar{x}_3 \bar{x}_2 \bar{x}_1 x_0 + x_3 x_2 x_1 x_0$

$= x_0 (\bar{x}_3 \bar{x}_2 \bar{x}_1 + x_3 x_2 x_1)$

$$Y_1 = \bar{x}_3 \bar{x}_2 x_1 x_0 + x_3 x_2 x_1 x_0$$

$$Y_1 = (x_3 \oplus x_2)\, x_1 x_0$$

Flash type AD require $2^n - 1$ comparator and 2^n equal value of resister

It is fastest type of A/D converter

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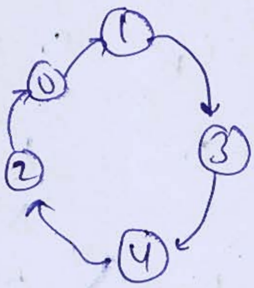
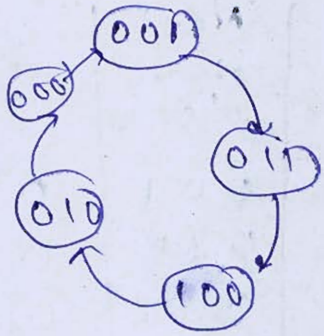
3 (b) Design a sequential circuit using J-K flip flop followed by T-FF followed by S-R flip flop to count a sequence $1 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 0 \rightarrow 1$ (Use J-K FF for MSB).

[20 marks]

State diagram





for 3 bits No. of FFs required is 3

~~[illegible]~~

J-K — MSB bit FF — 2

K — Middle bit FF — 1

SR — LSB bit FF — 0

excitation table of JK FF is —

Q_n	Q_{n+1}	J K
0	0	0 X
0	1	1 X
1	0	X 1
1	1	X 0

excitation table of T FF is —

Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

excitation table of SR FF is—

Q_n	Q_{n+1}	S R
0	0	0 X
0	1	1 0
1	0	0 1
1	1	X 0

using excitation table of JK, T and SR FF — ~~clear~~ —

Present state $Q_2\ Q_1\ Q_0$	Next state $Q_2'\ Q_1'\ Q_0'$	Flip flop input J_2	K_2	T_1	S_0	R_0
0 0 0	0 0 1	0	X	0	1	0
0 0 1	0 1 1	0	X	1	X	0
0 1 0	0 0 0	0	X	1	0	X
0 1 1	1 0 0	1	X	1	0	1
1 0 0	0 1 0	X	1	1	0	X
1 0 1	X X X	X	X	X	X	X
1 1 0	X X X	X	X	X	X	X
1 1 1	X X X	X	X	X	X	X

For state 5, 6, 7 we consider invalid state (don't care)

solving the reduced ~~express~~ expression of FF inputs using K-map —

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0			1	
1	X	X	X	X

$J_2 = Q_1 Q_0$

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	X	X	X	X
1	1	X	X	X

$K_2 = 1$

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0		1	1	1
1	1	X	X	X

$T_1 = Q_0 + Q_1 + Q_2 \bar{Q}_1$

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	1	X	0	0
1	0	X	X	X

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	0	0	1	X
1	X	X	X	X

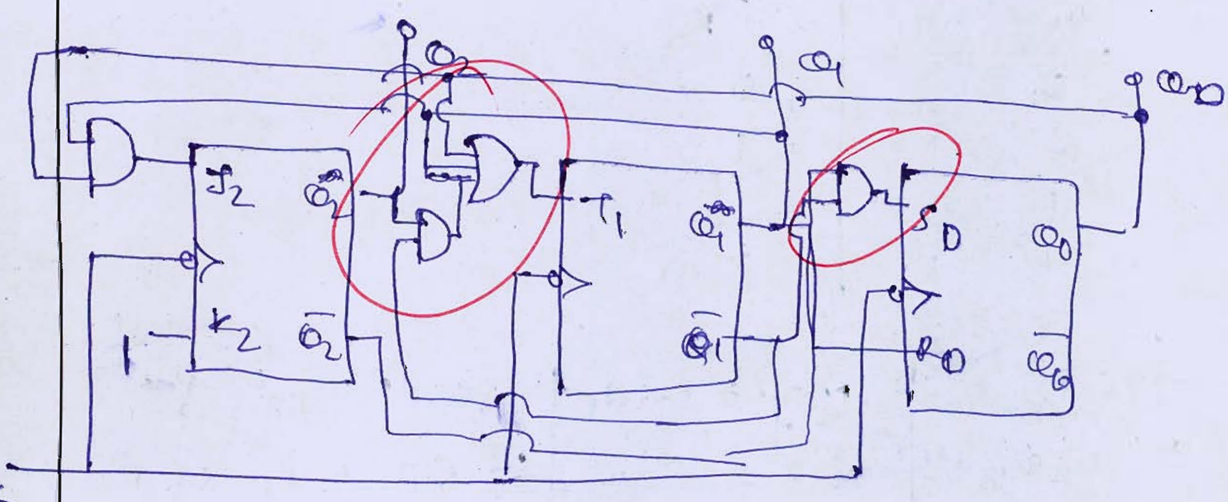
$S_0 = \overline{Q_2}\,\overline{Q_1}$ $\qquad$ $R_0 = Q_1$

Hence the flip flops input expression -

$J_2 = Q_1 Q_0 \qquad K_2 = 1$

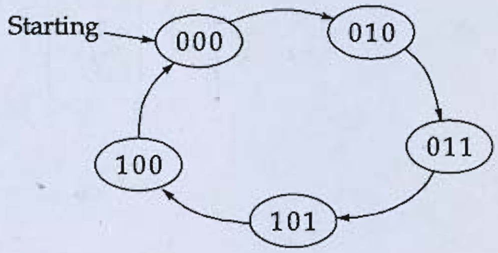
$T_1 = Q_0 + Q_1 + Q_2 \overline{Q_1}$

$S_0 = \overline{Q_2}\,\overline{Q_1} \qquad R_0 = Q_1.$



6

Q.3 (c) (i) The state transition diagram of a synchronous counter is given in figure. Design the counter circuit using S-R flip flops.



(ii) Draw EX-OR gate using transmission gates.

[14 + 6 marks]

(i) No. of FFs required for N = 5 states
= 3

excitation Table of SR FF is –

Q_n	Q_{n+1}	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

Present state $Q_2\ Q_1\ Q_0$	next state $Q_2'\ Q_1'\ Q_0'$	$S_2\ R_2$	$S_1\ R_1$	$S_0\ R_0$
0 0 0	0 1 0	0 X	1 0	0 X
0 1 0	0 1 1	0 X	X 0	1 0
0 1 1	1 0 1	1 0	0 1	X 0
1 0 1	1 0 0	X 0	0 X	0 1
1 0 0	0 0 0	0 1	0 X	0 X

(flip flop input)

For 001, 110 and 111 state we consider invalid state (don't care)

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	0	X	1	0
1	0	X	X	X

$\boxed{S_2 = Q_0}$

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	X	X	0	X
1	1	0	X	X

$\boxed{R_2 = \overline{Q_0}}$

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	1	X	0	X
1	0	0	X	X

$S_1 = \overline{Q_2}\,\overline{Q_1}$

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	0	X	1	0
1	X	X	X	X

$R_1 = Q_0$

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	0	X	X	1
1	0	0	X	X

$S_0 = Q_1$

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	X	X	0	0
1	X	1	X	X

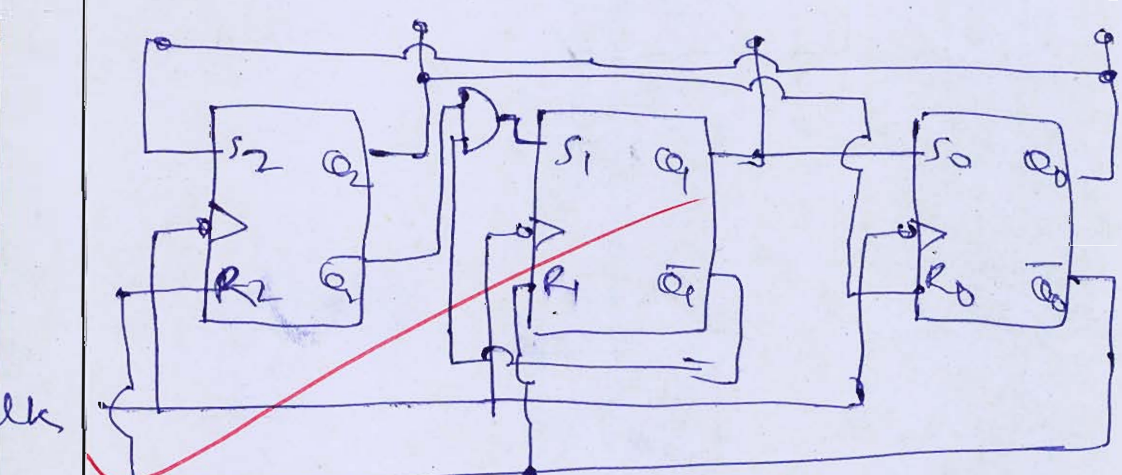
$R_0 = Q_2$

Hence, the reduced expression —

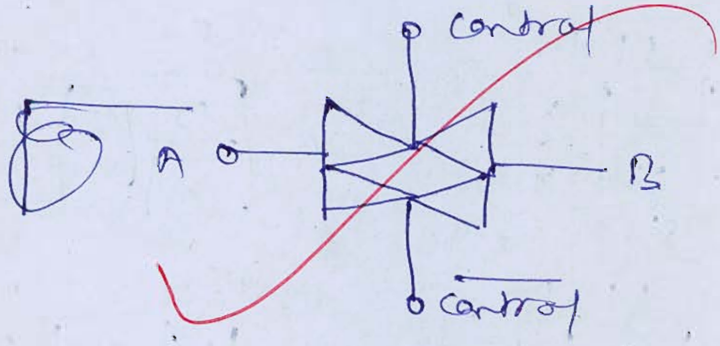
$S_2 = Q_0 \qquad R_2 = \overline{Q_0}$

$S_1 = \overline{Q_2}\,\overline{Q_1} \qquad R_1 = Q_0$

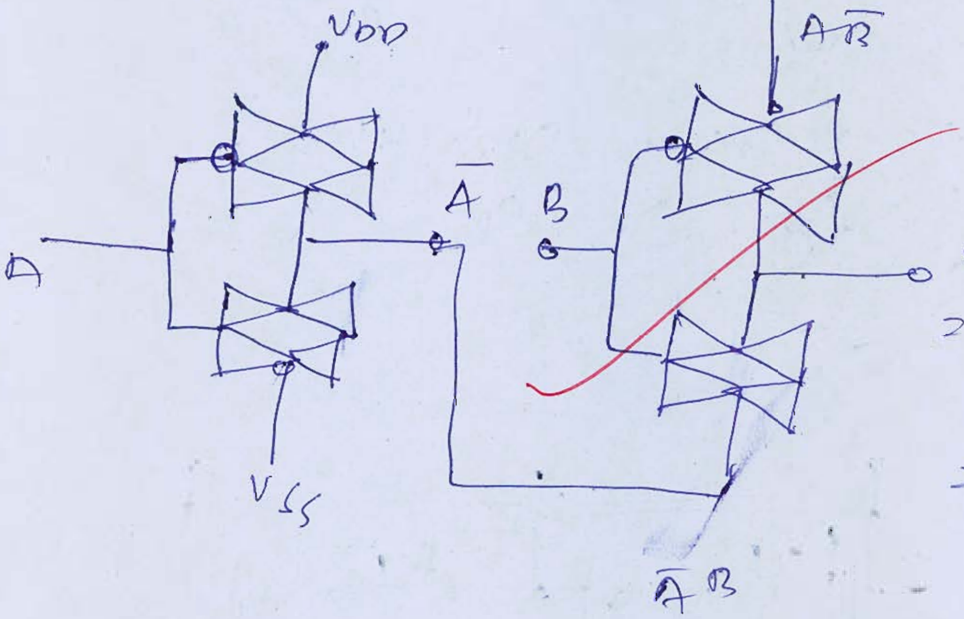
$S_0 = Q_1 \qquad R_0 = Q_2$



(ii) Transmission gate - it a bidirectional electronic switch that control the flows of signals between an input and output



EXOR gate using transmission gate

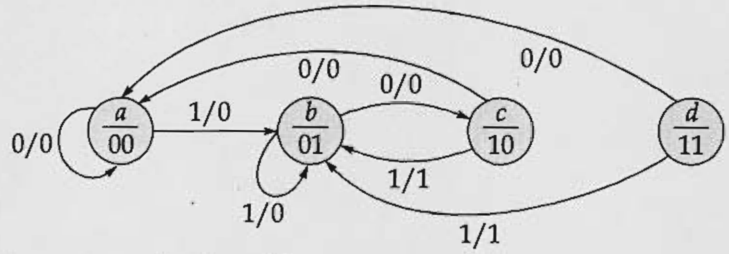


$= A\bar{B} + \bar{A}B$

$= A \oplus B$

16

4 (a) Consider the state diagram shown below:

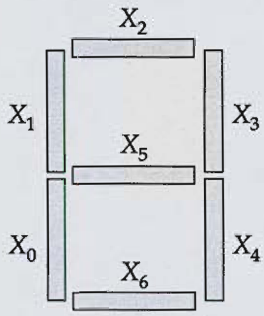


(i) Obtain the state reduction diagram.

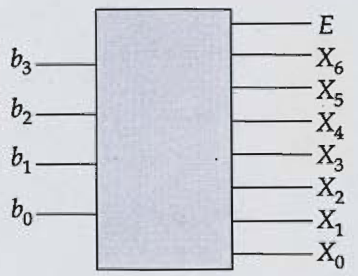
(ii) Design a clocked sequential machine using JK FFs for the given sequential circuit.

[8 + 12 marks]

Q.4 (b) A seven segment display consists of seven light-emitting diodes (LEDs) as shown below:



7-Segment display



where, $E = 0$ for valid digit with the display showing the corresponding digit.

$E = 1$ for invalid digit with the display showing the symbol 'E'.

Implement the above given seven segment display using a ROM.

[20 marks]

.4 (c) For an 8085 microprocessor, assume a sensor connected at port 30H. Read 10 bytes of data from port and if the value is +ve then store the corresponding bytes from memory location 4000H. If the data is –ve then switch 'ON' LED by sending FFH to port 50H. Write an Assembly Language Program for above condition.

[20 marks]

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Section B : Systems and Signal Processing

Q.5 (a) A stable system with zero initial conditions is described by the difference equation $y(n) = x(n) - 2x(n-1) + x(n+1)$.

(i) Find the impulse response $h(n)$ of the system.

(ii) Plot the output $y[n]$ for an input, $x[n] = u[n-2]$

(iii) Find the value of $\sum_{n=-\infty}^{\infty} y[n]$ for input $x[n] = u[n-2]$.

[4 + 4 + 4 marks]

$y(n) = x(n) - 2x(n-1) + x(n+1)$ — ①

(i) $\therefore$ $y(n) \xleftrightarrow{ZT} Y(z)$

$x(n) \longleftrightarrow X(z)$

$x(n-1) \longleftrightarrow z^{-1}X(z) + x(-1)$

$x(n-1) \longleftrightarrow z^{-1}X(z)$ $[x(-1) = 0]$

$x(n+1) \longleftrightarrow zX(z) - x(0)z$ $\{x(0) = 0\}$

$x(n+1) \longleftrightarrow zX(z)$

$\therefore$ Taking z-transform of given difference equation we get →

$Y(z) = X(z) - 2z^{-1}X(z) + zX(z)$

$= X(z)[1 - 2z^{-1} + z]$

$\frac{Y(z)}{X(z)} = H(z) = 1 - 2z^{-1} + z$ — ②

Now, taking inverse z-transform of above equation we get the impulse response h(n) of the system —

$\boxed{h(n) = \delta(n) - 2\delta(n-1) + \delta(n+1)}$

(ii) $x(n) = u(n-2)$

~~x(z)~~ $u(n) \leftrightarrow \frac{z}{z-1}$

$u(n-2) \leftrightarrow \frac{z^{-1}}{z-1}$

$\therefore X(z) = \frac{z^{-1}}{z-1} = \frac{1}{z(z-1)}$

$$\frac{Y(z)}{X(z)} = H(z) = 1 - 2z^{-1} + z = 1 - \frac{2}{z} + z$$

$$Y(z) = \left[\frac{z - 2 + z^2}{z}\right] \times \frac{1}{z(z-1)}$$

$$Y(z) = \frac{z^2 + z - 2}{z^2(z-1)} = \cancel{\frac{A}{z} + \frac{B}{z^2} + \frac{C}{z-1}}$$

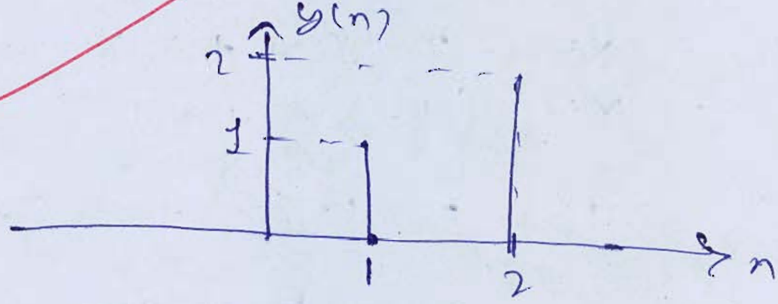
$$\cancel{B = \left.\frac{z^2+z-2}{(z-1)}\right|_{z=0} = \frac{-2}{-1} = 2}$$

$$\cancel{C = \left.\frac{z^2+z-2}{z^2}\right|_{z=1} = \frac{1+1-2}{1} =}$$

$$Y(z) = \frac{(z-1)(z+2)}{z^2(z-1)} = \frac{z+2}{z^2}$$

$$Y(z) = \frac{1}{z} + \frac{2}{z^2} = z^{-1} + 2z^{-2}$$

$$\therefore \boxed{y(n) = \delta(n-1) + 2\delta(n-2)}$$



(iii)

$$\sum_{n=-\infty}^{\infty} y(n) = \sum_{n=-\infty}^{\infty} [\delta(n-1) + 2\delta(n-2)]$$

$$= 1 + 2$$

$$\boxed{\sum_{n=-\infty}^{\infty} y(n) = 3}$$

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Good Approach

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Q.5 (b) Determine the inverse z-transform of the following $X(z)$ by the partial fraction expansion method,

$$X(z) = \frac{z+2}{2z^2-7z+3}$$

if the ROCs are (a) $|z| > 3$, (b) $|z| < 1/2$ and (c) $1/2 < |z| < 3$.

[12 marks]

$$X(z) = \frac{z+2}{2z^2-7z+3}$$

$$= \frac{z+2}{(2z-1)(z-3)}$$

$$= \frac{A}{2z-1} + \frac{B}{z-3}$$

$$A = \left.\frac{z+2}{z-3}\right|_{z=1/2} = \frac{1/2+2}{1/2-3} = -1$$

$$B = \left.\frac{z+2}{2z-1}\right|_{z=3} = \frac{5}{5} = 1$$

$$X(z) = \frac{-1}{2z-1} + \frac{1}{z-3}$$

$$= z^{-1}\left[\frac{-z}{2z-1} + \frac{z}{z-3}\right]$$

$$= z^{-1}\left[\frac{-1}{2(1-\frac{1}{2}z^{-1})} + \frac{1}{1-3z^{-1}}\right]$$

(a) $\therefore$ $|z| > 3$ — Right sided signal

taking inverse z-transform we get —

$$x(n) = -\frac{1}{2}\left(\frac{1}{2}\right)^{n-1} u(n-1) + (3)^{n-1} u(n-1)$$

$$\frac{1}{1-\frac{1}{2}z^{-1}} \leftrightarrow \left(\frac{1}{2}\right)^n u(n) \quad ; \quad |z| > \frac{1}{2}$$

$$\frac{1}{1-3z^{-1}} \leftrightarrow (3)^n u(n) \quad ; \quad |z| > 3$$

(b) $|z| < \frac{1}{2}$ - left sided signal

$$\frac{1}{1 - \frac{1}{2}z^{-1}} \leftrightarrow -\left(\frac{1}{2}\right)^n u(-n-1) \; ; \; |z| < \frac{1}{2}$$

$$\frac{1}{1-3z^{-1}} \leftrightarrow -(3)^n u(-n-1) \; ; \; |z| < 3$$

due to z^{-1} it is shifted by 1 in ~~left~~ right side

$$\therefore x(n) = \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} u(-n) - (3)^{n-1} u(-n)$$

(c) $\frac{1}{2} < |z| < 3$

$$\frac{1}{1-\frac{1}{2}z^{-1}} \leftrightarrow \left(\frac{1}{2}\right)^n u(n) \; ; \; |z| > \frac{1}{2}$$

$$\frac{1}{1-3z^{-1}} \leftrightarrow -(3)^n u(-n-1) \; ; \; |z| < 3$$

$$\therefore x(n) = -\frac{1}{2}\left(\frac{1}{2}\right)^{n-1} u(n-1) - (3)^{n-1} u(-n)$$

10

Q.5 (c) Determine the z-transform and the ROC of the signal, $x(n) = a^n u(n) - a^n u(n-1)$.

[12 marks]

$$x(n) = \underbrace{a^n u(n)}_{x_1} - \underbrace{a^n u(n-1)}_{x_2}$$

$$x_1(n) = a^n u(n)$$

$$X_1(z) = \sum_{n=-\infty}^{\infty} x_1(n)\, z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

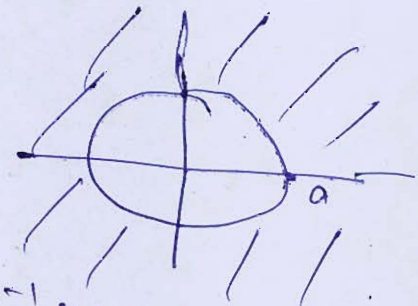
$$= \sum_{n=0}^{\infty} (a z^{-1})^n$$

$$= \frac{1}{1 - a z^{-1}} \quad ; \quad |a z^{-1}| < 1$$

$$|z| > |a|$$

$$\therefore \quad a^n u(n) \longleftrightarrow \frac{1}{1 - a z^{-1}}$$

ROC : $|z| > |a|$



$$x_2(n) = a^n u(n-1) = a \cdot a^{n-1} u(n-1)$$

$$X_2(z) = \sum_{n=-\infty}^{\infty} x_2(n)\, z^{-n}$$

$$= a \sum_{n=1}^{\infty} a^{n-1} z^{-n}$$

$$= \sum_{n=1}^{\infty} (a z^{-1})^n$$

$$= \frac{a z^{-1}}{1 - a z^{-1}} \quad ; \quad |a z^{-1}| < 1$$

$$|z| > |a|$$

$$a^n u(n-1) \longleftrightarrow \frac{a z^{-1}}{1 - a z^{-1}}$$

ROC : $|z| > |a|$

$\therefore X(z) = X_1(z) - X_2(z)$

$= \frac{1}{1-az^{-1}} - \frac{az^{-1}}{1-az^{-1}}$

$\boxed{X(z) = 1}$

ROC : $|z| > |a|$

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Good Approach

Q.5 (d) An LTI system has a unit step response given by $s(t) = (1 - e^{-t} - te^{-t})\,u(t)$. For a certain input $x(t)$, the output is observed to be equal to $y(t) = (2 - 3e^{-t} + e^{-3t})\,u(t)$. What is $x(t)$?

[12 marks]

unit step response

$$s(t) = (1 - e^{-t} - t\,e^{-t})\,u(t)$$

unit impulse response

$$h(t) = \frac{d\,s(t)}{dt} = \frac{d}{dt}\left[1 - e^{-t} - t\,e^{-t}\right)$$

$$= 0 + e^{-t} - \left[-t\,e^{-t} + e^{-t}\right]$$

$$= e^{-t} + t\,e^{-t} - e^{-t}$$

$$h(t) = t\,e^{-t}$$

Taking

∴ Laplace transform we get—

$$H(s) = \frac{1}{(s+1)^2}$$

Output

$$y(t) = (2 - 3e^{-t} + e^{-3t})\,u(t)$$

Laplace transform

$$e^{-t}u(t) \longleftrightarrow \frac{1}{s+1} \qquad u(t) \longleftrightarrow \frac{1}{s}$$

$$e^{-3t}u(t) \longleftrightarrow \frac{1}{s+3}$$

$$\therefore Y(s) = \frac{2}{s} - \frac{3}{s+1} + \frac{1}{s+3}$$

$$= \frac{2(s+1)(s+3) - 3s(s+3) + s(s+1)}{s(s+1)(s+3)}$$

$$= \frac{2s^2 + 8s + 6 - 3s^2 - 9s + s^2 + s}{s(s+1)(s+3)}$$

$$Y(s) = \frac{6}{s(s+1)(s+3)}$$

$$\therefore \quad \frac{Y(s)}{X(s)} = H(s)$$

$$\therefore \quad X(s) = \frac{Y(s)}{H(s)}$$

$$= \frac{6}{s(s+1)(s+3)} \times (s+1)^2$$

$$X(s) = \frac{6(s+1)}{s(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+3}$$

$$A = \left.\frac{6(s+1)}{s+3}\right|_{s=0} = \frac{6}{3} = 2$$

$$B = \left.\frac{6(s+1)}{s}\right|_{s=-3} = \frac{6\times(-2)}{-3} = 4$$

$$X(s) = \frac{2}{s} + \frac{4}{s+3}$$

Taking inverse laplace transform we get

$$x(t) = 2u(t) + 4e^{-3t}u(t)$$

$$= [2 + 4e^{-3t}]\,u(t)$$

Hence, input

$$\boxed{x(t) = (2 + 4e^{-3t})\,u(t)}$$

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Good Approach

Q.5 (e) Consider the signal $y(t) = e^{-2t}u(t)$ is the output of a causal all-pass system for which the system function is

$$H(s) = \frac{s-1}{s+1}$$

(i) Find and sketch at least two possible inputs $x(t)$ that could produce $y(t)$.

(ii) From the solutions obtained in part (i), what is the input $x(t)$ if it known that a stable system exists that will have $x(t)$ as an output and $y(t)$ as the input? Find the impulse response $h(t)$ for this system.

[12 marks]

$$H(s) = \frac{s-1}{s+1}$$

(i) $y(t) = e^{-2t}\,u(t)$

taking laplace transform -

$$Y(s) = \frac{1}{(s+2)}$$

$$\frac{Y(s)}{X(s)} = H(s) = \left(\frac{s-1}{s+1}\right)$$

$$X(s) = \frac{Y(s)}{H(s)} = \frac{1}{(s+2)} \times \left(\frac{s+1}{s-1}\right)$$

$$= \frac{s+1}{(s+2)(s-1)}$$

$$= \frac{A}{s+2} + \frac{B}{s-1}$$

$$A = \left.\frac{s+1}{s-1}\right|_{s=-2} = \frac{-2+1}{-2-1} = 1/3$$

$$B = \left.\frac{s+1}{s+2}\right|_{s=1} = \frac{1+1}{1+2} = 2/3$$

$$X(s) = \frac{1}{3(s+2)} + \frac{2}{3(s-1)}$$

if ROC $\sigma > -2$

$$x(t) = \frac{1}{3}e^{-2t}u(t) + \frac{2}{3}e^{t}u(t)$$

if ROC $\sigma < 1$

$x(t) = -\frac{1}{3} e^{-2t} u(-t) - \frac{2}{3} e^{+t} u(-t)$

ROC $-2 < \sigma < 1$

$x(t) = \frac{1}{3} e^{-2t} u(t) - \frac{2}{3} e^{+t} u(-t)$

(ii) $x(t)$ is stable if ROC $-2 < \sigma < 1$

$x(t) = \frac{1}{3} e^{-2t} u(t) - \frac{2}{3} e^{+t} u(-t)$

$X(s) = \frac{s+1}{(s+2)(s-1)}$

$y(t) = e^{-2t} u(t)$

$Y(s) = \frac{1}{s+2}$

$H(s) = \frac{X(s)}{Y(s)} = \frac{s+1}{(s+2)(s-1)} \times (s+2)$

$= \frac{s+1}{s-1}$

$= \frac{s-1+2}{s-1}$

$= 1 + \frac{2}{s-1}$

$-2 < \sigma < 1$

$h(t) = \delta(t) - 2e^{t} u(-t)$

impulse response of the system

$\boxed{h(t) = \delta(t) - 2e^{t} u(-t)}$

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Good approach

Q.6 (a) (i) Compute: $y(t) = x(t) * h(t)$ where, $x(t) = \text{sinc}\,\alpha t$, $h(t) = \text{sinc}\,\beta t$; $\alpha > \beta$

(ii) Compute : $y(t) = x(t) * h(t)$

$$x(t) = \text{rect}\left(\frac{t-6}{2}\right), h(t) = \text{rect}\left(\frac{t}{2} - 3\right)$$

[10 + 10 marks]

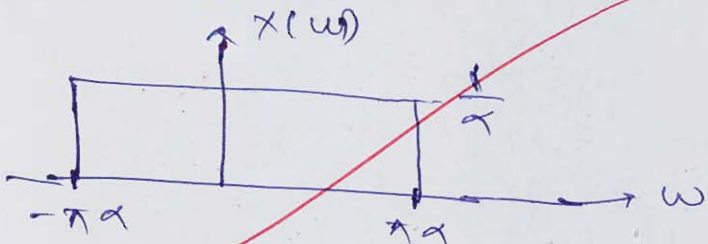
(i) $y(t) = x(t) * h(t)$

$x(t) = \text{sinc}(\alpha t)$

$= Sa(\pi \alpha t)$

$x(t) = \dfrac{\sin(\pi \alpha t)}{\pi \alpha}$

Fourier transform of $x(t)$ is –

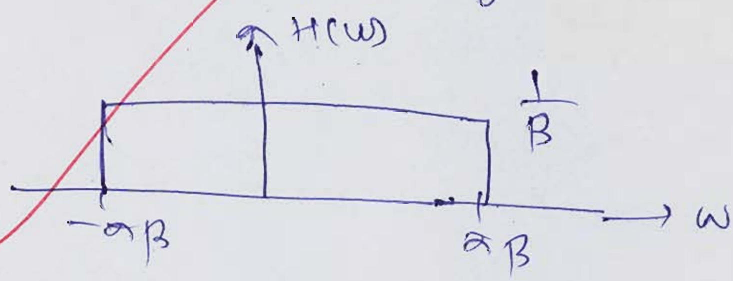


$h(t) = \text{sinc}(\beta t)$

$= Sa(\pi \beta t)$

$= \dfrac{\sin(\pi \beta t)}{\pi \beta t}$

Fourier transform of $h(t) \rightarrow$



$y(t) = x(t) * h(t)$

Taking Fourier transform –

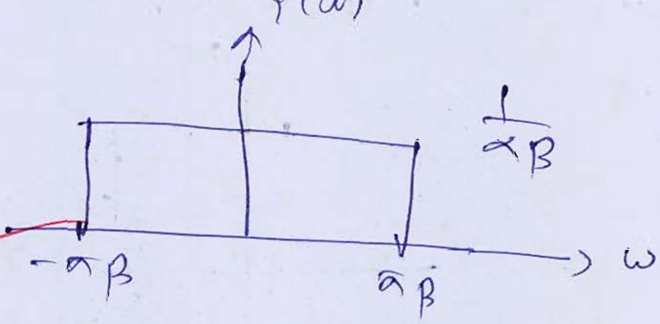
$Y(\omega) = X(\omega) \cdot H(\omega)$

$Sa(W\delta t)$

$\dfrac{A\sigma\pi}{\pi} = \dfrac{1 \times \pi}{\pi 2}$

since, $\alpha > \beta$

$Y(\omega) =$



Taking inverse fourier transform we get

$y(t) = \frac{2\pi}{\alpha} Sa(\alpha\beta t)$ $\quad \frac{1}{\alpha\beta} \times 2\pi\beta$

$$\boxed{y(t) = \frac{2\pi}{\alpha} \operatorname{sinc}(\beta t)}$$

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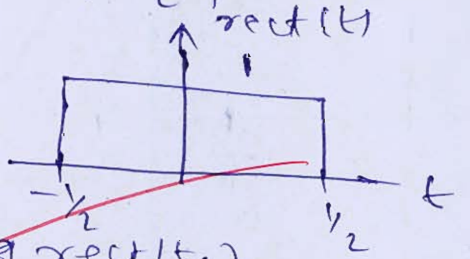
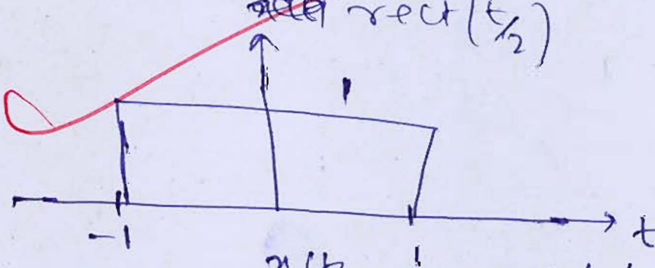
Good Approach

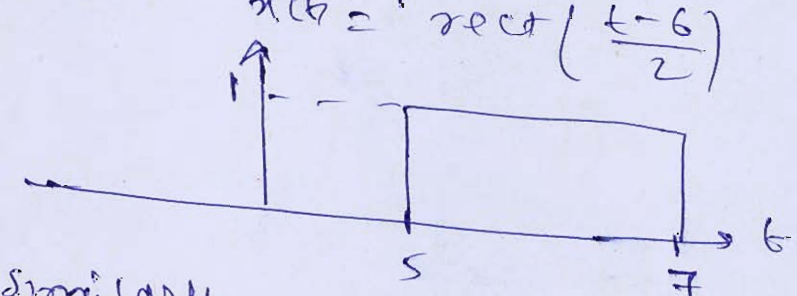
(ii) $x(t) = rect\left(\frac{t-6}{2}\right)$

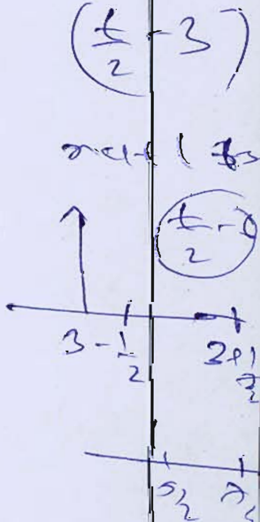
$rect(t) =$





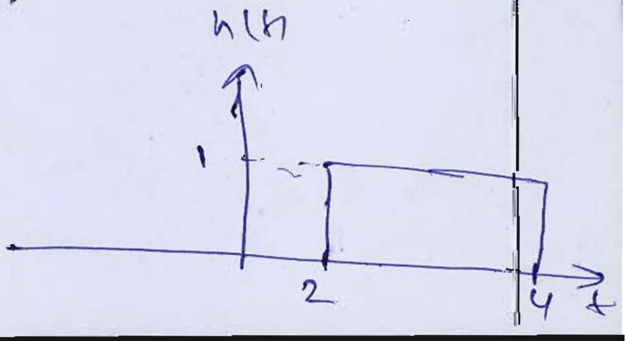
$x(t) = rect\left(\frac{t-6}{2}\right)$





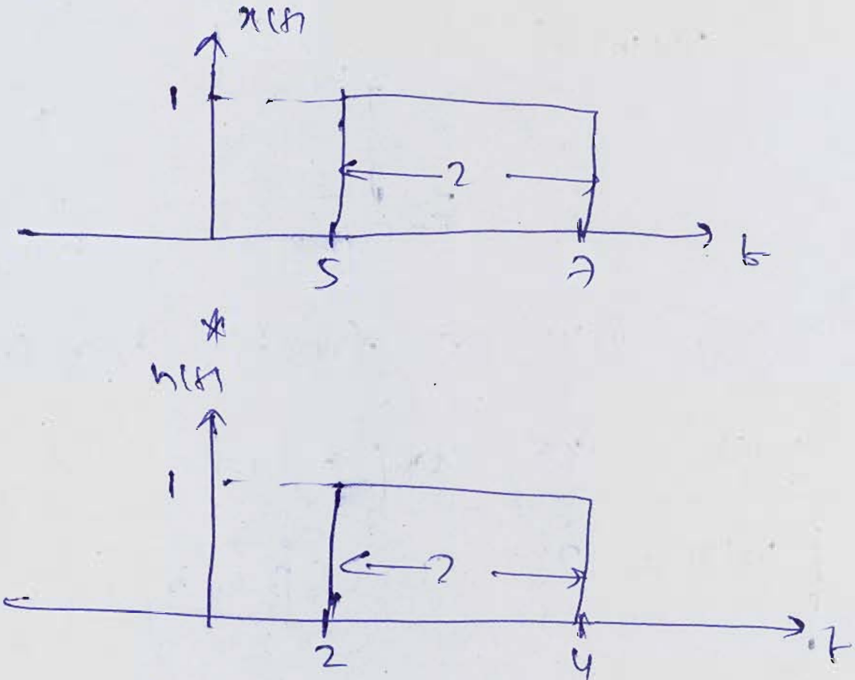
Similarly

$h(t) = rect\left(\frac{t}{2} - 3\right)$



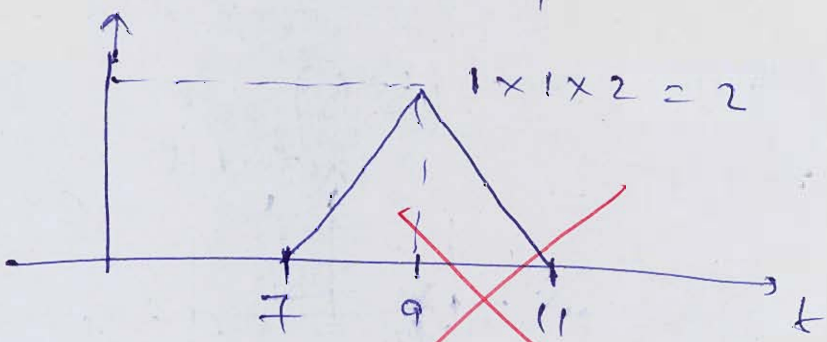
$y(t) = x(t) * h(t)$

$=$



3

$y(t) =$



$$y(t) = \begin{cases} t-7 & 7 \le t \le 9 \\ 11-t & 9 < t \le 11 \end{cases}$$

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.6 (b) Given below are the impulse response of FIR filter. Identify the type of filter implemented using this impulse response.

(i) $h[n] = 5\delta[n] - 7\delta[n-1] + 7\delta[n-3] - 5\delta[n-4]$

(ii) $h[n] = 5[\delta[n] - \delta[n-2]]$

[10 + 10 marks]

(i) $h(n) = 5\delta(n) - 7\delta(n-1) + 7(\delta - 3) - 5\delta(n-4)$

taking z-transform

$H(z) = 5 - 7z^{-1} + 7z^{-3} - 5z^{-4}$

$z = e^{jw}$

$H(e^{jw}) = 5 - 7e^{-jw} + 7e^{-3jw} - 5e^{-4jw}$

$w = 0$

$H(e^{j0}) = 5 - 7e^{0} + 7e^{0} - 5e^{0}$

$= 0$

$w = \pi$

$H(e^{j\pi}) = 5 - 7e^{-j\pi} + 7e^{-3\pi} - 5e^{-4\pi}$

$= 0$

$w = \pi/2$

$H(e^{j\pi/2}) = 5 - 7e^{-j\pi/2} + 7e^{-3\pi/2} - 5e^{-2\pi}$

$= 14j$

$\therefore$ Hence it is <u>Band pass filter</u> with cut off frequency

$\boxed{w_c = \pi \ r/s}$

(ii) $h(n) = 5\,[\delta(n) - \delta(n-2)]$

Taking z-transform -

$H(z) = 5\,(1 - z^{-2})$

$z = e^{j\omega}$

$H(e^{j\omega}) = 5\,(1 - e^{-2j\omega})$

$\omega = 0$

$H(e^{j0}) = 5\,(1 - e^{-2\times 0}) = 0$

$\omega = \pi$

$H(e^{j\pi}) = 5\,(1 - e^{-j2\times\pi}) = 0$

$\omega = \pi/2$

$H(e^{j\pi/2}) = 5\,(1 - e^{-j\pi}) = 10$

Hence it is band pass filter with cutoff frequency ~~$\omega_c = 2\pi$~~ r

$\omega_c = \pi$ r/s

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Q.6 (c) (i) Using impulse invariant method, convert the given analog filter transfer function

$H(s) = \dfrac{1}{s^2 + \sqrt{2}s + 1}$ to digital transfer function with a sampling period 1 sec.

(ii) Find the Fourier transform of Gaussian modulated signal $x(t) = e^{-at^2} \cos \omega_c t$.

[10 + 10 marks]

(i) $H(s) = \dfrac{1}{s^2 + \sqrt{2}s + 1}$

$s = \dfrac{T}{2}\left(\dfrac{z-1}{z+1}\right)$

T = sampling period

$T = 1$

$s = \dfrac{1}{2}\left(\dfrac{z-1}{z+1}\right)$

$H(z) = \dfrac{1}{\left[\frac{1}{2}\left(\frac{z-1}{z+1}\right)\right]^2 + \sqrt{2} \times \frac{1}{2}\left(\frac{z-1}{z+1}\right) + 1}$

$= \dfrac{1}{\frac{1}{4}\left(\frac{z-1}{z+1}\right)^2 + \frac{\sqrt{2}}{2}\left(\frac{z-1}{z+1}\right) + 1}$

5

$= \dfrac{4(z+1)^2}{(z-1)^2 + 2\sqrt{2}\,(z-1)(z+1) + 4(z+1)^2}$

$= \dfrac{4(z^2+2z+1)}{z^2 - 2z + 1 + 2\sqrt{2}(z^2-1) + 4(z^2+2z+1)}$

$H(z) = \dfrac{4(z^2+2z+1)}{7.828z^2 + 3.171z + 2.171}$

(1)

$$x(t) = e^{-at^2} \cos \omega_c t$$

fourier transform of signal $x(t)$

$$X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-at^2} \cos \omega_c t\, e^{-j\omega t}\, dt$$

$\cos \omega_c t =$ Real part of $e^{j\omega_c t}$

= Real part of

$$\left[\int_{-\infty}^{\infty} e^{-at^2} \cdot e^{j\omega_c t} \cdot e^{j\omega t}\, dt \right]$$

$$= \int_{-\infty}^{\infty} e^{-at^2 + j\omega_c t + j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-a\left(at^2 + j\frac{(\omega_c + \omega)}{a} t\right)}\, dt$$

3

$$= \int_{-\infty}^{\infty} e^{-a\left[t + j\left(\frac{\omega_c + \omega}{2a}\right)\right]^2} \cdot e^{\left(\frac{\omega_c + \omega}{2a}\right)^2}\, dt$$

$$= e^{\left(\frac{\omega_c + \omega}{2a}\right)^2} \int_{-\infty}^{\infty} e^{-a\left(t + j\left(\frac{\omega_c + \omega}{2a}\right)\right)^2}\, dt$$

$$t + j\left(\frac{\omega_c + \omega}{2a}\right) = u$$

$$dt = du$$

$$= e^{\frac{(\omega_c+\omega)^2}{4a^2}} \int_{-\infty}^{\infty} e^{-au^2}\, du$$

$$au^2 = v$$

$$u^2 = \frac{v}{a}$$

$$u = \frac{v^{1/2}}{a^{1/2}}$$

$$du = \frac{1}{2}\frac{v^{-1/2}\,dv}{a^{1/2}} = \frac{1}{2\sqrt{a}\sqrt{v}}\,dv$$

$$= e^{\frac{(\omega_c+\omega)^2}{4a^2}} \int_{-\infty}^{\infty} e^{v} \cdot \frac{1}{2\sqrt{a}} \cdot v^{-1/2}\,dv$$

$$= \frac{1}{2\sqrt{2}} e^{\frac{(\omega_c+\omega)^2}{4a^2}} \int_{-\infty}^{\infty} e^{v} v^{-1/2}\,dv$$

$$\int_0^{\infty} e^{v} v^{-1/2}\,dv = \Gamma_{1/2} = \sqrt{\pi}$$

$$\boxed{X(\omega) = \frac{\sqrt{a}}{2\sqrt{2}}\, e^{\frac{(\omega_c+\omega)^2}{4a^2}}}$$

Q.7 (a) (i) Find the magnitude and phase response for the system characterized by the difference equation

$$y(n)=\frac{1}{6}x(n)+\frac{1}{3}x(n-1)+\frac{1}{6}x(n-2)$$

[8 marks]

.7 (a) **(ii)** A filter is to be designed with the following desired frequency response using rectangular window:

$$H_d(e^{j\omega}) = \begin{cases} 0, & -\pi/4 \le \omega \le \pi/4 \\ e^{-j2\omega}, & \pi/4 \le |\omega| \le \pi \end{cases}$$

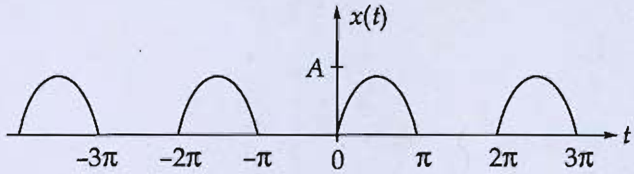
Determine the filter coefficients $h_d(n)$ if the window function is defined as

$$w(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Also, determine the frequency response $H(e^{j\omega})$ of the designed filter.

[12 marks]

.7 (b) Find the exponential Fourier series for the half wave rectified sine wave shown in figure.



[20 marks]

Q.7 (c) (i) The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation

$$\frac{dy(t)}{dt}+10y(t)=\int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau - x(t)$$

where, $z(t) = e^{-t}u(t) + \delta(t)$.

Determine the impulse response of the system.

[10 marks]

Q.7 (c) (ii) Convert the analog filter with system transfer function $H(s) = \dfrac{s+0.1}{(s+0.1)^2+9}$ into a digital IIR filter using bilinear transformation. The digital filter should have a resonant frequency of $\dfrac{\pi}{4}$.

[10 marks]

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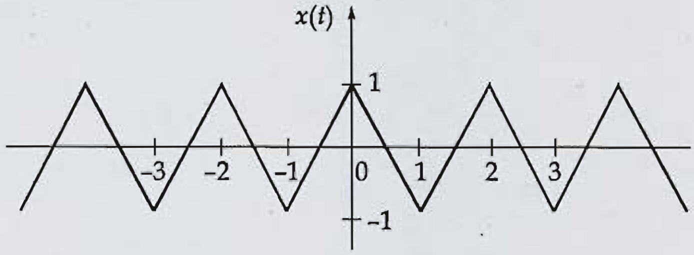
Q.8 (a) (i) Write down the exponential Fourier series representation of the signal $x(t)$.



(ii) Find the amount of power (in mW) contained in the 7th harmonic of the Fourier series representation of $x(t)$.

[10 + 10 marks]

(1) Exponential Fourier series of signal $x(t)$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j\omega n t}$$

where $C_n = \frac{1}{T}\int_0^T x(t) e^{-j\omega n t}\,dt$

where T = time period

$T = 2$

$$x(t) = \begin{cases} 2t+1 & -1 < t < 0 \\ -2t+1 & 0 < t < 1 \end{cases}$$

$$C_n = \frac{1}{2}\left[\int_{-1}^{0}(2t+1)e^{-j\omega n t}\,dt + \int_0^1 (1-2t)e^{j\omega n t}\,dt\right]$$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$$C_n = \frac{1}{2}\left[\int_{-1}^{0}(2t+1)e^{-j\pi n t} + \int_0^1 \left((1-2t)e^{j\pi n t}\right)dt\right]$$

$$= \frac{1}{2}\left[2\int_{-1}^{0} t e^{-jn\pi t} + \int_{-1}^{0} e^{-jn\pi t}\,dt + \int_{0}^{1} e^{jn\pi t} - 2\int_{0}^{1} t e^{jn\pi t}\,dt \right]$$

$$= \frac{1}{2}\left[2\left[t\frac{e^{-jn\pi t}}{-jn\pi} - \frac{e^{-jn\pi t}}{j^2n^2\pi^2} \right]_{-1}^{0} + \left(\frac{e^{-jn\pi t}}{-jn\pi}\right)_{-1}^{0} + \left(\frac{e^{jn\pi t}}{jn\pi}\right)_{0}^{1} - 2\left[t\frac{e^{jn\pi t}}{jn\pi} - \frac{e^{+jn\pi t}}{j^2n^2\pi^2} \right]_{0}^{1} \right]$$

$$= \frac{1}{2}\left[2\left(\frac{1}{n^2\pi^2} + \frac{e^{jn\pi}}{jn\pi} \right) + \frac{e^{jn\pi}}{j^2n^2\pi^2} + \frac{-e^{jn\pi}+1}{jn\pi} + \frac{e^{jn\pi}-1}{jn\pi} - 2\left[\frac{e^{jn\pi}}{jn\pi} - \frac{e^{jn\pi}}{j^2n^2\pi^2} + \frac{1}{j^2n^2\pi^2} \right] \right]$$

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$$= \frac{1}{2}\left[2\left(\frac{e^{jn\pi}-1}{jn\pi}\right)^{0} + 4\frac{e^{jn\pi}}{n^2\pi^2} \right]$$

$$= \frac{(-1)^n - 1}{jn\pi} + \frac{(-1)^n}{n^2\pi^2}$$

$$\boxed{C_n = \frac{2(-1)^n}{n^2\pi^2}}$$

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$$\therefore \; x(t) = \sum_{n=-\infty}^{\infty} \frac{2(-1)^n}{n^2\pi^2} e^{j\pi n t}$$

(ii) For $n = 7$

$$\text{Peak} = \frac{2}{7^2\pi^2} = \frac{2}{49\pi^2}$$

$$\text{Power} = \frac{V_p^2}{2} = \left[\frac{2}{49\pi^2}\right]^2$$

$$= \frac{4}{(49)^2\pi^4}$$

$$\text{Power} = 0.017 \text{ mW}$$

1

8 (b) Let $g(t) = x(t)\cos^2 t * \frac{\sin t}{\pi t}$, where $*$ is convolution. Assuming that $x(t)$ is real and $X(\omega) = 0$ for $|\omega| \geq 1$, show that there exists an LTI system 'S' such that

$$x(t) \longrightarrow \boxed{S} \longrightarrow g(t)$$

[20 marks]

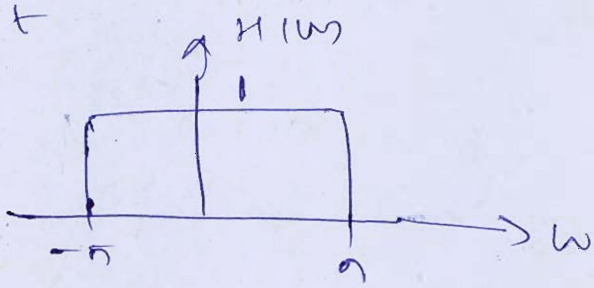
$$g(t) = \underbrace{x(t)\cos^2 t}_{x_1(t)} * \underbrace{\frac{\sin t}{\pi t}}_{h(t)}$$

$x(t)$ is real

$X(\omega) = 0 \quad |\omega| \geq 1$

$$h(t) = \frac{\sin \pi t}{\pi t} = sa(\pi t)$$

$H(\omega) =$



$$\cos^2 t = \frac{1+\cos 2t}{2}$$

$$= \frac{1 + \frac{e^{j2t} + e^{-j2t}}{2}}{2}$$

$$= \frac{1}{4} + \frac{1}{2}\left(e^{j2t} + e^{-j2t}\right)$$

$$\therefore \cos^2 t \xleftrightarrow{FT} \frac{1}{4} \times 2\pi\delta(\omega) + \frac{1}{2}\left\{\delta(\omega+2) + \delta(\omega-2)\right\}$$

$$x(t)\cos^2(t) \xleftrightarrow{FT} \frac{\pi}{2}X(\omega) + \frac{1}{2}\left[X(\omega+2) + X(\omega-2)\right]$$

~~$X(\omega) = 0 \quad |\omega| \geq 1$~~

$\frac{A\cdot\pi}{2}$

$\frac{1\times\pi}{\pi}$

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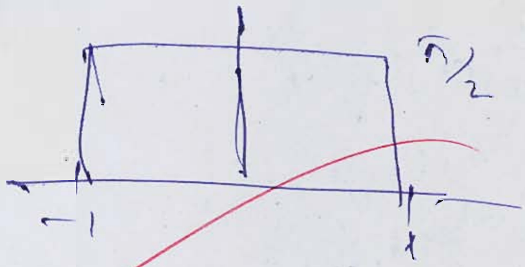
$g(t) = x(t) * h(t)$

$G(\omega) = X(\omega) \cdot H(\omega)$

$=$



$g(t) = \frac{\pi}{2} \times 2 \; Sa(t)$

$\boxed{g(t) = \pi \, Sa(t)}$

Hence, there exist LTI system 'S' such that

$x(t) \longrightarrow \boxed{S} \longrightarrow g(t)$

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3 (c) Determine the values of P_x and E_x for each of the following signals:

(i) $x_1(t) = e^{j(2t+\pi/4)}$

(ii) $x_2(n) = \left(\frac{1}{2}\right)^n u(n)$

(iii) $x_3(n) = e^{j\left(\frac{\pi}{2}n+\frac{\pi}{8}\right)}$

(iv) $x_4(n) = \cos\left(\frac{n\pi}{4}\right)$

(Where P_x = Power and E_x = Energy)

[20 marks]

(i)

$$x_1(t) = e^{j(2t+\pi/4)}$$

$$= e^{j2t} \cdot e^{j\pi/4}$$

it is periodic signal.

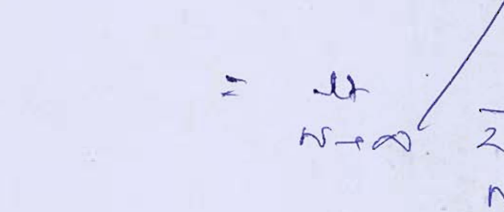
Hence power $= \frac{V_p^2}{2} = \frac{|e^{j\pi/4}|^2}{2}$

$$P_x = \frac{1}{2} = 0.5\,W$$

Energy $E_x = \lim_{t\to\infty} P_x \cdot t =$ infinite

(ii)

$$x_2(n) = \left(\frac{1}{2}\right)^n u(n)$$



$$\text{Power} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \times \frac{1}{1-\frac{1}{4}}$$

$$\text{Power} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{2}\right)^{2n}$$

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$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \times \left[\frac{1-\left(\frac{1}{4}\right)^N}{1-\frac{1}{4}} \right]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \times \frac{4}{3} \times \left[1-\left(\frac{1}{4}\right)^N\right]$$

$$\boxed{P = 0}$$

$$\text{Energy} = \sum_{n=-\infty}^{\infty} \left|x(n)\right|^2$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

(iii)

$$x_3(n) = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{8}\right)}$$

$$= e^{j\pi/2\, n} \cdot e^{j\pi/8}$$

it is periodic signal

$\therefore$ Hence it is power signal

$$P_x = \frac{\left|e^{j\pi/8}\right|^2}{2} = \frac{1}{2}$$

Energy = infinity

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(v) $x_4(n) = \cos\left(\frac{n\pi}{4}\right)$

It is periodic signal, hence power signal

$P_x = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

Energy E_x = infinite

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Space for Rough Work

Space for Rough Work

Space for Rough Work